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Modified cosmology through Barrow entropy and its thermodynamic implications

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Outline

- The standard model of cosmology
- Holographic dark energy models
- Barrow Holographic dark energy
- Analytical formulation for Hubble parameter
- Observational constraints
- Evolution of cosmological parameters
- Thermodynamics of the Barrow entropic universe
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- References

The standard model of cosmology

- Edwin Hubble, (1929) - Expanding Universe.
- Perlmutter et.al - High-redshift Supernova Search Team & Riess et.al - Supernova Cosmology Project Team (1988) [1, 2]

$$\Downarrow$$

The current universe is accelerating

- Thus the accelerated evolution,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \Longrightarrow \quad P < -\frac{\rho}{3} \tag{1}$$

- Dark energy (DE) - An exotic component, causes accelerated expansion.
- ΛCDM - The standard model of cosmology - Cosmological constant (Λ) as dark energy.
- Most accepted model and consistent with the observational data.
- Shortcomings : ⋆ Cosmological constant problem. ⋆ Coincidence problem.
- Possibilities of proposing new model,

$\rightsquigarrow$ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, Modified gravity theories.

$\rightsquigarrow$ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, Dynamical DE models.

- Holographic Dark energy models $\rightarrow$ based on the idea of Holographic principle in cosmology [3]

  "The degrees of freedom of a spatial region reside not in the interior but on the surface of the region"

- *Entropy (or energy) of region of spacetime is less than or equal to entropy (or energy) of a black hole of same size, [4]*

$$L^3\Lambda^4 \leq LM_p^2 \qquad L^3\Lambda^3 \leq S_{BH}^{3/4}. \tag{2}$$

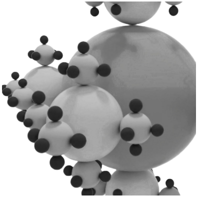
- Holographic dark energy (HDE) density [5],

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}$$

Barrow holographic dark energy model

- Barrow entropy relation,[6]

$$S = \left(\frac{A}{A_0}\right)^{1+\Delta/2} \qquad where, 0 \leq \Delta \leq 1$$



Fractal structure

- Barrow holographic dark energy (BHDE) density

$$\rho_\Lambda = 3CL^{\Delta-2} \tag{3}$$

- IR cut-off for the HDE model proposed by Granda and Oliveros(GO) as,[7]

$$L^{-2} = (\alpha_1 H^2 + \beta_1 \dot{H}) \tag{4}$$

- BHDE density with GO length scale as IR cut-off

$$\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H})^{\frac{2-\Delta}{2}} \tag{5}$$

- A flat homogeneous and isotropic universe evolves as,

$$3H^2 = \rho_m + \rho_\Lambda \tag{6}$$

- The dark sectors are assumed to be interacting

$$Q = 3bH\rho_m$$

- Q determines the energy flow,

$Q < 0$ - dark matter $\rightarrow$ BHDE $\qquad$ $Q > 0$ - BHDE $\rightarrow$ dark matter

- conservation equation with interacting term takes the differential form,

$$\frac{d\Omega_\Lambda}{dx} = -3b\Omega_m \quad ; \quad \frac{d\Omega_m}{dx} = -3(1-b)\Omega_m. \tag{7}$$

- Evolution equation corresponding to the weighted Hubble parameter $\bar{H} = \frac{H}{H_0}$,$(x = lna)$

$$\bar{H}^2 = \frac{\Omega_{m_0}}{1-b} e^{-3(1-b)x} - \frac{\Gamma_1}{3} e^{-3x} + \Gamma_2 \tag{8}$$

Γ_1 and Γ_2 are constant coefficients

- $x \to -\infty$,

$$\bar{H}^2 \to \left(\frac{\Omega_{m_0}}{1-b}a^b - \frac{\Gamma_1}{3}\right) a^{-3} \qquad \textit{(matter dominated decelerating phase)} \tag{9}$$

- $x \to +\infty$,

$$\bar{H}^2 \to \Gamma_2 \qquad \textit{(De-Sitter phase)} \tag{10}$$

- Reduces to ΛCDM-like behaviour, when $b = 0$

$$\bar{H}^2 = \bar{\Omega}_{m_0} e^{-3x} + 1 - \bar{\Omega}_{m_0} \tag{11}$$

- The effective mass density parameter for non-relativistic matter, $\bar{\Omega}_{m_0}$ and the constant $1 - \bar{\Omega}_{m_0}$ takes the form,

$$\bar{\Omega}_{m_0} = \frac{2\alpha}{3\beta}\left(1 - \frac{(\Omega_{\Lambda_0} H_0^{\Delta})^{\frac{2}{2-\Delta}}}{\alpha}\right) \tag{12}$$

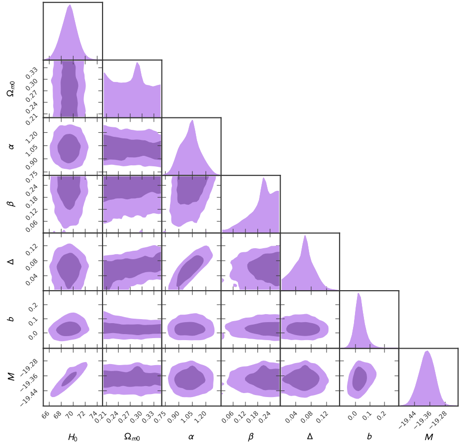
$$1 - \bar{\Omega}_{m_0} = \bar{\Omega}_{\Lambda_0} \tag{13}$$

The model approaches to the standard model only when ignore the interaction.

