# Texture specific mass matrices and mixing angles

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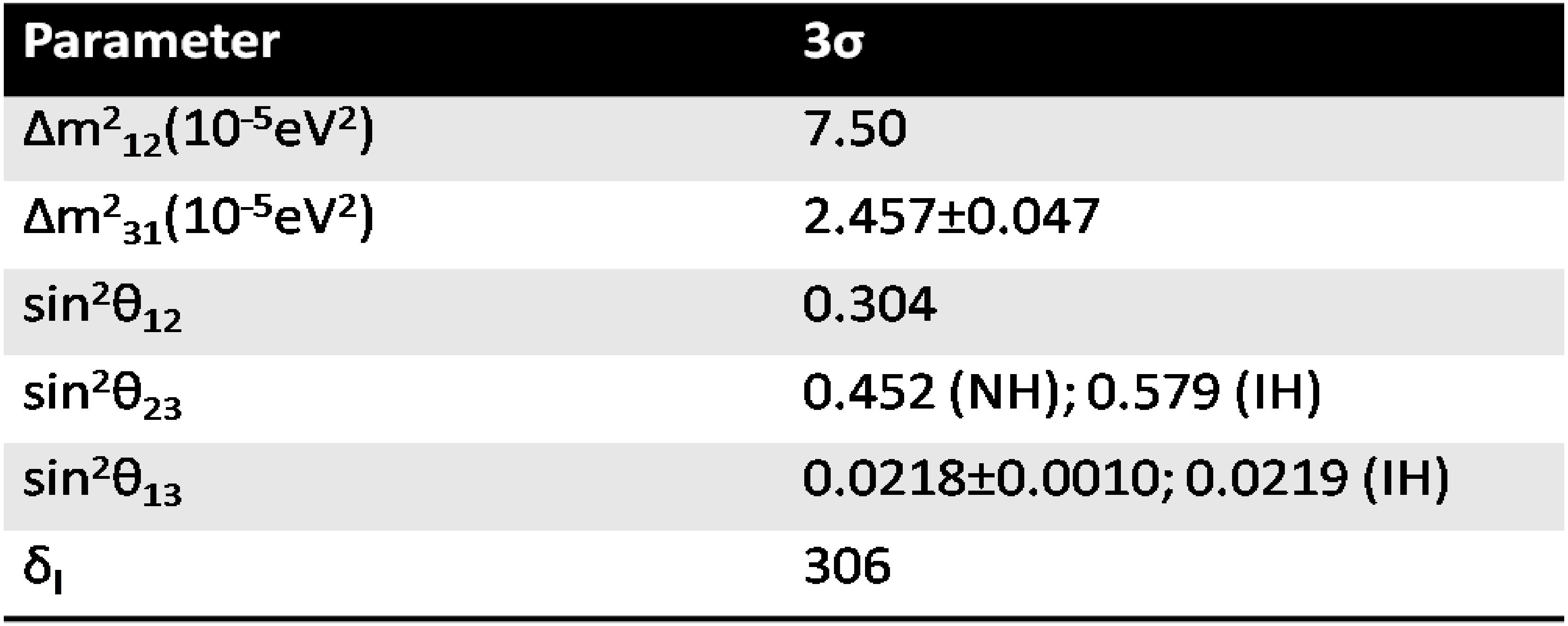
- **Standard Model (SM) is a gauge theory with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and is in excellent agreement up to energies of 100 GeV.**

- **Experimental observation of nonzero masses of neutrinos has provided a signal to go beyond SM.**

- **The Daya Bay and RENO neutrino oscillation experiments concludes non zero and relatively large value of mixing angle $\theta_{13}$.**

- **The results from long baseline accelerator neutrino experiments T2K and NovA explain the nearly maximal mixing angle $\theta_{23} \geq 45^o$ and $\delta_l \sim 270^o$ as well as statistical analysis of the cosmological data gives the value of neutrinoless double beta decay $\langle m_{ee} \rangle$ less than 16meV , indicates the preference for normal ordering of neutrino masses.**

# ❖ Experimental Data



| Parameter | 3σ |
|---|---|
| $\Delta m^2_{12}(10^{-5}eV^2)$ | 7.50 |
| $\Delta m^2_{31}(10^{-5}eV^2)$ | 2.457±0.047 |
| $\sin^2\theta_{12}$ | 0.304 |
| $\sin^2\theta_{23}$ | 0.452 (NH); 0.579 (IH) |
| $\sin^2\theta_{13}$ | 0.0218±0.0010; 0.0219 (IH) |
| $\delta_l$ | 306 |

