About generating small Yukawa couplings naturally from trans-Planckian asymptotic safety

Soumita Pramanick

Physical Research Laboratory (PRL), Ahmedabad, India

Based on

JHEP 08, 262 (2022) [arXiv:2204.00866 [hep-ph]]

by Kamila Kowalska, Soumita Pramanick, and Enrico Maria Sessolo

Three flavour mixing

Neutrinos are massive and they mix !!

Three flavors: $\boldsymbol{\nu_e, \nu_\mu, \nu_\tau}$

Oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j} \sin^2 \left(\frac{\pi L}{\lambda_{ij}} \right)$$

2 independent Δm^2, 3 mixing angles, 1 phase

$$U^T M_\nu U = \mathrm{diag}(m_1, m_2, m_3)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

The Pontecorvo, Maki, Nakagawa, Sakata – PMNS – matrix

A measure of CP-violation is given by the basis-independent leptonic Jarlskog(J) parameter:

$$J = \mathrm{Im}[U_{e1} U_{\mu 2} U^*_{e2} U^*_{\mu 1}]$$

Parametrizing the PMNS matrix:

Define: $X = |U_{e1}|^2$, $Y = |U_{e2}|^2$, $Z = |U_{\mu 1}|^2$, $W = |U_{\mu 2}|^2$ **such that**

$$\begin{aligned}
\theta_{12} &= \arctan\sqrt{\frac{Y}{X}} \\
\theta_{13} &= \arccos\sqrt{X+Y} \\
\theta_{23} &= \arcsin\sqrt{\frac{1-W-Z}{X+Y}}\,,
\end{aligned}$$

and

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}\,.$$

Thus the matrix of squared PMNS elements is given by:

Here we are using the same parametrization for the PMNS as was introduced for the CKM mixing matrix for the quark sector in :
R. Alkofer, A. Eichhorn, A. Held, C. M. Nieto, R. Percacci and M. Schröfl, Annals Phys. 421, 168282 (2020) [arXiv:2003.08401 [hep-ph]].

$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}\,.$$

The allowed 3σ ranges of the parameters X, Y, Z, and W [NuFIT 5.1 (2021)]:

$$X \in [0.64 - 0.71] \quad Y \in [0.26 - 0.34] \quad Z \in [0.05 - 0.26] \quad W \in [0.21 - 0.48]\,.$$

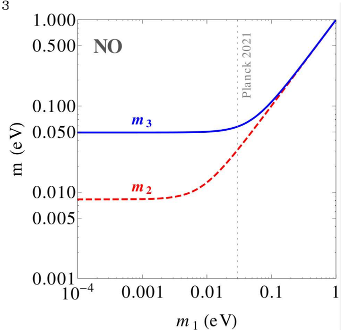
Current status of neutrino masses:

The current 1σ ranges of the mass splittings [NuFIT 5.1 (2021)]:

$$\begin{aligned} \Delta m^2_{21} &= 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2, \\ \text{NO:}\quad \Delta m^2_{31} &= 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\mathrm{eV}^2, \\ \text{IO:}\quad \Delta m^2_{32} &= -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\mathrm{eV}^2. \end{aligned}$$

From Planck data:

$$\sum_{i=1,2,3} m_i < 0.12\,\mathrm{eV}.$$



Note:

1. Degenerate regime i.e., $m_1 \approx m_2 \approx m_3$, is excluded at 95% C.L. by Planck data for both NO and IO.
2. The allowed largest neutrino masses at upper end:

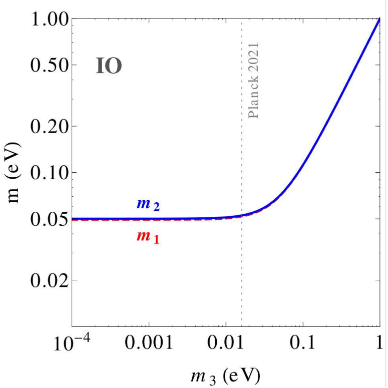
$$\text{NO}: \qquad m_1 = 0.030\,\mathrm{eV}, \quad m_2 = 0.031\,\mathrm{eV}, \quad m_3 = 0.058\,\mathrm{eV},$$

