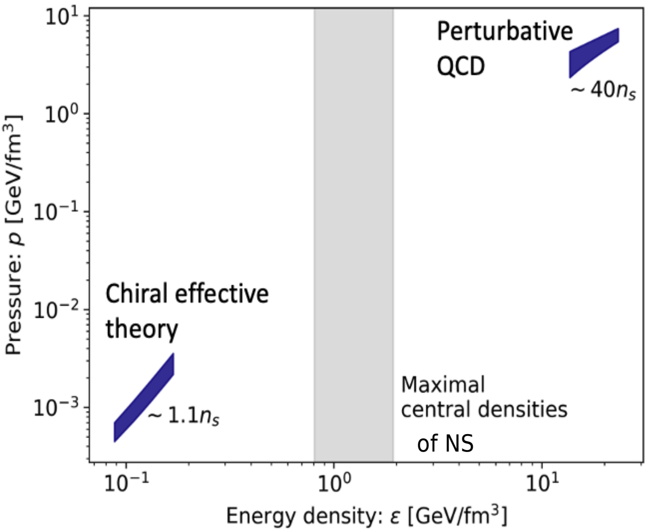
Reconstruction of nuclear matter parameters in a Bayesian approach

Based on : https://doi.org/10.1103/PhysRevC.105.015806

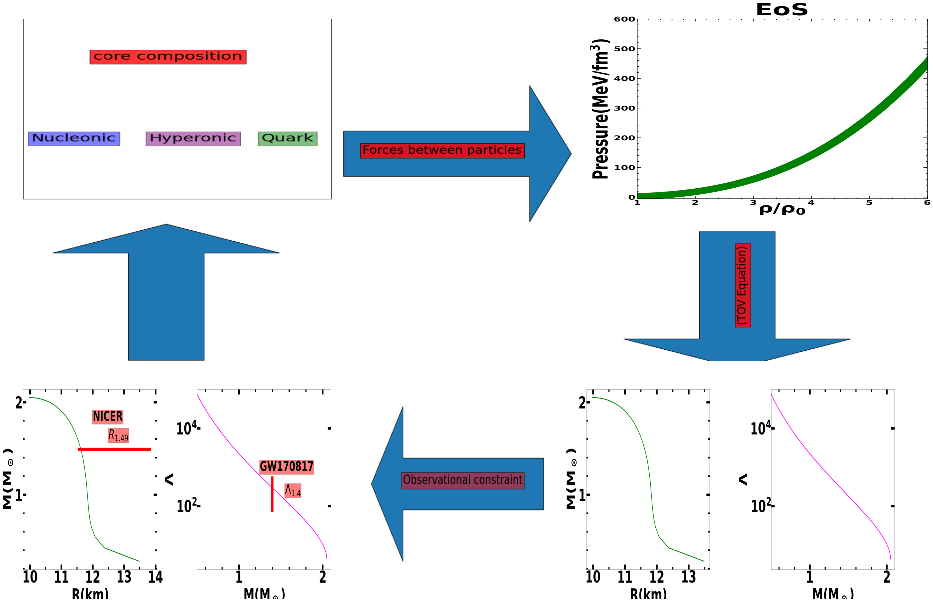
Sk Md Adil Imam

Co-authors : N. K. Patra, T. Malik,C. Mondal, B.K. Agrawal

Introduction



Introduction



Equation of State

- We consider n,p,e,μ in the core part of neutron star

Equation of State

- We consider n,p,e,μ in the core part of neutron star
- Conservation of baryon number :$\rho = \rho_n + \rho_p$

Equation of State

- We consider n,p,e,μ in the core part of neutron star
- Conservation of baryon number :$\rho = \rho_n + \rho_p$
- charge neutrality : $\rho_p = \rho_e + \rho_\mu$

Equation of State

- We consider n,p,e,μ in the core part of neutron star
- Conservation of baryon number :$\rho = \rho_n + \rho_p$
- charge neutrality : $\rho_p = \rho_e + \rho_\mu$
- β-equllibrium :$\mu_n = \mu_p + \mu_e$
  $\mu_e = \mu_\mu$

Equation of State

- We consider n,p,e,μ in the core part of neutron star
- Conservation of baryon number :$\rho = \rho_n + \rho_p$
- charge neutrality : $\rho_p = \rho_e + \rho_\mu$
- β-equllibrium :$\mu_n = \mu_p + \mu_e$
  $\mu_e = \mu_\mu$
- using these equations we can calculate the particle fraction and $(\rho_n - \rho_p)/\rho = \delta$:Isospin asymmetry parameter

