A template-based search for exotic gravitational wave signals from astrophysical compact binaries

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*Ongoing work in collaboration with Dr. Soumen Roy, Dr. Anand Sengupta

Compact Binary Coalescences as testing laboratories for General Relativity

Einstein's General Relativity is the most successful theory of gravity.

Is GR the ultimate theory that explains the nature of gravity even in the highly relativistic and strong field limit?

Relativistic and strong field limit can be quantified in terms of **characteristic velocity** and **orbital compactness.**

Physical System	Characteristic Velocity: $\frac{v}{c}$	Orbital Compactness: $\frac{GM}{c^2R}$
Compact binary system before merger	> 0.2	> 0.4
Sun-Mercury system	$\approx 10^{-4}$	$\approx 10^{-8}$

image credit: LIGO/Caltech/MIT/R. Hurt (IPAC)

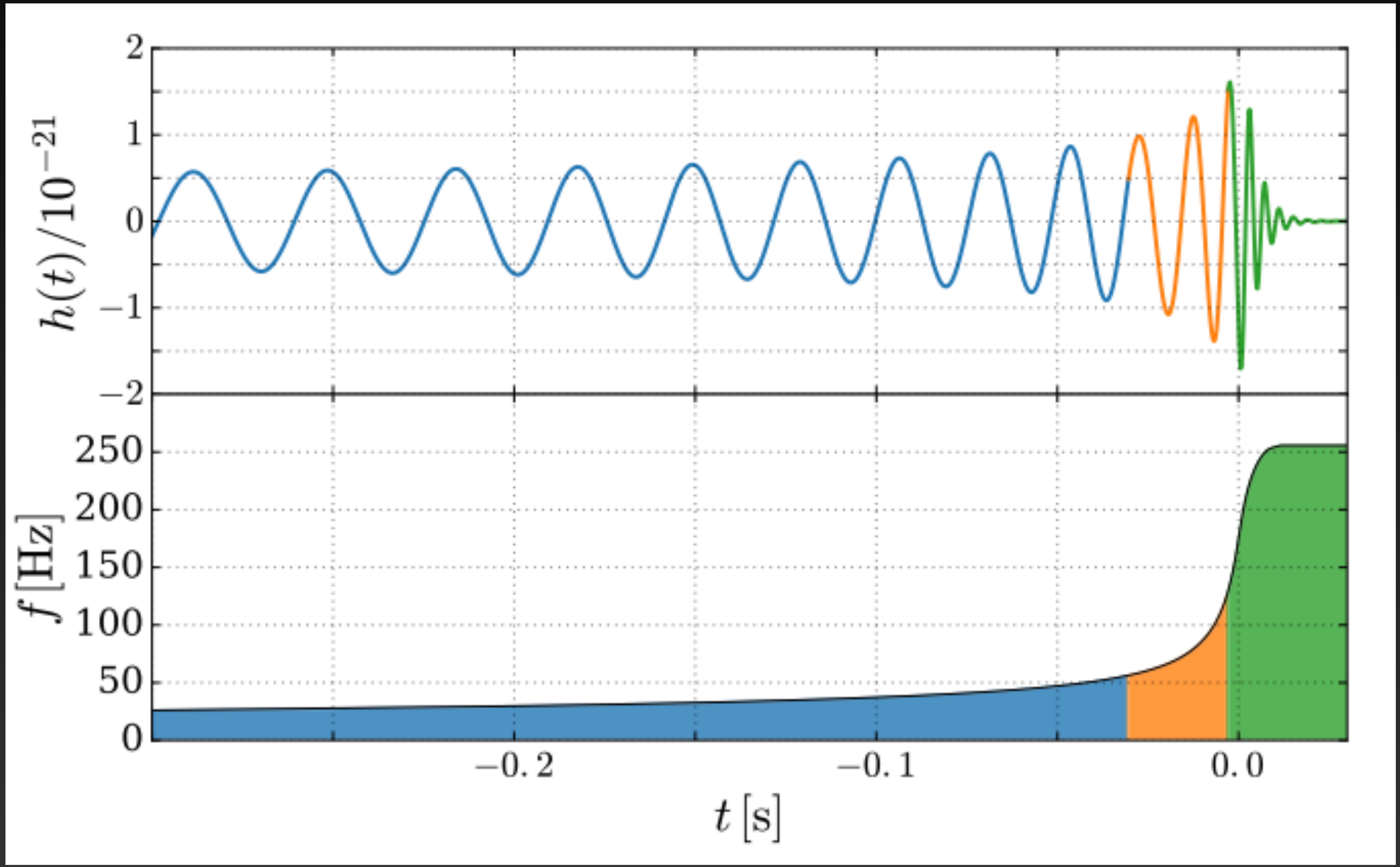
GWs from such systems (CBCs) can be used to test GR in **extreme environments.**

