Gravitational waves - a Quantum Field Theory perspective

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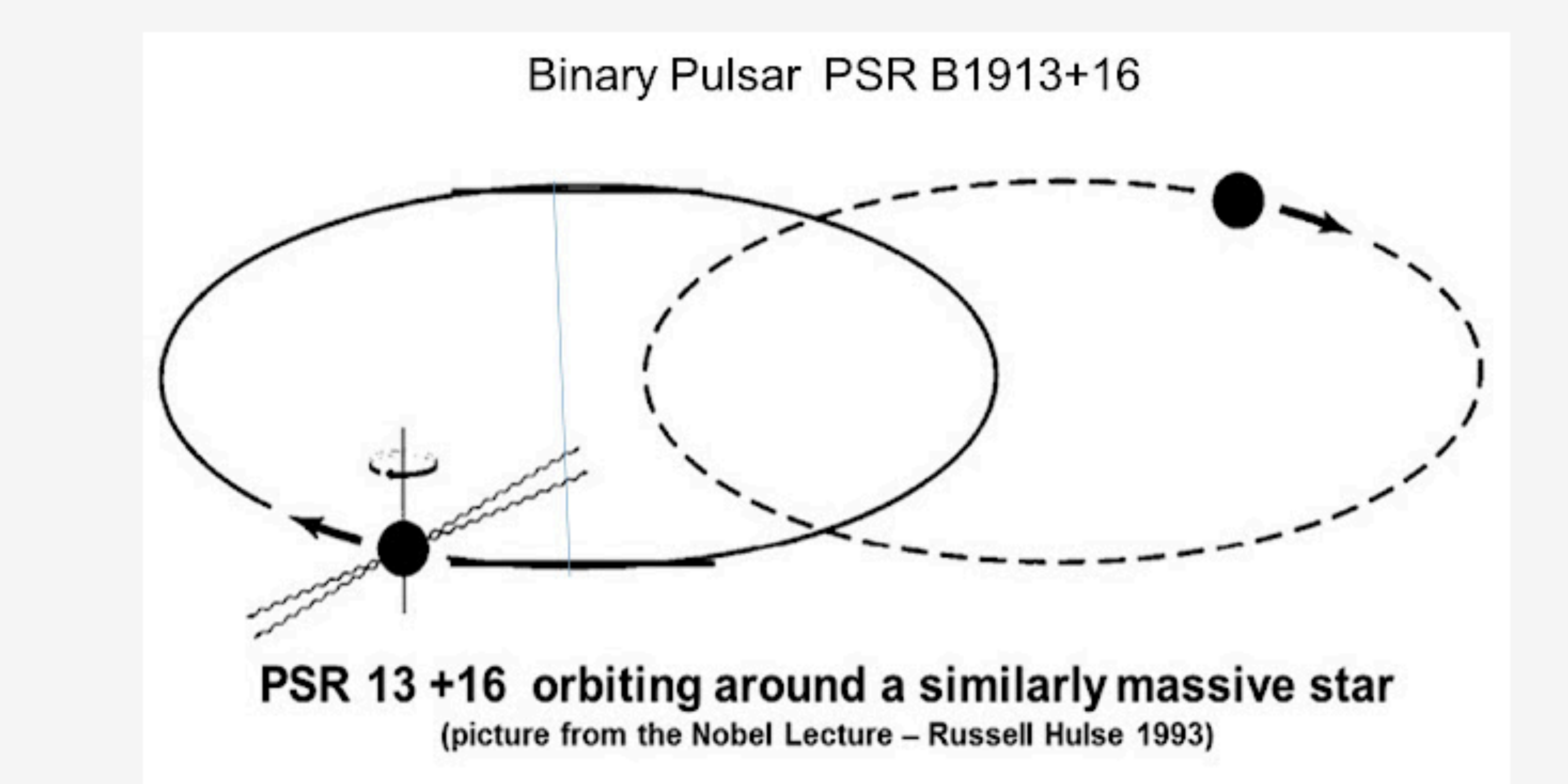
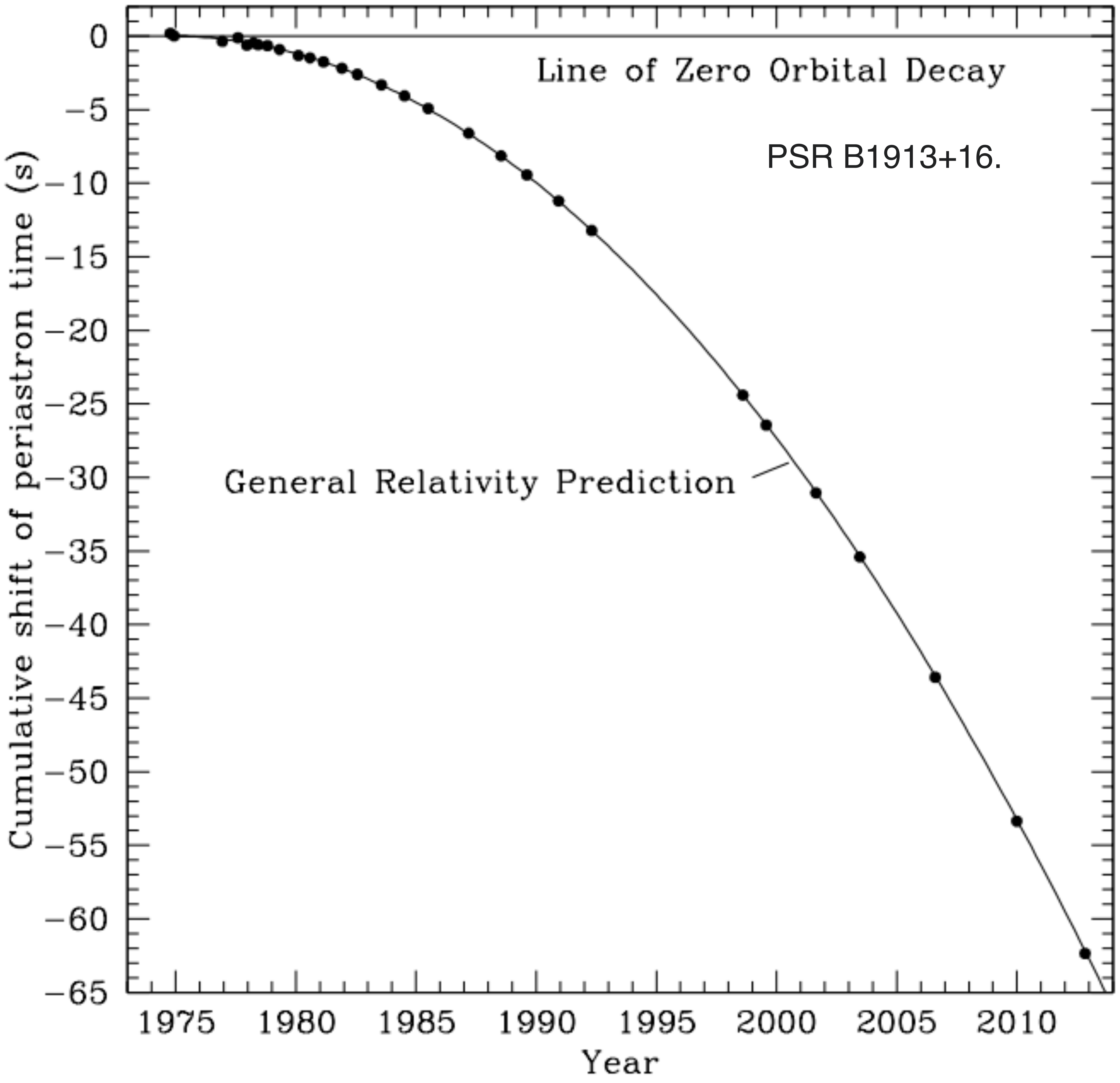


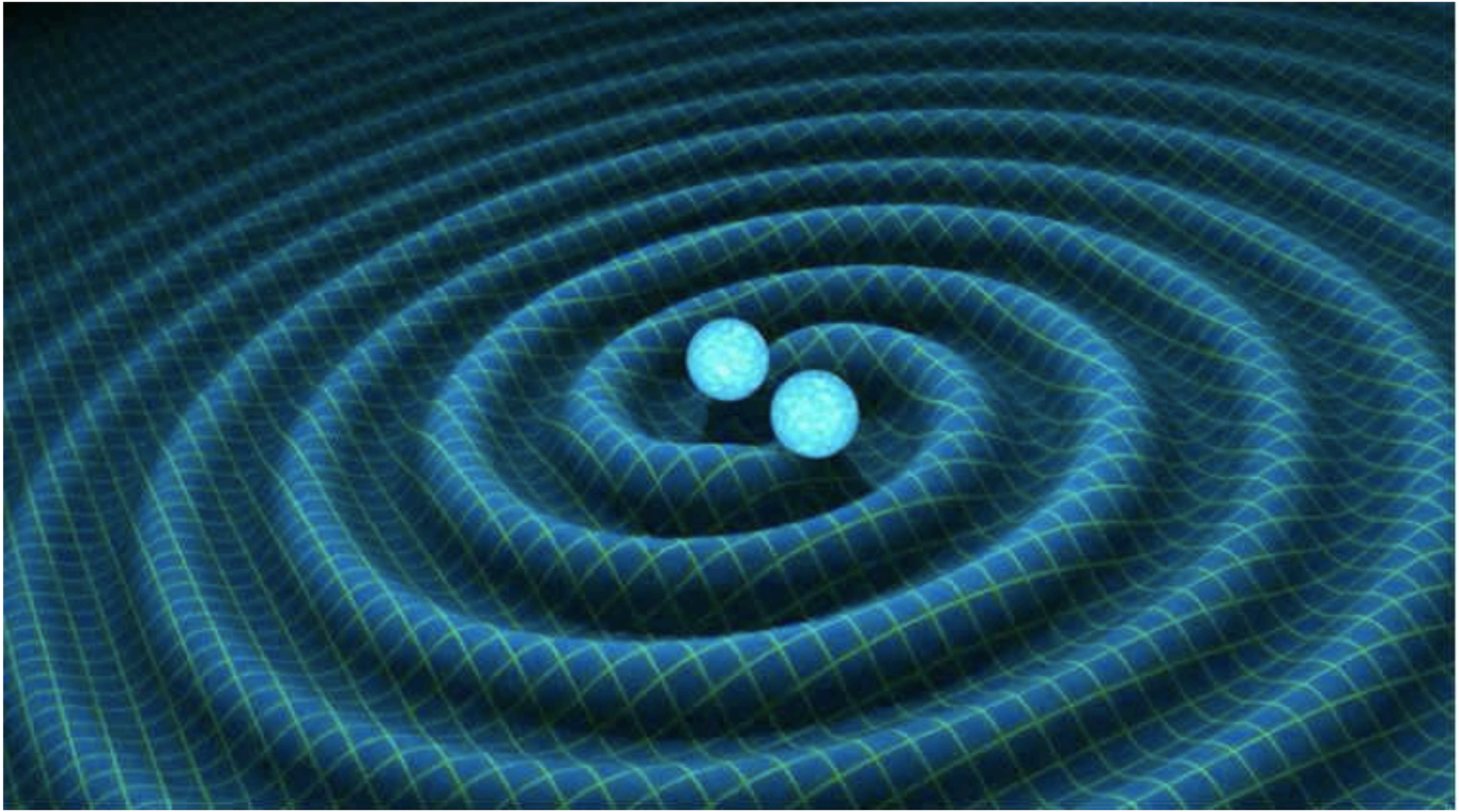
Energy loss by gravitational wave radiation in compact binaries

$$\left(\frac{\mathrm{d}E}{\mathrm{d}T}\right)^{\mathrm{GW}} = \frac{32G}{5}\Omega^6\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 a^4(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