Equation of State

- $\varepsilon(\rho, \delta) = \varepsilon(\rho, 0) + J(\rho)\delta^2$

Equation of State

- $\varepsilon(\rho,\delta) = \varepsilon(\rho,0) + J(\rho)\delta^2$
- $\varepsilon(\rho,0)$: SNM EoS

Equation of State

- $\varepsilon(\rho, \delta) = \varepsilon(\rho, 0) + J(\rho)\delta^2$
- $\varepsilon(\rho, 0)$: SNM EoS
- $J(\rho) = \varepsilon(\rho, 1) - \varepsilon(\rho, 0)$:Symmetry energy

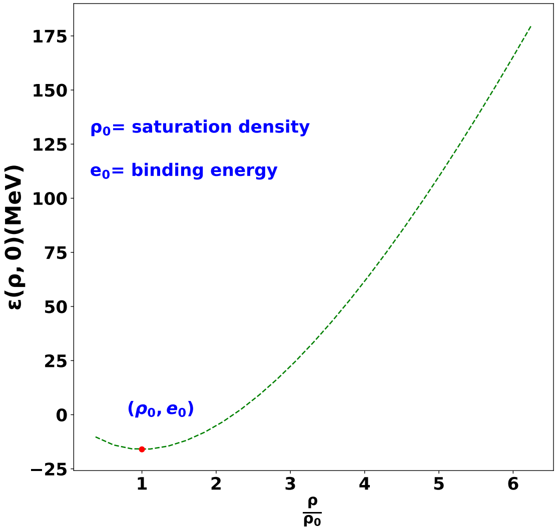
Equation of State

- $\varepsilon(\rho, \delta) = \varepsilon(\rho, 0) + J(\rho)\delta^2$
- $\varepsilon(\rho, 0)$: SNM EoS
- $J(\rho) = \varepsilon(\rho, 1) - \varepsilon(\rho, 0)$:Symmetry energy
- $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$:Isospin asymmetry

Equation of State

- $\varepsilon(\rho,\delta) = \varepsilon(\rho,0) + J(\rho)\delta^2$
- $\varepsilon(\rho,0)$: SNM EoS
- $J(\rho) = \varepsilon(\rho,1) - \varepsilon(\rho,0)$:Symmetry energy
- $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$:Isospin asymmetry
- $\rho_n(\rho_p)$:Neutron(Proton) density

Variation of energy of SNM with baryon density



Taylor Expansion

- $\varepsilon(\rho, 0) = \sum_n \frac{a_n}{n!} x^n$
- $x = \left(\frac{\rho - \rho_0}{3\rho_0}\right)$

Taylor Expansion

- $\varepsilon(\rho, 0) = \sum_n \frac{a_n}{n!} x^n$
- $x = \left(\frac{\rho - \rho_0}{3\rho_0}\right)$

- $J(\rho) = \sum_n \frac{b_n}{n!} x^n$

Taylor Expansion

- $\varepsilon(\rho, 0) = \sum_n \frac{a_n}{n!} x^n$
- $x = \left(\frac{\rho - \rho_0}{3\rho_0}\right)$

- $J(\rho) = \sum_n \frac{b_n}{n!} x^n$

- $\varepsilon(\rho, \delta) = \sum_n \frac{1}{n!}(a_n + b_n \delta^2) x^n$

Taylor Expansion

- **SNM parameters :**
- $a_0 = e_0 \equiv$Binding Energy at ρ_0
- $a_1 = 0$
- $a_2 = K_0 \equiv$ Incompressibility Coefficient at ρ_0
- $a_3(a_4) = Q_0(Z_0) \equiv$ Third(Fourth)order Derivative at ρ_0
- **Symmetry Energy parameters :**
- $b_0 = J_0 \equiv$Symmetry Energy at ρ_0
- $b_1 = L_0 \equiv$ Slope of Symmetry Energy at ρ_0
- $b_2 = K_{sym,0} \equiv$Symmetry Energy Curvature at ρ_0
- $b_3(b_4) = Q_{sym,0}(Z_{sym,0}) \equiv$ Third(Fourth)order Derivative at ρ_0

n/3 Expansion

- $\frac{n}{3}$ Expansion: $\varepsilon(\rho,\delta) = \sum_{n=2}^{6}(a'_n + b'_n\delta^2)(\frac{\rho}{\rho_0})^{n/3}$