- **On the theoretical side the flavor structure of SM is dictated by the fermion mass matrices, successful choice of mass matrix help to suggest an appropriate model for explaining the fermion mass spectrum.**

**Origin:** **Textures are imposed by some underlying symmetries to allow to derive some predictions which can be compared with experimental data.**

- **Phenomenologically 'Texture zero approach' has been considered wherein certain elements of mass matrices are zero and such mass matrices are referred as texture specific mass matrices.**

**Fritzsch type mass matrices**

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix},$$

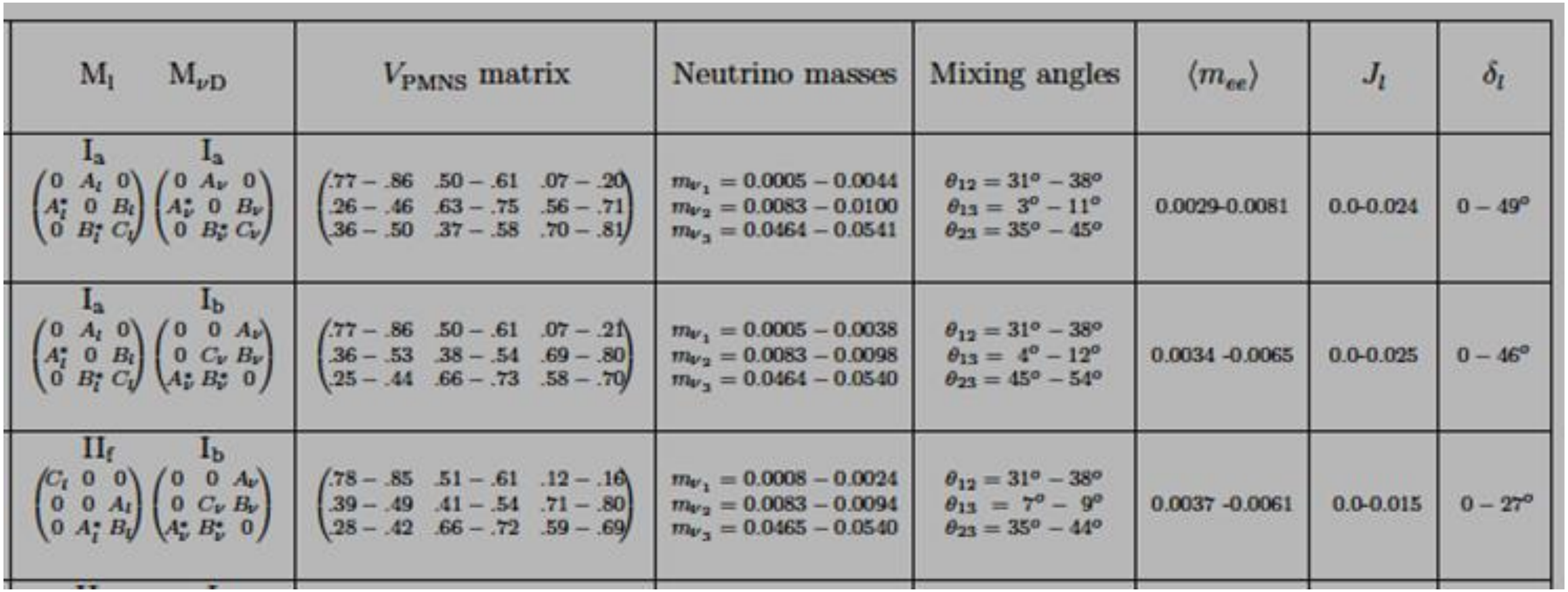
**The neutrino mass matrix $M_\nu$ is given by seesaw mechanism**

$$M_\nu = -M_{\nu D}^T \, (M_R)^{-1} \, M_{\nu D},$$

**Table I: Non Fritzsch type can be obtained by distributing nonzero elements in the possible 9 slots available keeping in mind the hermiticity of matrix. 12 possibilities of texture 3 zero classified into Class I and Class II and 15 possibilities of texture 2 classified into Class III, Class IV, Class V, Class VI.**