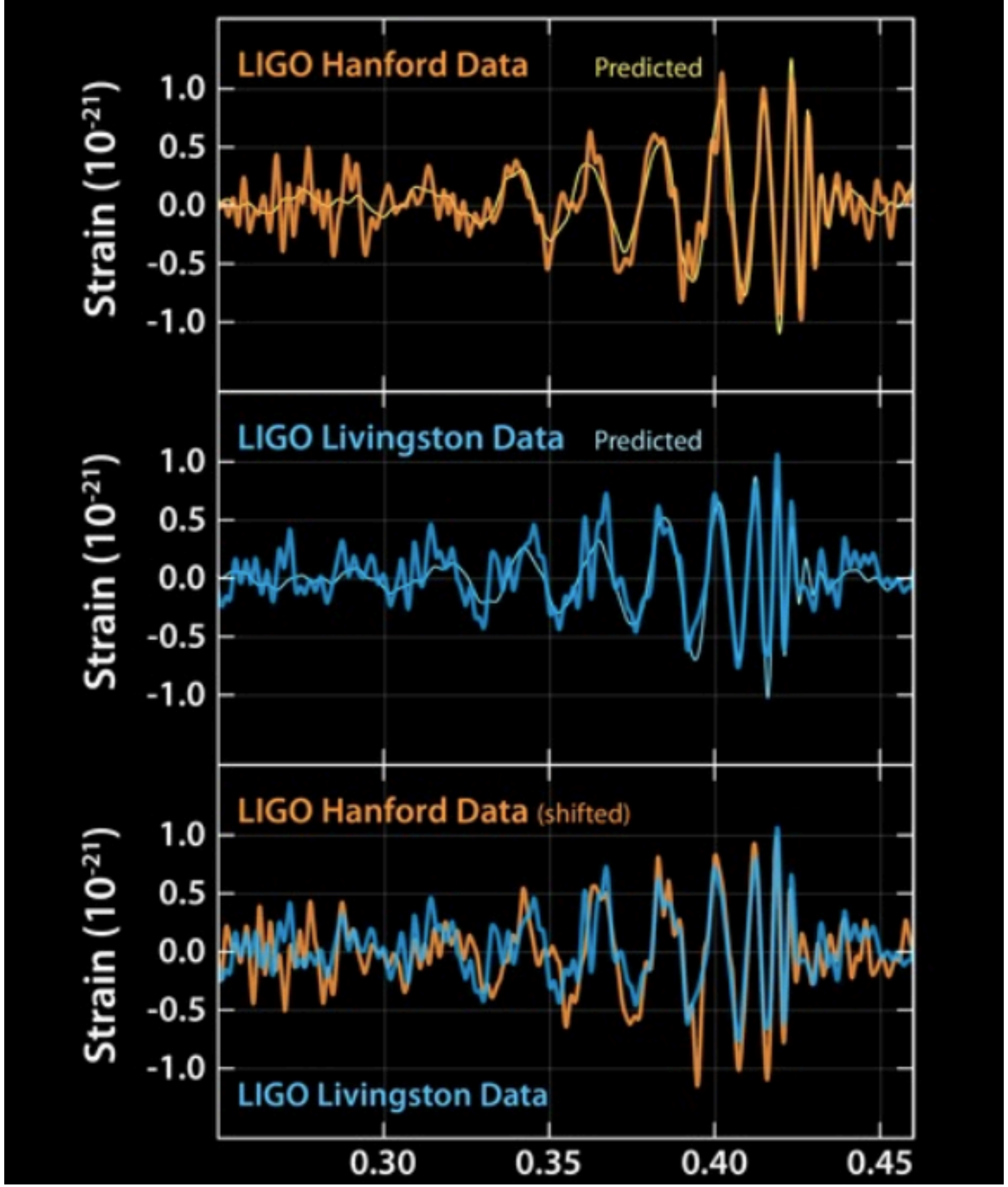
P.C Peters and J. Mathews (1963)

Binary pulsar tests of GR





LIGO Event 2016

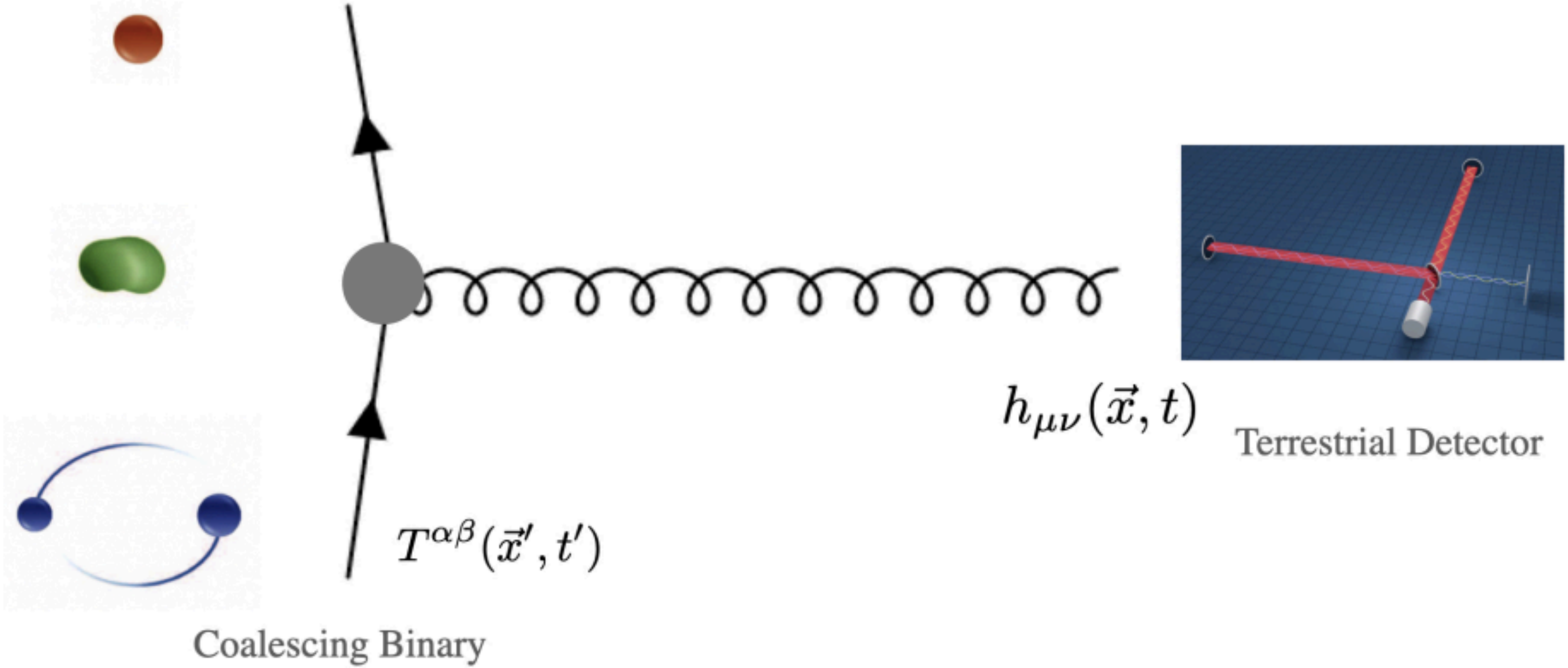


Energy radiated in gravitational waves

Einstein Quadrupole Radiation Formula (1918)

$$\left(\frac{\mathrm{d}E}{\mathrm{d}T}\right)^{\mathrm{GW}} = \frac{G}{c^5}\left\{\frac{1}{5}\frac{\mathrm{d}^3\mathrm{Q}_{ab}}{\mathrm{d}T^3}\frac{\mathrm{d}^3\mathrm{Q}_{ab}}{\mathrm{d}T^3} + \mathcal{O}\left(\frac{1}{c^2}\right)\right\}$$

Diagramatic calculation of GW emission



Linearised Gravity

$$S_{EH} = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\kappa = \sqrt{32\pi G}$$

$$S = S_{EH}(g_{\mu\nu}) + S_m(\phi_i, g_{\mu\nu})$$

Taylor expand matter Lagrangian around a background metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\begin{aligned}
\delta S_m &= \int d^4x \delta \left[\sqrt{-g}\mathcal{L}_m(\phi_i, g_{\mu\nu)})\right] \\
&= \int d^4x \sqrt{-g} \left[\frac{\delta \mathcal{L}_m}{\delta g_{\mu\nu}} - \frac{1}{2}\mathcal{L}_m\, g_{\mu\nu}\right]_{g_{\mu\nu}=\bar{g}_{\mu\nu}} \delta g_{\mu\nu} \\
&= \int d^4x \sqrt{-\bar{g}} \left(\frac{-1}{2}\right) T^{\mu\nu} \; (\kappa h_{\mu\nu})
\end{aligned}$$

$$T^{\mu\nu} = -2\left[\frac{\delta \mathcal{L}_m}{\delta g_{\mu\nu}} - \frac{1}{2}\mathcal{L}_m g_{\mu\nu}\right]_{g_{\mu\nu}=\bar{g}_{\mu\nu}}$$

Graviton-matter coupling

$$\mathcal{L}_{int} = -\frac{\kappa}{2} T^{\mu\nu} h_{\mu\nu}$$

Gauge symmetry

$$x^\mu \longrightarrow x^\mu + \xi^\mu(x),$$
$$h_{\mu\nu}(x) \longrightarrow h_{\mu\nu}(x) - \left(\partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)\right)$$

$$-\frac{\kappa}{2} T^{\mu\nu} h_{\mu\nu} \rightarrow -\frac{\kappa}{2} T^{\mu\nu} \left(h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu\right)$$
$$= -\frac{\kappa}{2} \left(T^{\mu\nu} h_{\mu\nu} + \partial_\mu T^{\mu\nu} \xi_\nu + \partial_\nu T^{\mu\nu} \xi_\mu\right)$$

Stress tensor conservation

$$\partial_\mu T^{\mu\nu} = \partial_\nu T^{\mu\nu} = 0$$

Treat the graviton as a Quantum Field

$$\hat{h}_{\mu\nu}(x) = \sum_{\lambda'} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_q}} \left[\epsilon^{\lambda'}_{\mu\nu}(\mathbf{q}) a_{\lambda'}(\mathbf{q}) e^{-iq\cdot x} + \epsilon^{*\lambda'}_{\mu\nu}(\mathbf{q}) a^{\dagger}_{\lambda'}(\mathbf{q}) e^{iq\cdot x} \right]$$

$$\left[a_\lambda(\mathbf{k}), a^{\dagger}_{\lambda'}(\mathbf{k}') \right] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')\delta_{\lambda\lambda'}$$

Energy radiated by binaries in circular orbit

$$\frac{dE_{gw}}{dt} = \frac{\kappa^2}{4}\frac{8\pi}{5}\int \left(\bar{T}_{ij}(\omega')\bar{T}^*_{ji}(\omega') - \frac{1}{3}|\bar{T}^i{}_i(\omega')|^2\right)\omega^3 2\pi\delta(\omega - 2\Omega)\,\frac{d\omega}{(2\pi)^3 2\omega}$$

$$= \frac{32G}{5}\mu^2 a^2 \Omega^4$$

$$= \frac{32G^4}{5}\frac{m_1^2 m_2^2(m_1+m_2)}{a^5}$$

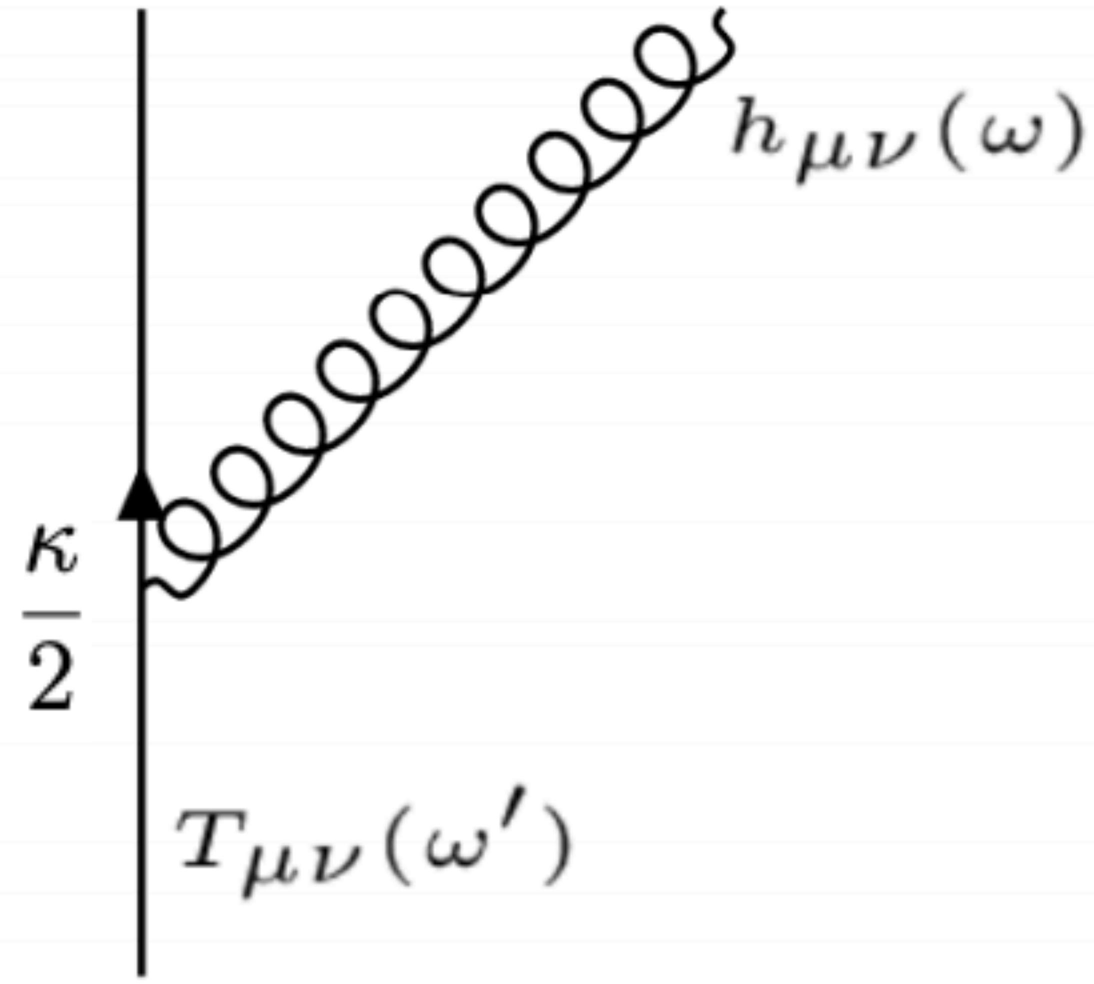
Gravitational radiation from elliptical orbits

