

# VISCOUS COSMOLOGICAL MODEL AND THE VALIDITY OF NEAR EQUILIBRIUM CONDITION IN THE CONTEXT OF $F(R,T)$ GRAVITY

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**6th International Workshop On  
AAPCOS-2023**

**SNIP Kolkata, India**



# OVERVIEW

**ACCELERATING UNIVERSE**

**BULK VISCOSITY IN COSMOLOGY**

**NEAR EQUILIBRIUM CONDITION (NEC)**

**NEC IN  $f(R,T)$  GRAVITY**

**TESTING VALIDITY OF NEC CONSTRAINTS**

**CONCLUSIONS**

**THANK YOU**

# ACCELERATING UNIVERSE

Observations suggests an accelerated expansion for the Universe.

- First evidence from S. Perlmutter, B. Schmidt and Adam. G. Riess through observations of distant supernovae.
- During this last decade several evidences have been collected in this behalf.



Saul Perlmutter

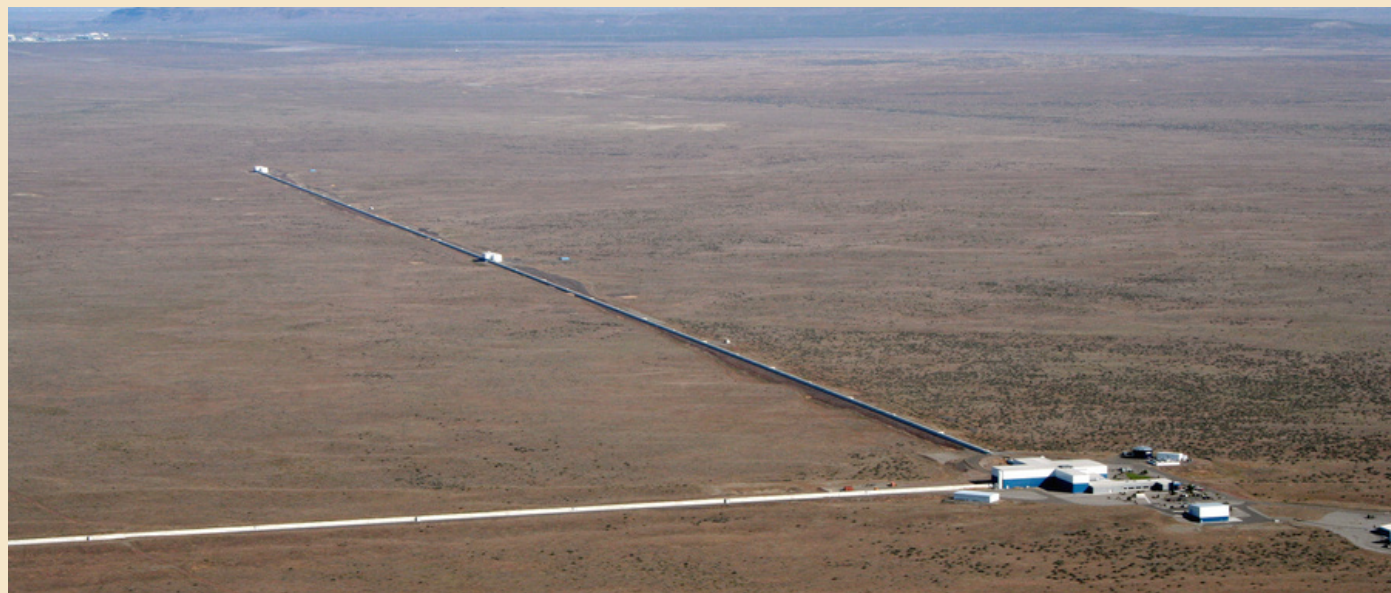


Brian Schmidt



Adam. G. Riess

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LIGO Hanford

Credits: Caltech/MIT/LIGO Lab

- Recent evidence in 2021 using gravitational wave measurement of Hubble constant.

[10.3847/1538-4357/abdc7](#)

According to which, hubble parameter is in the range:  $H_0 = 68.7^{+17.0}_{-7.8}$  km/s/Mpc.

**This accelerated expansion is best described by the  $\Lambda$ CDM model.**

- Model proposed cosmological constant as dark energy.
- Even though the model makes many successful predictions, it has several drawbacks some of which include,

- **Cosmological Constant Problem**

- **Hubble Tension**

- **Cosmic Coincidence Problem**

- **Modified Gravity ?**

- **Varying Dark Energy ?**

# BULK VISCOSITY IN COSMOLOGY

Viscosity as a natural way for explaining an accelerating universe without Dark energy.

