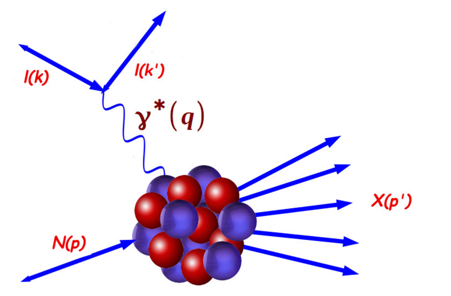
Nuclear medium effects in weak & electromagnetic induced DIS processes

F. Zaidi and H. Haider
Aligarh Muslim University, India

Introduction

We have studied nuclear medium effects in electromagnetic and charged current induced weak structure functions $F_{iA}(x,Q^2)$; $(i=1,2)$ for the deep inelastic scattering(DIS) processes in a wide range of x and Q^2 in different nuclear targets. For the determination of nuclear structure functions the effects due to Fermi motion, binding energy and nucleon correlations are incorporated using spectral function[1,2]. Furthermore, shadowing effect which is significant for $x \le 0.1$ and mesonic(π, ρ) cloud contribution which is significant for the low and intermediate $x(0.3 < x < 0.6)$ have also been included[1,2]. We have obtained the structure functions by using various PDFs for nucleons(CTEQ6.6[3] & MSTW[4]) as well as for pions (GRV[5], CTEQ5L[6] & Conway[6]) available in literature. For the ρ meson, we have used the same PDFs as in the case of pion. Results for the differential scattering cross sections are also presented for different energy ranges. The importance of kinematic limits of Q^2 and center of mass energy(W) corresponding to various experiments for obtaining the contribution from the DIS region will be presented and discussed. Specifically these calculations are performed in the kinematic range relevant for the current and future experiments at JLab, MINERνA, NOνA and DUNE[7]. The results are also compared with the existing experimental data of JLab, EMC & BCDMS[8] for electromagnetic interaction induced process and with the experimental data of NuTeV, CDHSW & CHORUS collaborations[9] for weak interaction induced process.

EM INTERACTIONS



For $l^\pm - A$ DIS process, basic reaction is given by

$$l^\pm(k) + A(p) \to l^\pm(k') + X(p'),$$

This process takes place via the exchange of mediating quanta carrying four momentum transfer $q(Q^2=-q^2)$ and invariant mass of final hadronic state is given by

$$W^2 = (p+q)^2$$

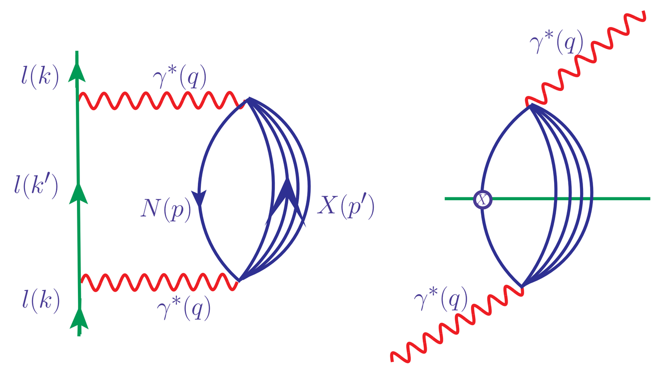
The cross section for an element of volume dV in the nucleus is related to the probability per unit time (Γ) of the lepton interacting with the nucleons:

$$d\sigma = \Gamma dt dS = \Gamma \frac{dt}{dl} dS dl = \Gamma \frac{1}{v} dV = \Gamma \frac{E_l}{|\mathbf{k}|} dV = \Gamma \frac{E_l}{|\mathbf{k}|} d^3r,$$

$dl \equiv$ length of the interaction
$v (= \frac{dl}{dt}) \equiv$ velocity of the incoming lepton and $\mathbf{k} = \mathbf{v} E_l$.