n/3 Expansion

- $\frac{n}{3}$ Expansion:$\varepsilon(\rho,\delta) = \sum_{n=2}^{6}(a'_n + b'_n\delta^2)(\frac{\rho}{\rho_0})^{n/3}$
- $$\begin{pmatrix} e_0 \\ 0 \\ K_0 \\ Q_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ -2 & 0 & 4 & 10 & 18 \\ 8 & 0 & -8 & -10 & 0 \\ -56 & 0 & 40 & 40 & 0 \end{pmatrix} \begin{pmatrix} a'_0 \\ a'_1 \\ a'_2 \\ a'_3 \\ a'_4 \end{pmatrix}$$
$$\begin{pmatrix} J_0 \\ L_0 \\ K_{\text{sym},0} \\ Q_{\text{sym},0} \\ Z_{\text{sym},0} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ -2 & 0 & 4 & 10 & 18 \\ 8 & 0 & -8 & -10 & 0 \\ -56 & 0 & 40 & 40 & 0 \end{pmatrix} \begin{pmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \end{pmatrix}.$$

Knowledge about the NMPs

- The parameters which are known within a few percent: e_0 and ρ_0.

Knowledge about the NMPs

- The parameters which are known within a few percent: e_0 and ρ_0.

- The parameters which are known within about 10 percent: J_0, K_0

Knowledge about the NMPs

- The parameters which are known within a few percent: e_0 and ρ_0.

- The parameters which are known within about 10 percent: J_0, K_0

- The parameters which are known within about 50 percent: L_0.

Knowledge about the NMPs

- The parameters which are known within a few percent: e_0 and ρ_0.
- The parameters which are known within about 10 percent: J_0, K_0
- The parameters which are known within about 50 percent: L_0.
- The parameters which are almost unknown: $Q_0, Z_0, K_{sym,0}, Q_{sym,0}, Z_{sym,0}$

Aim:

- $\varepsilon(\rho, \delta) = \varepsilon(\rho, 0) + J(\rho)\delta^2 = f(\theta)$

Aim:

- $\varepsilon(\rho, \delta) = \varepsilon(\rho, 0) + J(\rho)\delta^2 = f(\theta)$
- We want to reconstruct these parameters from a given EoS(known precisely)

Bayesian estimation

- This approach is mainly based on the Bayes theorem which states that,
- $P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}$
- $\boldsymbol{\theta}$:model parameters and D:Data
- $\mathcal{Z}$:Evidence

Bayesian estimation

- This approach is mainly based on the Bayes theorem which states that,
- $P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}$
- $\boldsymbol{\theta}$:model parameters and D:Data
- $\mathcal{Z}$:Evidence
- $P(\boldsymbol{\theta})$: Prior for the model parameters

Bayesian estimation

- This approach is mainly based on the Bayes theorem which states that,
- $P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}$
- $\boldsymbol{\theta}$:model parameters and D:Data
- $\mathcal{Z}$:Evidence
- $P(\boldsymbol{\theta})$: Prior for the model parameters
- $\mathcal{L}(D|\boldsymbol{\theta})$: Likelihood function=$\prod_j \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{1}{2}\left(\frac{d_j - m_j(\boldsymbol{\theta})}{\sigma_j}\right)^2}$

Bayesian estimation

- This approach is mainly based on the Bayes theorem which states that,
- $P(\boldsymbol{\theta}|D) = \frac{\mathcal{L}(D|\boldsymbol{\theta})P(\boldsymbol{\theta})}{\mathcal{Z}}$
- $\boldsymbol{\theta}$:model parameters and D:Data
- $\mathcal{Z}$:Evidence
- $P(\boldsymbol{\theta})$: Prior for the model parameters
- $\mathcal{L}(D|\boldsymbol{\theta})$: Likelihood function=$\prod_j \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{1}{2}\left(\frac{d_j - m_j(\boldsymbol{\theta})}{\sigma_j}\right)^2}$
- $P(\boldsymbol{\theta_i}|D)$: Marginalised posterior distribution of the parameters (θ_i) =$\int P(\boldsymbol{\theta}|D) \prod_{k\neq i} d\theta_k$

Bayesian estimation

- So for Bayesian parameter estimation we need:
- Data (D): EoS

Bayesian estimation

- So for Bayesian parameter estimation we need:
- Data (D): EoS
- Model$(M(\theta))$

