Introduction to perturbative QCD

Giancarlo Ferrera

giancarlo.ferrera@mi.infn.it

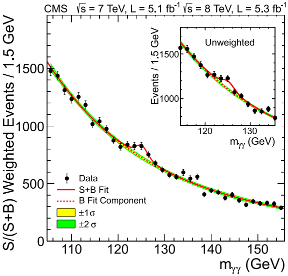
Università di Milano & INFN Milano

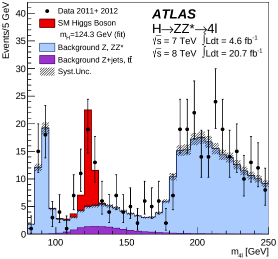
Saha Theory Workshop – SINP – Kolkata – 23/2/2016

Outline

1. PDFs
2. Partonic Hard Cross Section
3. Shower Monte Carlo
4. Jets
5. Conclusions

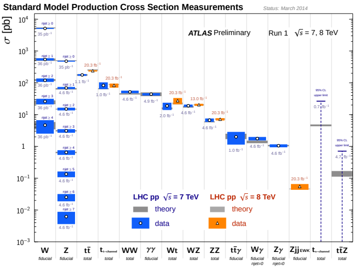
LHC key results: Higgs signal

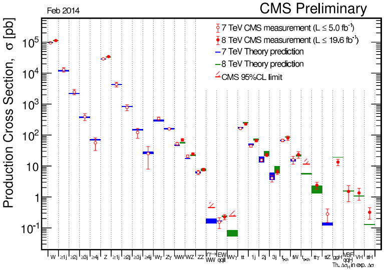




- Observation of the Higgs boson through sharp mass peak, $m_H \simeq 136 GeV$, in various channels.
- Its properties are consistent with the Standard Model (SM) predictions at the $\pm 20\%$ level: this accuracy can (must!) be further reduced with the help of more theoretical work.

LHC key results: and much more



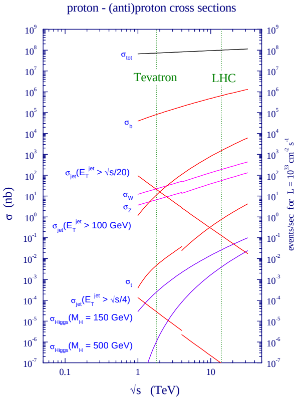


- Very good agreement between experimental results and SM theoretical predictions of the high-Q^2 processes

Motivations

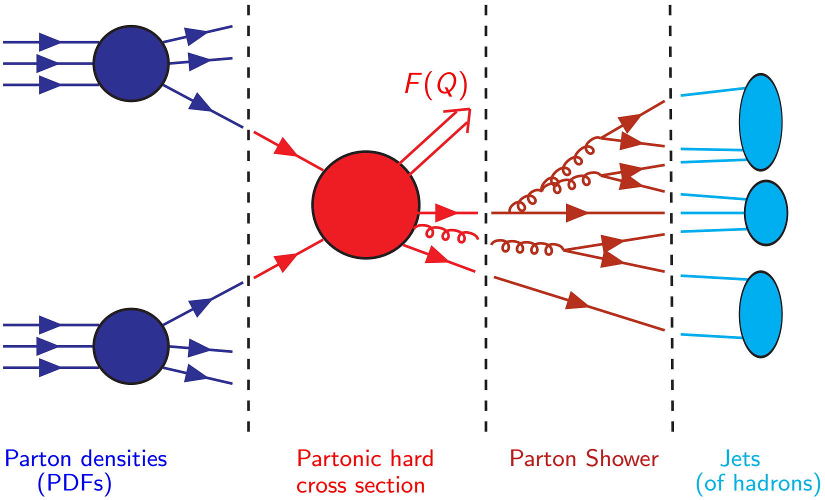
The LHC is a (large) hadron collider machine: all the interesting high-p_T reactions initiate by QCD hard scattering of partons.

To claim for new-physics signals a good control of the SM processes is necessary.

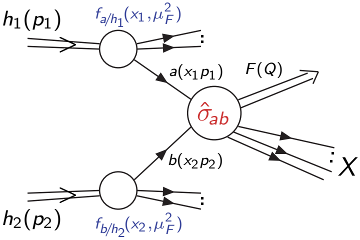


To fully exploit the information contained in the LHC experimental data, precise theoretical predictions of the SM cross sections are needed.

