# Generalised Tsallis Distributions in Leptonic & Hardon-Nucleus Collisions

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## Abstract

Particle collisions at relativistic high energy lead to production of large number of particles. Thermodynamics and statistics play an important role in studying the characteristic properties of charged particle production in leptonic and hadronic collisions at high energy.

Various thermo-dynamical models using standard statistical distributions have been developed to describe the multi particle production in a systematic way. Statistical equilibrium leads to non exponential behavior of transverse energy distribution of hadrons produced in high energy collisions. This deviation is related to power-like hadron spectra as hadrons produced from quarks and gluon fragmentation overcome the thermal hadron production which is exponential in nature. Distributions satisfying such criteria are required in investigating the multi particle production. We use different approaches to study the hadron production in particle collisions at high energies. Tsallis q-statistics have been considered for studying multiplicity distributions in $e^+e^-$ and hadron-nucleus collisions at high energies.

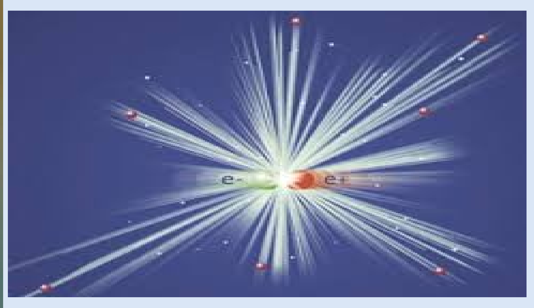
## Introduction

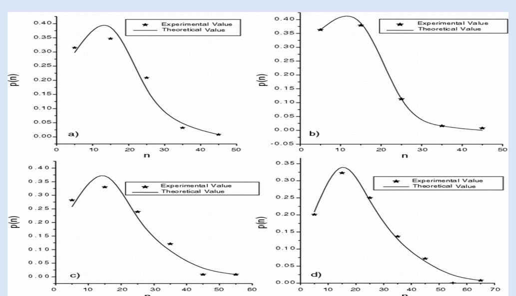
During last three decades or so, the multi particle production has been the study of great interest. The problem of multi particle production is one of the central problems in elementary particle physics. Modern accelerators have made possible the intensive and detailed investigations of multi particle production in a large energy interval. Though a precise understanding of the particle dynamics is still not achieved, yet a number of fundamental regularities and specific properties have been established for such processes . While dealing with the long range interactions, such as quark-quark interactions, one finds that standard statistical mechanics (Boltzmann Gibbs) becomes non extensive, specially quantities like energy, momentum and entropy. Multiparticle production in high energy collisions can result in any number of particles. Experimentally, we can measure the probability of production of particular number of particles, $P_N$. Various models are developed to have appropriate understanding and description of the distribution function for $P_N$ . Many Statistical thermo-dynamical Models have been developed to understand the high energy collisions by considering the produced particles as a gaseous system in which Entropy of the system is taken into consideration. As Entropy is directly related to the probability, by applying mathematical formulation, we can get appropriate partition function of the distribution followed by the system. In the present work we study the nature of entropy of the system, use this to understand the multi-particle production and exploit the non-extensive nature of the entropy for explanation.

## Physics Behind

In high energy collisions, both the colliding and produced objects can be hadrons, mesons or leptons . The multitude of particles produced in relativistic collisions follow certain production which rules can be predicted in terms of various models including the laws of statistical mechanics.

**MULTIPARTICLE PRODUCTION** **MULTIPLICITY DISTRIBUTION**





**EXPERIMENTAL MEASUREMENT**

The collision or interaction of two particles is generally described in terms of cross-section which is calculated by measuring the number of charged particles produced. These particles are detected with the help of high energy particle detectors. The cross section essentially gives the measure of the probability of production of particular number of particles.

**THEORETICAL ASPECT**

A large number of phenomenological approaches have been proposed to characterize the charged multiplicity distributions in high energy hadronic and leptonic processes. Some of the commonly used are Poisson distribution, KNO scaling, Negative Binomial distribution, modified Negative Binomial etc. Considering the system as a statistical thermo-dynamical canonical ensemble and studying its entropy subject to constraints, we formulate a partition function which leads to a particular distribution describing the relation between probability and charged multiplicity in terms of Tsallis distribution, modified Tsallis, modified NBD.