Different Phases of Compact binary evolution

Inspiral
$v/c \approx 0.1 - 0.3$

Late Inspiral/Merger
$v/c \approx 0.4 - 0.6$

Merger+Ringdown



PN-Approximation

NR simulations

BH perturbation Theory

*image source: Meidam, Tsang, Goldstein et al. PhysRevD.97.044033 (2018)

Post Newtonian-Approximation

Relevant physical quantities are expressed as a series in powers of the characteristic velocity, $v \sim (\pi M f)^{1/3}$.

$$Q = \sum_n \phi_n(\vec{\Theta})\, v^{2n}$$

- Gravitational wave flux
- Energy
- Waveform Phase

nth-PN coefficient

Source's intrinsic Parameters:
Masses, spins etc.

Waveform in restricted PN approximation

Frequency domain GW chirp signal:

$$\tilde{h}(f) = \frac{\mathscr{A}(\alpha, \delta, \iota, \psi, \mathscr{M}, \eta)}{D} f^{-7/6} e^{i\Phi\left(t_c, \phi_c, \mathscr{M}, \eta; f\right)}$$

GW phase is given by,

$$\Phi(f) = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \sum_{j=0}^{7} \left[\phi_j + \phi_j^{(l)} \ln f\right] f^{(j-5)/3}$$

$$\phi_j = \phi_j(\mathscr{M}, \eta, \chi_1, \chi_2) \qquad \mathscr{M} = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1 + m_2)^{\frac{1}{5}}} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\phi_0 = \frac{3}{128}(\pi\mathscr{M})^{-5/3}$$

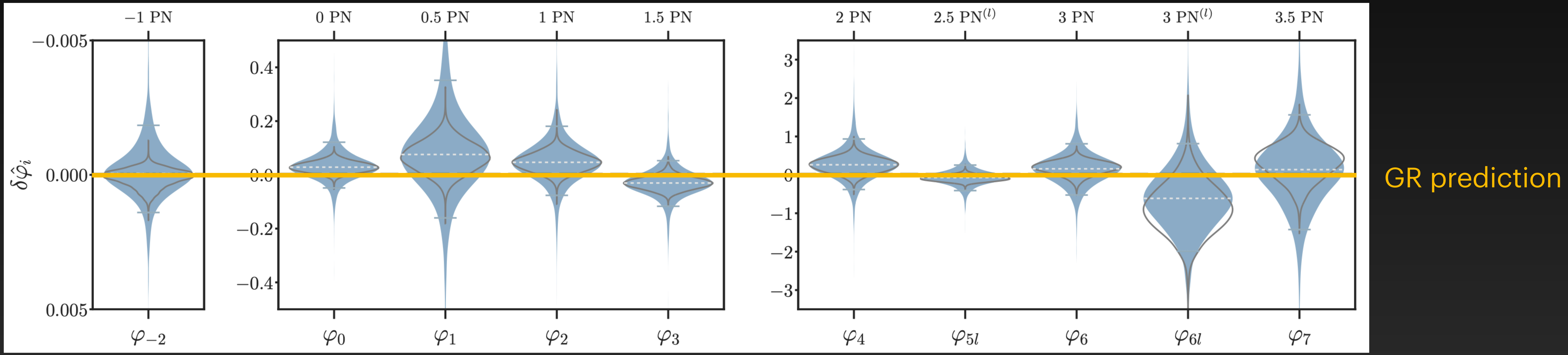
$$\phi_2 = \frac{5}{96\eta^{2/5}}\left(\frac{743}{336} + \frac{11}{4}\eta\right)(\pi\mathscr{M})^{-1}$$

$$\phi_3 = -\frac{3\pi}{8\eta^{3/5}}\left(1 - \frac{1}{4\pi}\beta\right)(\pi\mathscr{M})^{-2/3}$$

Results from GWTC-3

*R. Abbott et al. https://arxiv.org/abs/2112.06861 (2021)

GR can be tested by constraining the fractional deviations in PN coefficients, $\phi_j = (1 + \delta\phi_j)\phi_j^{GR}$



$\delta\phi = 0$ lies well inside the 90% credible interval, this implies GR is consistent with data.