We obtain cross section as:

$$d\sigma = \frac{-2m}{E_l(k)} \mathrm{Im}\Sigma(k) \frac{E_l(k)}{|\mathbf{k}|} d^3r,$$



Lepton self energy $\Sigma(k)$ is written as:

$$-i\Sigma(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(k)\, i e\gamma^\alpha\, i\frac{\not{k}'+m}{k'^2-m^2+i\epsilon} i e\gamma^\beta u_l(k) \frac{-ig_{\alpha\rho}}{q^2}(-i)\Pi^{\rho\sigma}(q)\frac{-ig_{\sigma\beta}}{q^2}$$

photon self-energy $\Pi^{\alpha\beta}(q)$ in the nuclear medium:

$$\Pi^{\alpha\beta}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^{N} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^\alpha|H >< X|J^\beta|H >^* (2\pi)^4 \delta^4(q+p-\sum_{i=1}^N p'_i)$$

Relativistic nucleon propagator in a nuclear medium:

$$G(p^0,\mathbf{p}) = \frac{M}{E(\mathbf{p})}\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\mathbf{p})}{p^0-\omega-i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\mathbf{p})}{p^0-\omega+i\epsilon}\right]$$

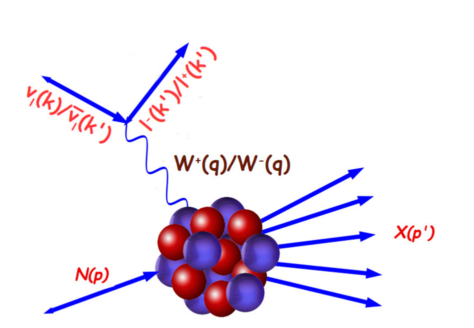
for $p^0 \le \mu$ hole spectral function

$$S_h(p^0,\mathbf{p}) = \frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})}\mathrm{Im}\Sigma(p^0,\mathbf{p})}{\left\{p^0-E(\mathbf{p})-\frac{M}{E(\mathbf{p})}\mathrm{Re}\Sigma(p^0,\mathbf{p})\right\}^2+\left\{\frac{M}{E(\mathbf{p})}\mathrm{Im}\Sigma(p^0,\mathbf{p})\right\}^2}$$

for $p^0 > \mu$ particle spectral function

$$S_p(p^0,\mathbf{p}) = -\frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})}\mathrm{Im}\Sigma(p^0,\mathbf{p})}{\left\{p^0-E(\mathbf{p})-\frac{M}{E(\mathbf{p})}\mathrm{Re}\Sigma(p^0,\mathbf{p})\right\}^2+\left\{\frac{M}{E(\mathbf{p})}\mathrm{Im}\Sigma(p^0,\mathbf{p})\right\}^2}$$

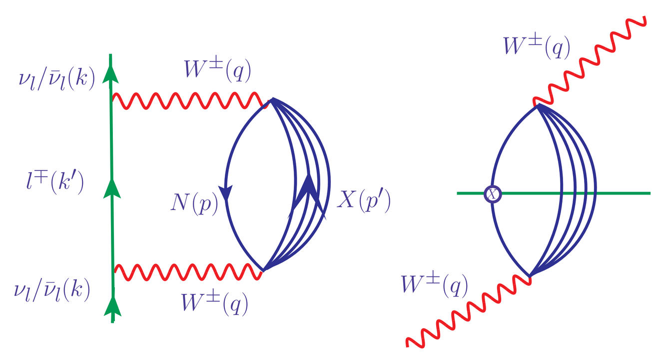
WEAK INTERACTIONS



For CC $\nu_l(\bar\nu_l) - A$ DIS scattering process, basic reaction is given by

$$\nu_l(\bar\nu_l)(k) + A(p) \to l^\mp(k') + X(p'),$$

Self energy $\Sigma(k)$ for neutrino/antineutrino and W^+/W^- is shown below:



$$\Sigma(k) = (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int \frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2-m^2+i\epsilon}\left(\frac{m_W}{q^2-m_W^2}\right)^2 L_{\alpha\beta}\Pi^{\alpha\beta}(q)$$