Motivation:

- Perfect fluid assumption in concordance model is an approximation
- Real fluids in nature possess dissipative nature.

First formalism for relativistic dissipative fluids developed by Carl Eckart in 1940.

$$T_{\mu\nu} = (\rho + P_{eff}) U_{\mu} U_{\nu} + g_{\mu\nu} P_{eff}$$

**Where,**  $P_{eff} = p_0 + \Pi$     **and**     $\Pi = -3\zeta H$

$\zeta$  : Coefficient of bulk viscosity.

$U_{\mu}$  : Four velocity of viscous fluid.

For a  
Homogeneous and  
Isotropic  
Universe



No shear viscosity

## MERITS

- Higher-order causal viscous theories reduce to Eckart's theory in their first-order approximation. Hence, Eckart theory is good for preliminary investigations of viscous effects in cosmology.
- Mathematical ease.

## DRAWBACKS

- Acausal nature.
- Unstable Equilibrium States.

# NEAR EQUILIBRIUM CONDITION (NEC)

Eckart's theory assumes NEC for viscous fluid

- Assumption : Throughout its the evolution, fluid remains in a near equilibrium state.

→ Deviations from this assumption can cause theory to breakdown.

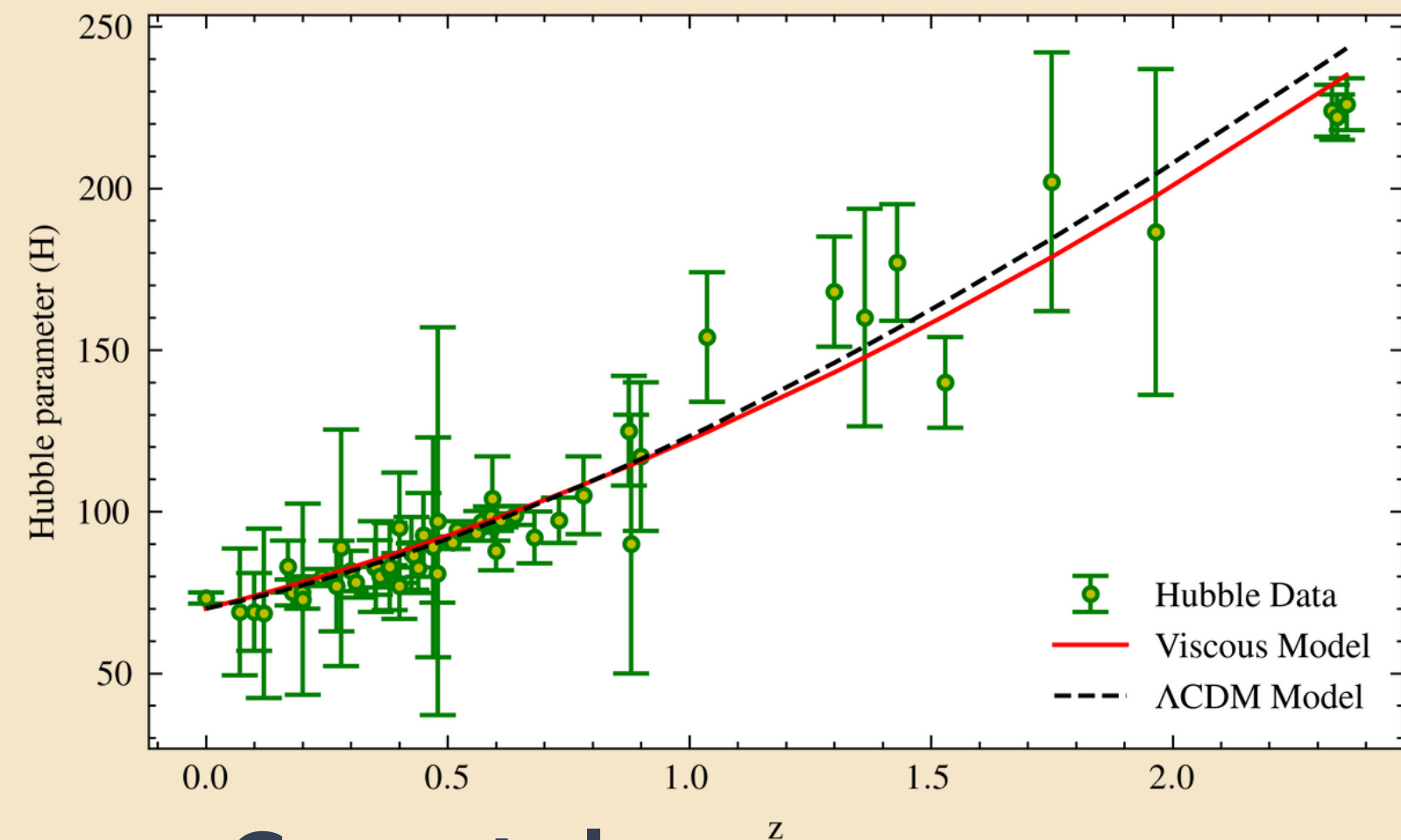
In mathematical terms, NEC means :  $\left| \frac{\Pi}{p_0} \right| \ll 1$