3. For hierarchical regime: the lightest neutrino can be much lighter than the other two (actually, it can even be massless), whose masses saturate at:

$$\text{NO}: \qquad m_2 = 0.009\,\mathrm{eV}, \quad m_3 = 0.050\,\mathrm{eV},$$

Inverted Ordering

Similarly for IO:



Note: 1. The allowed largest neutrino masses at upper end:

$$\text{IO}: \quad m_1 = 0.052\,\text{eV}, \quad m_2 = 0.052\,\text{eV}, \quad m_3 = 0.016\,\text{eV}.$$

2. Hierarchical regime:

$$\text{IO}: \quad m_1 = 0.049\,\text{eV}, \quad m_2 = 0.050\,\text{eV}.$$

Extension of SM : Adding 3 right-handed neutrinos

Neutrinos are massless within SM.

Popular ways of giving mass to neutrinos:
Tree Level : See-saw Mechanism
Loop level : Radiative neutrino mass models

In this work: To give masses to neutrinos $\Rightarrow$ Extend SM by 3 right-handed neutrinos $\nu_{R,i}$, $i = 1, 2, 3$ (singlets under the SM gauge group):

$$\mathcal{L}_D = -y_\nu^{ij} \nu_{R,i} \, (H^c)^\dagger L_j + \text{H.c.} \, ,$$

$L_j \rightarrow$ SM lepton $\text{SU}(2)_L$ doublets, $H \rightarrow$ the SM Higgs boson doublet under $\text{SU}(2)_L$ and $H^c \equiv i\sigma_2 H^* \rightarrow$ charged conjugate doublet.

Neutrino Dirac mass: $m_D \sim y_\nu v/\sqrt{2}$, where $v = 246\,\text{GeV}$.

To match experimental data:

$y_\nu \sim 10^{-13}$ **whereas Yukawa couplings of other SM fermions ranges between** 10^{-5} **to** 1.

Motivation $\Rightarrow$ Generate these small yukawa couplings naturally from trans-Planckian asymptotic safety.

General discussion of AS gravity

There are evidence from Quantum Gravity that there exists Fixed Points (FP) for gravity couplings

Reuter, Saueressig, hep-th/0110054

Gravity can couple with matter and its impact is captured in the new terms in the β functions

We assume: Above the Planck scale, the SM couples to quantum gravity or some other New Physics (NP) leading to generation of FP for beta functions of all dimensionless couplings.

So, above the Planck scale the beta functions ($\beta_x \equiv dx/d\log k$) gets modified:

$$\beta_g = \beta_g^{\mathrm{SM}} - g\, f_g\,,$$
$$\beta_y = \beta_y^{\mathrm{SM}} - y\, f_y\,,$$

Effects of the new AS trans-Planckian interactions are parametrized with f_g and f_y.

f_g and f_y $\Rightarrow$ Determined from gravitational dynamics.

We treat f_g and f_y as free parameters and determine them by matching the low-scale experimental constraints.

[Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig'09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18, ...]

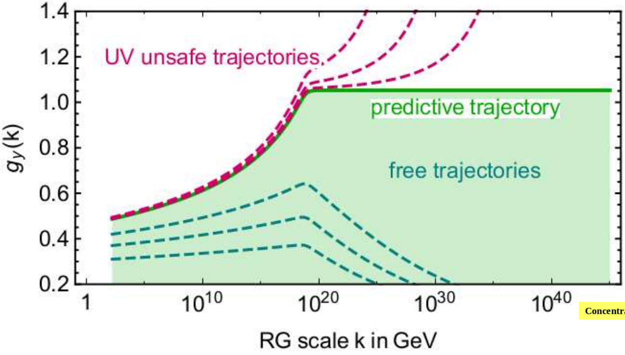
Fixed Points (FP)

FPs of gauge-Yukawa system is any set $\{g^*, y^*\}$ for which $\beta_g(g^*, y^*) = \beta_y(g^*, y^*) = 0$.