Theoretical predictions at the LHC



Theoretical predictions at the LHC



$$h_1(p_1) + h_2(p_2) \rightarrow F(Q) + X$$

The framework: **QCD factorization formula**

$$\sigma_{h_1 h_2}(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^p$$

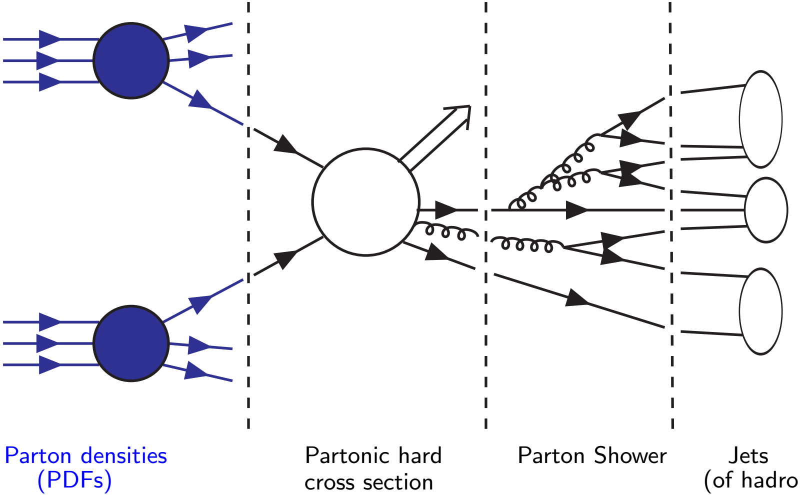
- $f_{a/h}(x, \mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim Q$. Measured from experiments at a given scale μ_0, Evolution to μ_F calculable in pQCD through DGLAP equation.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion i the strong coupling $\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda_{QCD}^2) \sim 0.1$ (for $Q = m_H, m_W, m_Z, m_t, p_T^{jet}, \cdots$).

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \alpha_S(\mu_R^2)\, \hat{\sigma}_{ab}^{(1)} + \alpha_S^2(\mu_R^2)\, \hat{\sigma}_{ab}^{(2)} + \mathcal{O}(\alpha_S^3)\,.$$

- $\left(\frac{\Lambda_{QCD}}{Q}\right)^p$ (with $p \geq 1$): Non perturbative power-corrections (higher-twist).

Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x, \mu_F^2)$

PDFs


Parton densities
(PDFs)
Partonic hard
cross section
Parton Shower
Jets
(of hadrons)

Fit of PDFs

- Method: typical parametrization of parton densities at input scale $\mu_0^2 \sim 1 \div 4 \; GeV^2$:

$$x\, f_a(x, \mu_0^2) \; = \; A_a \, x^{\lambda_a} \, (1-x)^{\eta_a} \, (1 \, + \, \epsilon_a \, \sqrt{x} \, + \, \gamma_a \, x + \cdots).$$

  Parameters constrained by imposing momentum sum rules: $\sum_a \int_0^1 dx \, x \, f_a(x, \mu_0^2) = 1$, then adjust parameters to fit data.

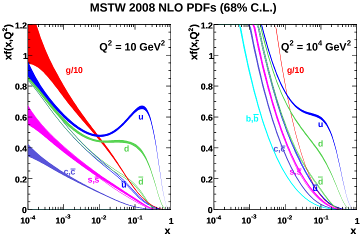
- Typical constraining process:
  - DIS (fixed target exp. and HERA): sensitive to quark densities.
  - Jet data (HERA and Tevatron): sensitive to high-x gluon density.
  - Drell-Yan (low energy and Tevatron data): sensitive to (anti-)quark densities.

- Evolution $\mu_0 \to \mu$ using DGLAP equations:

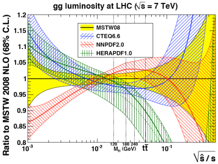
$$\frac{\partial f_a(x, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi) f_b(\xi, \mu^2)$$

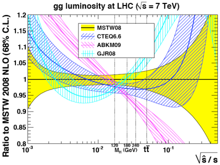
  AP kernels calculable in pQCD

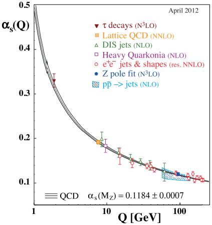
$$P_{ab}(z) = P_{ab}^{(0)}(z) + \frac{\alpha_S(\mu^2)}{2\pi} P_{ab}^{(1)}(z) + (\frac{\alpha_S(\mu^2)}{2\pi})^2 P_{ab}^{(2)}(z) + \cdots$$

