Overview on q_T resummation and q_T subtraction formalism

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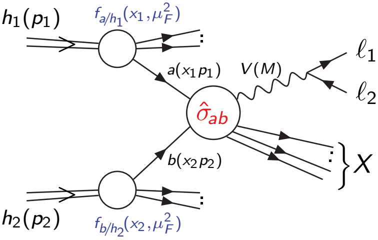
Drell–Yan q_T distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\to\; V(M) + X \;\to\; \ell_1 + \ell_2 + X}$$

where $\quad V = \gamma^*, Z^0, W^\pm \quad$ and $\quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$

pQCD *collinear* factorization formula ($M \gg \Lambda_{QCD}$):

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \overset{q_T \ll M}{\sim} 1 + \alpha_S\left[c_{12}\ln^2\frac{M^2}{q_T^2} + c_{11}\ln\frac{M^2}{q_T^2} + c_{10}\right] + \cdots$$

$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} \overset{q_T \to 0}{=} 0$$

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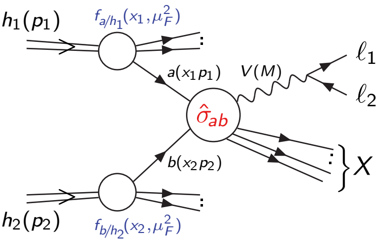

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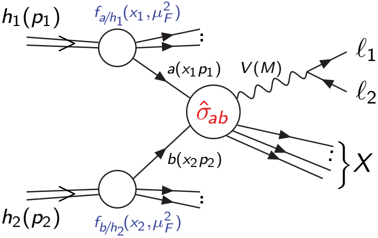

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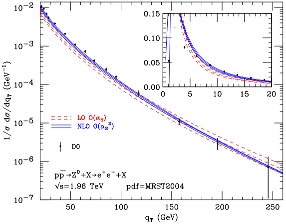

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Idea of (analytic) resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\alpha_S^2 L^2$	$\alpha_S^2 L$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\alpha_S^n L^{2n-2}$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	next-to-dominant logs	$\cdots$	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.
$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$
- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

$$\int d^2\mathbf{q_T}\ \exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta\left(\mathbf{q_T}-\sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

State of the art: q_T resummation

- Method to resum large q_T logarithms is known [Dokshitzer,Diakonov,Troian('78)], [Parisi,Petronzio('79)],[Kodaira,Trentadue('82)],[Collins,Soper,Sterman('85)], [Altarelli et al.('84)],[Catani,d'Emilio,Trentadue('88)],[Catani,de Florian, Grazzini('01)],[Catani,Grazzini('10)],[Catani,Grazzini,Torre('14)]
- Various phenomenological studies[ResBos:Balasz,Yuan,Nadolsky et al.('97,'02)], [Ellis et al.('97)],[Kulesza et al.('02)],[Guzzi,Nadolsky,Wang('13)].
- Results for q_T resummation in the framework of Effective Theories [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)],[Mantry,Petriello('10)],[Becher, Neubert('10)],[Echevarria,Idilbi,Scimemi('11)].
- Studies within transverse-momentum dependent (TMD) factorization and TMD parton densities[D'Alesio,Murgia('04)],[Roger,Mulders('10)],[Collins('11)], [D'Alesio,Echevarria,Melis,Scimemi('14)],[Ceccopieri,Trentadue('14)].
- Effective q_T-resummation obtained with Parton Shower algorithms POWHEG/MC@NLO [Barze et al.('12,'13)], [Hoeche,Li,Prestel('14)],[Karlberg,Re,Zanderighi('14)].

$\mathbf{q_T}$ resummation: $q\bar{q}$-annihilation processes

Hadroproduction of a system F of *colourless* particles initiated at Born level by $q_f \bar{q}_{f'} \to F$.