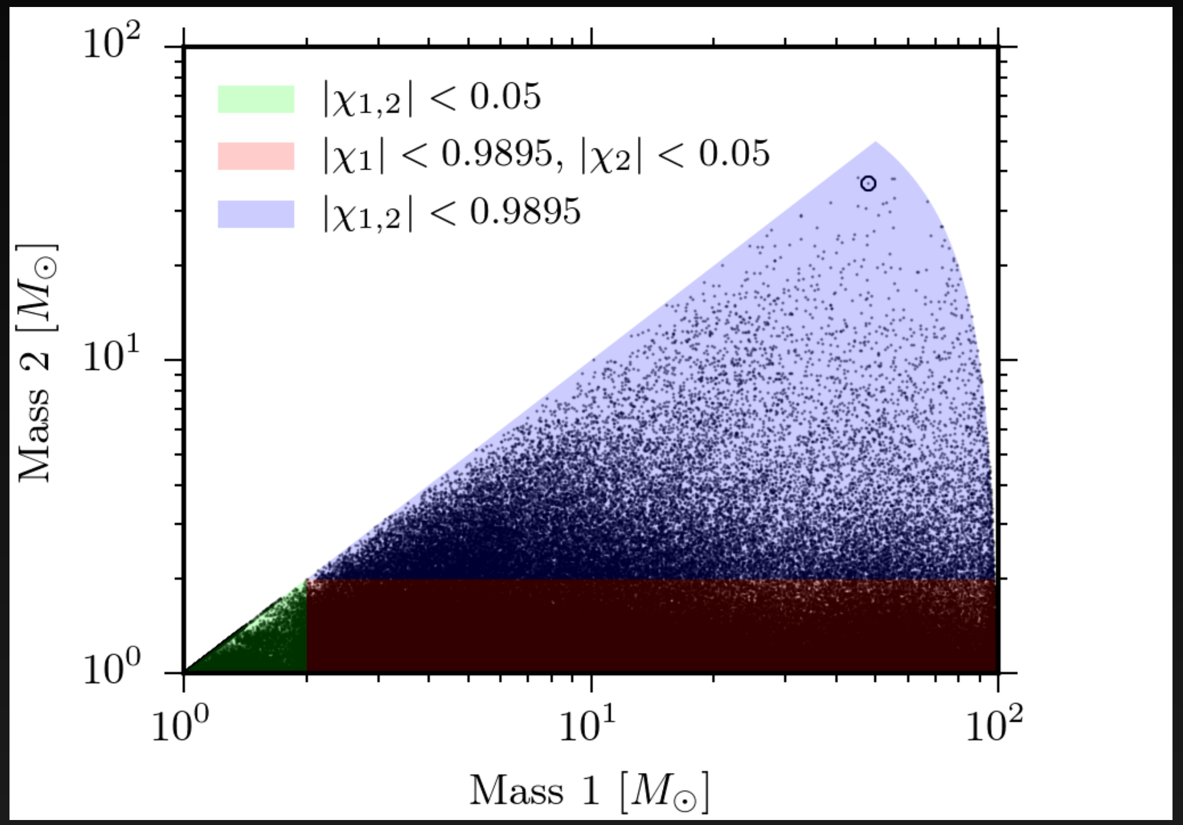
BUT: Plenty of wiggle room for **exotic signals**.

How does template-based search work?

Matched Filtering is used to search for the underlying GW signals in the noisy data.

The detector data is correlated with a collection of template waveforms and SNR is calculated.