$\Pi^{\alpha\beta}(q)$ is the W self-energy in the nuclear medium:

$$-i\Pi^{\alpha\beta}(q) = (-)\int \frac{d^4p}{(2\pi)^4} iG(p)\sum_X \sum_{s_p,s_l}\prod_{i=1}^{n}\int \frac{d^4p'_i}{(2\pi)^4}\prod_l iG_l(p'_l)\prod_j iD_j(p'_j)$$
$$\times \left(\frac{-G_F m_W^2}{\sqrt{2}}\right)\langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*(2\pi)^4\delta^4(q+p-\Sigma_{i=1}^n p'_i)$$

Nuclear Medium Effects

- **Fermi motion:** Inside the nucleus nucleons move with a finite momentum.
- **Pauli Blocking:** Nucleons obey the fundamental exclusion principle that states
  $$p \le p_F \quad ; \quad p' > p_F$$
- **Nucleon Correlations:** Bound nucleons interact among themselves via exchange of bosons like π, ρ, ω etc.

EM and WEAK INTERACTIONS

Differential Cross Section

$$\frac{d^2\sigma^A}{dxdy} \propto L^{\alpha\beta} W^A_{\alpha\beta}$$

Leptonic tensor

$$L^{\alpha\beta} = k^\alpha k'^\beta + k'^\alpha k^\beta - k\cdot k' g^{\alpha\beta} \pm \underbrace{i\epsilon^{\alpha\beta\rho\sigma}k_\rho k'_\sigma}_{0 \text{ for EM interaction}}$$

Nuclear hadronic tensor

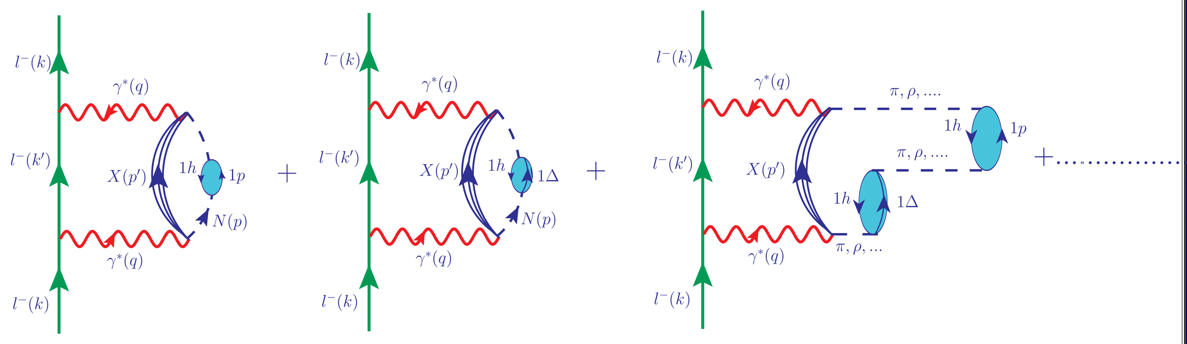
$$W^A_{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} dp^0 \frac{M}{E(\mathbf{p})} S_h(p^0,\mathbf{p},\rho(r)) W^N_{\alpha\beta}(p,q)$$

Electromagnetic and Weak Nuclear Structure Function

$$F_{1A}^{EM,WI}(x_A,Q^2) = 4AM\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \left[\frac{F_{1N}^{EM,WI}(x_N)}{M} + \frac{1}{M^2}p_x^2\frac{F_{2N}^{EM,Weak}(x_N)}{\nu}\right]$$

$$F_{2A}^{EM,WI}(x_A,Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \times \left[\frac{Q^2}{(q^z)^2}\left(\frac{|\mathbf{p}|^2-(p^z)^2}{2M^2}\right) + \frac{(p^0-p^z\gamma)^2}{M^2} \times \left(\frac{p^z Q^2}{(p^0-p^z\gamma)q^0q^z}+1\right)^2\right]\frac{M}{p^0-p^z\gamma}F_{2N}^{EM,WI}(x_N,Q^2),$$