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$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

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$$\frac{d\sigma_F^{(res)}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q}}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_q(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\;\;,$$

[Collins,Soper,Sterman('85)],

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $x_{1,2} = \frac{M}{\sqrt{s}}\,e^{\pm y}$, $L \equiv \ln Mb$ [Catani,de Florian,Grazzini('01)]

$$S_q(M,b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_q(\alpha_S(q^2))\;\ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2))\right]\right\}\;.$$

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2} = H_q^F(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S(M^2))\;\;C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;\;C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\;\;,$$

$$A_q(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)},\quad B_q(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)},$$

$$H_q^F(\alpha_S) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)},\qquad C_{qa}(z;\alpha_S) = \delta_{qa}\;\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\,.$$

LL$(\sim\alpha_S^n L^{n+1})$: $A_q^{(1)}$; NLL$(\sim\alpha_S^n L^n)$: $A_q^{(2)}, B_q^{(1)}, H_q^{F\,(1)}, C_{qa}^{(1)}$; NNLL$(\sim\alpha_S^n L^{n-1})$: $A_q^{(3)}, B_q^{(2)}, H_q^{F\,(2)}, C_{qa}^{(2)}$

$\mathbf{q_T}$ resummation: $q\bar{q}$-annihilation processes

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$$\times \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2, b_0^2/b^2) \;,$$

[Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]

$b_0 = 2e^{-\gamma_E}$ $(\gamma_E = 0.57\ldots)$, $x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$, $L \equiv \ln Mb$

$$S_q(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_q(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + B_q(\alpha_S(q^2))\right]\right\} \;.$$

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2} = H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2))\;\; C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;\; C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2)) \;,$$

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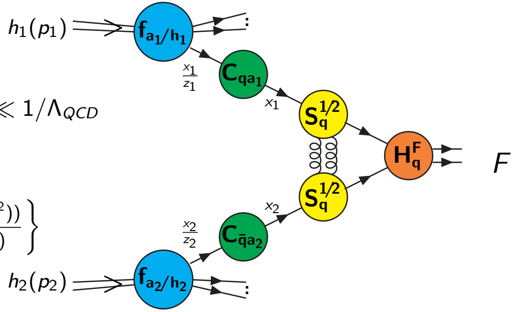
Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times \ \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right] H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{(2\pi)^2}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

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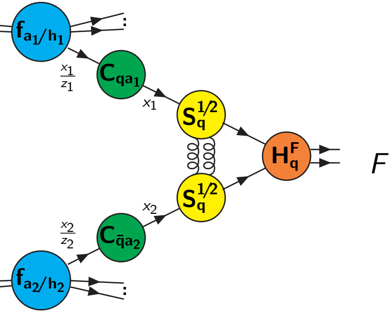

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$\mathbf{q_T}$ resummation: gluon fusion processes

In processes initiated at Born level by the gluon fusion channel ($gg \to F$), collinear radiation from gluons leads to spin and azimuthal correlations [Catani,Grazzini('11)].

$$\left[H^F C_1 C_2\right]_{gg;a_1 a_2} = H^F_{g;\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2))$$
$$\times \; C^{\mu_1\nu_1}_{ga_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \, C^{\mu_2\nu_2}_{ga_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2)) \,.$$

where $\quad H_g^{F\,\mu_1\nu_1,\mu_2\nu_2}(\alpha_S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2}$,

$$C^{\mu\nu}_{ga}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\,\mu\nu}(p_1, p_2)\; C_{ga}(z; \alpha_S) + D^{\,\mu\,\nu}(p_1, p_2; \mathbf{b})\; G_{ga}(z; \alpha_S) \;\; ,$$

$$d^{\,\mu\nu}(p_1, p_2) = -\, g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \;\; , \quad D^{\,\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\,\mu\nu}(p_1, p_2) - 2\, \frac{b^\mu\, b^\nu}{\mathbf{b}^2} \;\; ,$$

$$C_{ga}(z; \alpha_S) = \delta_{ga}\;\; \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C^{(n)}_{ga}(z)\,, \quad G_{ga}(z; \alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n G^{(n)}_{ga}(z) \;\; .$$