| Class | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| C-I | 11,22,13,31 | 11,12,21,33 | 11,22,23,32 | 13,31,22,33 | 11,23,32,33 | 12,21,22,33 |
| C-II | 11,13,31,23 | 11,12,21,23, 32 | 13,31,22,23 | 12,21,13,31, 33 | 12,23,32, 33 | 12,21,13,31, 22 |
| C-III | 11,13,31 | 11,12,21 | 22,23,32 | 13,31,33 | 23,32,33 | 12,21,22 |
| C-IV | 13,31,22 | 12,21,33 | 11,23,32 | - | - | - |
| C-V | 11,22 | 11,33 | 22,33 | - | - | - |
| C-VI | 12,21,13,31 | 12,21,23,32 | 13,31,23,32 | - | - | - |

# Table III: Calculated Phenomenological Quantities for Texture 6 zero combinations (NH)



| $M_l$ $M_{\nu D}$ | $V_{PMNS}$ matrix | Neutrino masses | Mixing angles | $\langle m_{ee}\rangle$ | $J_l$ | $\delta_l$ |
|---|---|---|---|---|---|---|
| $\mathrm{I_a}$ $\mathrm{I_a}$<br>$\begin{pmatrix}0 & A_l & 0\\ A_l^* & 0 & B_l\\ 0 & B_l^* & C_l\end{pmatrix}\begin{pmatrix}0 & A_\nu & 0\\ A_\nu^* & 0 & B_\nu\\ 0 & B_\nu^* & C_\nu\end{pmatrix}$ | $\begin{pmatrix}.77-.86 & .50-.61 & .07-.20\\ .26-.46 & .63-.75 & .56-.71\\ .36-.50 & .37-.58 & .70-.81\end{pmatrix}$ | $m_{\nu_1}=0.0005-0.0044$<br>$m_{\nu_2}=0.0083-0.0100$<br>$m_{\nu_3}=0.0464-0.0541$ | $\theta_{12}=31^o-38^o$<br>$\theta_{13}=3^o-11^o$<br>$\theta_{23}=35^o-45^o$ | 0.0029-0.0081 | 0.0-0.024 | $0-49^o$ |
| $\mathrm{I_a}$ $\mathrm{I_b}$<br>$\begin{pmatrix}0 & A_l & 0\\ A_l^* & 0 & B_l\\ 0 & B_l^* & C_l\end{pmatrix}\begin{pmatrix}0 & 0 & A_\nu\\ 0 & C_\nu & B_\nu\\ A_\nu^* & B_\nu^* & 0\end{pmatrix}$ | $\begin{pmatrix}.77-.86 & .50-.61 & .07-.21\\ .36-.53 & .38-.54 & .69-.80\\ .25-.44 & .66-.73 & .58-.70\end{pmatrix}$ | $m_{\nu_1}=0.0005-0.0038$<br>$m_{\nu_2}=0.0083-0.0098$<br>$m_{\nu_3}=0.0464-0.0540$ | $\theta_{12}=31^o-38^o$<br>$\theta_{13}=4^o-12^o$<br>$\theta_{23}=45^o-54^o$ | 0.0034 -0.0065 | 0.0-0.025 | $0-46^o$ |
| $\mathrm{II_f}$ $\mathrm{I_b}$<br>$\begin{pmatrix}C_l & 0 & 0\\ 0 & 0 & A_l\\ 0 & A_l^* & B_l\end{pmatrix}\begin{pmatrix}0 & 0 & A_\nu\\ 0 & C_\nu & B_\nu\\ A_\nu^* & B_\nu^* & 0\end{pmatrix}$ | $\begin{pmatrix}.78-.85 & .51-.61 & .12-.16\\ .39-.49 & .41-.54 & .71-.80\\ .28-.42 & .66-.72 & .59-.69\end{pmatrix}$ | $m_{\nu_1}=0.0008-0.0024$<br>$m_{\nu_2}=0.0083-0.0094$<br>$m_{\nu_3}=0.0465-0.0540$ | $\theta_{12}=31^o-38^o$<br>$\theta_{13}=7^o-9^o$<br>$\theta_{23}=35^o-44^o$ | 0.0037 -0.0061 | 0.0-0.015 | $0-27^o$ |