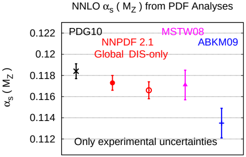


PDFs



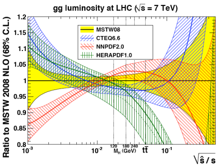


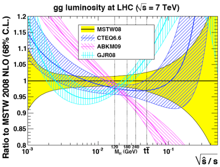


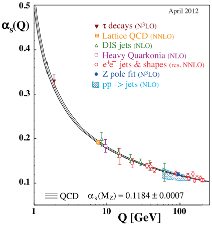


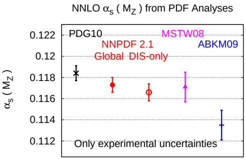
- Several PDFs sets available:
  MSTW, NNPDF, CTEQ/CT, GJR, ABKM, HERAPDF.
  MSTW: ∼ 3000 data pts., ∼ 50 free param.
  NNPDF: ∼ 3000 data pts., ∼ 250 free param.
- Differences among sets include:
  data set in the fit, parton parametrization, statistical treatment, perturbative accuracy (NLO,NNLO), value of α_S.
- PDFs sets can be combined using the "PDF4LHC recommendation" to obtain a central value and an estimate of the uncertainty.
- Simultaneous extraction of $\alpha_S(m_Z)$ from NNLO fits lead to some tension:
  World avg.'12 $\alpha_S(m_Z) = 0.1184 \pm 0.0007$
  ABKM11 $\alpha_S(m_Z) = 0.1135 \pm 0.0014$
  MSTW08 $\alpha_S(m_Z) = 0.1171 \pm 0.0014$
  NNPDF2.1 $\alpha_S(m_Z) = 0.1173 \pm 0.0011$
  JR09 $\alpha_S(m_Z) = 0.124 \pm 0.002$

PDFs



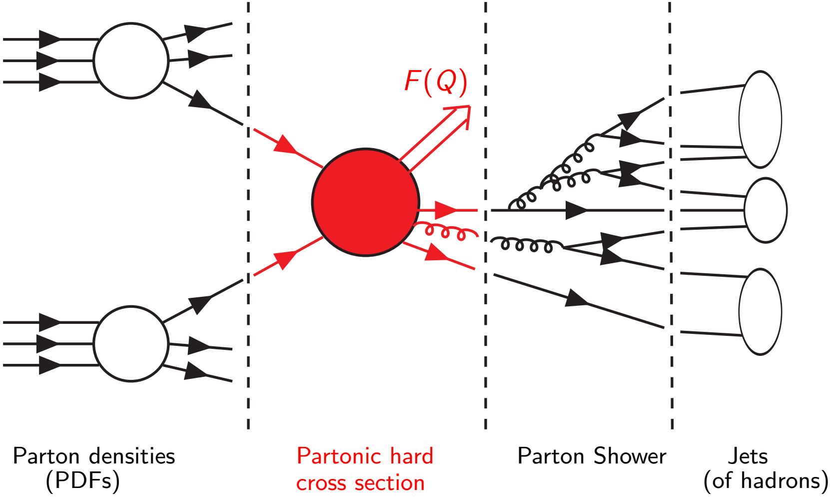






- Several PDFs sets available:
  MSTW, NNPDF, CTEQ/CT, GJR, ABKM, HERAPDF.
  MSTW: $\sim$ 3000 data pts., $\sim$ 50 free param.
  NNPDF: $\sim$ 3000 data pts., $\sim$ 250 free param.
- Differences among sets include:
  data set in the fit, parton parametrization, statistical treatment, perturbative accuracy (NLO,NNLO), value of α_S.
- PDFs sets can be combined using the "PDF4LHC recommendation" to obtain a central value and an estimate of the uncertainty.
- Simultaneous extraction of $\alpha_S(m_Z)$ from NNLO fits lead to some tension:
  World avg.'12 $\alpha_S(m_Z) = 0.1184 \pm 0.0007$
  ABKM11 $\alpha_S(m_Z) = 0.1135 \pm 0.0014$
  MSTW08 $\alpha_S(m_Z) = 0.1171 \pm 0.0014$
  NNPDF2.1 $\alpha_S(m_Z) = 0.1173 \pm 0.0011$
  JR09 $\alpha_S(m_Z) = 0.124 \pm 0.002$

Partonic Hard Cross Section



Higher orders: NLO

- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations, once the QCD amplitudes are available

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \; + \; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \; + \; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

Higher orders: NLO

- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \; + \; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \; + \; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