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\kappa = \sqrt{32\pi G}$$

Interaction vertex:

$$\frac{\kappa}{2} T_{\mu\nu}(k') \epsilon_\lambda^{\mu\nu}(k)$$



$$T_{\mu\nu}(x') = \mu \delta^3(\mathbf{x}' - \mathbf{x}(t)) U_\mu U_\nu$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad U_\mu = (1, \dot{x}, \dot{y}, 0)$$

Keplerian orbit $\quad x = a(\cos\xi - e), \quad y = a\sqrt{(1-e^2)}\sin\xi, \quad \Omega t = \xi - e\sin\xi, \quad \Omega = \left[G\frac{(m_1+m_2)}{a^3}\right]^{\frac{1}{2}}$

Energy-momentum tensor for Elliptical Keplerian orbit

$$T_{xx}(\omega') = -\frac{\mu\omega'^2 a^2}{4n}\left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right]$$

$$T_{yy}(\omega') = \frac{\mu\omega'^2 a^2}{4n}\left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{4}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right]$$

$$T_{xy}(\omega') = \frac{-i\mu\omega'^2 a^2}{4n}(1-e^2)^{\frac{1}{2}}\left[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)\right]$$

Due to eccentricity of the orbit all harmonics of fundamental frequency of the orbit appear

$$\omega' = n\Omega$$

Energy loss from elliptical binaries in gravitational wave radiation

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2} \int \frac{8\pi}{5} \left[T_{ij}(\omega')T^*_{ji}(\omega') - \frac{1}{3}|T^i{}_i(\omega')|^2\right] \delta(\omega - \omega')\omega^2 d\omega$$

$$= \frac{32G}{5}\Omega^6 \mu^2 a^4 \sum_{n=1}^{\infty} n^6 f(n, e)$$

$$= \frac{32G}{5}\Omega^6 \left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 a^4 \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

SM & P.K.Panda Phys Rev D (1996)

Matches Peters-Mathews formula (1963) from classical general relativity

Energy loss of binary orbit by scalar wave emission

$$\frac{dE_s}{dt} = \frac{1}{24\pi}\left(\frac{N_1}{m_1} - \frac{N_2}{m_2}\right)^2 M^2\, g_s^2\, \Omega^4\, a^2\, \frac{(1+e^2/2)}{(1-e^2)^{5/2}}$$

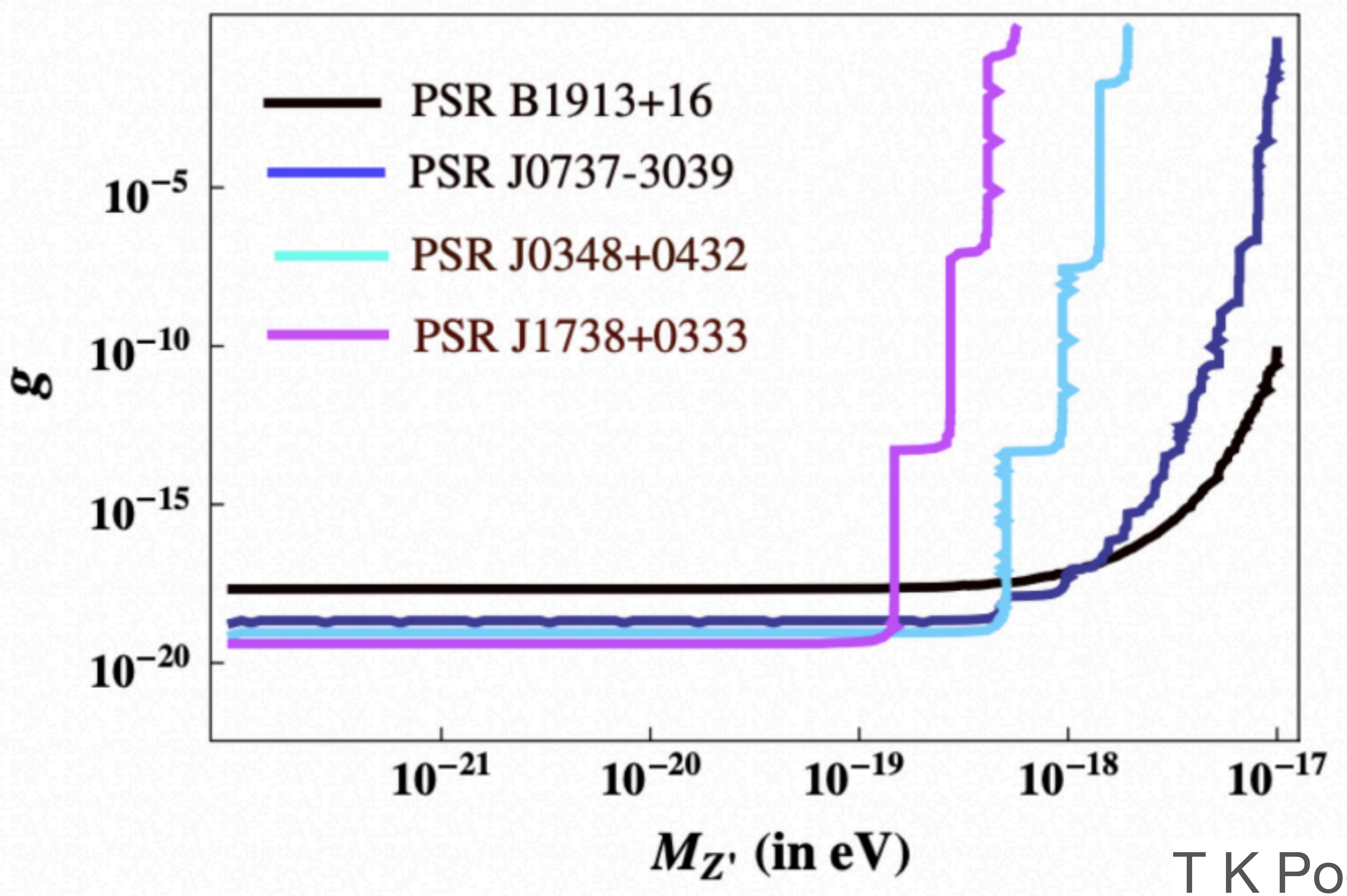
SM & P.K.Panda Phys Rev D (1996)

Ultra-light vectors with coupling to muons

$$\frac{dE_V}{dt} = \frac{g^2}{6\pi} a^2 M^2 \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)^2 \Omega^4 \frac{(1+\frac{e^2}{2})}{(1-e^2)^{\frac{5}{2}}}$$

T K Poddar, S M, and S Jana
Phys. Rev. D **100**, 123023 – 2019

$$\mathcal{L}_I = gJ^\mu Z'_\mu + \frac{M_{Z'}^2}{2} Z'_\mu Z'^\mu$$



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Phys. Rev. D **100**, 123023 – 2019

Gravitational radiation from binary systems in f(R) gravity - 2211.12947

Ashish Narang, Soumya Jana, SM.

$$S = \int \sqrt{-g} \mathrm{d}^4 \mathrm{x} \left[-\frac{1}{16\pi G} f(R) + \mathcal{L}_M \right]$$

Jordan Frame

Conformal transformation

$$\begin{aligned} g_{\mu\nu} &= A^2(\phi) \tilde{g}_{\mu\nu}, \\ A^2(\phi) &= \frac{1}{f'(R)}, \\ f'(R) &= \frac{df(R)}{dR}, \end{aligned}$$

Einstein Frame

$$S = \int \sqrt{-\tilde{g}} \mathrm{d}^4 \mathrm{x} \left[-\frac{\tilde{R}}{16\pi G} - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + V(\phi) \right] + S_M[A^2(\phi)\tilde{g}_{\mu\nu}, \Psi],$$

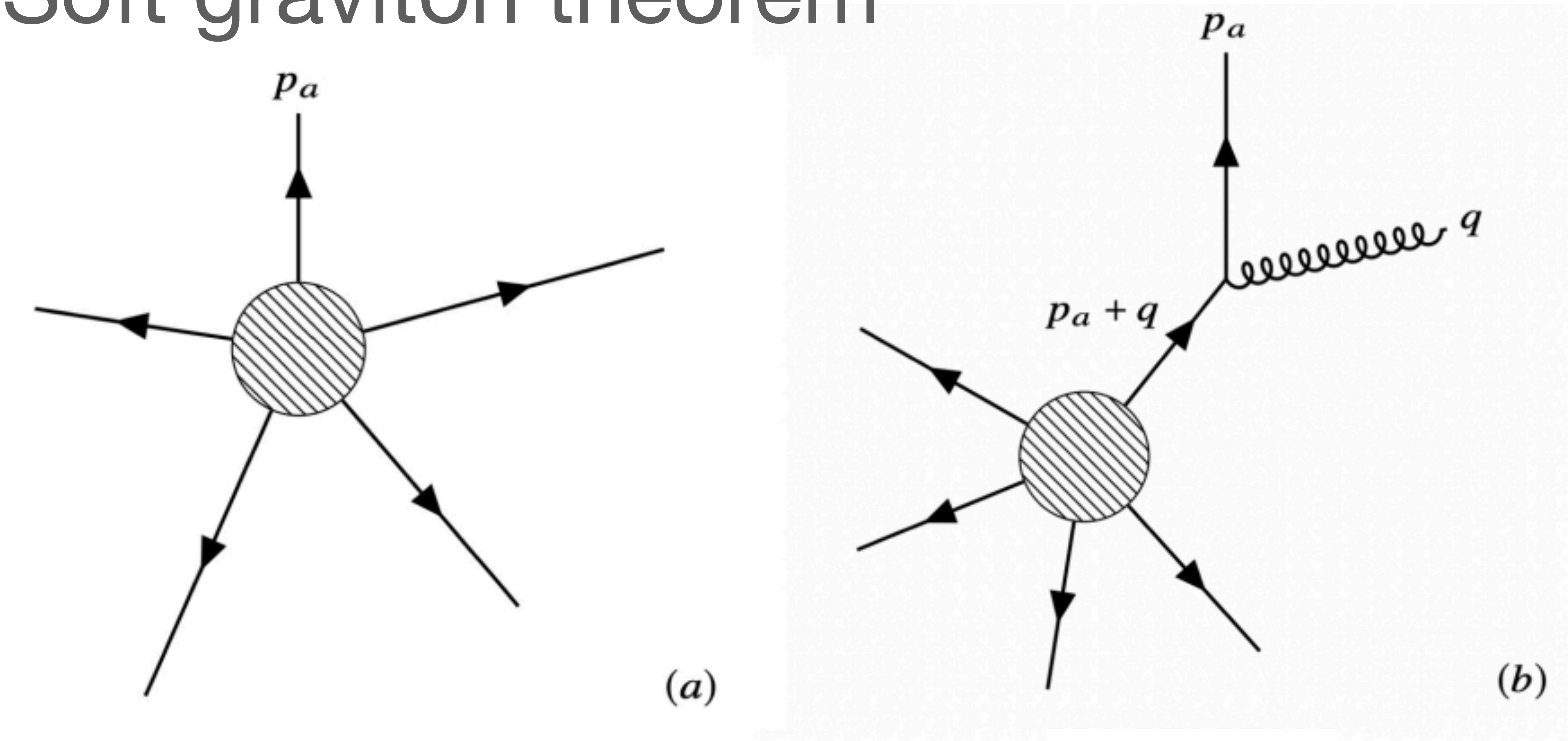
$$\phi = -\sqrt{\frac{3}{16\pi G}} \ln f'(R),$$

$$V(\phi) = \frac{R f'(R) - f(R)}{16\pi G f'(R)^2}.$$

Binary orbits radiate graviton + scalar

Constraints on Hu-Sawicki and Starobinsky Dark energy models cometetive with Solar system constraints.