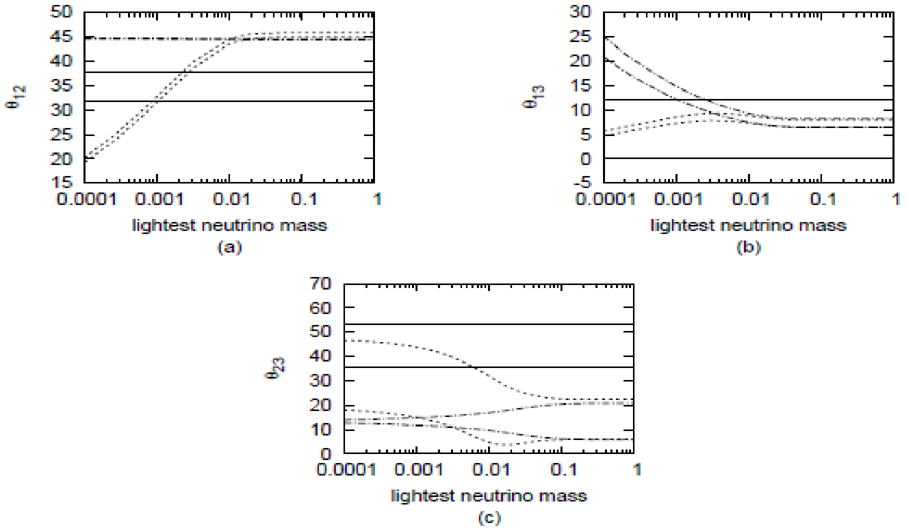
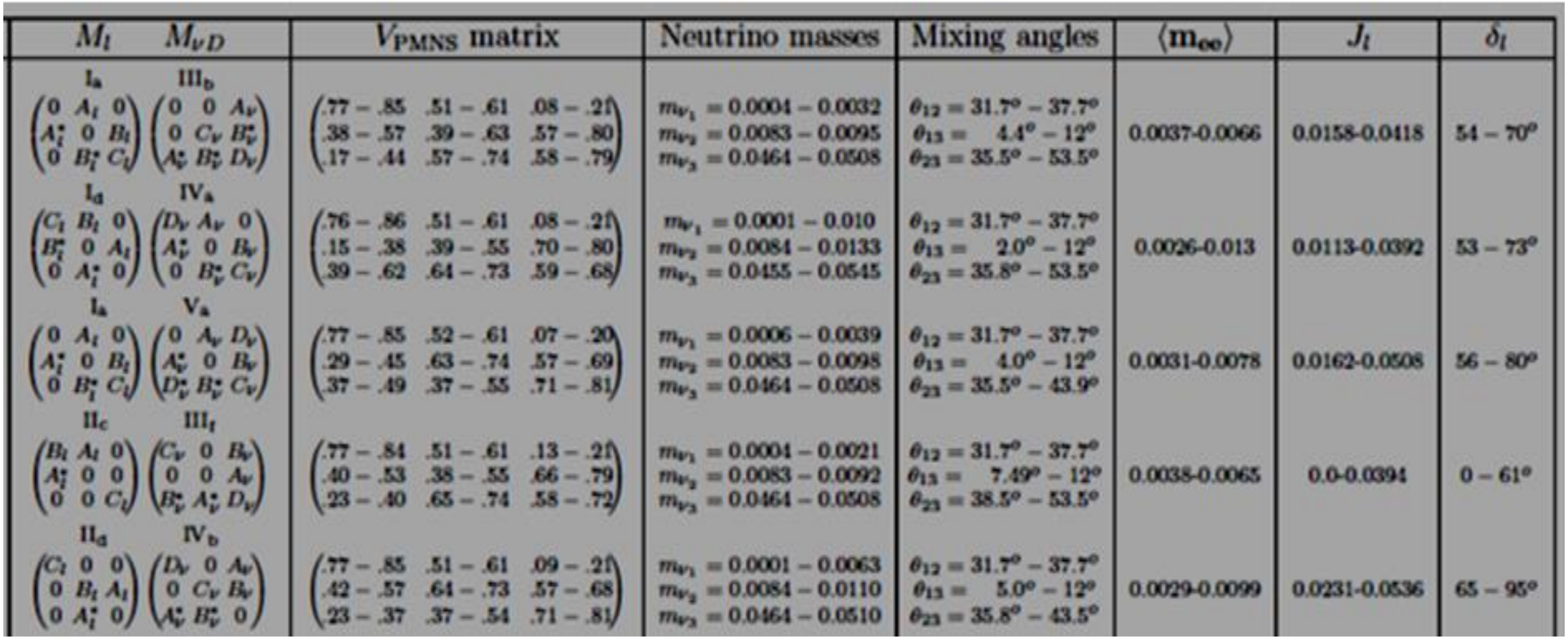