## Tsallis Approach

**Extensive Entropy** : Additive in Nature $S = S_A + S_B$

$$S = -\sum p \log_n(p).$$

Probability in extensive case is,

$$P_N = (N+K+1)!\ X\ (N_{avg}/K) / (N!)(K-1)!\ x\ (1+N_{avg}/K)^{N+K}$$

**Non-Extensive Entropy** :

Introduction of q index- q factor is introduced in order to elaborate the tail part of distribution.

$S_q = (1-\sum p) / (q-1)$ , q = any real number (should be greater than 1)

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$$

This entropy leads to **Tsallis Distribution**.

Probability $P_N$ is defined by using the partition function $P_N = Z / \sum Z$ where $Z= (1/N!)\ x\ (nV-nv_0)^N$

N = number of charged particle produced($\sim V/v_0$)

n= number density (N/V)

V=Adjusted Volume of Real Gas

$v_0$ = Excluded volume of particle produced ($v_0 = 16\pi r^3 / 3$) , r is radius of particle produced

After calculations we get the value of q with

$$N_{avg} = Vn + (q-1)\lambda^2V^2n^2 - (q-1)nv\lambda - 2v_0n^2V$$
$$K = N_{avg}^2/(D^2 - N_{avg})$$
$$1/K = (q-1)\lambda^2 - (2v_0/n)$$

Upper limit on q : $q < 1 + v_0/3V$ , this limits comes in picture due to limit on N [ $N<1/(3q-3)$ ]

l When $q = 1$ ,& $v_0 = 0$ all formulation leads back to Negative binomial distribution.

## Modified Tsallis Distribution

- High energy data reflect the presence of shoulder structure in multiplicity distributions
- Appearance of one or more jets in final state is a result of radiation of hard gluons.
- Probability of such distribution is defined as