This collection of template waveforms ~ **Template Bank**



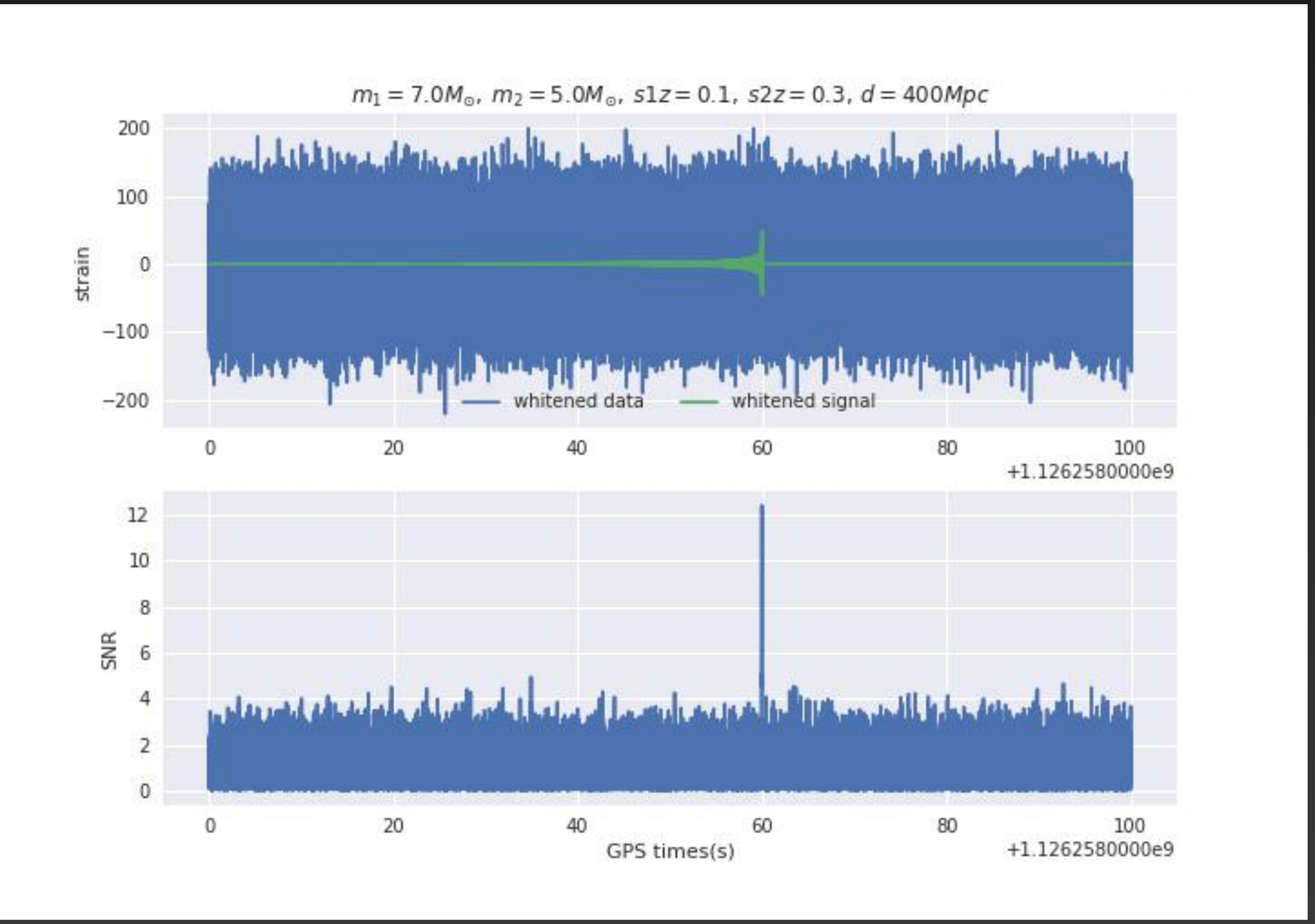
B. P. Abbott et al. PhysRevD.93.122003

$$d(t) = n(t) + h(t)$$

$u(t)$ = template waveform

$$\text{SNR} = \rho = \frac{(d\,|\,u)}{rms(n\,|\,u)}$$

$$(a\,|\,b)(t) = 4\Re \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)} e^{-i2\pi f t} df$$



Template-based search for the exotic signals

In a recent study by, **Narola et al**. Phys. Rev. D 107, 024017 (2023) demonstrated that if an astrophysical signal carrying departure from GR is present in detector data, then GR template bank would fail to detect such a signal.

The authors have constructed a new non-GR template bank with the following parameter space boundaries,

Parameter	Limits
Compononent masses	$m_{1,2} \in [3,\ 100] M_\odot$
Total mass	$M \in [6,\ 100] M_\odot$
Mass ratio	$m_1/m_2 \in [1,\ 10]$
Component spins	$\chi_{1,2} \in [-0.9899,\ +0.9899]$

PN Deviation Parameter	Limits
0.5PN	$\delta\hat{\varphi}_1 \in [-0.345,\ 0.037]$
1.0PN	$\delta\hat{\varphi}_2 \in [-0.254,\ 0.011]$
1.5PN	$\delta\hat{\varphi}_3 \in [-0.071,\ 0.118]$
2.0PN	$\delta\hat{\varphi}_4 \in [-1.216,\ 0.456]$

The new template bank improves the sensitivity of the search as shown by the numerical experiments in the study.