Gaussian FP: If the FP value of the coupling is zero at FP i.e., $(\alpha_i^* = 0)$ where $\{\alpha_i\} \equiv \{g, y\}$.

Interactive FP: If the FP value of the coupling is nonzero at FP i.e., $(\alpha_i^* \neq 0)$ where $\{\alpha_i\} \equiv \{g, y\}$.

No f_g, f_y **With f_g, f_y**



Concentrate on the predictive trajectory

This figure is taken from:
A. Eichhorn and F. Versteegen, JHEP 01, 030 (2018) [arXiv:1709.07252 [hep-th]].

Stability Matrix:

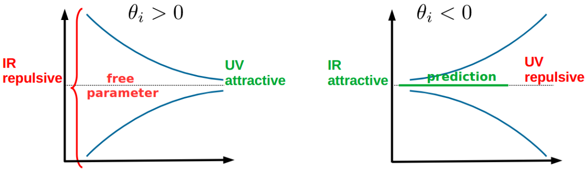
Stability Matrix: In order to determine FP structure, the RGEs system of the couplings can be linearized around the FP and the stability matrix M_{ij} can be defined as:

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}, \quad \text{where} \quad \{\alpha_i\} \equiv \{g, y\}.$$

Critical Exponents (θ_i): Opposite of the Eigenvalues of the stability matrix M_{ij}

Note:

$\theta_i > 0 \Rightarrow$ Corresponding eigendirection is **relevant and UV-attractive i.e., IR-repulsive.**
$\theta_i < 0 \Rightarrow$ Corresponding eigendirection is **irrelevant and UV-repulsive i.e., IR-attractive.**
$\theta_i = 0 \Rightarrow$ **Marginal Eigendirection**



Relevant couplings are free parameters of the theory

Irrelevant couplings provide predictions

Note: This figure is taken from a slide of a talk by Kamila Kowalska at Portoroz 2021: Physics of the flavourful Universe (September, 2021)

Determining f_g and f_y:

f_g and f_y $\Rightarrow$ Determined from gravitational dynamics.

[Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso et al. '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, ...]

Calculations with functional RG in AS quantum gravity has shown $f_g > 0$ irrespective of chosen RG scheme and $f_g > 0$ is required to ensure asymptotic freedom in gauge sector.

No general results and definite conclusions about size and sign of f_y available.

In this work we treat f_g and f_y as free parameters and determine them by matching the low-scale experimental constraints.

Determining f_g:

The RGE (one-loop) of the $U(1)_Y$ gauge coupling g_Y with trans-Planckian correction included:

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y$$

This develops an interactive FP and matching the low-scale value of g_Y gives:

$$\boxed{g_Y^* = 0.47 \text{ and } f_g = 0.0096 \text{ for } M_{\rm Pl} = 10^{19}\ \text{GeV.}}$$

Determining f_y:

Similarly, f_y is determined by y_t:

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + \ldots\ldots - \frac{17}{12}g_Y^2\right] - f_y\, y_t$$

Matching the low-scale value of y_t determines f_y.

For earlier works: see e.g. Eichhorn, Held, 1707.01107, 1803.04027; Reichert, Smirnov, 1911.00012; Alkofer et al. 2003.08401; Kowalska, Sessolo and Yamamoto, 2007.03567, 2012.15200, ...

Toy Model (top/neutrino scenario)

Consider a toy model consisting of the SM top quark and neutrino Yukawa couplings.
Assume, all SM Yukawa couplings, other than y_t and y_ν, originate from relevant Gaussian fixed-point directions in the UV.