Mesonic Contributions



Nucleon propagator → Meson propagator

In the nuclear medium meson propagator is given by

$$D_i(p) = [p_0^2 - \mathbf{p}^2 - m_i^2 - \Pi_i(p_0,\mathbf{p})]^{-1}$$

where meson self energy

$$\Pi_i = \frac{\left(\frac{f^2}{m_\pi^2}\right) c' F_{iNN}^2(p)\mathbf{p}^2\Pi^*}{1-\frac{f^2}{m_\pi^2}V'_j\Pi^*}$$

with iNN form factor

$$F_{iNN}(p) = \frac{(\Lambda^2-m_i^2)}{(\Lambda^2-\mathbf{p}^2)} \quad ; \Lambda = 1 \text{ GeV}, f = 1.01.$$

$V'_j = V'_L$ for pion and $V'_j = V'_T$ for rho meson and respectively stands for longitudinal and transverse spin-isospin interaction.
Π^* is the irreducible pion self energy that contains the contribution of particle - hole and delta - hole excitations.

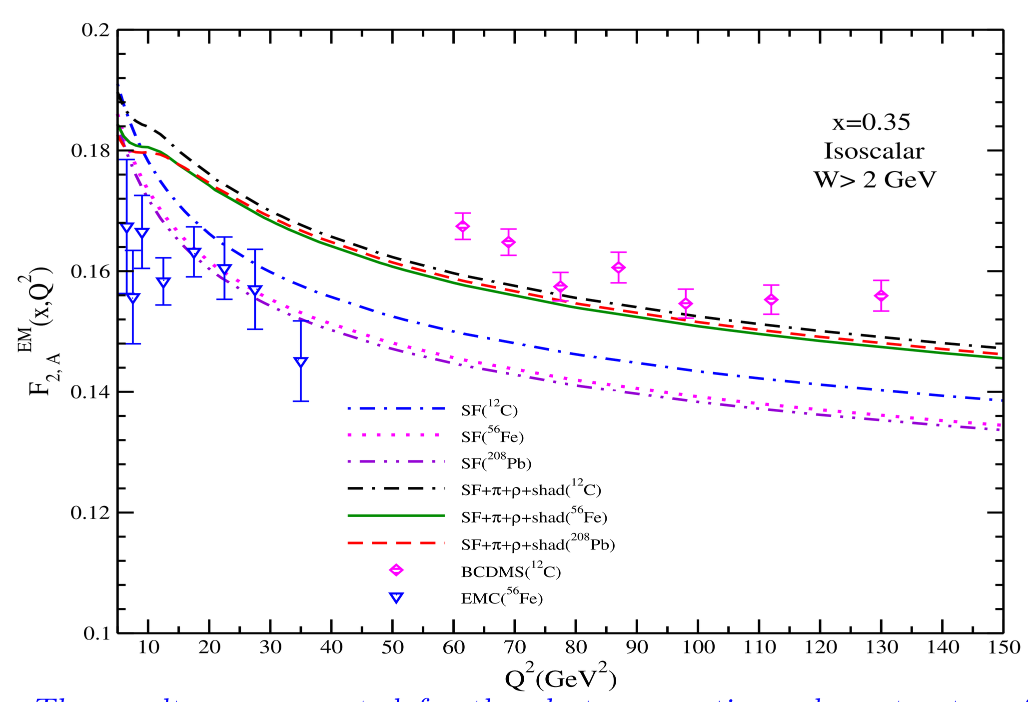
$$F_{1A,i}^{EM,WI}(x,Q^2) = -6\times\eta\times AM_N\int d^3r\int\frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta\mathrm{Im}D_i(p)\ 2m_i \times\left[\frac{F_{1i}^{EM,WI}(x_i)}{m_i}+\frac{|\mathbf{p}|^2-p_z^2}{2(p_0q_0-p_zq_z)}\frac{F_{2i}^{EM,WI}(x_i)}{m_i}\right],$$

$$F_{2A,i}^{EM,WI}(x,Q^2) = -6\times\eta\int d^3r\int\frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta\mathrm{Im}D_i(p)\ 2m_i\left(\frac{m_i}{p_0-p_z\gamma}\right)\times \left[\frac{Q^2}{(q_z)^2}\left(\frac{|\mathbf{p}|^2-(p_z)^2}{2m_i^2}\right)+\frac{(p_0-p_z\gamma)^2}{m_i^2}\left(\frac{p_z Q^2}{(p_0-p_z\gamma)q_0q_z}+1\right)^2\right]F_{2i}^{EM,WI}(x_i),$$

$i=\pi$, ρ and $\eta=1$ for pion & $\eta=2$ for rho meson.