**Figure depict that mixing angles are well within experimental data but IH is completely out of range.**

# Table II: Viable texture 6 zero & 5 zero lepton mass matrices for NH and IH

| $M_l$, $M_{\nu D}$ | NH | IH |
|---|---|---|
| I, I | Aa,bb,cc,dd,ee,ff,ab,ba,cf,fc,de,ed | - |
| II, II | - | - |
| I, II | - | - |
| II, I | fb, da, db, fa | - |
| I, III | aa, bb, cc, dd, ee, ff, ab, ba, cf, fc,de, ed | ac,ae,bc,be,ca,cd,db,df,eb,ef,fa,fd |
| III, I | aa,bb,cc,dd,ee,ff,ab,ba,cf, de,ed,fc | - |
| I, IV | da, db, ea, eb, fa, fb | - |
| IV, I | ad,ae,bc,bf,cc | |
| I, V | aa, ab, ba, bb, cc, fc | - |
| V, I | aa,ab,bb,ba,cc,cf | - |
| II, III | aa,bb, cc, ee, ab,ba, cf, da, fa, fb | ac,ae,bb,bf,ca,ce,dc,de,eb,ef,fc,fe |
| III, II | aa,ba,cc,fc | - |
| II, IV | db, fb | - |
| IV, II | - | - |
| II, V | da, ba, da, fc | - |
| V, II | - | - |