Bayesian estimation

- So for Bayesian parameter estimation we need:
- Data (D): EoS
- Model$(M(\theta))$
- Prior $(P(\theta))$

Model

- Taylor Expansion: $\varepsilon(\rho, \delta) = \sum_{n=0}^{4} \frac{1}{n!}(a_n + b_n\delta^2)x^n$

Model

- Taylor Expansion:$\varepsilon(\rho, \delta) = \sum_{n=0}^{4} \frac{1}{n!}(a_n + b_n\delta^2)x^n$
- $\frac{n}{3}$ Expansion:$\varepsilon(\rho, \delta) = \sum_{n=2}^{6}(a'_n + b'_n\delta^2)(\frac{\rho}{\rho_0})^{n/3}$

Construction of EoS(Data)

- we choose a fixed set of parameters to create the mock data

Construction of EoS(Data)

- we choose a fixed set of parameters to create the mock data

Table: The values of nuclear matter parameters (in MeV) which are employed to construct various pseudo data using the Taylor and $\frac{n}{3}$ expansions.

N	Symmetric nuclear matter		Symmetry energy	
0	e_0	-16.0	J_0	32.0
1			L_0	50.0
2	K_0	230	$K_{\mathrm{sym},0}$	-100
3	Q_0	-400	$Q_{\mathrm{sym},0}$	550
4	Z_0	1500	$Z_{\mathrm{sym},0}$	-750

- Thermodynamic Stability

Construction of EoS(Data)

- we choose a fixed set of parameters to create the mock data

Table: The values of nuclear matter parameters (in MeV) which are employed to construct various pseudo data using the Taylor and $\frac{n}{3}$ expansions.

-

N	Symmetric nuclear matter		Symmetry energy	
0	e_0	-16.0	J_0	32.0
1			L_0	50.0
2	K_0	230	$K_{\mathrm{sym},0}$	-100
3	Q_0	-400	$Q_{\mathrm{sym},0}$	550
4	Z_0	1500	$Z_{\mathrm{sym},0}$	-750

- Thermodynamic Stability
- Symmetry Energy,$J(\rho)$ should not be negative

Construction of EoS(Data)

- we choose a fixed set of parameters to create the mock data

Table: The values of nuclear matter parameters (in MeV) which are employed to construct various pseudo data using the Taylor and $\frac{n}{3}$ expansions.

N	Symmetric nuclear matter		Symmetry energy	
0	e_0	-16.0	J_0	32.0
1			L_0	50.0
2	K_0	230	$K_{\rm sym,0}$	-100
3	Q_0	-400	$Q_{\rm sym,0}$	550
4	Z_0	1500	$Z_{\rm sym,0}$	-750

- Thermodynamic Stability
- Symmetry Energy, $J(\rho)$ should not be negative
- Causality($\frac{dP}{d\epsilon} \leq 1$)

Construction of EoS(Data)

- we choose a fixed set of parameters to create the mock data

Table: The values of nuclear matter parameters (in MeV) which are employed to construct various pseudo data using the Taylor and $\frac{n}{3}$ expansions.

N	Symmetric nuclear matter		Symmetry energy	
0	e_0	-16.0	J_0	32.0
1			L_0	50.0
2	K_0	230	$K_{\mathrm{sym},0}$	-100
3	Q_0	-400	$Q_{\mathrm{sym},0}$	550
4	Z_0	1500	$Z_{\mathrm{sym},0}$	-750

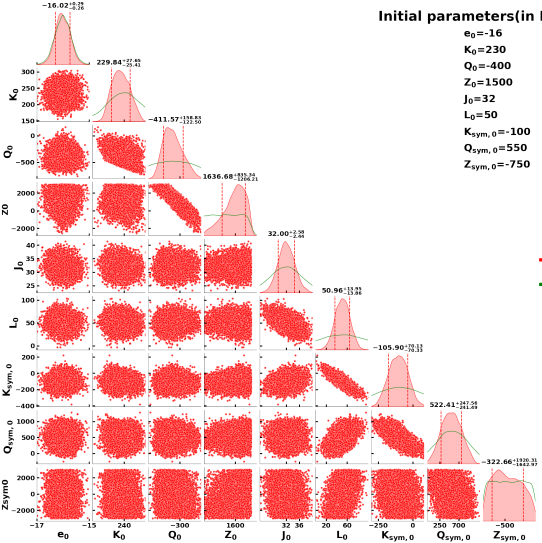
- Thermodynamic Stability
- Symmetry Energy, $J(\rho)$ should not be negative
- Causality($\frac{dP}{d\epsilon} \leq 1$)
- Maximum Mass for that EoS $\geq 2M_{\odot}$

Prior

Table: Two different sets P1 and P2 for the prior distributions of the nuclear matter parameters (in MeV). The saturation density ρ_0 is taken to be 0.16 fm^{-3}.