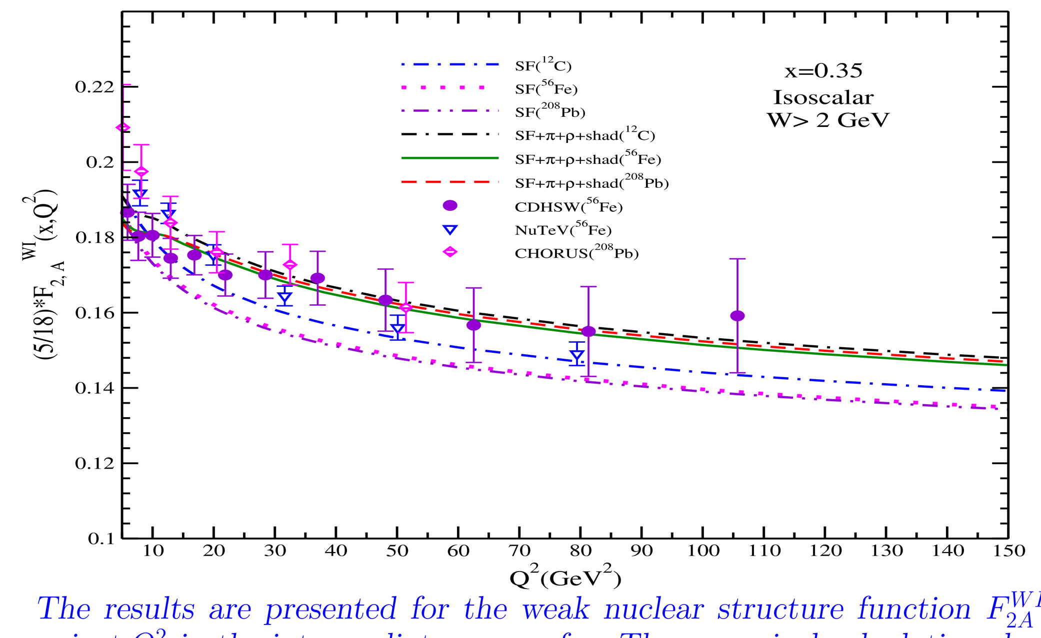
Results & Discussion

F_{2A}^{EM} vs Q^2 at x=0.35; A= ^{12}C, ^{56}Fe & ^{208}Pb



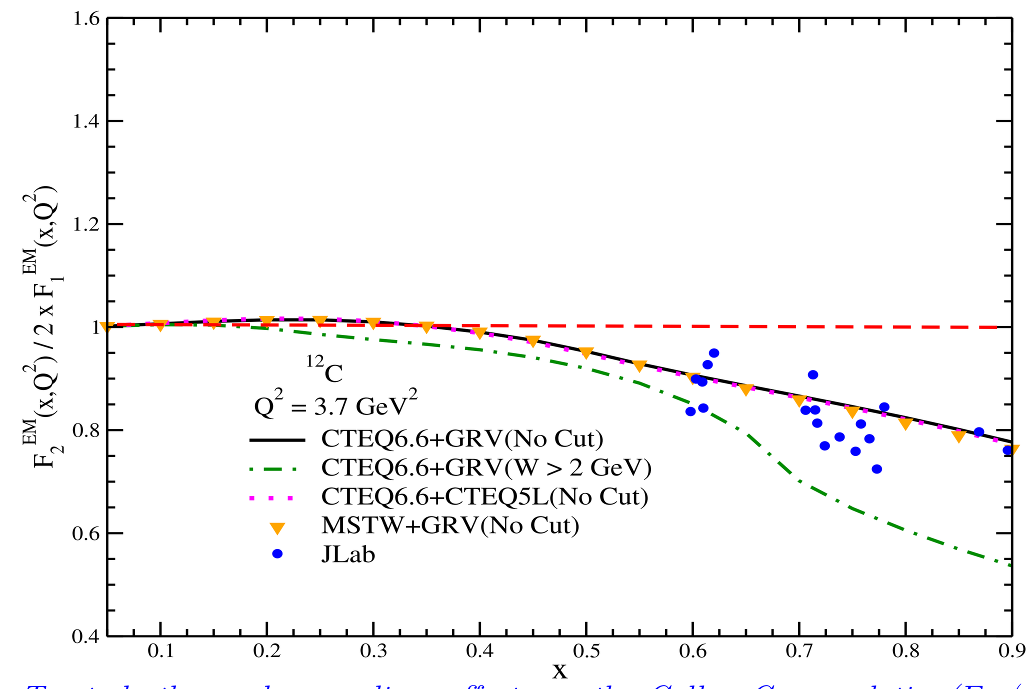
The results are presented for the electromagnetic nuclear structure function $F_{2A}^{EM}(x,Q^2)$ against Q^2 in the intermediate range of x. The numerical calculations have been performed with the constrained that invariant mass of hadronic state should be > 2 GeV. We have highlighted the nuclear medium effects by comparing the results obtained with spectral function only i.e. labeled by SF with the results obtained including mesonic cloud contribution and shadowing effect along with it(SF+π+ρ+shad). Furthermore, from the figure it may be observed that nuclear medium effects become more pronounced with the increase in mass number of nuclear target. For example, the difference between the results for SF and the results with mesonic & shadowing effect is 6% for carbon, 8% for iron and 10% for