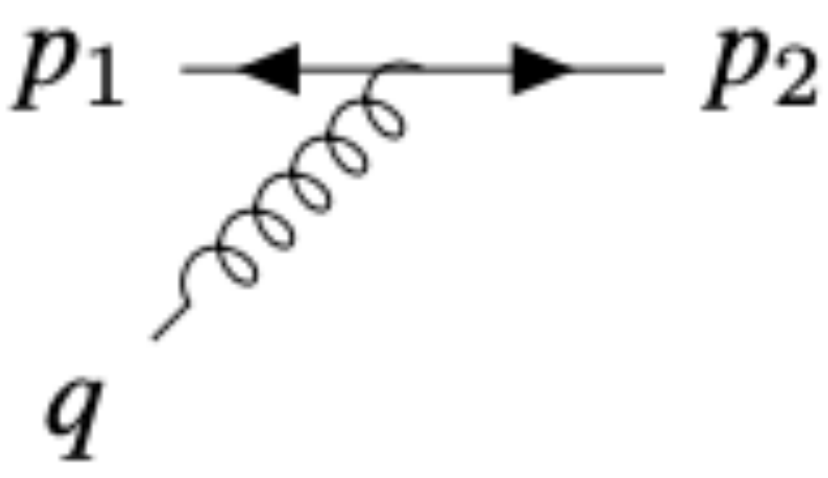
Soft graviton theorem



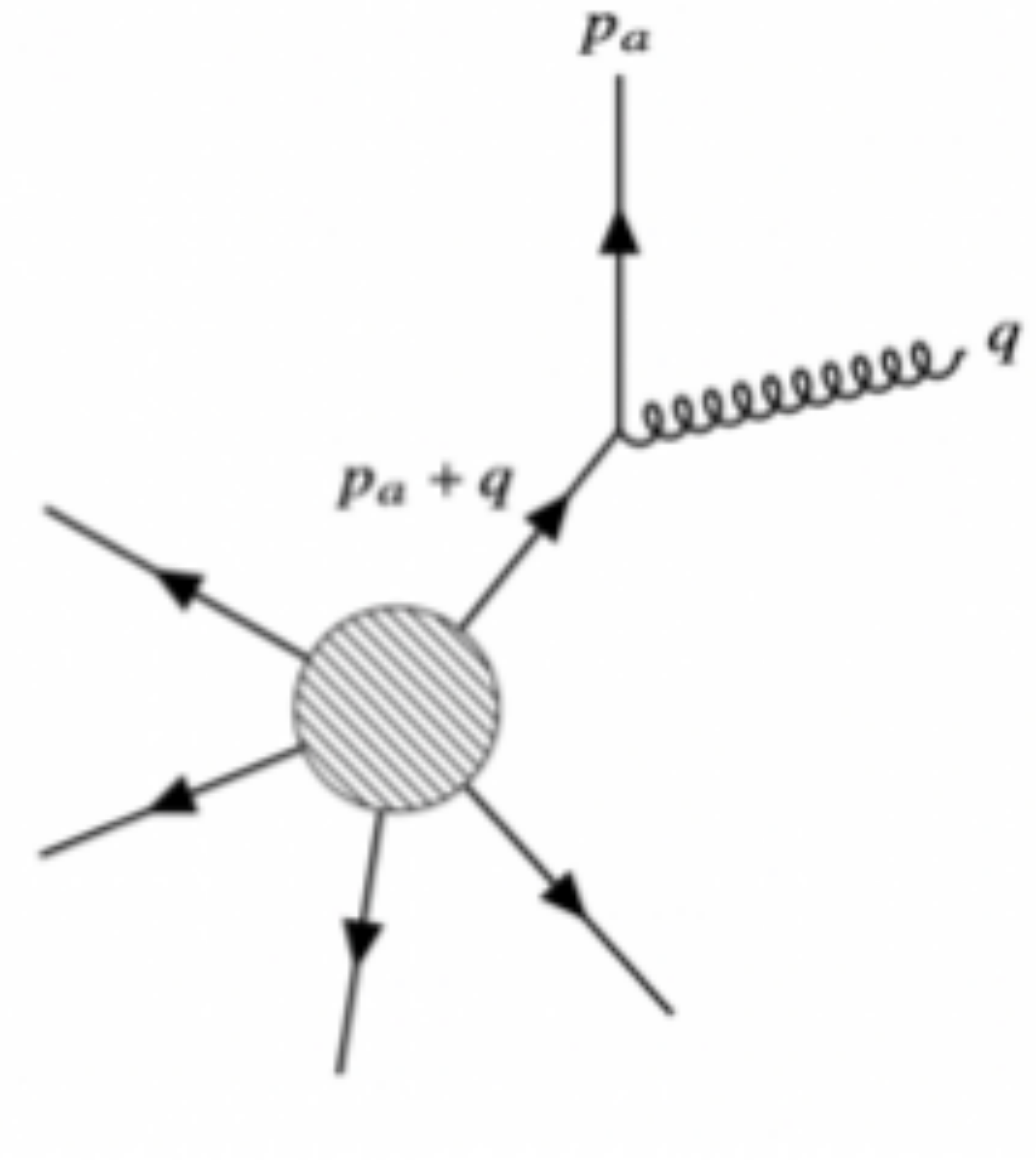
$\mathcal{A}_n(p_a)$ $\mathcal{A}_{n+1}$

$$\mathcal{A}_{n+1}(p_a, q) = \frac{\kappa}{2}\epsilon^{*\mu\nu}(q)\sum_{a=1}^{n}\frac{p_{a\mu}p_{a\nu}}{p_a \cdot q}\mathcal{A}_n(p_a)$$

For each graviton vertex



$$= \frac{i\kappa}{2}\epsilon_\lambda^{*\mu\nu}(q)\Big[p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - \eta_{\mu\nu}(p_1 \cdot p_2 - m^2)\Big]$$



$$\frac{\kappa}{2}\frac{\epsilon^{*\mu\nu}(q)}{(p_a+q)^2 - m_a^2}\Big[(p_{a\mu} + q_\mu)p_{a\nu} + (p_{a\nu} + q_\nu)p_{a\mu} - \eta_{\mu\nu}\big((p_a + q)\cdot p_a - m_a^2\big)\Big]$$

$$= \frac{\kappa}{2}\frac{\epsilon^{*\mu\nu}(q)}{2p_a \cdot q}\Big[2p_{a\mu}p_{a\nu} + q_\mu p_{a\nu} + q_\nu p_{a\mu} - \eta_{\mu\nu}p_a \cdot q\Big]$$

Tree level NLO terms

$$\mathcal{A}_{n+1}(p_a, q) = \Big(S_0 + S_1 + S_2\Big)\mathcal{A}_n(p_a)$$

$$S_0 = \frac{\kappa}{2}\sum_{a=1}^{n} \frac{\epsilon_\lambda^{*\mu\nu} p_{a\mu} p_{a\nu}}{p_a \cdot q},$$

$$S_1 = -i\frac{\kappa}{2}\sum_{a=1}^{n} \frac{\epsilon^*_{\lambda\mu\nu} p_a^\mu q_\beta J_a^{\beta\nu}}{p_a \cdot q},$$

$$S_2 = -\frac{\kappa}{2}\sum_{a=1}^{n} \frac{\epsilon^*_{\lambda\mu\nu} q_\alpha q_\beta J_a^{\alpha\mu} J_a^{\beta\nu}}{p_a \cdot q}$$

Amplitude of graviton emission is related to the gravitational wave signal

$$\tilde{h}_{\alpha\beta}(\mathbf{n}r, t) = \frac{\kappa}{4\pi r} \int \frac{dk_0}{(2\pi)} \sum_{\lambda=1}^{2} \epsilon^{\lambda}_{\alpha\beta}(\mathbf{n}) \mathcal{A}_{\lambda}(k_0, \mathbf{n}k_0) e^{-ik_0(t-r)}$$

$$\Box h_{\mu\nu} = -\frac{\kappa}{2} T_{\mu\nu}$$

$$h_{\alpha\beta}(x) = -\frac{\kappa}{2} \int d^4x' \, G_{\alpha\beta,\mu\nu}(x, x') \, T^{\mu\nu}(x')$$

$$G_{\alpha\beta,\mu\nu}(x,x') = -\frac{1}{4\pi}\frac{\delta\left(t' - (t - |\mathbf{x} - \mathbf{x}'|)\right)}{|\mathbf{x} - \mathbf{x}'|}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})$$

$$\tilde{h}_{\alpha\beta}(\mathbf{n}r,t) = \frac{\kappa}{4\pi r}\int\frac{dk_0}{(2\pi)}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\mathcal{A}_{\lambda}(k_0,\mathbf{n}k_0)e^{-ik_0(t-r)}$$

Using the soft-theorem to calculate the amplitude of emission

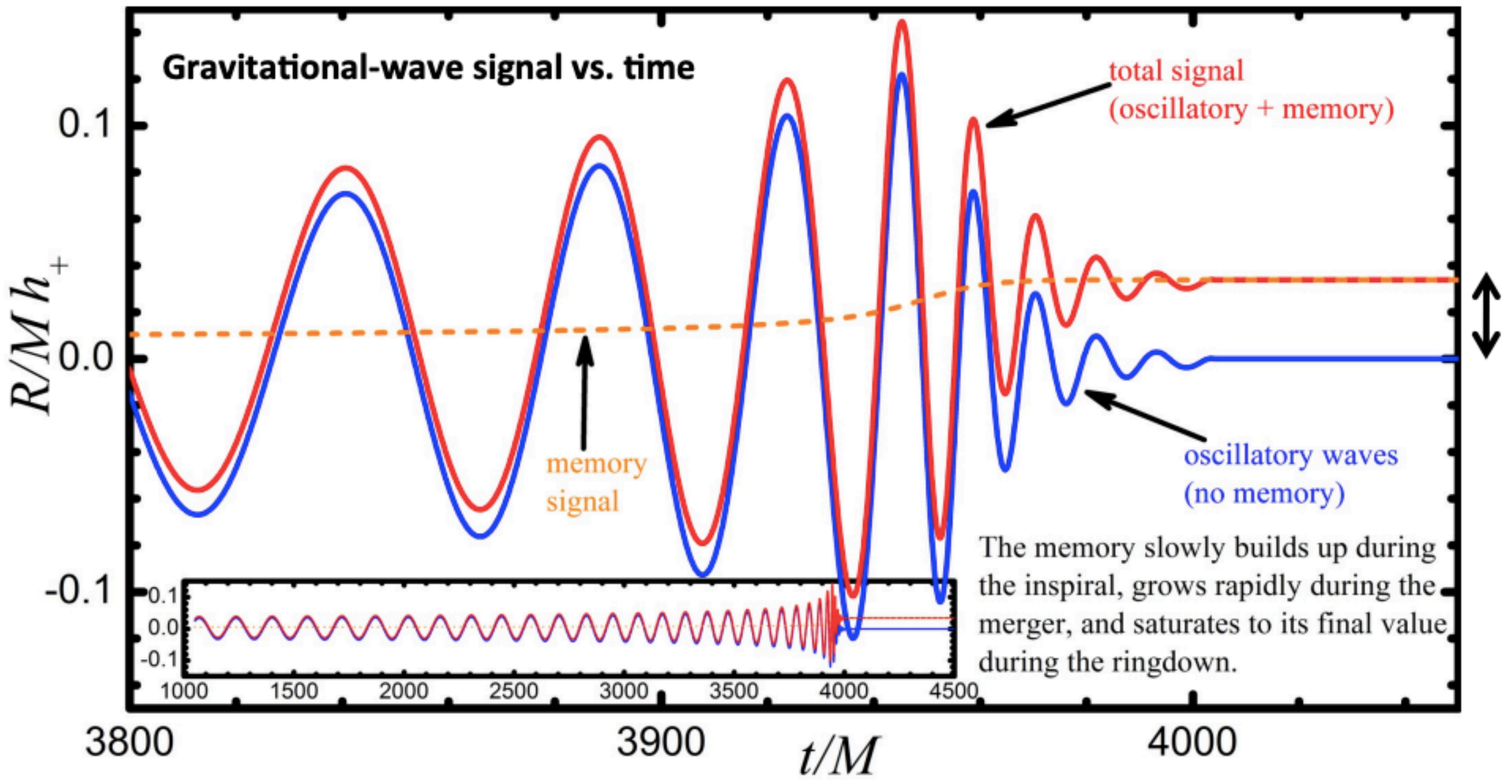
$$\tilde{h}_{\alpha\beta}(\mathbf{n}r,t) = \frac{\kappa}{4\pi r}\int \frac{dk_0}{(2\pi)}\frac{\kappa}{2}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\epsilon^{*\mu\nu}_{\lambda}(\mathbf{n})\sum_{a=1}^{n}\frac{p_{a\mu}p_{a\nu}}{p_a\cdot k}e^{-ik_0(t-r)}$$

$$\tilde{h}^{\mathrm{TT}}_{ij}(\mathbf{n}r,t) = \frac{4G}{r}\int \frac{dk_0}{2\pi i k_0}\sum_{a=1}^{n}\frac{1}{\sqrt{1-v_a^2}}\left[\frac{v_{ai}v_{aj}}{(1-\mathbf{v}_a\cdot\mathbf{n})}\right]^{\mathrm{TT}}e^{-ik_0(t-r)}$$

$$\int \frac{dk_0}{2\pi i k_0} e^{ik_0(t-r)} = \Theta(t-r)$$

The step function gives the DC signal in gravitational waves and this is called the memory effect.