Search sensitivity as a function of iFAR

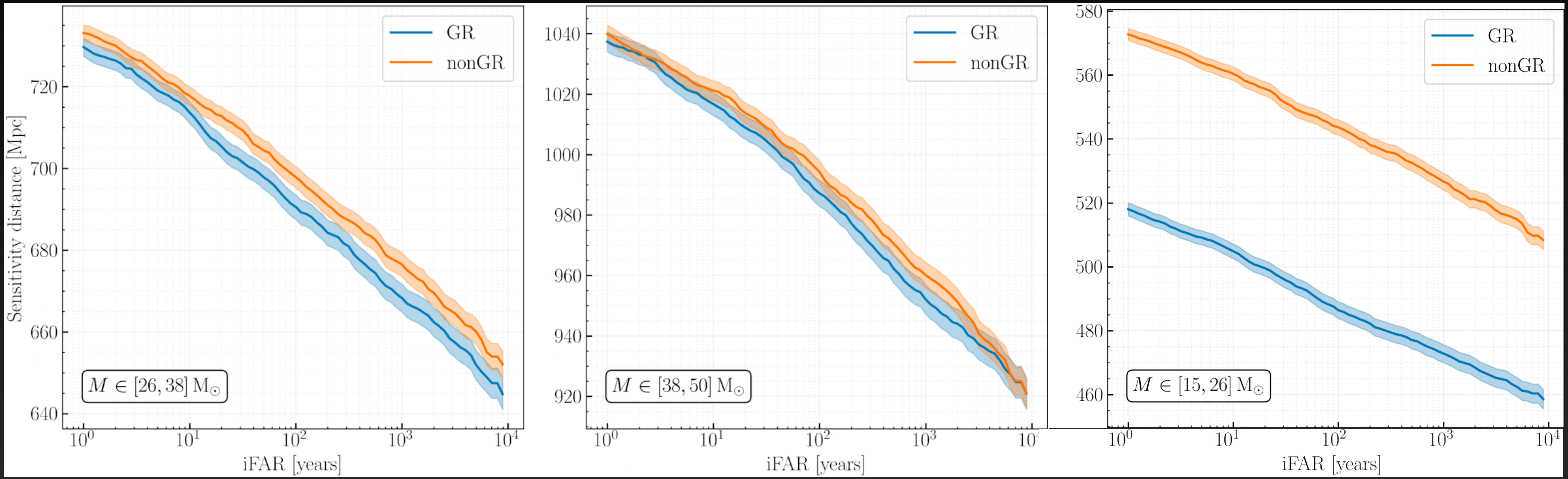


*Image source: Harsh Narola, Soumen Roy, Anand Sengupta PhysRevD.107.024017 (2023)

Inference: Improvement in sensitivity is more prominent in the **low total mass range**.

Template-based search for exotic (non-GR) signals in BNS mass range.

Due to longer in-band duration of the low mass signals, the effect of deviation parameters will be more prominent in BNS systems.

Therefore, we aim to perform a similar analysis in the BNS mass range, considering all the deviation parameters upto 3.5PN.

The parameter space boundaries considered to construct the non-GR template bank:

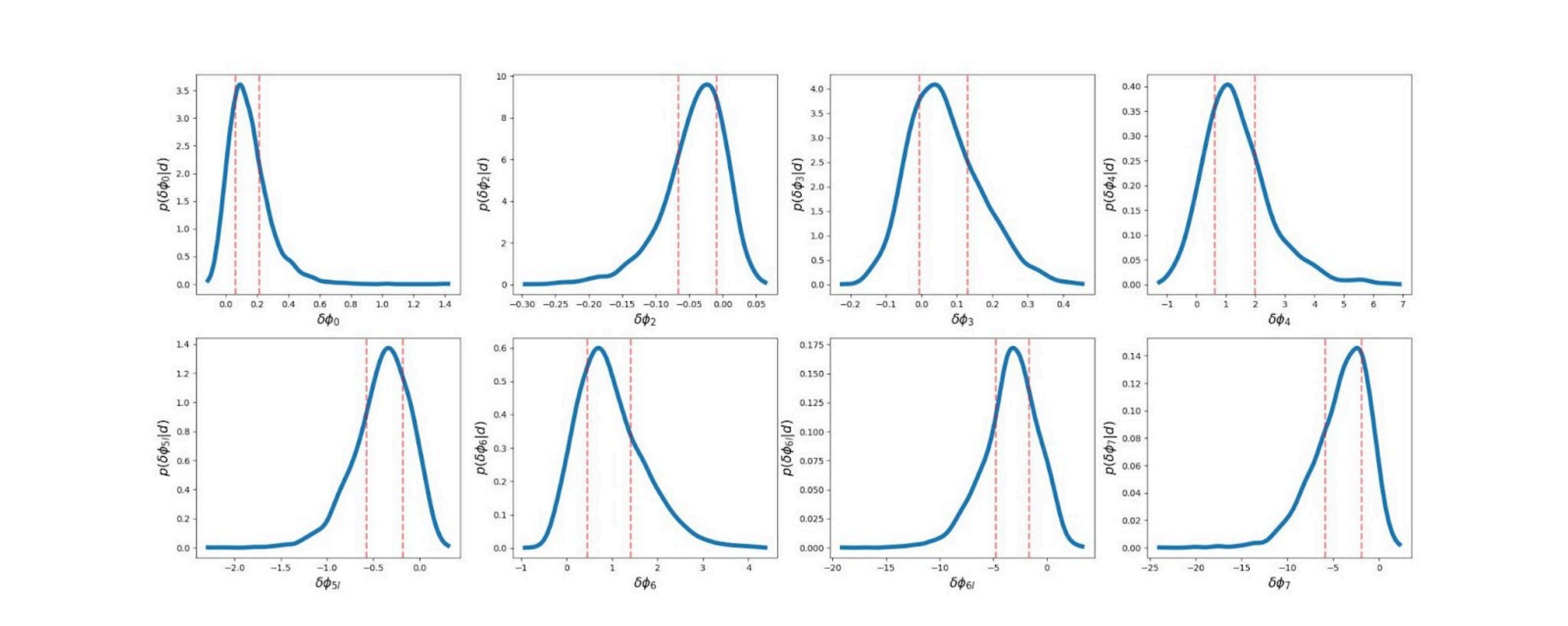
Parameter	Limits
Component masses, $m_{1,2}$	$[1,\ 3]\ M_{\odot}$
Total mass, M	$[2,\ 6]\ M_{\odot}$
Mass ratio, m_1/m_2	$[1,\ 3]$
Component spins, $\chi_{1,2}$	$[-\ 0.05,\ 0.05]$

PN Deviation Parameter	Limits
0PN, $\delta\phi_0$	$[0.058,\ 0.213]$
1PN, $\delta\phi_2$	$[-\ 0.066,\ \ -\ 0.009]$
1.5PN, $\delta\phi_3$	$[-\ 0.006,\ 0.129]$
2PN, $\delta\phi_4$	$[0.607,\ 1.974]$
2.5PN, $\delta\phi_{5l}$	$[-\ 0.576,\ \ -\ 0.181]$
3PN, $\delta\phi_6$	$[0.448,\ 1.408]$
3PN, $\delta\phi_{6l}$	$[-\ 4.798,\ \ -\ 1.652]$
3.5PN, $\delta\phi_7$	$[-\ 5.872,\ \ -\ 1.903]$

Choice of deviation parameter space

The boundaries of deviation parameter space ~ **50% credible intervals** of posteriors obtained from **GW170817.**

*B. P. Abbott et al. PhysRevLett.123.011102



Template Bank Construction

Match between two template waveforms: $M\left(h_1, h_2\right) = \max_{\phi_c, t_c}\left(\hat{h}_1 \mid \hat{h}_2\left(\phi_c, t_c\right)\right)$, where $\left(h_1 \mid h_2\right) = 4\,\mathrm{Re}\int_0^{\infty} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}df$

The aim is to construct a non-GR template bank such that:

- It contains the **minimum number of templates**.
- And for any **plausible** exotic signal, there **must exist at least one** template that has match value ≥ 0.97.

We follow **Duncan A. Brown et al.** PhysRevD.86.084017 to place templates in the parameter space.

Metric in parameter space:

$$M(h(\boldsymbol{\theta}), h(\boldsymbol{\theta} + \delta\boldsymbol{\theta})) = 1 - \sum_{ij} g_{ij}(\boldsymbol{\theta})\delta\theta^i\delta\theta^j \qquad g_{ij}(\boldsymbol{\theta}) = -\frac{1}{2}\frac{\partial^2 M}{\partial\delta\theta^i\partial\delta\theta^j} = \left(\frac{\partial h(\boldsymbol{\theta})}{\partial\theta^i}\middle|\frac{\partial h(\boldsymbol{\theta})}{\partial\theta^j}\right)$$

Template Bank Construction

$$\Phi(f) = 2\pi f t_c - \varphi_c - \pi/4 + \frac{3}{128\eta}(\pi M f)^{-5/3} \sum_{i=0}^{7} \phi_i(\vec{\theta})(\pi M f)^{i/3}$$

The metric is obtained in the coordinate system composed of 8 PN phasing coefficients.