Parameters	P1			P2		
	Pr-Dist	μ min	σ max	Pr-Dist.	μ min	σ max
e_0	G	-16	0.3	G	-16	0.3
K_0	G	240	100	G	240	50
Q_0	U	-2000	2000	G	-400	400
Z_0	U	-3000	3000	U	-3000	3000
J_0	G	32	5	G	32	5
L_0	U	20	150	G	50	50
$K_{\mathrm{sym},0}$	U	-1000	1000	G	-100	200
$Q_{\mathrm{sym},0}$	U	-2000	2000	G	-550	400
$Z_{\mathrm{sym},0}$	U	-3000	3000	U	-3000	3000

Posterior Distribution of the Parameters (n/3 model,P2):



Initial parameters(in MeV)

- e_0=-16
- K_0=230
- Q_0=-400
- Z_0=1500
- J_0=32
- L_0=50
- $K_{sym,0}$=-100
- $Q_{sym,0}$=550
- $Z_{sym,0}$=-750

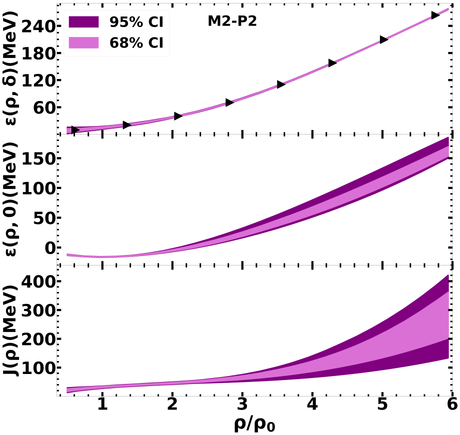
posterior

prior

Results

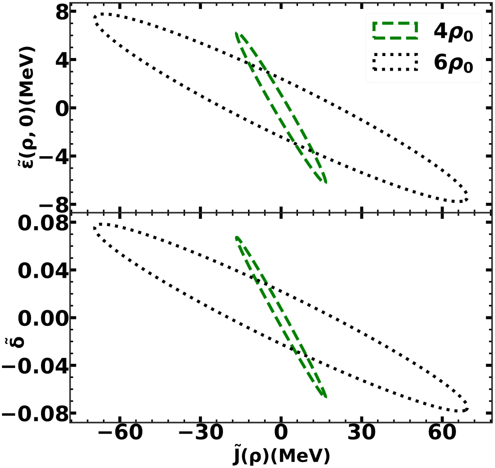
$$\varepsilon(\rho,\delta) = \varepsilon(\rho,0) + J(\rho)\delta^2$$

- **Posterior distribution of the EoS (n/3 model,P2):**



Results

- **Confidence ellipse (n/3 model, P2) :**



Summary

- we have used Bayesian Inference to reconstruct NMPs from EoS of NS matter.

Summary

- we have used Bayesian Inference to reconstruct NMPs from EoS of NS matter.
- The median values of second or higher order NMPs show sizeable deviations from their true values and associated uncertainties are larger.

Summary

- we have used Bayesian Inference to reconstruct NMPs from EoS of NS matter.

- The median values of second or higher order NMPs show sizeable deviations from their true values and associated uncertainties are larger.

- The sources of these uncertainties are intrinsic in nature, identified as:

Summary

- we have used Bayesian Inference to reconstruct NMPs from EoS of NS matter.
- The median values of second or higher order NMPs show sizeable deviations from their true values and associated uncertainties are larger.
- The sources of these uncertainties are intrinsic in nature, identified as:
- (i) Correlations among various NMPs and

Summary

- we have used Bayesian Inference to reconstruct NMPs from EoS of NS matter.
- The median values of second or higher order NMPs show sizeable deviations from their true values and associated uncertainties are larger.
- The sources of these uncertainties are intrinsic in nature, identified as:
- (i) Correlations among various NMPs and
- (ii) The balance between the EoS of symmetric nuclear matter, symmetry energy, and the neutron-proton asymmetry

Thank You!

For Your Attention