The Trans-Planckian RGEs for this system:

$$\begin{aligned}
\frac{dg_Y}{dt} &= \frac{g_Y^3}{16\pi^2}\,\frac{41}{6} - f_g\, g_Y \\
\frac{dy_t}{dt} &= \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t \\
\frac{dy_\nu}{dt} &= \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu \,.
\end{aligned}$$

Several FPs exists (Asterisk indicates non-zero value)

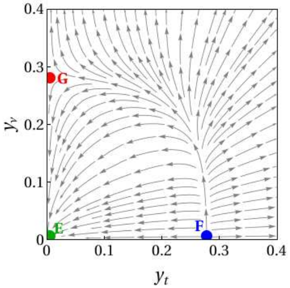
	g_Y^*	y_t^*	y_ν^*	θ_Y	θ_t	θ_ν
$\mathrm{FP_A}$	0	0	0	+	+	+
$\mathrm{FP_B}$	0	0	*	+	+	−
$\mathrm{FP_C}$	0	*	0	+	−	+
$\mathrm{FP_D}$	0	*	*	+	−	−
$\mathrm{FP_E}$	*	0	0	−	+	+
$\mathbf{FP_F}$	*	*	0	−	−	−
$\mathrm{FP_G}$	*	0	*	−	+	−

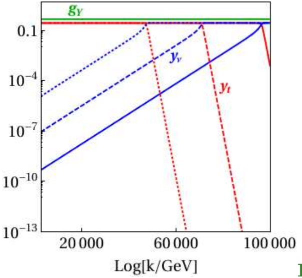
Note: $\mathbf{FP_F}$ is irrelevant in all directions.

Toy Model (top/neutrino scenario) continued ....

The real FPs of Toy Model (Asterisk indicates non-zero value):

	g_Y^*	y_t^*	y_ν^*	θ_Y	θ_t	θ_ν
$\mathrm{FP_A}$	0	0	0	+	+	+
$\mathrm{FP_B}$	0	0	*	+	+	−
$\mathrm{FP_C}$	0	*	0	+	−	+
$\mathrm{FP_D}$	0	*	*	+	−	−
$\mathrm{FP_E}$	*	0	0	−	+	+
$\mathrm{FP_F}$	*	*	0	−	−	−
$\mathrm{FP_G}$	*	0	*	−	+	−





$\mathrm{FP_F}$ **$\mathrm{FP_G}$**

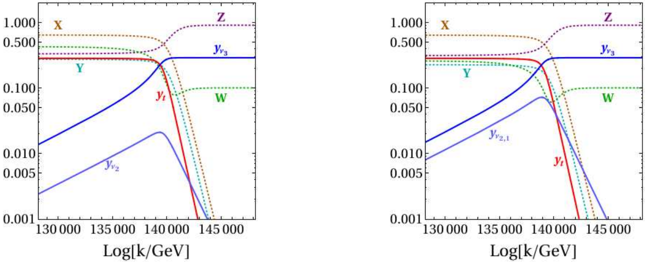
Left panel: Phase diagram for $\mathrm{FP_E}$, $\mathrm{FP_F}$, $\mathrm{FP_G}$. RG flow directions point towards UV. Right panel: Flow of top (red) and neutrino (blue) Yukawa coupling for three different choices (solid, dashed, dotted) of y_t joining the UV FP (FP_G) to the IR FP (FP_F). g_Y is in solid green. Here, $f_g = 0.0096$ and $f_y = 0.0002$.

SM + 3 right-handed neutrinos

Coming back to the SM + 3 right-handed neutrinos:
Assume neutrinos are Dirac.

	X^*	Y^*	Z^*	W^*	θ_X	θ_Y	θ_Z	θ_W
$\mathrm{FP}_{\mathrm{IR},1}$	$1-Y^*$	ind.	Y^*	$1-Y^*$	0	−	−	−
$\mathrm{FP}_{\mathrm{IR},2}$	0	1	1	0	−	−	−	−
FP_3	0	1	ind.	0	−	−	+	0
FP_4	0	0	ind.	1	+	+	+	−
FP_5	0	0	$1-W^*$	ind.	+	+	+	0
FP_6	0	0	1	0	+	+	−	+
FP_7	0	1	0	0	−	0	+	0
FP_8	0	0	0	1	+	+	+	−
FP_9	0	0	1	1	+	+	−	−

FPs with $y^*_{\nu 3} \neq 0$ and all other lepton Yukawa couplings set at zero. "ind." indicates an indeterminate value of the corresponding mixing parameter.