Advantage: The metric is flat!

Parameter space is very thin along many directions.

We assess the effective dimensionality of our parameter space using PCA.

$$(1 + \delta\phi_j)\phi_j = \tilde{\phi}_j$$

Rotation + Rescaling

$$\mu_i = \sum_j \left(V_{ij}\sqrt{E_i}\right) \tilde{\phi}^j$$

PCA

$$\xi_i = \sum_j C_{ij}\mu^j$$

Template Bank Construction

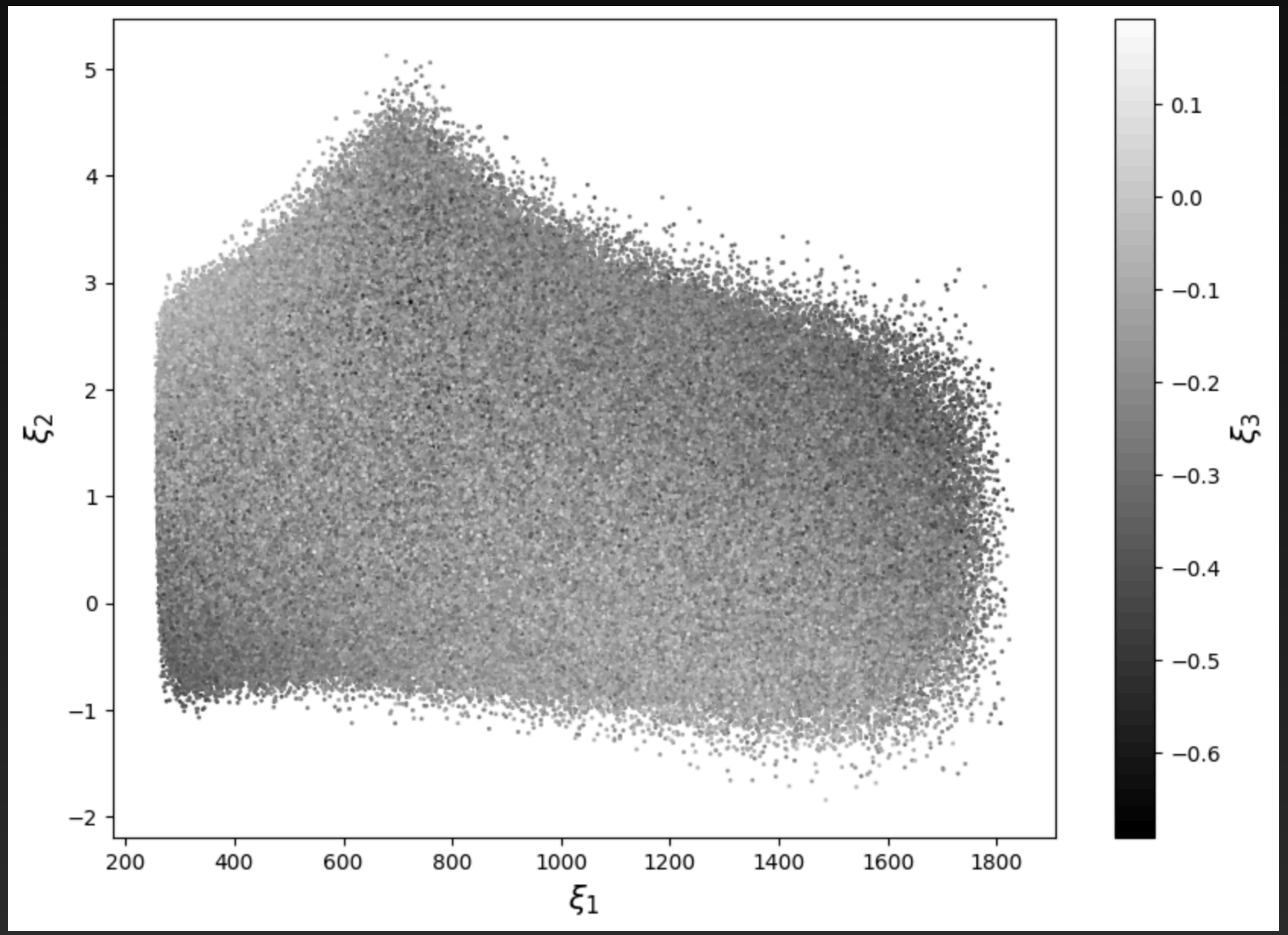
Effective dimensions of the parameter space = 3.

A_3^* (truncated octahedron) lattice is used to place templates in the ξ coordinates.