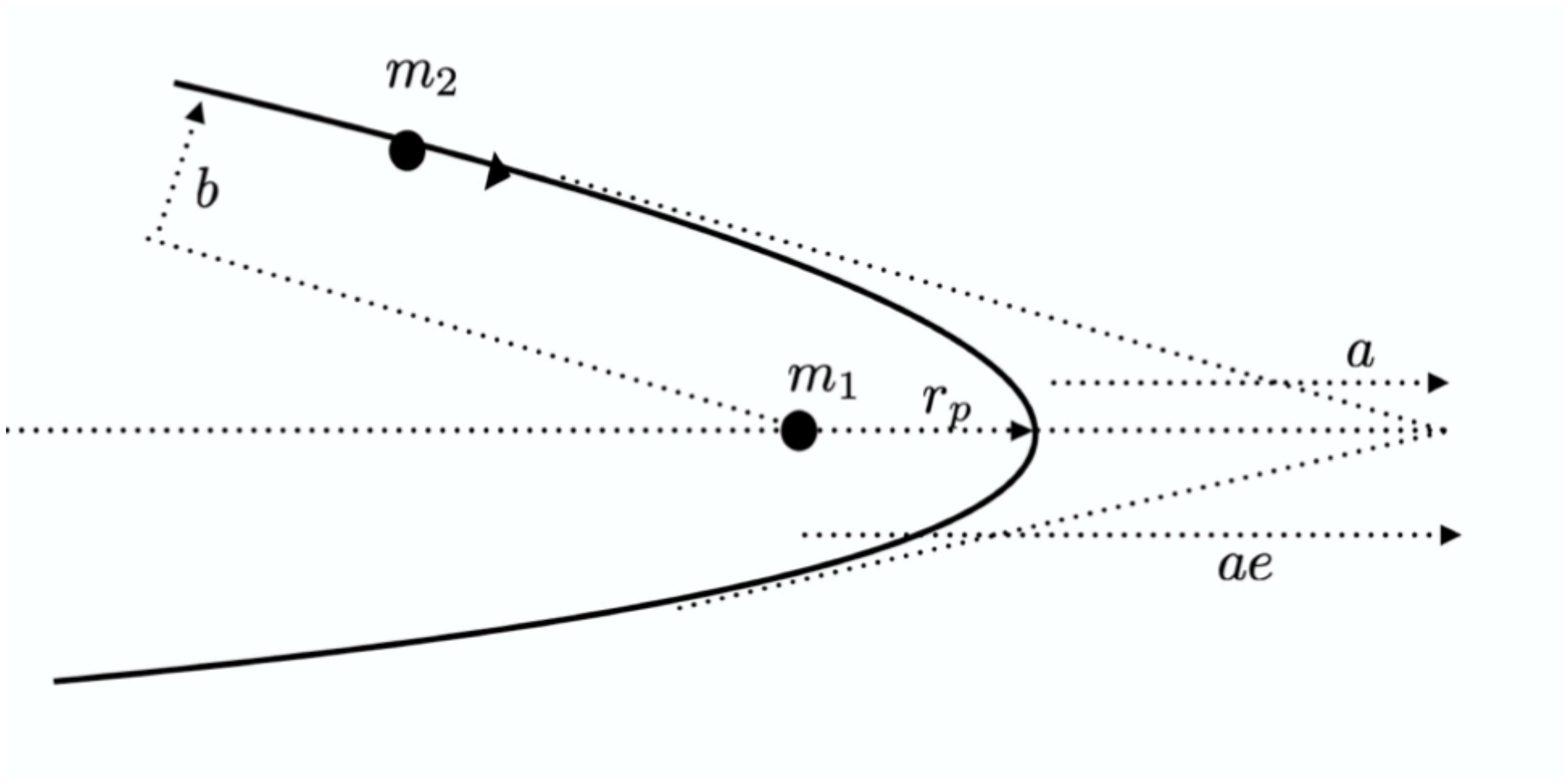
Gravitational memory



Marc Favata (ICTS talk)

Suraj Prakash, Arpan Hait, SM - Linear and non-linear memory from eccentric orbits

2211.13120



Hyperbolic orbits

Parametric form of Hyperbolic orbits

$$x(\xi) = a\,(e - \cosh\xi), \qquad y(\xi) = b\,\sinh\xi, \qquad z(\xi) = 0, \qquad \frac{\omega'}{\nu} t = \Omega\, t = (e\sinh\xi - \xi)$$

$$\Omega = \left(\frac{G\,(m_1 + m_2)}{a^3}\right)^{1/2} \qquad \nu \in (0, \infty)$$

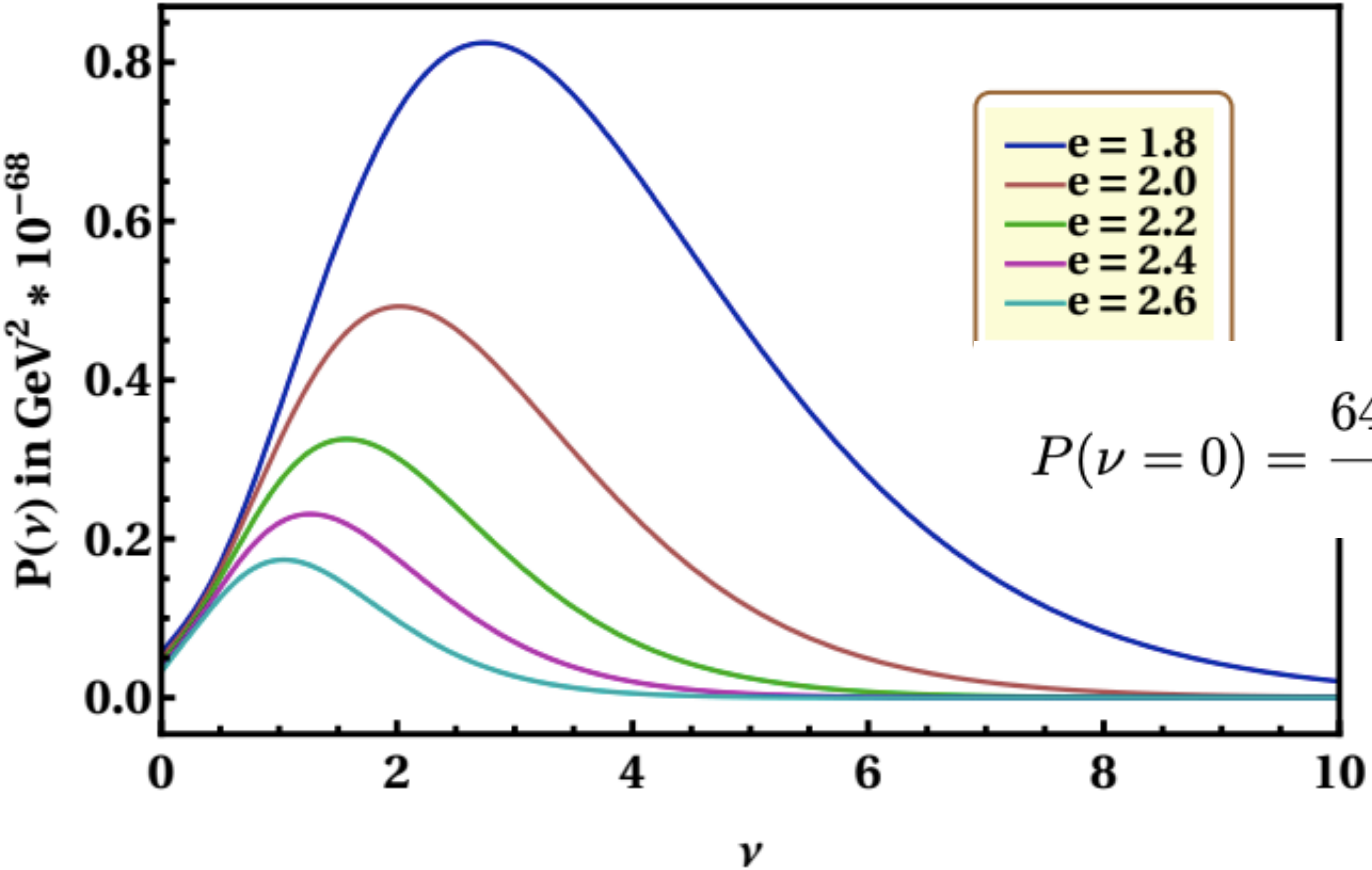
Stress tensor components

$$T_{xx}(\nu) = \mu\,\nu\,\Omega\,a^2\pi\left[\frac{\iota}{\nu e^2} H^{(1)}_{\iota\nu}(ie\nu) - \left(e - \frac{1}{e}\right) H^{(1)\prime}_{\iota\nu}(\iota e\nu)\right]$$

$$T_{yy}(\nu) = \mu\,\nu\,\Omega\,a^2(e^2 - 1)\,\pi\left[\frac{\iota}{\nu e^2} H^{(1)}_{\iota\nu}(\iota e\nu) + \frac{1}{e} H^{(1)\prime}_{\iota\nu}(\iota e\nu)\right]$$

$$T_{xy}(\nu) = -\mu\,\nu\,\Omega\,a^2\sqrt{e^2 - 1}\,\pi\left[\left(\frac{1}{e^2} - 1\right) H^{(1)}_{\iota\nu}(\iota e\nu) + \frac{\iota}{\nu e} H^{(1)\prime}_{\iota\nu}(\iota e\nu)\right]$$

Spectrum of energy radiated as gravitational waves



$$P(\nu = 0) = \frac{64\,G}{5}\mu^2\,a^4\,\Omega^4\,\frac{(e^2-1)}{e^4}.$$

Non-zero power radiated at zero frequency.

$$\omega' = \nu\,\Omega$$

Waveform from amplitudes

$$\mathcal{A}_\lambda(k_0, \mathbf{n}k_0) = -i\frac{\kappa}{2}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\tilde{T}^{\mu\nu}(k_0, \mathbf{n}k_0)$$

$$\Box h_{\mu\nu} = -\frac{\kappa}{2}T_{\mu\nu}$$

$$h_{\alpha\beta}(x) = -\frac{\kappa}{2}\int d^4x'\, G_{\alpha\beta,\mu\nu}(x, x')\, T^{\mu\nu}(x')$$

$$G_{\alpha\beta,\mu\nu}(x, x') = -\frac{1}{4\pi}\frac{\delta\left(t' - (t - |\mathbf{x} - \mathbf{x}'|)\right)}{|\mathbf{x} - \mathbf{x}'|}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})$$

$$\tilde{h}_{\alpha\beta}(\mathbf{x}, t) = \frac{\kappa}{4\pi r}\int\frac{dk_0}{(2\pi)}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\mathcal{A}_\lambda(k_0, \mathbf{n}k_0)e^{-ik_0(t-r)}$$

$$
\begin{aligned}
\tilde{h}_{\alpha\beta}(\mathbf{x},t) &= -\frac{\kappa^2}{8\pi r}\int \frac{dk_0}{2\pi}\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\alpha\beta}(\mathbf{n})\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\tilde{T}^{\mu\nu}(k_0,\mathbf{n}k_0)e^{-ik_0(t-r)} \\
&= -\frac{4G}{r}\int \frac{dk_0}{2\pi}\left(\tilde{T}_{\alpha\beta}(k_0,\mathbf{n}k_0) - \frac{1}{2}\eta_{\alpha\beta}\tilde{T}^{\mu}_{\mu}(k_0,\mathbf{n}k_0)\right)e^{-ik_0(t-r)}
\end{aligned}
$$