## Table IV: Calculated phenomenological quantities for viable 5 zero combinations



| $M_l$ | $M_{\nu D}$ | $V_{\text{PMNS}}$ matrix | Neutrino masses | Mixing angles | $\langle \mathbf{m_{ee}} \rangle$ | $J_l$ | $\delta_l$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{I_a}$<br>$\begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix}$ | $\mathbf{III_b}$<br>$\begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu^* \\ A_\nu^* & B_\nu^* & D_\nu \end{pmatrix}$ | $\begin{pmatrix} .77-.85 & .51-.61 & .08-.21 \\ .38-.57 & .39-.63 & .57-.80 \\ .17-.44 & .57-.74 & .58-.79 \end{pmatrix}$ | $m_{\nu_1} = 0.0004 - 0.0032$<br>$m_{\nu_2} = 0.0083 - 0.0095$<br>$m_{\nu_3} = 0.0464 - 0.0508$ | $\theta_{12} = 31.7^\circ - 37.7^\circ$<br>$\theta_{13} = 4.4^\circ - 12^\circ$<br>$\theta_{23} = 35.5^\circ - 53.5^\circ$ | 0.0037-0.0066 | 0.0158-0.0418 | $54 - 70^\circ$ |
| $\mathbf{I_d}$<br>$\begin{pmatrix} C_l & B_l & 0 \\ B_l^* & 0 & A_l \\ 0 & A_l^* & 0 \end{pmatrix}$ | $\mathbf{IV_a}$<br>$\begin{pmatrix} D_\nu & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix}$ | $\begin{pmatrix} .76-.86 & .51-.61 & .08-.21 \\ .15-.38 & .39-.55 & .70-.80 \\ .39-.62 & .64-.73 & .59-.68 \end{pmatrix}$ | $m_{\nu_1} = 0.0001 - 0.010$<br>$m_{\nu_2} = 0.0084 - 0.0133$<br>$m_{\nu_3} = 0.0455 - 0.0545$ | $\theta_{12} = 31.7^\circ - 37.7^\circ$<br>$\theta_{13} = 2.0^\circ - 12^\circ$<br>$\theta_{23} = 35.8^\circ - 53.5^\circ$ | 0.0026-0.013 | 0.0113-0.0392 | $53 - 73^\circ$ |
| $\mathbf{I_a}$<br>$\begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix}$ | $\mathbf{V_a}$<br>$\begin{pmatrix} 0 & A_\nu & D_\nu \\ A_\nu^* & 0 & B_\nu \\ D_\nu^* & B_\nu^* & C_\nu \end{pmatrix}$ | $\begin{pmatrix} .77-.85 & .52-.61 & .07-.20 \\ .29-.45 & .63-.74 & .57-.69 \\ .37-.49 & .37-.55 & .71-.81 \end{pmatrix}$ | $m_{\nu_1} = 0.0006 - 0.0039$<br>$m_{\nu_2} = 0.0083 - 0.0098$<br>$m_{\nu_3} = 0.0464 - 0.0508$ | $\theta_{12} = 31.7^\circ - 37.7^\circ$<br>$\theta_{13} = 4.0^\circ - 12^\circ$<br>$\theta_{23} = 35.5^\circ - 43.9^\circ$ | 0.0031-0.0078 | 0.0162-0.0508 | $56 - 80^\circ$ |
| $\mathbf{II_c}$<br>$\begin{pmatrix} B_l & A_l & 0 \\ A_l^* & 0 & 0 \\ 0 & 0 & C_l \end{pmatrix}$ | $\mathbf{III_f}$<br>$\begin{pmatrix} C_\nu & 0 & B_\nu \\ 0 & 0 & A_\nu \\ B_\nu^* & A_\nu^* & D_\nu \end{pmatrix}$ | $\begin{pmatrix} .77-.84 & .51-.61 & .13-.21 \\ .40-.53 & .38-.55 & .66-.79 \\ .23-.40 & .65-.74 & .58-.72 \end{pmatrix}$ | $m_{\nu_1} = 0.0004 - 0.0021$<br>$m_{\nu_2} = 0.0083 - 0.0092$<br>$m_{\nu_3} = 0.0464 - 0.0508$ | $\theta_{12} = 31.7^\circ - 37.7^\circ$<br>$\theta_{13} = 7.49^\circ - 12^\circ$<br>$\theta_{23} = 38.5^\circ - 53.5^\circ$ | 0.0038-0.0065 | 0.0-0.0394 | $0 - 61^\circ$ |
| $\mathbf{II_d}$<br>$\begin{pmatrix} C_l & 0 & 0 \\ 0 & B_l & A_l \\ 0 & A_l^* & 0 \end{pmatrix}$ | $\mathbf{IV_b}$<br>$\begin{pmatrix} D_\nu & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}$ | $\begin{pmatrix} .77-.85 & .51-.61 & .09-.21 \\ .42-.57 & .64-.73 & .57-.68 \\ .23-.37 & .37-.54 & .71-.81 \end{pmatrix}$ | $m_{\nu_1} = 0.0001 - 0.0063$<br>$m_{\nu_2} = 0.0084 - 0.0110$<br>$m_{\nu_3} = 0.0464 - 0.0510$ | $\theta_{12} = 31.7^\circ - 37.7^\circ$<br>$\theta_{13} = 5.0^\circ - 12^\circ$<br>$\theta_{23} = 35.8^\circ - 43.5^\circ$ | 0.0029-0.0099 | 0.0231-0.0536 | $65 - 95^\circ$ |

## Table reveals

- *The spectrum of neutrino masses following NH, ruling out degenerate neutrinos.*
- *$\vartheta_{1\,2}$ is spanning its entire expt range.*
- *The magnitude of $J_l$ ($\sim 10^{-2}$) can reach the percentage level and might be detected in the upcoming long baseline neutrino oscillation experiments.*
- *The magnitude of effective mass $<m_{ee}>$ to be measured in the neutrino less double beta decay ($\sim 10^{-3}$) is too small to be experimentally accessible in foreseeable experiments.*

## Texture 6 zero lepton mass matrices

- 16 combinations out of 144 compatible  for NH.
- None combination compatible for IH and degenerate Majorana neutrinos

## Texture 5 zero lepton mass matrices

- 66 combinations out of 288 compatible for NH.
- 24 combinations out  of 288 compatible for IH.
- Degenerate neutrinos ruled out.

**Texture specific mass matrices have provided valuable clues in understanding fermion masses and mixing in a unified manner. For all the allowed cases we are able to get the ranges of effective neutrino mass and CP violating phase which lie in the sensitivity limits of the experimental data. More refined measurements of mixing angles, effective neutrino mass and CP violating phase in leptonic sector can help us to locate unique texture structure for charged leptons and neutrinos and thus they are important for building more ambitious theories.**