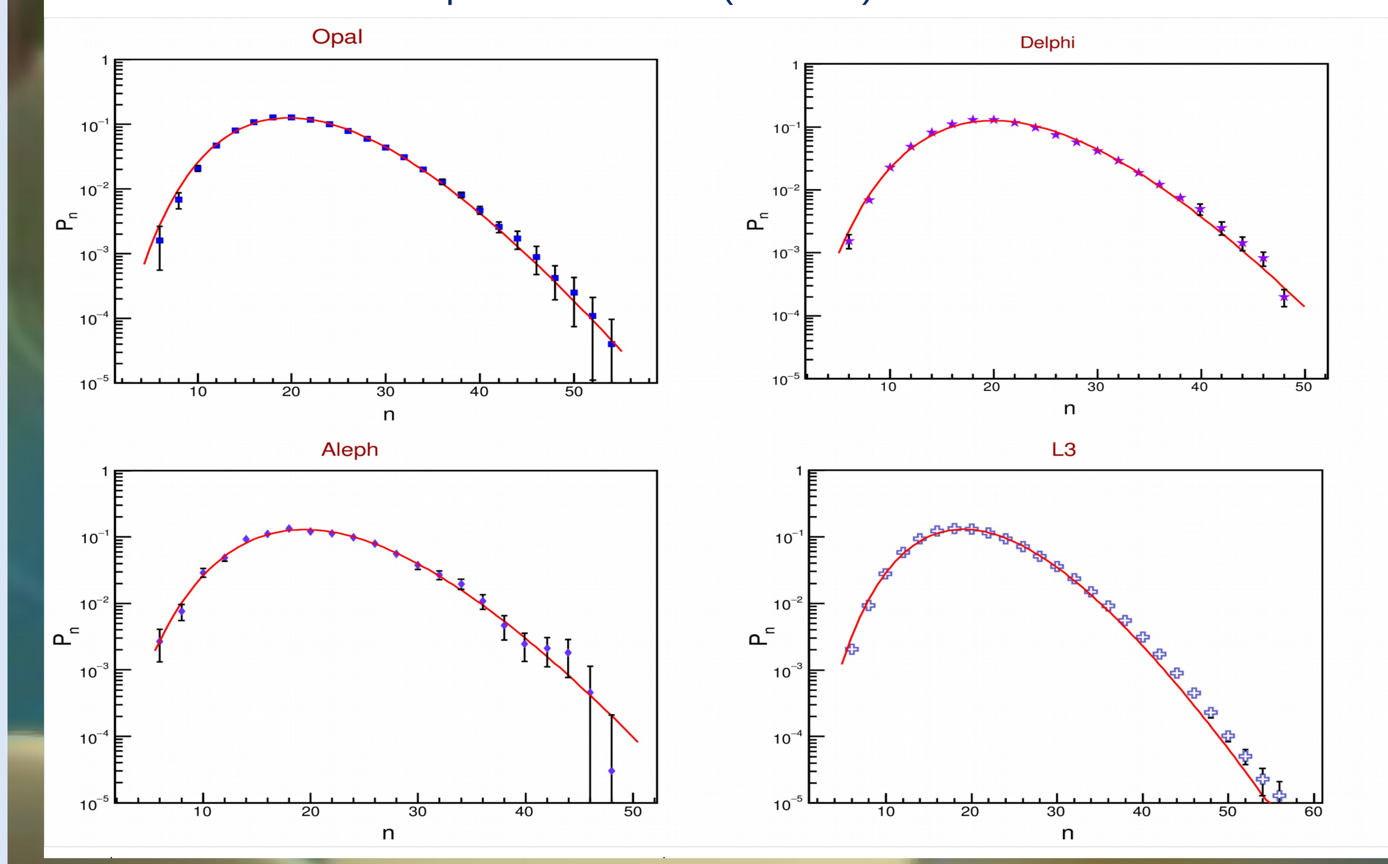
$$P_N = \alpha P_n + (1-\alpha)P_n$$

where $\alpha$ is the weight factor of modified Tsallis distribution.

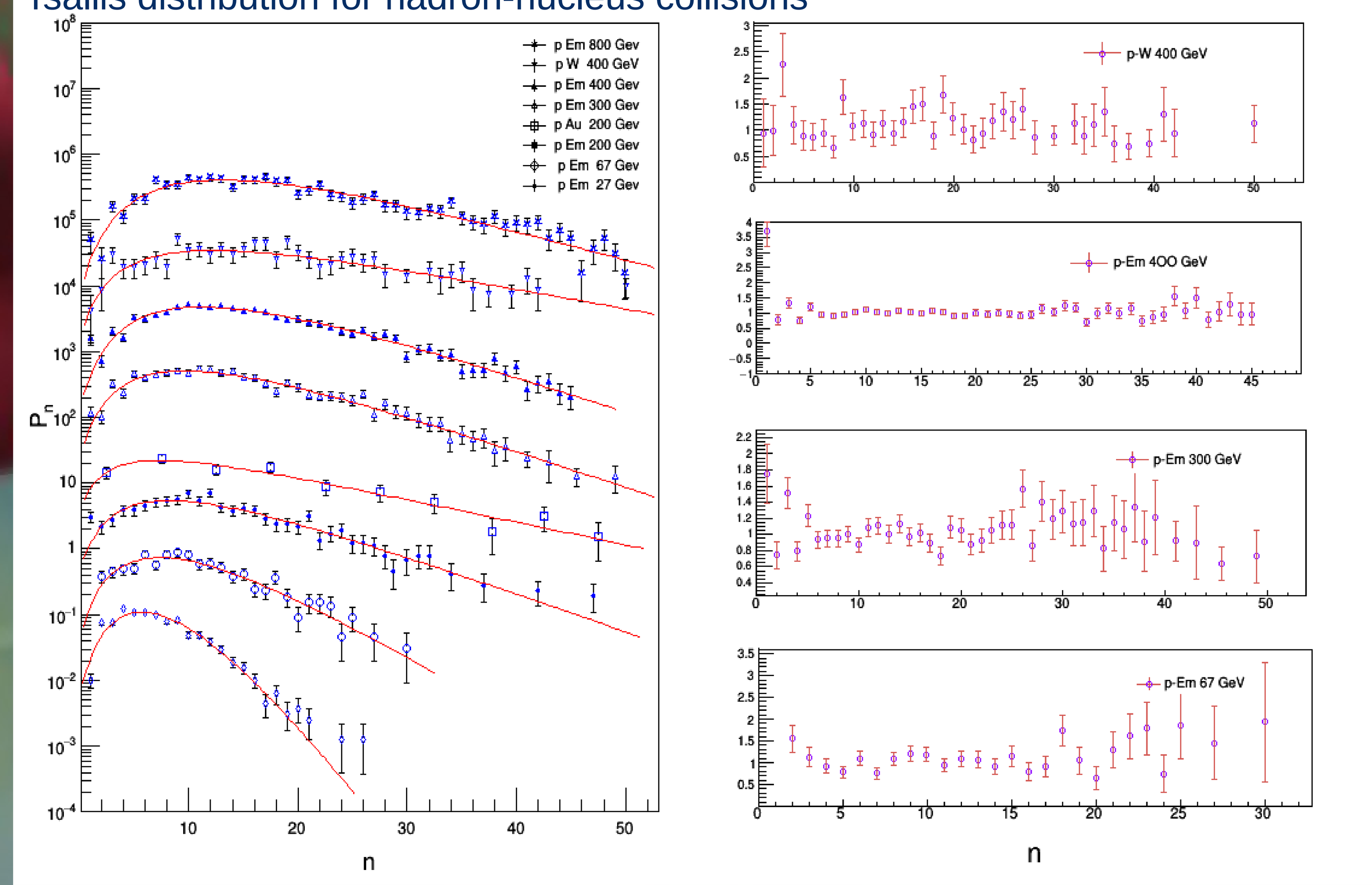
The weight factor $\alpha$ gives the 2- jet event fraction.