- Unlike $q\bar{q}$ annih. $\left[H^F C_1 C_2\right]$ does depend on the azimuthal angle $\phi(\mathbf{b})$, this leads to azimuthal correlations with respect to the azimuthal angle $\phi(\mathbf{q_T})$ (consistent with [Mulders,Rodrigues('00)],[Henneman et al.('02)]).
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Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani,de Florian,Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

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- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1\,,\ \ H_g^H(\alpha_S) \equiv 1\,.$
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Hard-collinear coefficients at NNLO

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- Explicit NNLO *analytic* calculations of the q_T cross section (at small-q_T): *(i)* SM Higgs boson production [Catani,Grazzini('07,'12)] and *(ii)* DY process [Catani,Cieri,de Florian,G.F.,Grazzini('09,'12)].
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- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
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renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon) ,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

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$$H_q^{\gamma\gamma(1)} = \frac{C_F}{2}\left\{(\pi^2-7) + \frac{\left((1-v)^2+1\right)\ln^2(1-v) + v(v+2)\ln(1-v) + (v^2+1)\ln^2 v + (1-v)(3-v)\ln v}{(1-v)^2+v^2}\right\} .$$

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Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$.
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2b^2) \to \ln(Q^2b^2) + \ln(M^2/Q^2)$$

- Perturbative unitarity constraint:

$$\ln(Q^2b^2) \to \widetilde{L} \equiv \ln(Q^2b^2+1)$$

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- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2 b^2\right) \to \ln\left(Q^2 b^2\right) + \ln\left(M^2/Q^2\right)$$

- Perturbative unitarity constraint:

$$\ln\left(Q^2 b^2\right) \to \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right)$$

- avoids unjustified higher-order contributions in the small-b region.
- recover *exactly* the total cross-section (upon integration on q_T)

The q_T resummation formalism

Distinctive features of the formalism [Catani at al ('01)], [Bozzi et al.('03,'06)]:

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HqT/DYqT: q_T-resummation at NNLL

- We have performed the resummation up to NNLL(NNLO)+NLO. It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders (i.e. $\mathcal{O}(\alpha_S^n L^{n-1})$ in the exponent);
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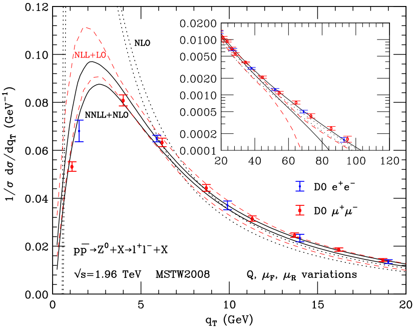
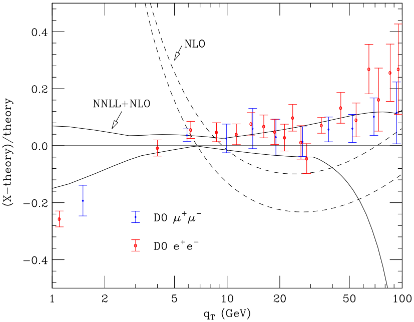
DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:
  $\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
- Significant reduction of scale dependence from NLL to NNLL for all q_T.
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
  The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

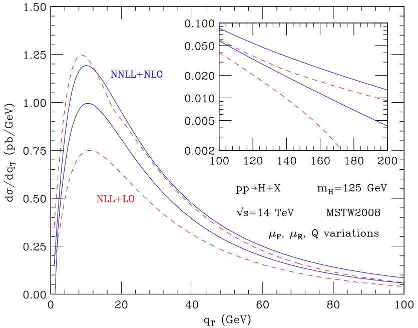
DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.