- Pointed out by Marteens in the context of dissipative inflation.

**Problem : In Einstein's Gravity NEC is violated if one tries to explain accelerated expansion as caused by viscous matter.**

- According to him, either one should postulate that Eckart's theory can hold even if fluid is far from equilibrium or one should respect NEC and give up the idea of explaining acceleration as caused by viscous matter alone.

- Nevertheless, several models were proposed using this assumption, many of which lead to predictions of cosmological parameters in an acceptable range.



- Recently it was pointed out by Norman Cruz et.al.,

[N. Cruz, E. González, and J. Jovel \(2022\), 2207.14244.](#)

**Problem : In Einstein's Gravity NEC is achieved only in the presence of cosmological constant.**



**Cosmological constant is necessary**

**Which in turn reduced the attractive feature of viscosity as an alternative to dark energy.**

**Is NEC satisfied in modified gravities without  
Cosmological constant ??**

# NEC IN $f(R,T)$ GRAVITY

$f(R,T)$  gravity is a generalization of  $f(R)$  gravity

- Developed by : Harko, Lobo, Nojiri and Odintsov in 2011. .

T. Harko, F. S. Lobo, S. Nojiri, and S. D. Odintsov, Physical Review D 84, 024020 (2011).

Ricci Scalar ( $R$ ) is coupled to trace of energy momentum tensor ( $T$ ).

Gravitational Lagrangian changes to  $L_g = f(R, T)\sqrt{-g}$

We investigated,  $f(R, T) = R + 2\lambda T$ , where  $\lambda$  is the coupling parameter.

- Aim is to develop constraints for model parameters using  $\lambda$

Such that, NEC is satisfied during accelerated expansion without the need of cosmological constant.

**Friedmann Equations in  $f(R, T) = R + 2\lambda T$  gravity.**

$$3H^2 = \rho + \lambda (3\rho - P_{eff})$$

$$2\dot{H} + 3H^2 = - [ P_{eff} + \lambda (3P_{eff} - \rho) ]$$

$$P_{eff} = p_0 + \Pi$$

$$p_0 = \omega\rho \quad \text{and} \quad \Pi = -3\zeta H$$

## Derived constraints are independent of form of bulk viscosity

- For a generic viscous fluid whose EoS,  $\omega \in [-1, 1]$  but not zero.

### WDM fluid

$$\omega \in (0, 1]$$

$$\lambda < -\frac{3}{8} \quad \bullet \quad \left| \frac{\Pi}{\omega\rho} \right| \ll 1$$

$$-1 < \frac{1}{\omega(3+8\lambda)} < 0, \text{ for } \Pi < 0$$

$$\frac{\Pi}{3H^2} < \frac{-1}{\lambda} \quad \text{and} \quad (3-\omega) < \frac{-1}{\lambda}$$

### Dark fluid

$$\omega \in [-1, 0)$$

$$\lambda > -\frac{3}{8} \quad \bullet \quad \left| \frac{\Pi}{\omega\rho} \right| \ll 1$$

$$-1 < \frac{1}{\omega(3+8\lambda)} < 0, \text{ for } \Pi > 0$$

$$\frac{\Pi}{3H^2} > \frac{-1}{\lambda} \quad \text{and} \quad (3-\omega) > \frac{-1}{\lambda}$$

# TESTING VALIDITY OF NEC CONSTRAINTS

Model analysis is carried out by assuming  $\zeta = \zeta_0 H$ .