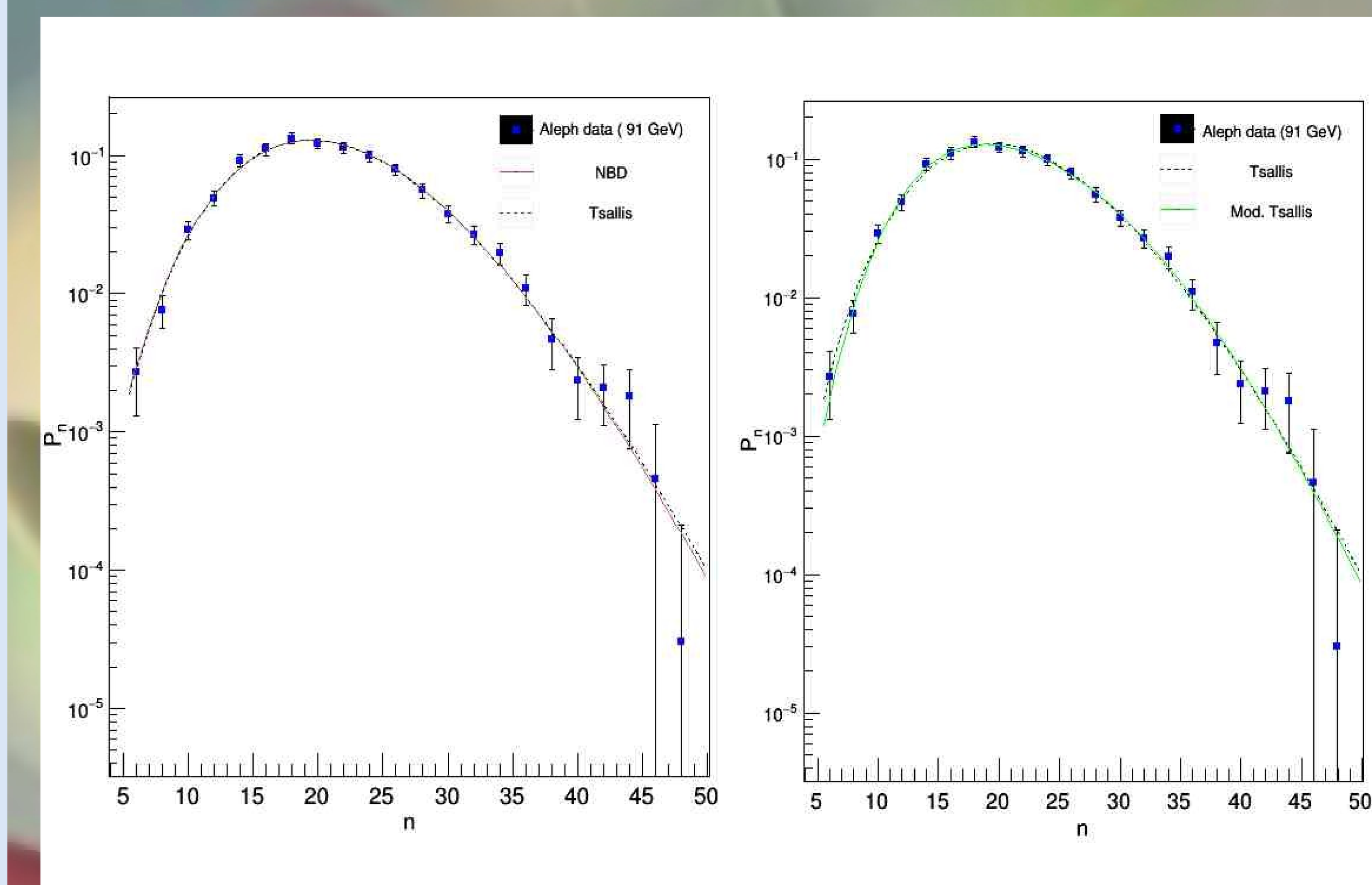
- In this case multiplicity is parameterised in terms of superposition of two Tsallis distributions that arises due to 2 jet and multi jet fraction.
- Total number of charged particles are even due to charge conservation.
- The weight factor, alpha depends upon jet algorithm, Duhram in present case.
- This distribution is named as modified Tsallis distribution.