HqT results: q_T spectrum of H boson at the LHC $\sqrt{s} = 14$ TeV



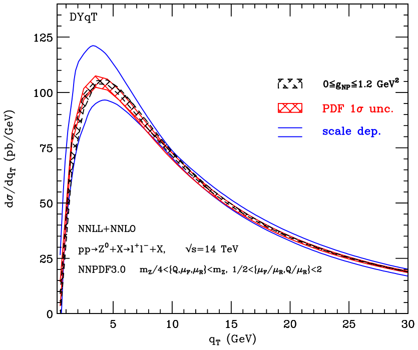
Higgs q_T spectrum for $m_H = 125\ GeV$ at LHC.

- Uncertainty bands obtained as before:

$1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$

- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all q_T.
- Good convergence of resummed results: NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).

PDFs uncertainties and NP effects: DYqT



NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14\ TeV$. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\ GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\ S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T. Non trivial interplay of perturbative and NP effects (higher-order contributions at small q_T can be mimicked by NP effects).
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3GeV$ (i.e. below the peak).

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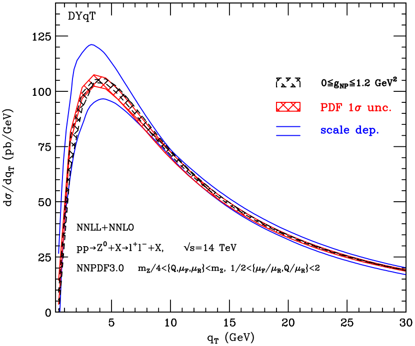

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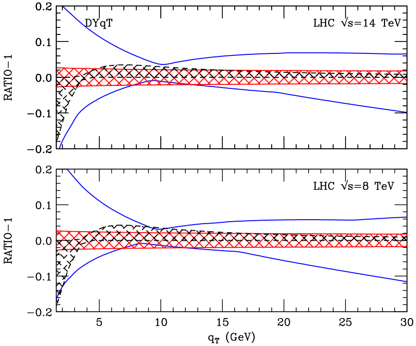

NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14$ TeV (up) $\sqrt{s} = 8$ TeV (down). Perturbative scale dependence, PDF uncertainties and impact of NP effects normalized to central NNLL+NNLO prediction.

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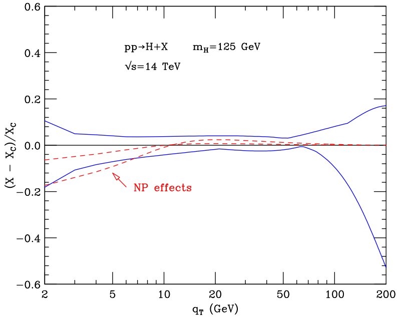
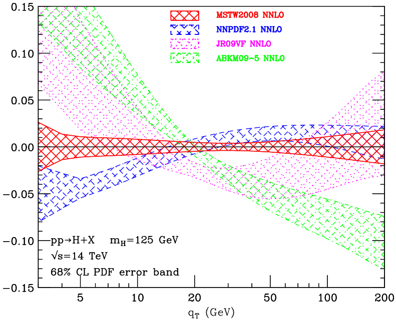
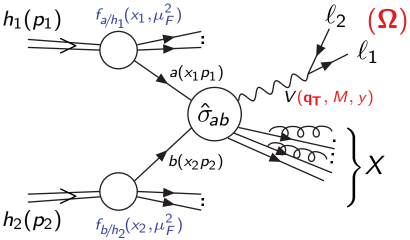


Uncertainties in the normalized q_T spectrum of the Higgs boson at the LHC. NNLL+NLO uncertainty bands (solid) compared to an estimate of NP effects with smearing parameter $g_{NP} = 1.67 - 5.64\ GeV^2$ (dashed).



The q_T spectrum has a strong sensitivity from collinear PDFs (especially from the gluon density).