But, there are two challenges:

1. The boundaries in these coordinates are non-trivial.
2. Inverse mapping to the physical coordinates is not known.

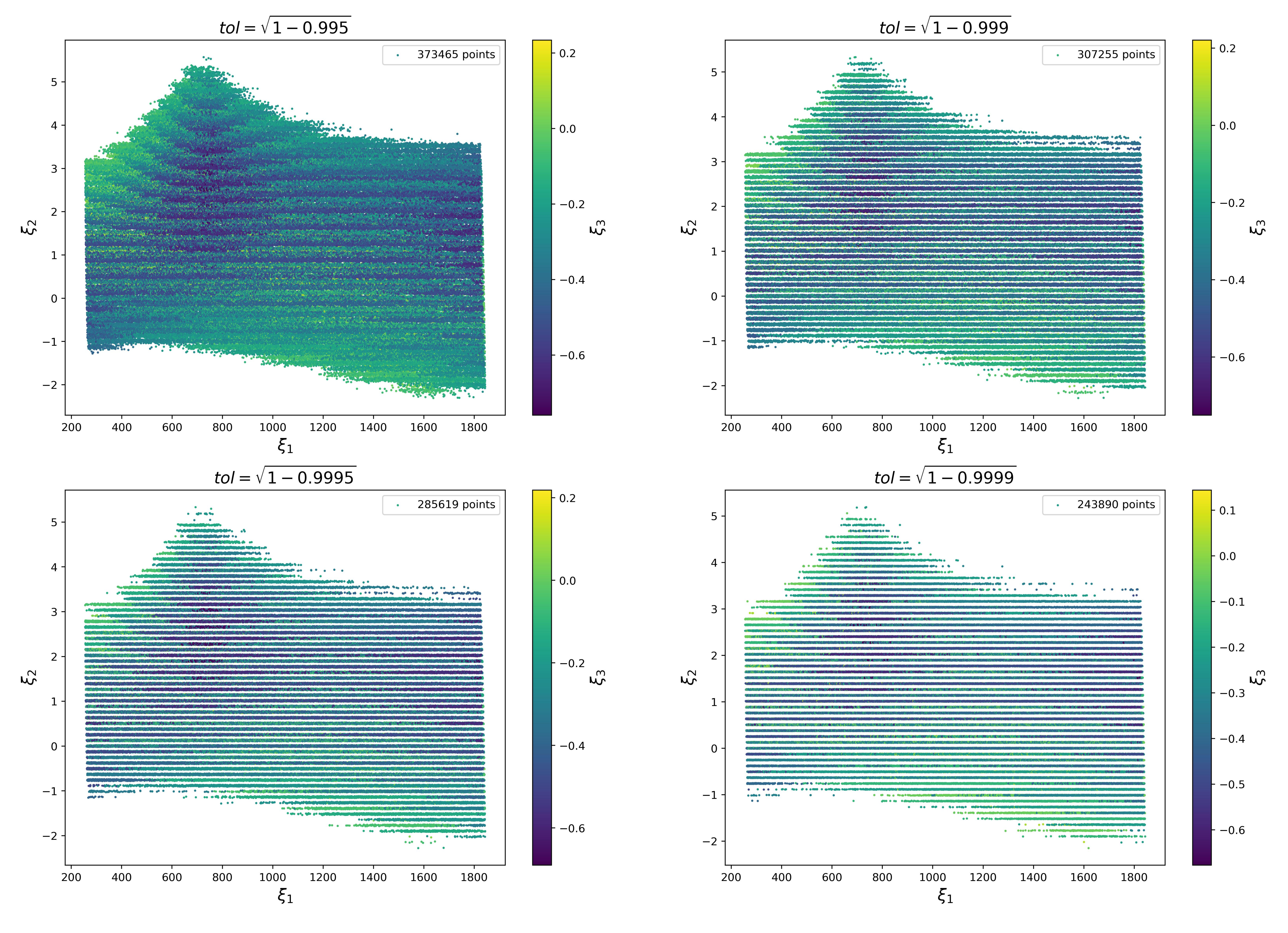
Therefore, We adopt brute force method.



We generate a lattice in a cuboidal boundary described by extreme values of ξ coordinates.

We generate a large number of random points in ξ coordinates and look for nearest neighbours of lattice points.

Early results



Future Plans

- Validation studies for the template bank.
- Numerical experiments to assess the search sensitivity of the non-GR template bank.
- We target to search the LIGO, Virgo O1 and O2 data for exotic signals!

Thank You!