lead, respectively at $Q^2 = 20$ GeV^2. This difference gets saturated with the increase in Q^2. Numerical results are also compared with the experimental data of EMC and BCDMS.

F_{2A}^{WI} vs Q^2 at x=0.35; A= ^{12}C, ^{56}Fe & ^{208}Pb



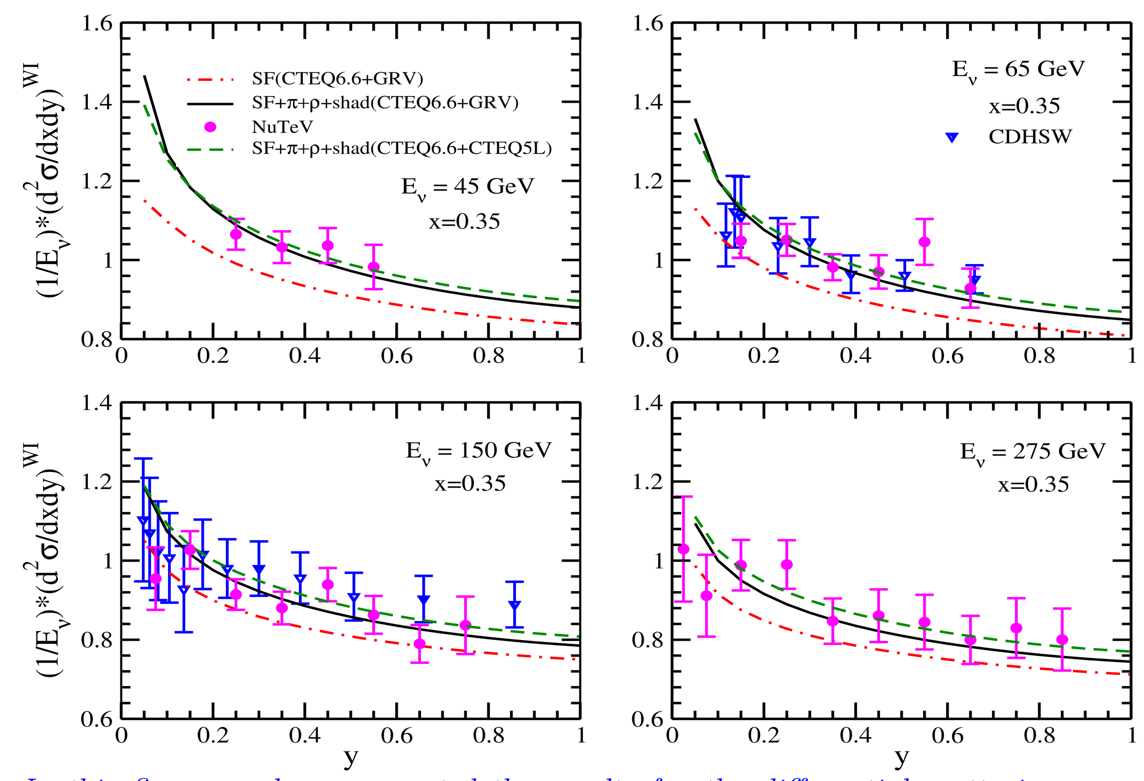
The results are presented for the weak nuclear structure function $F_{2A}^{WI}(x,Q^2)$ against Q^2 in the intermediate range of x. The numerical calculations have been performed with the same constraints as in the case of EM interaction. This kinematic region is relevant to MINERνA. The difference between the results with spectral function and the results including mesonic contribution & shadowing effect with it is $3-5\%$ at $Q^2 = 10$ GeV^2 and $6-10\%$ at $Q^2 = 20$ GeV^2, respectively. This difference due to medium effects get saturated with the increase in Q^2. Numerical results are compared with the available experimental data of NuTeV, CDHSW and CHORUS and are agreed well.