DYRes/HRes: q_T resummation and decay



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- Full kinematical dependence on decay products: possible to apply cuts.
- To construct the "finite" part we rely on the fully-differential NNLO result from the codes DYNNLO/HNNLO [Catani,Cieri,de Florian,G.F., Grazzini('09)], [Catani,Grazzini('07)].
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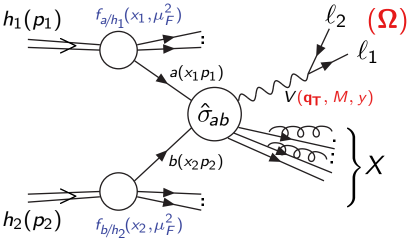

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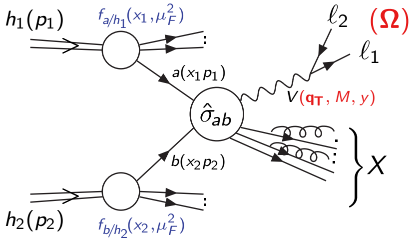

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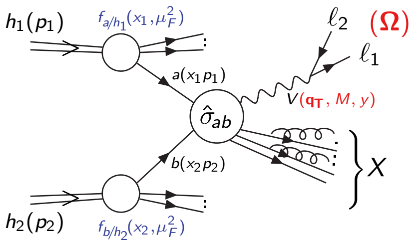

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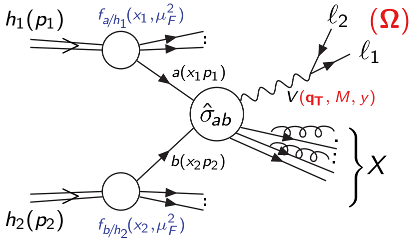

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$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable $\mathbf{\Omega}$ is

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) \;\;,\;\; \text{with} \;\; \int d\mathbf{\Omega}\, F(\mathbf{q_T}/M; \mathbf{\Omega}) = 1\;.$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

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  - After integration over leptonic variable $\mathbf{\Omega}$ the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

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  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

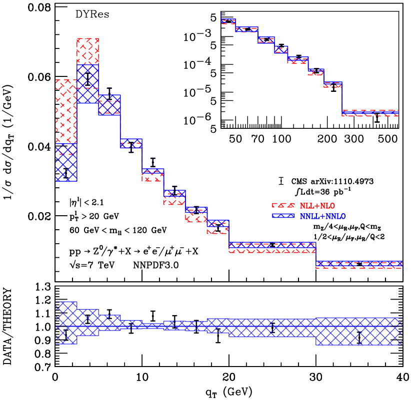
$$F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) = F(\mathbf{0}/M; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T}/M) \;\; ,$$

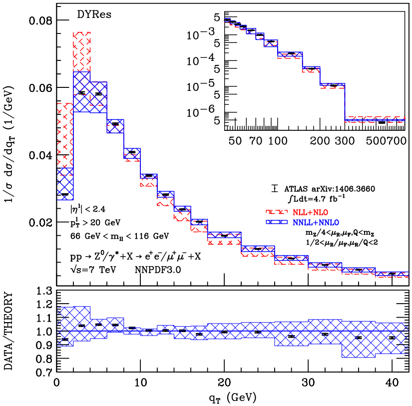
  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T}/M)$ ambiguity start at $\mathcal{O}(\alpha_S^3)$ $\left(\mathcal{O}(\alpha_S^2)\right)$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable $\mathbf{\Omega}$ the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

DYRes results: q_T spectrum of Z boson at the LHC

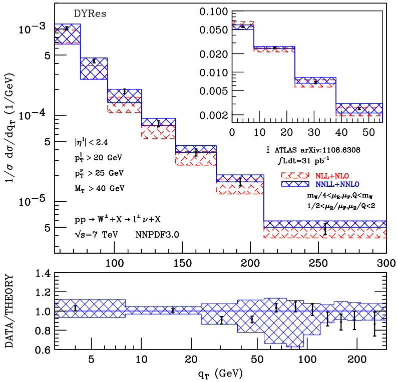




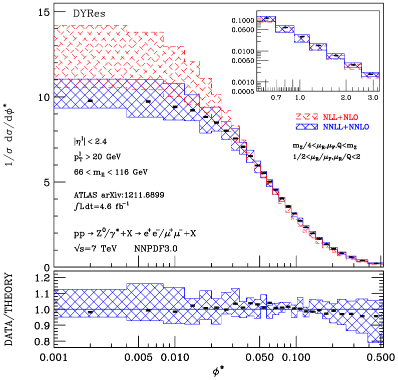
NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with CMS (left) and ATLAS (right) data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