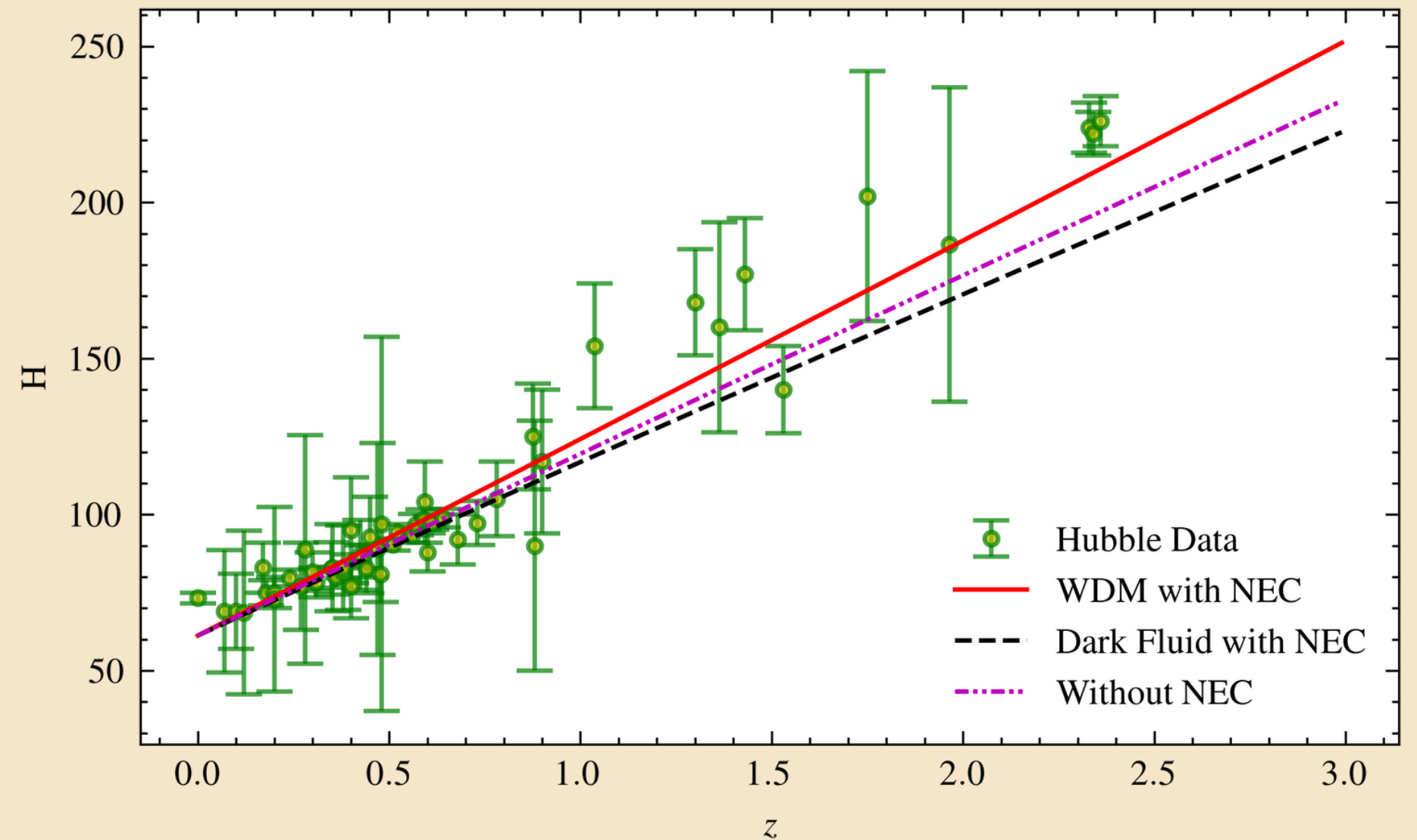
- We chose simplest model just for testing the developed constraints,