Observational constraints

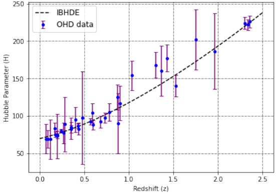
Table 1. Observational constraints on model parameters using OHD36 + SN Ia dataset.

H_0 kms^{-1} Mpc^{-1}	Ωm_0	α	β	Δ	b
$69.256^{+1.228}_{-1.228}$	$0.281^{+0.050}_{-0.050}$	$1.030^{+0.116}_{-0.116}$	$0.211^{+0.063}_{-0.063}$	$0.063^{+0.029}_{-0.029}$	$0.026^{+0.030}_{-0.030}$

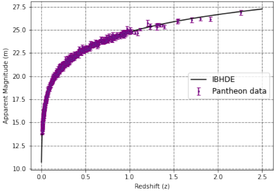


Corner plot of two-dimensional contours with confidence level and marginalized posterior distributions of model parameters for the combined dataset *OHD*36 + *SNIa*

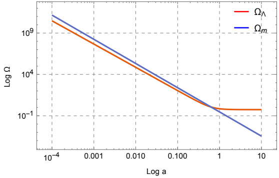
Evolution of cosmological parameters

- 

- Evolution of Hubble parameter for IBHDE model with best estimated model parameters from SN Ia+OHD36 combined dataset and Observational Hubble data.

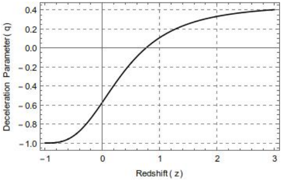
- 

- Comparison plot of apparent magnitude m(z) for IBHDE model with the best estimated model parameters from SN Ia+OHD36 combined dataset and Observational supernovae data.



- Evolution of density parameter corresponds to non-relativistic matter and IBHDE in logarithmic scale.

$$\Omega_m = \Omega_{m_0} e^{-3(1-b)x}, \quad \Omega_\Lambda = \frac{1}{H_0^2}\left(\alpha H^2 + \frac{\beta}{2}\frac{dH^2}{dx}\right)^{\frac{2-\Delta}{2}}$$



- Behaviour of q parameter vs z with transition redshift, $z_T = 0.767$
- Present value $q_0 = -1 + \frac{1}{\beta}[\alpha - ((1 - \Omega_{m_0})H_0^{\Delta})^{\frac{2}{2-\Delta}}] = -0.572$

- Age of the universe,

$$t_0 - t_B = \int_0^1 \frac{1}{aH(a)} da$$

- 13.958 Gyr

Thermodynamics of the Barrow entropic universe

- For entropy evolves and to be maximized [8, 9],

$$\dot{S} \geq 0 \quad \text{for always} \quad ; \quad \ddot{S} < 0 \quad \text{for atleast later time of evolution.}$$

- Entropy corresponds to non-relativistic matter, $P_m = 0$, $\rho = \rho_m$.

$$S_m = 8\pi^2 c^3 H_0^2 \frac{\Omega_{m_0} a^{-3(1-b)}}{H^4} \tag{14}$$

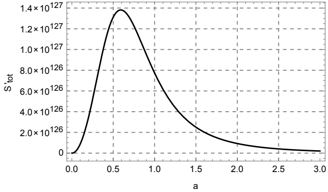
- Entropy corresponds to the horizon is assumed as the Barrow entropy,

$$S_h = \left(\frac{\pi c^5}{\hbar G H^2}\right)^{1+\Delta/2} k_B \tag{15}$$

- Derivative of total entropy with respect to x, where $x = lna$,

$$S'_{tot} = S'_m + S'_h \tag{16}$$

- Evolution of s'_{tot} vs a



- The total entropy is increasing during the evolution of the universe.

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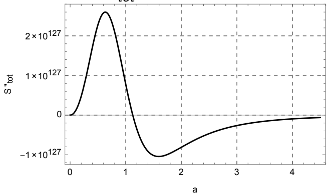
Generalised second law of thermodynamics is satisfied.

- Evolution of s''_{tot} vs a



- The second derivative of total entropy is satisfies the convexity condition.

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Entropy maximization holds.
entropy will get maximized at least for the asymptotic limit, thus entropy grows bounded.

Conclusion

- HDE model is one of the strong consideration among decaying models
- IBHDE model is constructed for a flat universe with interacting components and Barrow entropy in cooperate with the holographic principle with GO length scale as IR cut off.
- Asymptotic behaviour of the model is studied and is analogous to standard model once we ignore the coupling constant.
- The solution with best estimated value exactly fit with the observational data and shows the expansion history
- GSL is validated for the proposed IBHDE model with best fit model parameter.
- Entropy maximization also hold, implies the universe evolve towards a de Sitter epoch corresponding to maximum entropy.

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Thank you...!