Higher orders: NLO

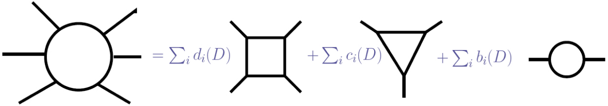
- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \quad + \quad \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \quad + \quad \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

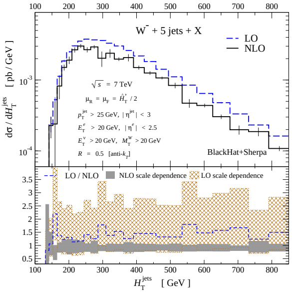
NLO: virtual amplitudes

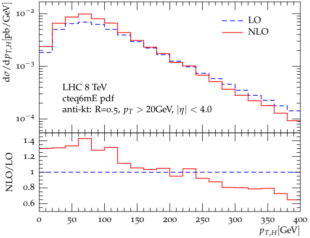
- The paradigm for the calculation of one-loop diagram is [Passarino,Veltman('79)].
- Any one-loop amplitude can be written as a linear sum of scalar box-, triangle-, bubble- and tadpole-integrals.



- Analytic results for these scalar integrals are known [Ellis,Zanderighi('08)].
- The traditional approach is not adequate when the number of external legs increase $2 \to 3, 4, 5, \ldots$ (factorial growth of diagrams).
- Recently advances in multi-leg one-loop amplitudes calculations thanks to
  - New semi-numerical methods based on on-shell recursion relations and unitarity: isolate coefficients by cutting propagators [Bern,Dixon,Dunbar, Kosower('94)],[Britto,Cachazo,Feng('04)].
  - Tensor integrals to scalar master integrals reduction performed numerically at the integrand level in a algorithmic way [Ossola,Papadopoulos, Pittau('06)].

NLO: automation





- NLO calculations are now highly automated. Virtual corrections can be combined with real corrections (based on CS or FKS subtraction formalism).
- HELAC-NLO [Bevilacqua,Czakon,Garzelli, van Hameren,Kardos,Papadopoulos, Pittau,Worek].
- BlackHat+Sherpa [Berger,Bern,Dixon,Cordero, Forde,Gleisberg,Ita,Kosower,Maitre].
- MadLoop+MadFKS [Hirschi,Frederix,Frixione, Garzelli,Maltoni,Pittau].
- Rocket [Ellis,Giele,Kunszt,Melnikov,Zanderighi].
- GoSam [Cullen,Greiner,Heinrich,Luisoni, Mastrolia,Ossola,Reiter,Tramontano].
- OpenLoops [Cascioli,Maierhöfer,Pozzorini].
- NLO dedicated calculations also available: MCFM, VBFNLO,NLOJet++,. . . .

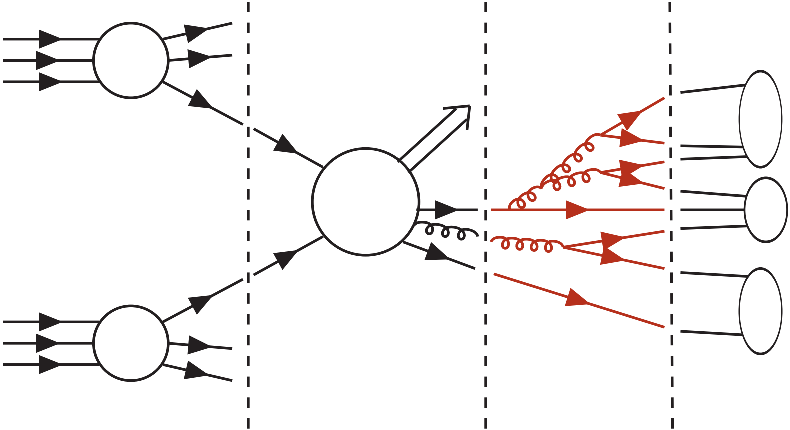
Higher orders: fully-exclusive NNLO calculation

- NNLO corrections allow a good control of theoretical uncertainties: important to provide exclusive prediction to implement the experimental cuts.
- NNLO computations in hadronic collisions cumbersome, anyway several calculations with few alernative methods performed:
  - Sector decomposition: [Binoth,Heinrich('00)], [Anastasiou,Melnikov,Petriello('04)]
  - q_T-subtraction: [Catani,Grazzini('07)]
  - Antenna subtraction: [Gehrmann,Gehrmann-De Ridder,Glover('05)]
  - Non-linear mapping: [Anastasiou,Herzog,Lazopoulos('10)]
  - Sector-improved subtraction: [Czakon('10)]