RGE trajectories from FP_5, producing neutrino Yukawa couplings and mixing parameters for NO in the IR for hierarchical (left panel) and nearly degenerate (right panel) situations. Here, $f_g = 0.0096$, $f_y = 0.00025$.

Conclusions

- Neutrino masses and mixing in general

- Generation of small Yukawa couplings naturally from trans-Planckian asymptotic safety:

  (1) General idea about AS gravity

  (2) Toy Model (top/neutrino scenario)

  (3) SM + 3 right-handed neutrinos: For one of the FPs (FP_5), flowing into the fully irrelevant $FP_{IR,1}$ for NO both in hierarchical and nearly degenerate situations correct low-scale values of neutrino mass and mixing could be achieved. No such solution for IO.

  (4) The same mechanism can be extended to explain the feeble Yukawa couplings needed to generate the correct relic abundance for a sterile-neutrino dark matter via freeze-in.

Thank you

Backup Slides

Sterile-neutrino dark matter : a freeze-in scenario

Consider: A scalar field S that decays to a sterile neutrino ν_R (only one generation is considered for simplicity) dark matter that freezes in.

A. Kusenko, Phys. Rev. Lett. 97, 241301 (2006) [arXiv:hep-ph/0609081 [hep-ph]];
K. Petraki and A. Kusenko, Phys. Rev. D 77, 065014 (2008) [arXiv:0711.4646 [hep-ph]];
M. Frigerio and C. E. Yaguna, Eur. Phys. J. C 75, no.1, 31 (2015) [arXiv:1409.0659 [hep-ph]].

$$\boxed{\mathcal{L} \supset -Y_S\, S\, \nu_R \nu_R - Y_L\, S \chi\, \xi + \text{H.c.}\,,}$$

where, additional Yukawa interaction of S with two NP Weyl spinors ξ, χ of opposite dark (or SM) charge $-Q_D, Q_D$, which may be the (left-chiral) components of a Dirac fermion or else (given by the Y_L term) can also be present.

$$\text{Relic Abundance: } \Omega h^2 \approx 0.12 \left(\frac{Y_S}{10^{-8}}\right)^2 \left(\frac{M_N}{10^{-8}\, m_S}\right).$$

$m_S \to$ mass of scalar S and $M_N \to$ the Majorana mass

RGEs (1-loop) for this system:

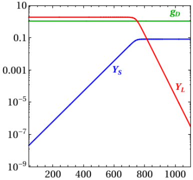
$$\frac{dg_D}{dt} = \frac{g_D^3}{16\pi^2}\frac{4}{3}Q_D^2 - f_g\, g_D$$

$$\frac{dY_S}{dt} = \frac{Y_S}{16\pi^2}\left(Y_L^2 + 6\, Y_S^2\right) - f_y\, Y_S$$

$$\frac{dY_L}{dt} = \frac{Y_L}{16\pi^2}\left(2\, Y_S^2 + 2\, Y_L^2 - 6\, Q_D^2\, g_D^2\right) - f_y\, Y_L\,,$$

Here: $g_D \to$ Dark $U(1)_D$ gauge coupling.

Small Y_S is generated naturally if the trans-Planckian RGEs admit an IR-attractive fixed point, $g_D^* \neq 0$, $Y_L^* \neq 0$, $Y_S^* = 0$. Recall the general discussion with two Yukawas (y_X and y_Z) and identify: $X = L$, $Y = D$, $Z = S$.