Wave-form at zero frequency $\nu e \to 0$

$$H_{\iota\nu}^{(1)}(\iota e\nu) \simeq \frac{2\iota}{\pi}\ln(\nu e), \qquad H_{\iota\nu}^{(1)\prime}(\iota e\nu) \simeq \frac{2}{\pi\nu e}.$$

$$T_{xx} = -\frac{2\,\mu\,a^2\,\Omega}{e^2}\left[\ln(\nu e) + (e^2-1)\right],$$

$$T_{yy} = -\frac{2\,\mu\,a^2\,\Omega}{e^2}(e^2-1)\left[\ln(\nu e) - 1\right],$$

Log terms in addition to poles

$$T_{xy} = 2\,\iota\,\mu\,\nu\,\Omega\,a^2\,\sqrt{e^2-1}\left[\frac{(e^2-1)}{e^2}\ln(\nu e) - \frac{1}{\nu^2\,e^2}\right]$$

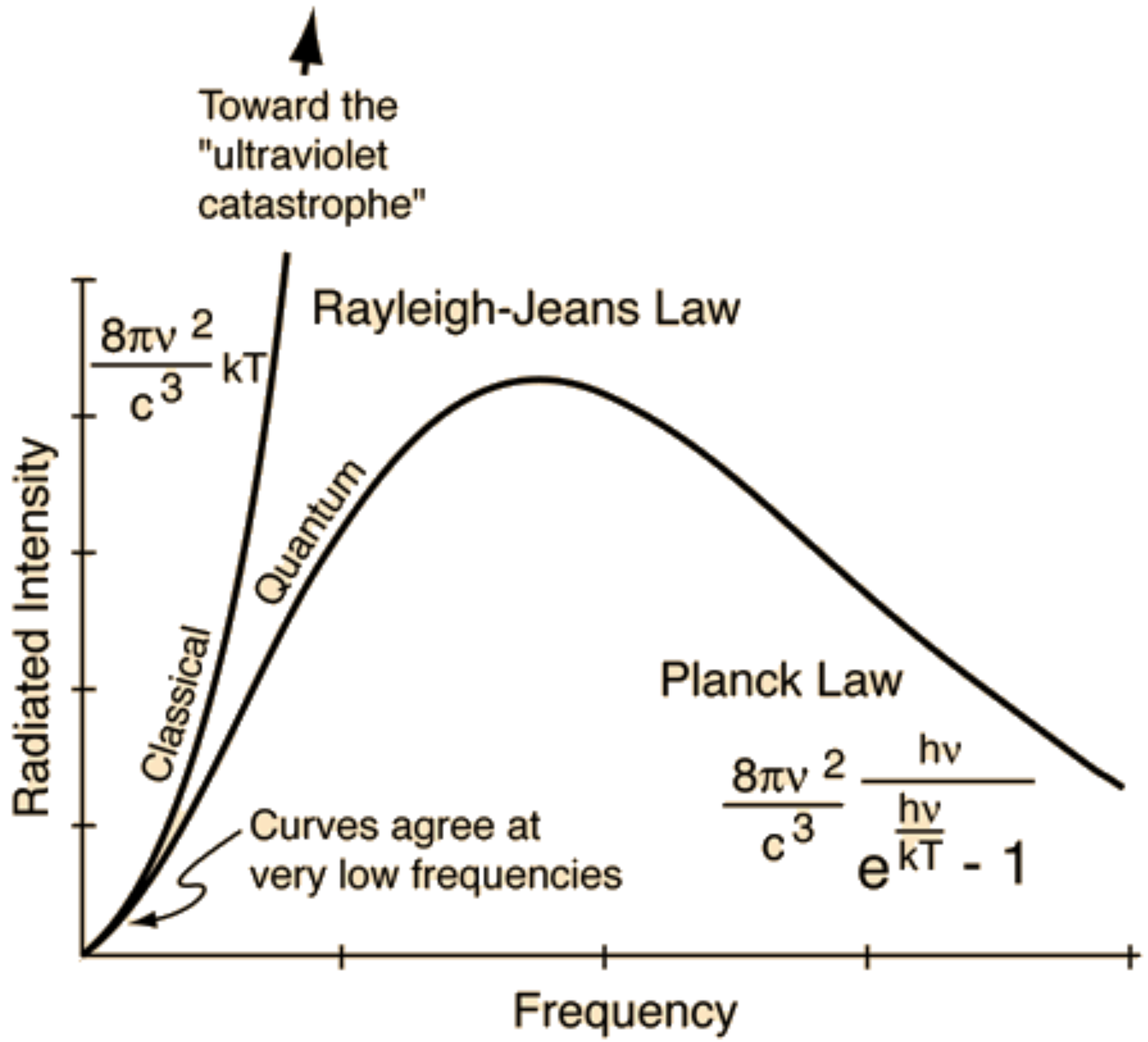
Laddha, Sen, Sahoo: 1804.09193 , 1808.03288, 1806.01872

Soft-graviton amplitude

$$
\begin{aligned}
S_{\rm gr} \;=\; & \sum_a \frac{\varepsilon_{\mu\nu} p_a^\mu p_a^\nu}{p_a.k} - i \ln\omega^{-1} \sum_a \frac{\varepsilon_{\mu\nu} p_a^\nu k_\rho}{p_a.k} \sum_{\substack{b\neq a\\ \eta_a\eta_b=1}} \frac{q_a q_b}{4\pi} \frac{m_a^2 m_b^2 \{p_b^\rho p_a^\mu - p_b^\mu p_a^\rho\}}{\{(p_b.p_a)^2 - m_a^2 m_b^2\}^{3/2}} \\
& + \frac{i}{4\pi} (\ln\omega^{-1} + \ln R^{-1}) \sum_{\substack{b\\ \eta_b=-1}} k.p_b \sum_a \frac{\varepsilon_{\mu\nu} p_a^\mu p_a^\nu}{p_a.k} \\
& + \frac{i}{8\pi} \ln\omega^{-1} \sum_a \frac{\varepsilon_{\mu\nu} p_a^\nu k_\rho}{p_a.k} \sum_{\substack{b\neq a\\ \eta_a\eta_b=1}} \frac{p_b.p_a}{\{(p_b.p_a)^2 - m_a^2 m_b^2\}^{3/2}} (p_b^\rho p_a^\mu - p_b^\mu p_a^\rho) \{2(p_b.p_a)^2 - 3m_a^2 m_b^2\}
\end{aligned}
$$

Our exact calculation in the low frequency limit gives the matches the log terms of Sen and Sahu exactly.

Planck’s Radiation Law, 1900



["Planck's law and the hypothesis of light quanta"](#)

Zeitschrift für Physik

S.N.Bose, 1924

Gravitational wave radiation as graviton emission in a thermal bath

$$\langle a^\dagger_{k'} a_k \rangle_\beta = \bar{n}(\omega_k)\delta^3(\mathbf{k} - \mathbf{k}'),$$
$$\langle a_k a^\dagger_{k'} \rangle_\beta = (1 \pm \bar{n}(\omega_k))\delta^3(\mathbf{k} - \mathbf{k}')$$

$$S^T_{if} = -i\langle n_k + 1| \int d^4x \, \mathcal{L}_{int}(x) |n_k\rangle$$

Can we show experimentally that Gravitational waves are Gravitons?

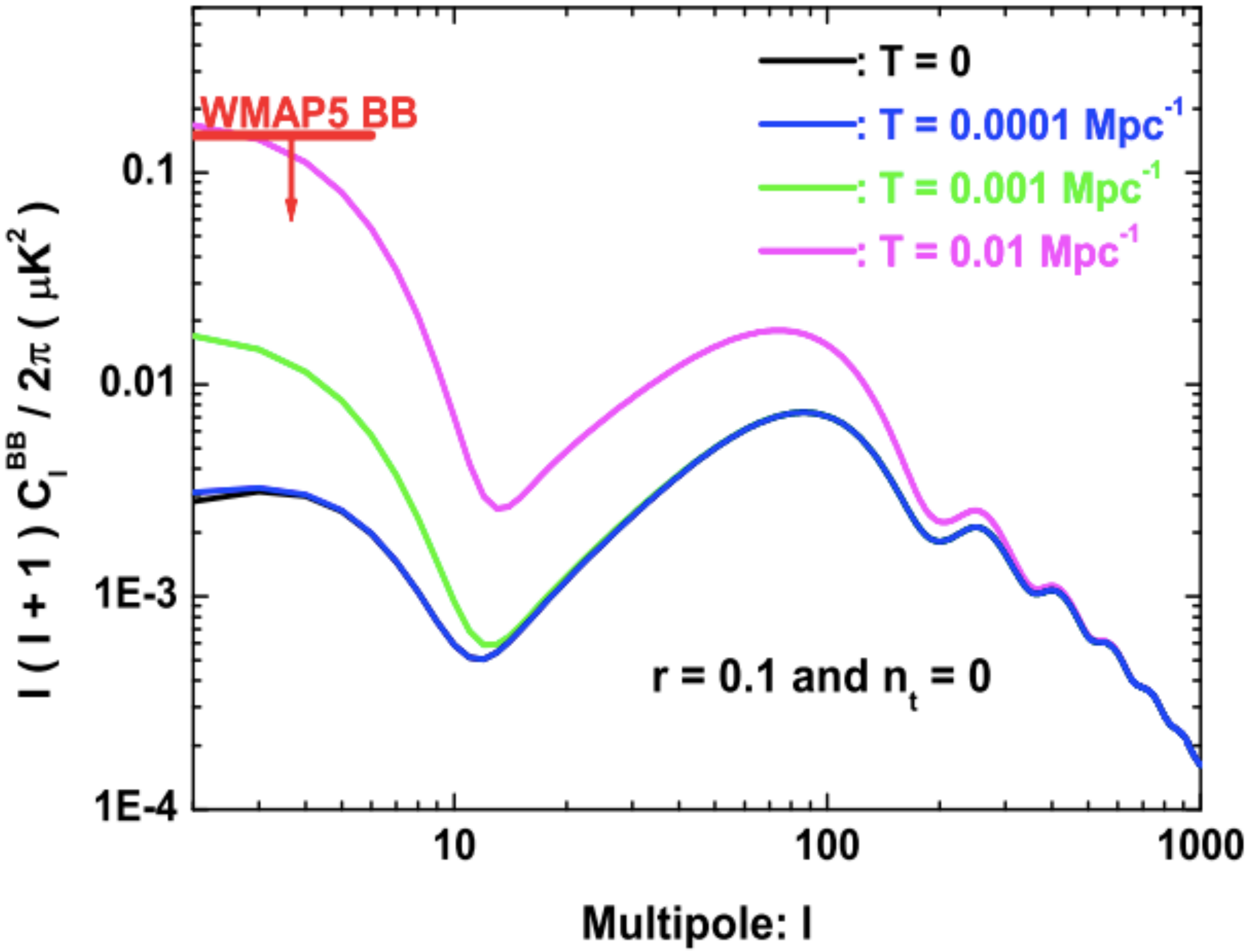
Inflation: Zero point fluctuations in the fields become classical.