$$H = H_0 a^{-\beta}$$



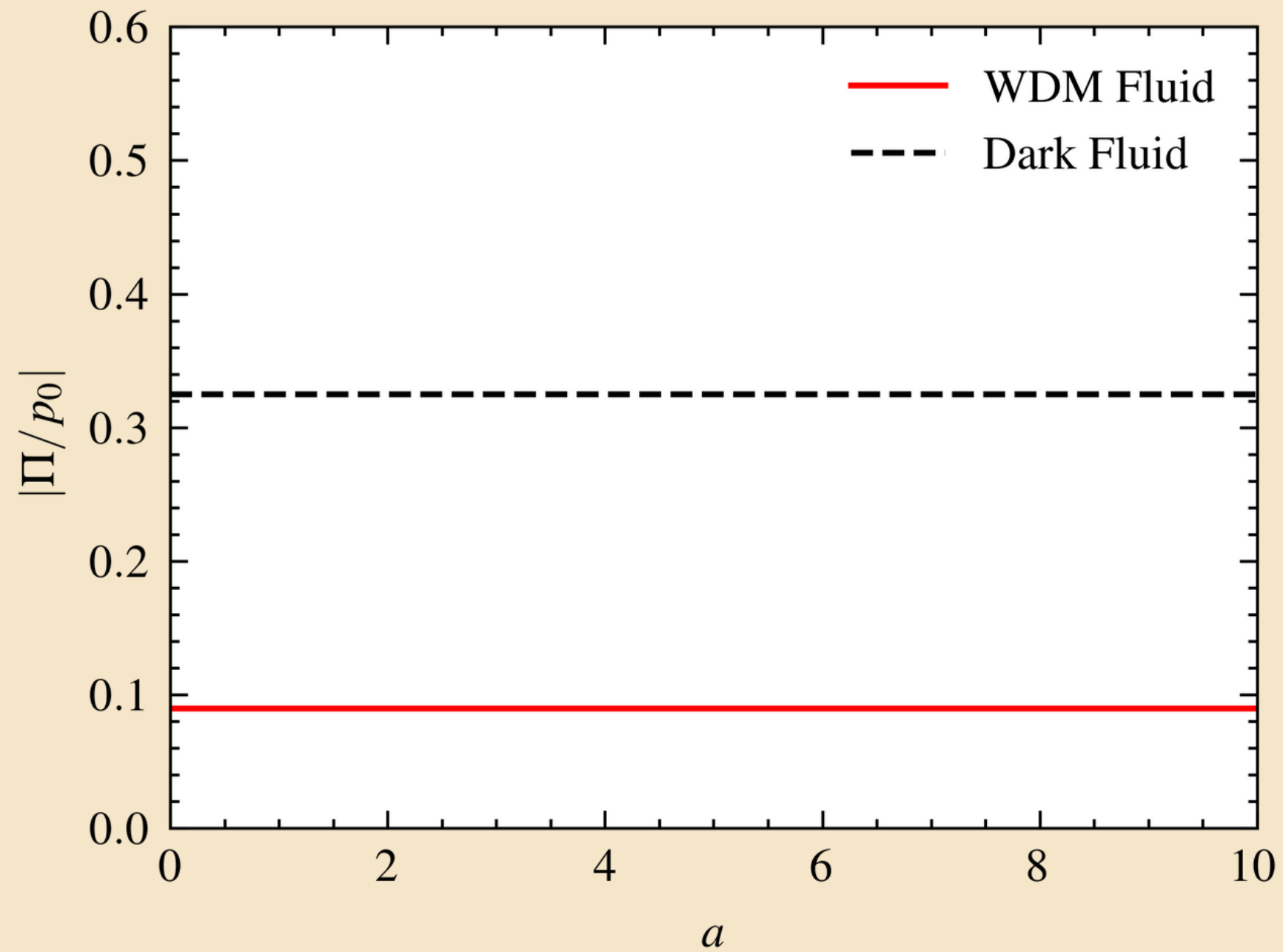
For accelerated expansion,  $\beta < 2$

$$\beta = \frac{3(2\lambda + 1) ((\zeta_0(4\lambda + 1)) - (\omega + 1))}{2(1 + \lambda(3 - \omega))}$$