$\Rightarrow Q_D = 1,\ f_g = 0.0096,\ f_y = 0.00025.$

Extension of SM : Adding 3 right-handed neutrinos

Neutrinos are massless within SM.
To give masses to neutrinos ⇒ Extend SM by 3 right-handed neutrinos $\nu_{R,i}$, $i = 1, 2, 3$ (singlets under the SM gauge group):

$$\mathcal{L}_D = -y_\nu^{ij} \nu_{R,i} (H^c)^\dagger L_j + \text{H.c.} ,$$

$L_j \to$ SM lepton $\text{SU}(2)_L$ doublets, $H \to$ the SM Higgs boson doublet under $\text{SU}(2)_L$ and $H^c \equiv i\sigma_2 H^* \to$ charged conjugate doublet.
Neutrino Dirac mass: $m_D \sim y_\nu v/\sqrt{2}$, where $v = 246\,\text{GeV}$.
To match experimental data:
$y_\nu \sim 10^{-13}$ **whereas Yukawa couplings of other SM fermions ranges between** 10^{-5} **to** 1.
Motivation ⇒ Generate these small yukawa couplings naturally from trans-Planckian asymptotic safety.

RGEs for this system

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2} \frac{41}{6} - f_g \, g_Y$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2} \frac{19}{6} - f_g \, g_2$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2} 7 - f_g \, g_3$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2} \left[\frac{9}{2} y_t^2 + \frac{3}{2} y_b^2 - \left(\frac{17}{12} g_Y^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right] - f_y \, y_t$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2} \left[\frac{9}{2} y_b^2 + \frac{3}{2} y_t^2 - \left(\frac{5}{12} g_Y^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right] - f_y \, y_b \,.$$

RGEs for SM+3 right-handed neutrinos continued .. ...

$$
\begin{aligned}
\frac{dy_e}{dt} &= \frac{y_e}{16\pi^2}\left\{\frac{3}{2}y_e^2 - \frac{3}{2}\left[X y_{\nu1}^2 + Y y_{\nu2}^2 + (1-X-Y)y_{\nu3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\
&- \left.\left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_e\\
\frac{dy_\mu}{dt} &= \frac{y_\mu}{16\pi^2}\left\{\frac{3}{2}y_\mu^2 - \frac{3}{2}\left[Z y_{\nu1}^2 + W y_{\nu2}^2 + (1-Z-W)y_{\nu3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\
&- \left.\left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\mu\\
\frac{dy_\tau}{dt} &= \frac{y_\tau}{16\pi^2}\left\{\frac{3}{2}y_\tau^2 - \frac{3}{2}\left[(1-X-Z)y_{\nu1}^2 + (1-Y-W)y_{\nu2}^2 + (X+Y+Z+W-1)y_{\nu3}^2\right]\right.\\
&+ \left. y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2 - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\tau\\
\frac{dy_{\nu1}}{dt} &= \frac{y_{\nu1}}{16\pi^2}\left\{\frac{3}{2}y_{\nu1}^2 - \frac{3}{2}\left[X y_e^2 + Z y_\mu^2 + (1-X-Z)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\
&- \left.\left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu1}\\
\frac{dy_{\nu2}}{dt} &= \frac{y_{\nu2}}{16\pi^2}\left\{\frac{3}{2}y_{\nu2}^2 - \frac{3}{2}\left[Y y_e^2 + W y_\mu^2 + (1-Y-W)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\
&- \left.\left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu2}\\
\frac{dy_{\nu3}}{dt} &= \frac{y_{\nu3}}{16\pi^2}\left\{\frac{3}{2}y_{\nu3}^2 - \frac{3}{2}\left[(1-X-Y)y_e^2 + (1-Z-W)y_\mu^2 + (X+Y+Z+W-1)y_\tau^2\right]\right.\\
&+ \left. y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2 - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu3}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dX}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2 + y_\mu^2}{y_e^2 - y_\mu^2}\right)\left\{(y_{\nu1}^2 - y_{\nu3}^2)XZ + \frac{(y_{\nu3}^2 - y_{\nu2}^2)}{2}[W(1-X) + X - (1-Y)(1-Z)]\right\}\right.\\
&+ \left(\frac{y_e^2 + y_\tau^2}{y_e^2 - y_\tau^2}\right)\left\{(y_{\nu1}^2 - y_{\nu3}^2)X(1-X-Z) + \frac{(y_{\nu3}^2 - y_{\nu2}^2)}{2}[(1-Y)(1-Z) - X(1-2Y) - W(1-X)]\right\}\\
&+ \left(\frac{y_{\nu1}^2 + y_{\nu2}^2}{y_{\nu1}^2 - y_{\nu2}^2}\right)\left\{(y_e^2 - y_\tau^2)XY + \frac{(y_\tau^2 - y_\mu^2)}{2}[W(1-X) + X - (1-Y)(1-Z)]\right\}\\
&+ \left.\left(\frac{y_{\nu1}^2 + y_{\nu3}^2}{y_{\nu1}^2 - y_{\nu3}^2}\right)\left\{(y_e^2 - y_\tau^2)X(1-X-Y) + \frac{(y_\tau^2 - y_\mu^2)}{2}[(1-Y)(1-Z) - X(1-2Z) - W(1-X)]\right\}\right]
\end{aligned}
$$