Inflation

$$\mathcal{P}_T \sim \frac{1}{2} + \frac{1}{e^{k/T} - 1} = \coth\left(\frac{k}{2T}\right)$$

Bhattacharya, Mohanty, Nautiyal, Phys Rev Letters, 2006

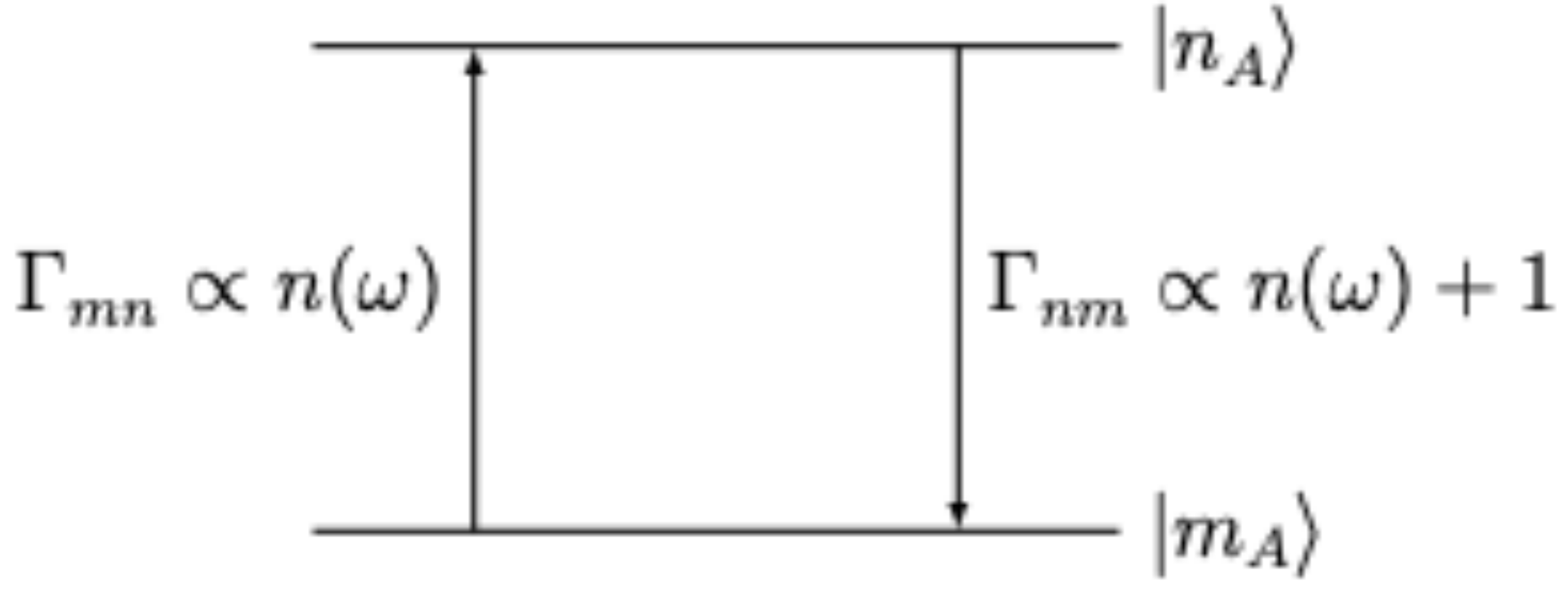
Evidence of stimulated emission will be proof that gravitational waves are gravitons.



Bhattacharya, Mohanty, Nautiyal (2006)

Zhao, Baskaran, Coles (2009)

Atomic transitions in a heat bath



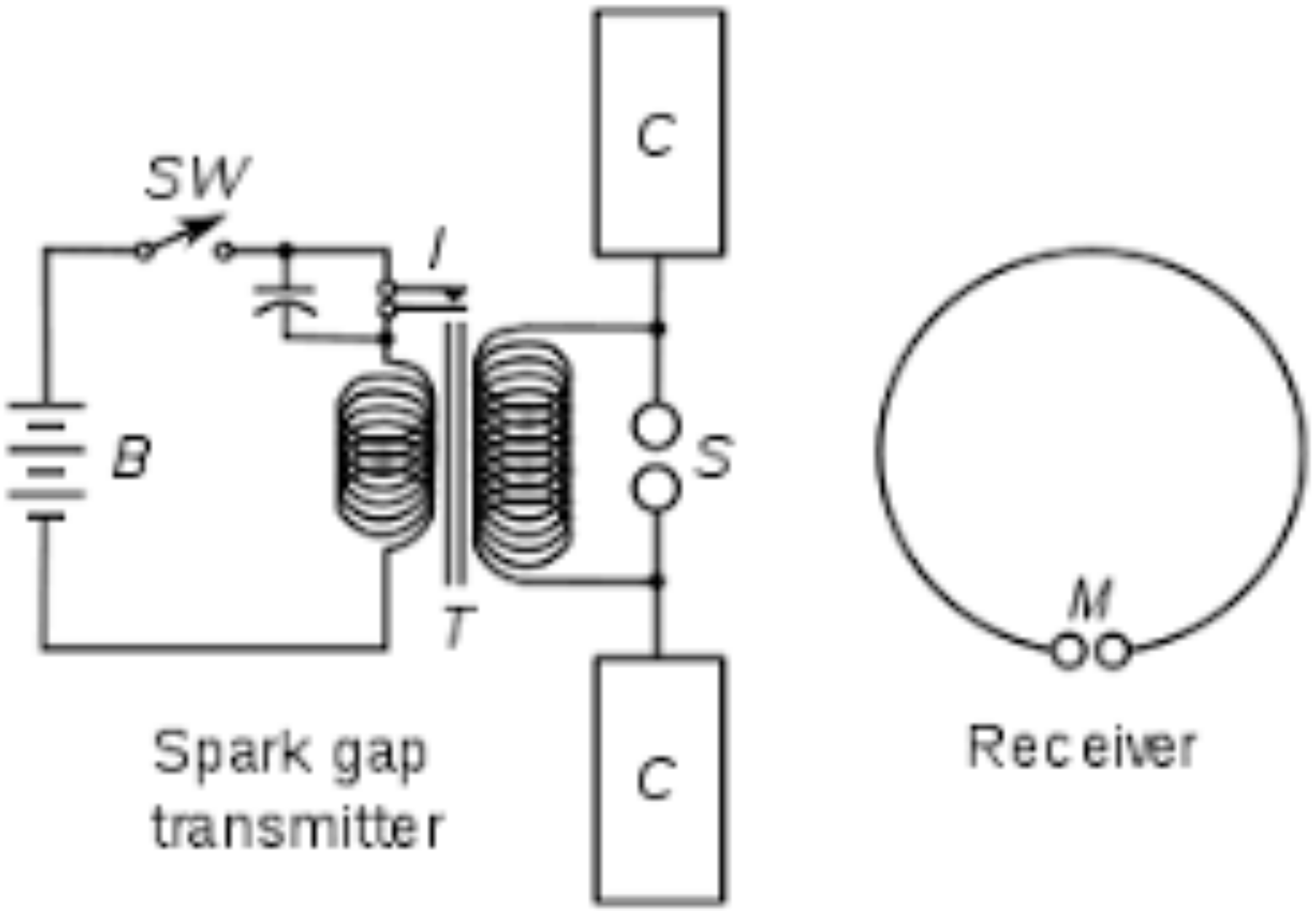
$$\Gamma_{nm} = \begin{cases} \dfrac{\omega_{nm}^3 |\boldsymbol{d}_{nm}|^2}{3\pi\varepsilon_0 \hbar c^3}\left[n(\omega_{nm}) + 1\right] \\ \dfrac{\omega_{mn}^3 |\boldsymbol{d}_{nm}|^2}{3\pi\varepsilon_0 \hbar c^3}\left[n(\omega_{mn})\right] \end{cases}$$

Emission rate

Absorption rate

Is there a similar stimulated emission of gravitons from binary pulsars or black-holes in a stochastic background of gravitons?

Thank You



"It's of no use whatsoever… this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic wavesthat we cannot see with the naked eye. But they are there."

Heinrich Hertz, 1887

Net emission rate

$$\Gamma^{emi}_{m\rightarrow n} = A_{m\rightarrow n} + B_{m\rightarrow n} u(\omega)$$
$$= 8\pi\omega^3 B_{m\rightarrow n} \left(1 + n(\omega)\right)$$

Net absorption rate

$$\Gamma^{abs}_{n\rightarrow m} = B_{m\rightarrow n} u(\omega)$$
$$= 8\pi\omega^3 \, B_{m\rightarrow n} \, n(\omega) \, .$$

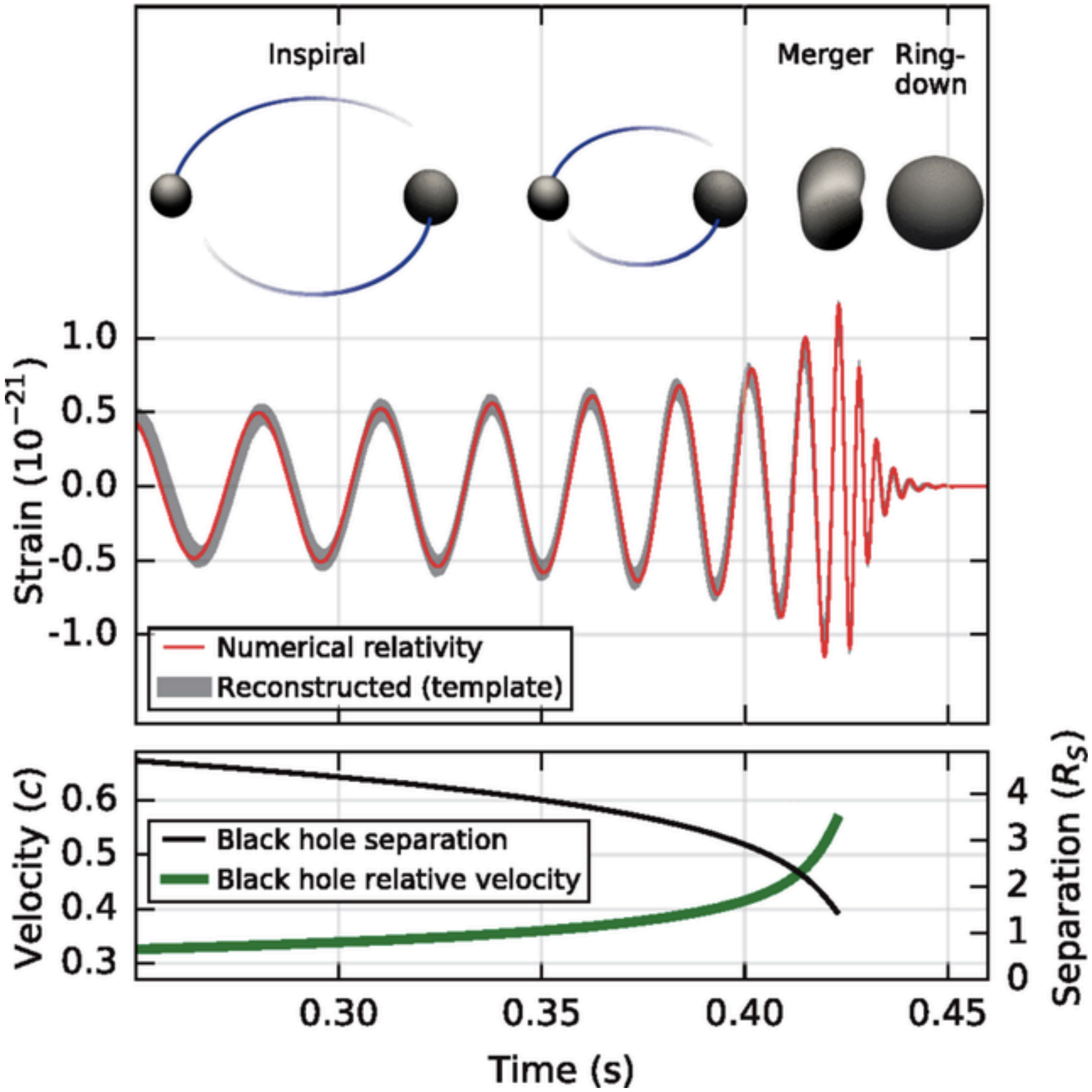
Bose-Einstein distribution

$$n(\omega) = \frac{1}{e^{\omega/T} - 1}$$

At low frequencies, $\omega \ll T$

$$1 + n(\omega) = \frac{1}{2} + \frac{T}{\omega} + \frac{1}{12}\frac{\omega}{T} - \frac{1}{720}\left(\frac{\omega}{T}\right)^3 + \cdots$$

Huge enhancement in the gravitational emission rate from binaries if there is a thermal background of gravitons.