Program performances: for high statistic runs (i.e. few per mille accuracy on cross sections) *on a single CPU*: $\sim$ 1day at full NLL, $\sim$ 3days at full NNLL.

DYRes results: q_T spectrum of W and ϕ^* spectrum of Z boson at the LHC

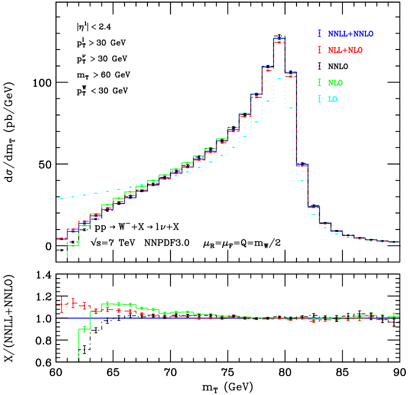


NLL+NLO and NNLL+NNLO bands for $W^\pm$ q_T spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.



NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

DYRes **results:** lepton kinematical distributions from W decay



Effect of q_T resummation on the transverse mass (m_T) for W^- production at the LHC. NLL+NLO and NNLL+NNLO results compared with LO, NLO and NNLO results. Lower panel: ratio between various results and NNLL+NNLO result.

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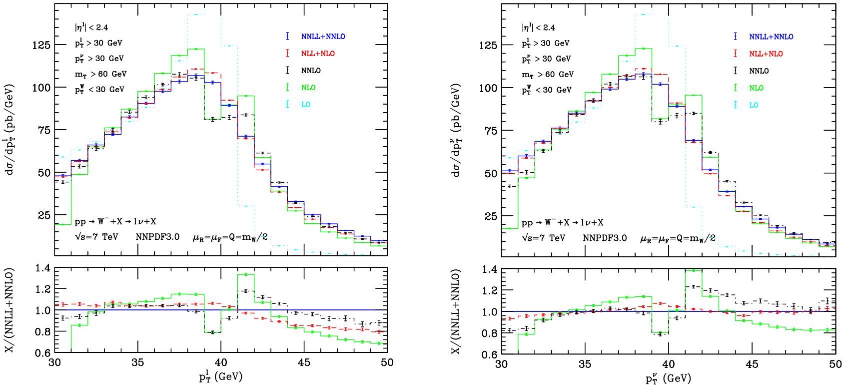
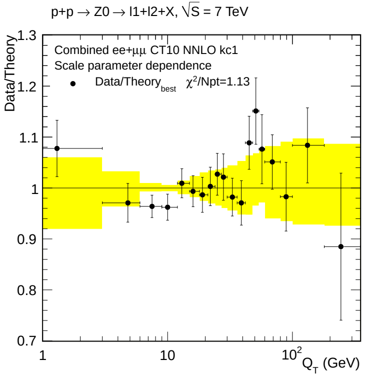
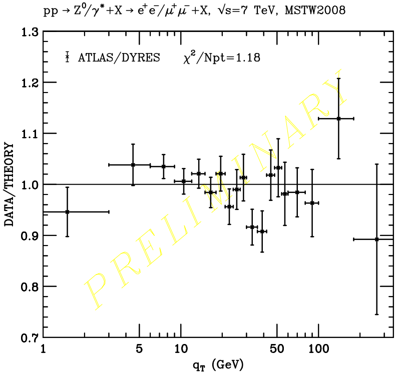


Effect of q_T resummation on lepton p_T (left) and missing p_T distribution for W^- production at the LHC. NLL+NLO and NNLL+NNLO results compared with LO, NLO and NNLO results.