More RGEs for SM+3 right-handed neutrinos.. ...

$$\frac{dY}{dt} = -\frac{3}{(4\pi)^2}\Bigg[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]+(y_{\nu2}^2-y_{\nu3}^2)YW\right\}$$
$$+\left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[(1-Y)(1-Z)-W(1-X)-X(1-2Y)]+(y_{\nu2}^2-y_{\nu3}^2)Y(1-Y-W)\right\}$$
$$+\left(\frac{y_{\nu2}^2+y_{\nu1}^2}{y_{\nu2}^2-y_{\nu1}^2}\right)\left\{(y_e^2-y_\tau^2)XY+\frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}$$
$$+\left(\frac{y_{\nu2}^2+y_{\nu3}^2}{y_{\nu2}^2-y_{\nu3}^2}\right)\left\{(y_e^2-y_\tau^2)Y(1-X-Y)+\frac{(y_\mu^2-y_\tau^2)}{2}[W(1-X-2Y)+X-(1-Z)(1-Y)]\right\}\Bigg]$$

$$\frac{dZ}{dt} = -\frac{3}{(4\pi)^2}\Bigg[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)XZ+\frac{(y_{\nu3}^2-y_{\nu2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}$$
$$+\left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)Z(1-X-Z)+\frac{(y_{\nu2}^2-y_{\nu3}^2)}{2}[W(1-X-2Z)+X-(1-Y)(1-Z)]\right\}$$
$$+\left(\frac{y_{\nu1}^2+y_{\nu2}^2}{y_{\nu1}^2-y_{\nu2}^2}\right)\left\{\frac{(y_e^2-y_\tau^2)}{2}[(1-Y)(1-Z)-X-W(1-X)]+(y_\mu^2-y_\tau^2)ZW\right\}$$
$$+\left(\frac{y_{\nu1}^2+y_{\nu3}^2}{y_{\nu1}^2-y_{\nu3}^2}\right)\left\{\frac{(y_\tau^2-y_e^2)}{2}[(1-Z)(1-Y)-W(1-X)-X(1-2Z)]+(y_\mu^2-y_\tau^2)Z(1-Z-W)\right\}\Bigg]$$

$$\frac{dW}{dt} = -\frac{3}{(4\pi)^2}\Bigg[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu2}^2-y_{\nu3}^2)WY+\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}$$
$$+\left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu2}^2-y_{\nu3}^2)W(1-Y-W)+\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Z)]\right\}$$
$$+\left(\frac{y_{\nu2}^2+y_{\nu1}^2}{y_{\nu2}^2-y_{\nu1}^2}\right)\left\{(y_\mu^2-y_\tau^2)WZ+\frac{(y_\tau^2-y_e^2)}{2}[(1-X)W+X-(1-Y)(1-Z)]\right\}$$
$$+\left(\frac{y_{\nu2}^2+y_{\nu3}^2}{y_{\nu2}^2-y_{\nu3}^2}\right)\left\{(y_\mu^2-y_\tau^2)W(1-Z-W)+\frac{(y_\tau^2-y_e^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Y)]\right\}\Bigg].$$