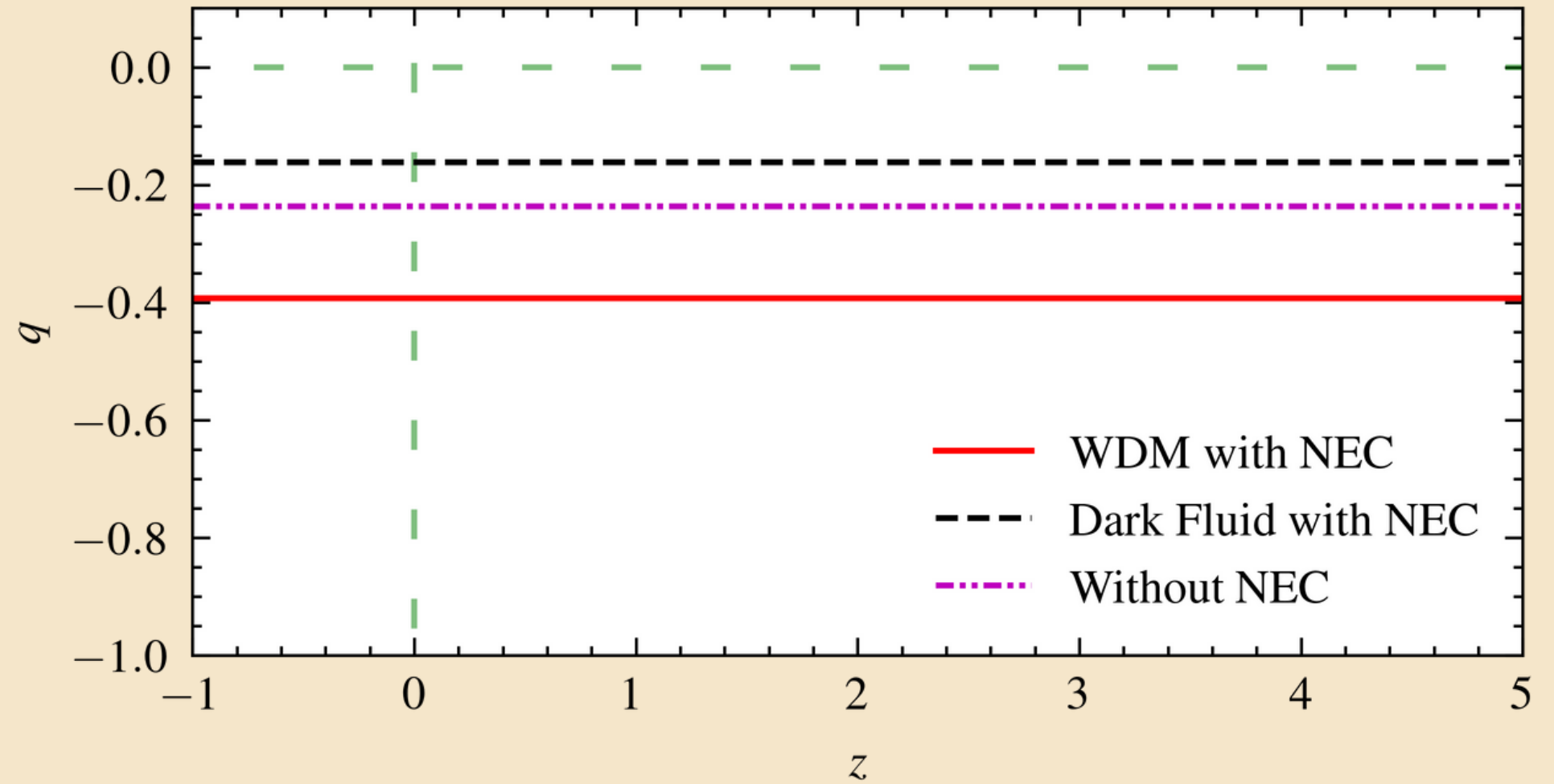


Fluid Model	$H_0$	$\lambda$	$\zeta_0$	$\omega$	$\chi^2_{min}$	$\chi^2_{dof}$
Dark	$61.20 \pm 0.84$	$0.23 \pm 0.39$	$-0.22 \pm 0.19$	$-0.63 \pm 0.27$	145.86	3.04
WDM	$61.25 \pm 0.92$	$-0.62 \pm 0.03$	$0.49 \pm 0.06$	$0.04 \pm 0.03$	93.47	1.95
Without NEC	$61.22 \pm 0.82$	$0.29 \pm 0.36$	$0.32 \pm 0.26$	$0.39 \pm 0.33$	116.63	2.43

Hubble parameter vs redshift for best estimates of model parameters.



**Plot of NEC vs scale factor.**



**Deceleration parameter vs redshift.**

NEC is satisfied during accelerated expansion without cosmological constant.

**CONSTRAINTS WORKED !**

# CONCLUSIONS

- Constraints are model independent as far as functional form of viscosity is concerned.
- Viscous fluids can achieve NEC in  $f(R,T)$  gravity while explaining an accelerated expansion in the absence of cosmological constant.
- Developed bounds for coupling parameter and then verified the constraints through model analysis.

**THANK YOU !**