Inspiral
Merger
Ring-down
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
Numerical relativity
Reconstructed (template)
Velocity (c)
0.6
0.5
0.4
0.3
Black hole separation
Black hole relative velocity
Separation (R_S)
4
3
2
1
0
0.30
0.35
0.40
0.45
Time (s)

Quasi-Normal Modes

Regge-Wheeler equation for the odd parity metric perturbations

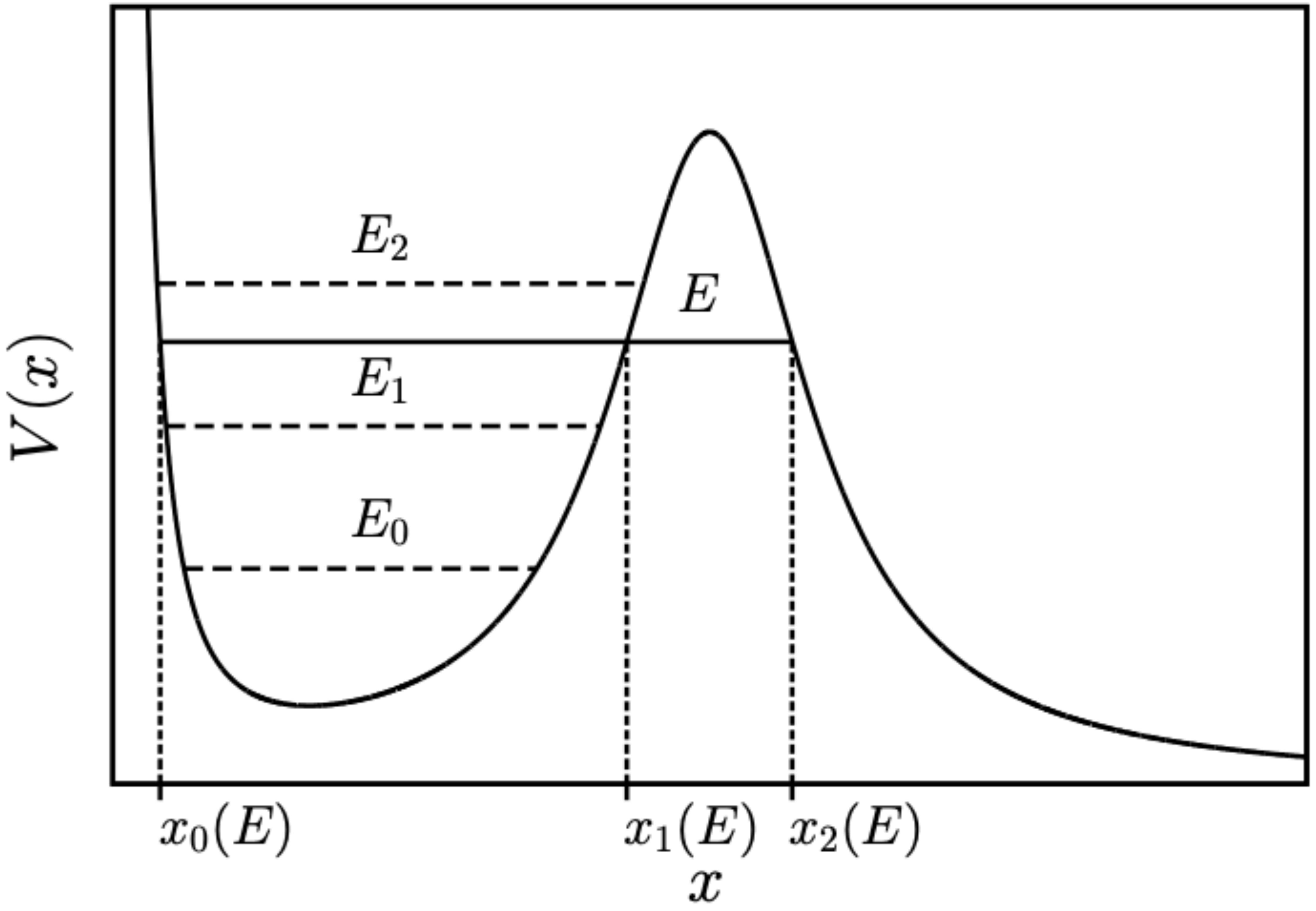
$$\partial_t^2 \Psi^{(\mathrm{o})} - \partial_{r_*}^2 \Psi^{(\mathrm{o})} + V_\ell^{(\mathrm{o})} \Psi^{(\mathrm{o})} = S^{(\mathrm{o})}$$

$$r_* \equiv r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

$$V_\ell^{(\mathrm{o})} \equiv \left(1 - \frac{2M}{r}\right)\left(\frac{\Lambda}{r^2} - \frac{6M}{r^3}\right)$$

$$\Lambda = l(l+1)$$

Volkel (2020)



$$T_H = \frac{1}{8\pi G M}$$

$$3\sqrt{3}M\omega_n \approx \ell + \frac{1}{2} - i\left(n + \frac{1}{2}\right)$$

What is the effect of Hawking temperature in the tunnelling probability

Gravitational wave propagator in classical GR

$$D^{(0)}_{\mu\nu\alpha\beta}(k) = -\frac{i}{k^2}\left(\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}\right)$$

Graviton as a Quantum Field

$$\hat{h}_{\mu\nu}(x) = \sum_{\lambda}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left[\epsilon^{\lambda}_{\mu\nu}(k)a_{\lambda}(k)e^{-ik\cdot x} + \epsilon^{*\lambda}_{\mu\nu}(k)a^{\dagger}_{\lambda}(k)e^{ik\cdot x}\right]$$

Graviton propagator in QFT

$$D^{(0)}_{\mu\nu\alpha\beta}(x-y) \equiv \langle 0|T(\hat{h}_{\mu\nu}(x)\hat{h}_{\alpha\beta}(y))|0\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{1}{-k^2+i\epsilon}e^{ik(x-y)}\sum_{\lambda}\epsilon^{\lambda}_{\mu\nu}(k)\epsilon^{*\lambda}_{\alpha\beta}(k)$$

Polarisation sum

$$\sum_{\lambda=1}^{2}\epsilon^{\lambda}_{\mu\nu}(k)\epsilon^{*\lambda}_{\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

Graviton emission rate

$$d\Gamma = \frac{\kappa^2}{4}\sum_{\lambda=1}^{2} |T_{\mu\nu}(k')\epsilon_{\lambda}^{\mu\nu}(k)|^2 2\pi\delta(\omega-\omega')\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}$$

$$= \frac{\kappa^2}{8(2\pi)^2}\sum_{\lambda=1}^{2}\left(T_{\mu\nu}(k')T^{*}_{\alpha\beta}(k')\epsilon_{\lambda}^{\mu\nu}(k)\epsilon_{\lambda}^{*\alpha\beta}(k)\right)\frac{d^3k}{\omega}\delta(\omega-\omega')$$

Polarisation sum of massless gravitons

$$\sum_{\lambda=1}^{2}\epsilon_{\mu\nu}^{\lambda}(k)\epsilon_{\alpha\beta}^{*\lambda}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

Graviton emission rate

$$d\Gamma = \frac{\kappa^2}{8(2\pi)^2} \sum_{\lambda=1}^{2} \Big(T_{\mu\nu}(k') T^*_{\alpha\beta}(k') \epsilon^{\mu\nu}_{\lambda}(k) \epsilon^{*\alpha\beta}_{\lambda}(k) \Big) \frac{d^3k}{\omega} \delta(\omega - \omega')$$

$$= \frac{\kappa^2}{8(2\pi)^2} \int \Big[T_{\mu\nu}(k') T^*_{\alpha\beta}(k') \Big] \Big[\frac{1}{2} (\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}) \Big] \frac{d^3k}{\omega} \delta(\omega - \omega')$$

$$= \frac{\kappa^2}{8(2\pi)^2} \int \Big[|T_{\mu\nu}(k')|^2 - \frac{1}{2} |T^{\mu}{}_{\mu}(k')|^2 \Big] \delta(\omega - \omega') \omega d\omega d\Omega_k$$

Energy radiated through gravitons

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2} \int \Big[|T_{\mu\nu}(k')|^2 - \frac{1}{2} |T^{\mu}{}_{\mu}(k')|^2 \Big] \delta(\omega - \omega') \omega^2 d\omega d\Omega_k$$

Massive gravity theory (Fierz-Pauli 1932)

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$S = \int d^4x \Big[-\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + \frac{1}{2}(\partial_\mu h)^2 - (\partial_\mu h)(\partial^\nu h^\mu_\nu) + (\partial_\mu h_{\nu\rho})(\partial^\nu h^{\mu\rho}) \;\; + \frac{1}{2} m_g^2 \Big(h_{\mu\nu} h^{\mu\nu} - h^2 \Big) + \frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu} \Big]$$

Propagator

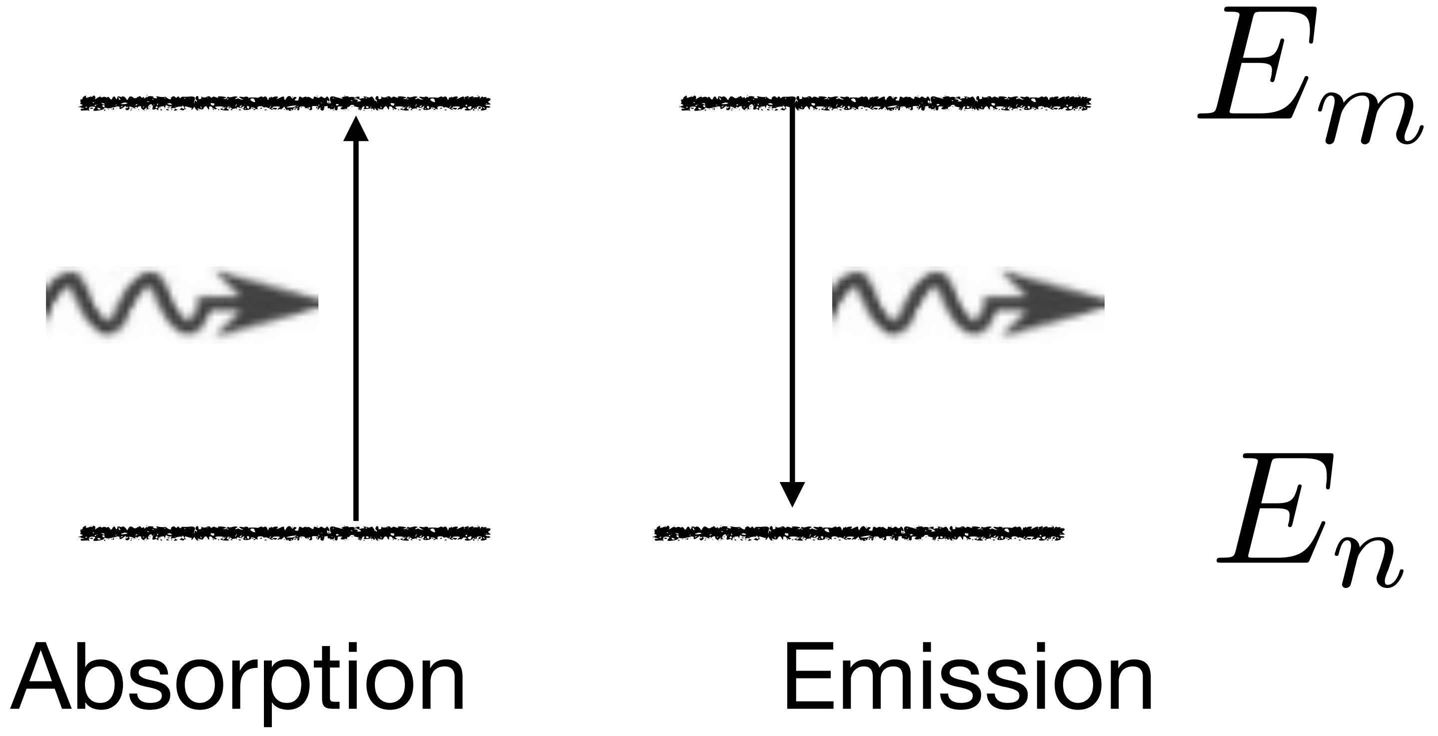
$$D^{(m)}_{\mu\nu\alpha\beta}(p) = \frac{1}{-k^2 + m_g^2} \left(\frac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu}) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\mu\nu} + (k\text{—dependent terms}) \right)$$

Polarisation sum

$$\sum_\lambda \epsilon^\lambda_{\mu\nu}(k) \epsilon^{*\lambda}_{\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta}) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\mu\nu}$$

There is a contribution from the scalar part of graviton even in the zero mass limit : zDVZ discontinuity.

Spontaneous and stimulated emissions (Einstein 1916)



$$\frac{dN_n}{dt} = A_{m\to n}\, N_m + B_{m\to n}\, N_m\, u(\omega) - B_{n\to m}\, N_n\, u(\omega)$$

Emission Absorption

Thermal Equilibrium

$$\frac{N_n}{N_m} = e^{-(E_n - E_m)/T} = e^{-\omega/T} \quad \text{and} \quad \frac{dN_n}{dt} = 0$$

$$A_{m\to n} = B_{m\to n}\left(e^{\omega/T-1} - 1\right) u(\omega)$$

$$A_{m\to n} = 8\pi\omega^3 B_{m\to n} \quad \Leftrightarrow \quad u(\omega) = \frac{8\pi\omega^3}{e^{\omega/T} - 1}$$