$\frac{F_{2A}(x,Q^2)}{2xF_{1A}(x,Q^2)}$ vs x; A= ^{12}C



To study the nuclear medium effects on the Callan-Gross relation($F_{2N}(x,Q^2) = 2xF_{1N}(x,Q^2)$) we have presented the results for the ratio $\frac{F_{2A}(x,Q^2)}{2xF_{1A}(x,Q^2)}$. It may be observed from the figure that the ratio is different than unity which is an evidence that Callan-Gross relation deviates inside the nucleus. Another point is that ratio obtained for EM interaction overlaps to the ratio obtained for weak interaction(not shown here). Furthermore, it is also noticeable that nucleon as well as pion PDFs does not make any difference to the result while there is difference of $2\% - 3\%$ between pion PDFs given by Gluck et al.(GRV) and Wijesooriya et al.(CTEQ5L) in the intermediate range of x at small value of Q^2. Pion PDFs given by Conway et al.(not shown here) agrees well with the PDFs given by Wijesooriya et al.

$\frac{d^2\sigma}{dxdy}$ vs y at x=0.35 for WI; A= ^{56}Fe



In this figure we have presented the results for the differential scattering cross section in iron showing nuclear medium effects. There is difference of $7-10\%$ at x=0.2 and $\approx 5\%$ at x=0.8 from SF to our full model for the energy values shown above. All the nuclear targets for the present study are treated as isoscalar. Results are presented obtained using different pion PDFs like GRV and CTEQ5L and CTEQ6.6 is used for nucleon PDFs here. It is found that there is difference of $3\% - 4\%$ between these pion parameterizations.

Conclusion

- *We have studied nuclear medium effects in electromagnetic and weak nuclear structure functions of nucleons in ^{12}C, ^{56}Fe, ^{208}Pb. The results for differential scattering cross section are also presented for neutrino induced CC DIS process in iron.*
- *For the nuclear medium effects, we took into account Fermi motion, nuclear binding, nucleon correlations, effect of meson degrees of freedom, and shadowing effects. The calculations are performed at NLO.*
- *We have shown the effect of different nucleon and pion PDFs on nuclear structure functions. These results may be useful for JLab, MINERνA, DUNE and NOνA experiments.*
- *The theoretical expressions for $F_1^A(x,Q^2)$ and $F_2^A(x,Q^2)$ have been obtained without assuming Callan-Gross relation at nuclear level.*

Where to read more?

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