Lower panel: ratio between various results and NNLL+NNLO result.

PDFs uncertainties and NP effects: DYRes

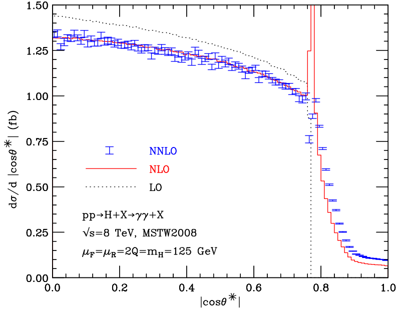




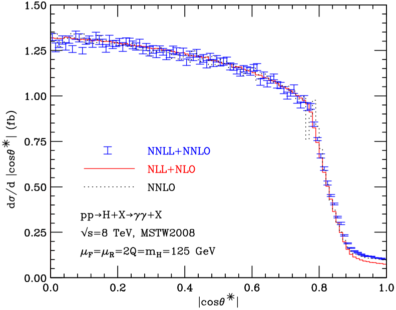
ATLAS ('11) data for the Z q_T spectrum compared with ResBos predictions with a Non Perturbative smearing parameter $g_{NP} = 1.1\, GeV^2$ [Guzzi,Nadolsky,Wang('13)].

ATLAS ('11) data for the Z q_T spectrum compared with DYRES predictions without Non Perturbative smearing ($g_{NP} = 0$).

HRes results: q_T-resummation with H boson decay

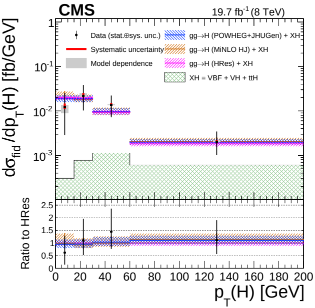


Fixed order results for $|\cos\theta^*| = \sqrt{1-4p_{T,\gamma}^2/m_H^2}$ distribution at the LHC.

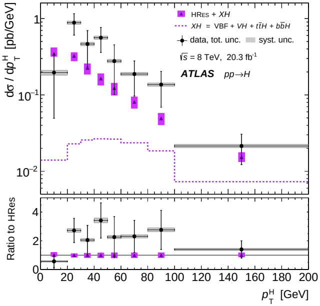


Resummed results for $|\cos\theta^*| = \sqrt{1-4p_{T,\gamma}^2/m_H^2}$ distribution at the LHC.

HRes results: q_T-resummation with H boson decay



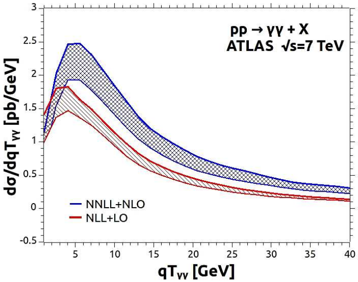
Higgs boson p_T spectrum measured by CMS Coll. ($H \to ZZ \to 4l$ decay) compared with HRes prediction.



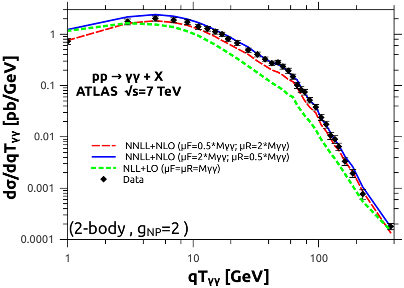
Higgs boson p_T spectrum measured by ATLAS Coll. (combining $H \to \gamma\gamma$ and $H \to ZZ \to 4l$ decays) compared with HRes prediction.

2γRes results: q_T-resummation for diphoton production.

[Cieri,Coradeschi,de Florian('15)].