Tsallis distribution for leptonic collisions (91 GeV)




Tsallis distribution for hadron-nucleus collisions




## Comparison NBD, Tsallis & modified Tsallis

| Exp. | Energy (GeV) | NBD | Tsallis | Modified Tsallis | α | nv01 | nV02 | q1 | q2 | $nV_0$ | q |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Modified Tsallis | | | | | Tsallis | |
| AMY | 57 | 5.82/16 | 3.42/14 | 1.45/12 | 0.723 | -0.355 ± 0.03 | -0.165 ± 0.06 | 1.004 ± 0.0003 | 1.014 ± 0.006 | -0.13± 0.012 | 1.001 ± 0.002 |
| ALEPH | 91 | 9.22/19 | 8.76/20 | 6.48/18 | 0.657 | -0.305 ±0.004 | -0.302 ± 0.037 | 1.007 ± 0.003 | 1.001 ± 0.0009 | -0.291 ± 0.008 | 1.010 ± 0.004 |
| DELPHI | 91 | 49.79/20 | 42.03/19 | 7.85/17 | 0.657 | -0.345 ± 0.008 | -0.342 ± 0.007 | 1.009 ± 0.001 | 1.002 ± 0.0003 | -0.329 ± 0.005 | 1.010 ± 0.002 |
| OPAL | 91 | 33.15/22 | 25.64/21 | 0.70/19 | 0.657 | -0.102 ± 0.007 | -0.225 ± 0.003 | 1.006 ± 0.002 | 1.002 ± 0.0008 | -0.306 ± 0.002 | 1.015 ± 0.004 |
| L3 | 91 | 538/25 | 403.63/14 | 2.86/22 | 0.657 | -0.321 ± 0.007 | -0.140 ± 0.021 | 1.017 ± 0.005 | 1.001 ± 0.0007 | -0.337 ± 0.014 | 1.009 ± 0.005 |
| OPAL | 161 | 5.81/22 | 3.69/20 | 2.12/18 | 0.635 | -0.249 ± 0.011 | -0.051 ± 0.005 | 1.003 ± 0.0017 | 1.006 ± 0.002 | -0.337 ± 0.014 | 1.049 ± 0.026 |

## Literature Cited

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## Conclusions

**❖All the q values greater than 1 in Tsallis distribution confirms that non extensive q statistics is applicable in the multiplicity distributions.**

**❖ A convolution of the two Tsallis distribution corresponding to two jet and multi jet events improves the chi square value significantly.**

**❖ There is small increases in q values with increase in $\sqrt{s}$**

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