Higher orders: fully-exclusive NNLO calculation

- NNLO corrections allow a good control of theoretical uncertainties: important to provide exclusive prediction to implement the experimental cuts.
- NNLO computations in hadronic collisions cumbersome, anyway several calculations with few alernative methods performed:
  - Sector decomposition: [Binoth,Heinrich('00)], [Anastasiou,Melnikov,Petriello('04)]
  - q_T-subtraction: [Catani,Grazzini('07)]
  - Antenna subtraction: [Gehrmann,Gehrmann-De Ridder,Glover('05)]
  - Non-linear mapping: [Anastasiou,Herzog,Lazopoulos('10)]
  - Sector-improved subtraction: [Czakon('10)]

Shower Monte Carlo

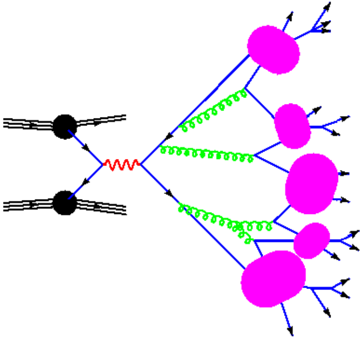


Parton densities (PDFs) | Partonic hard cross section | Parton Shower | Jets (of hadrons)

Shower Monte Carlo

Complete description of a hadron scattering event.

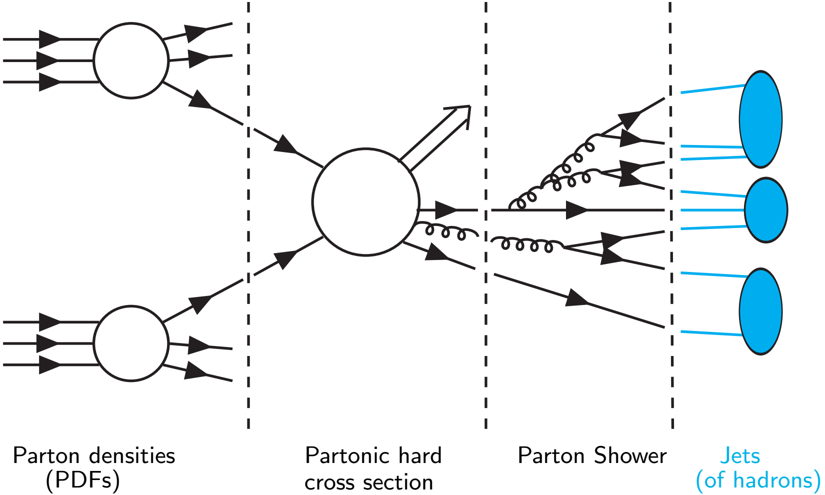
- QCD parton shower (PS): Starting from LO QCD, inclusion of dominant collinear and soft-gluon emissions (by angular ordering thanks to color coherence) to all order in a approximate way as a Markov process (probabilistic picture).
- No analytic solution but simple iterative structure of coherent parton branching.
- QCD accuracy analogous to LL (plus part of NLL) Sudakov resummation.
- Implemented in numerical Monte Carlo programs.



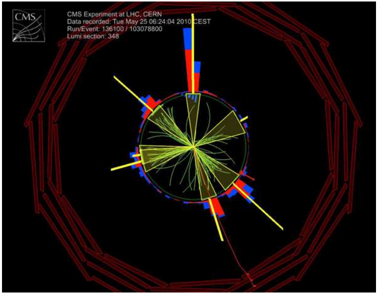
Scheme of QCD Parton Shower and hadronization from final states.

- QCD parton cascade matched with hadronization model for conversion of partons into hadrons (and model for resonance decay) ⇒ QCD event generators (Herwig/PYTHIA/Sherpa).
- Possible to consistently combine Parton Shower with high multiplicity tree-level matrix elements: CKKW/MLM matching.

Jets


Parton densities
(PDFs)
Partonic hard
cross section
Parton Shower
Jets
(of hadrons)

Jets



- QCD matrix elements enhanced in the collinear regions, this give rise to Jets: collimated spray of high-energy hadrons (ubiquitous at the LHC).
- Jet clustering algorithms (k_T,anti-k_T,C/A): iterative procedure to cluster particles in experimental analysis and in theoretical calculations (infrared/coll. safe).
  (i) Define a “distance” d_{ij} between particles.
  (ii) Merge particles with minimum d_{ij} until a fixed resolution d_{cut} is reached.

Conclusions

- This talk is an overview on some selected topic in pQCD.
- Main message: pQCD is always involved in the description of the LHC processes.
  To fully exploit the information contained in the LHC data precise pQCD predictions are needed.