Diphoton q_T spectrum at NNLL+NLO and NLL+LO for $q_T < 40$ GeV.



Full diphoton q_T spectrum at NNLL+NLO and NLL+LO compared with data from ATLAS.

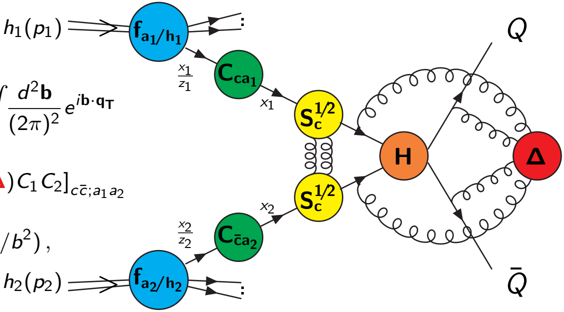
$\mathbf{q_T}$ resummation for heavy-quark hadroproduction

[Catani,Grazzini,Torre('14)]

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\boldsymbol{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c}}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\cdot\mathbf{q_T}}$$

$$\times\; S_c(M,b)\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\,[(\mathbf{H}\boldsymbol{\Delta})C_1C_2]_{c\bar{c};a_1a_2}$$

$$\times\; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)\,,$$



- Main difference with colourless case: soft factor (colour matrix) $\boldsymbol{\Delta}(\mathbf{b}, M; \boldsymbol{\Omega})$ which embodies soft (wide-angle) emissions from $Q\bar{Q}$ and from initial/final-state interferences (no collinear emission from heavy-quarks). Its contribution starts at NLL.
- Soft radiation produce colour-dependent azimuthal correlations at small-q_T entangled with the azimuthal dependence due to gluonic collinear radiation.
- Explicit results for coefficients obtained up NLO and NNLL accuracy.
- Soft-factor $\boldsymbol{\Delta}(\mathbf{b}, M; \boldsymbol{\Omega})$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins,Qiu('07)].

Higher orders: NLO and NNLO

- Calculations at LO give the order of magnitude of cross sections, at NLO reliable predictions, at NNLO reliable estimate of uncertainty.
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- Subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission. Fully formalized at NLO [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations (once the QCD amplitudes are available). Fully general formalism to perform NNLO calculations still lacking.

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \quad + \quad \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \quad + \quad \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

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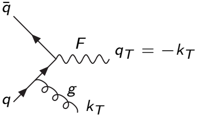
q_T-subtraction method at NNLO [Catani,Grazzini('07)]

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$

F is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,...) [Catani,Grazzini('07)].



- Key point I: at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO} \;,$$

  for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method (e.g. with dipole formalism [Catani,Seymour('98)] as in MCFM).

- The only remaining NNLO singularities are associated with the $q_T \to 0$ limit.
- Key point II: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{F}_{N^nLO} \overset{q_T \to 0}{\longrightarrow} d\sigma^{F}_{LO} \otimes \Sigma(q_T/M) dq_T^2 = d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

$$d\sigma^{CT} \overset{q_T \to 0}{\longrightarrow} d\sigma^{F}_{LO} \otimes \Sigma(q_T/M) dq_T^2$$

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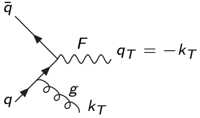

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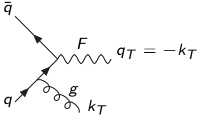

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$$\textit{where} \quad \mathcal{H}^F_{NNLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{F(2)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^F_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
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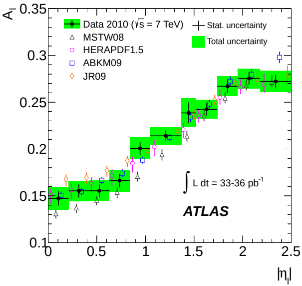
Drell–Yan and Higgs boson production



Lepton charge asymmetry from W decay compared with ATLAS data.

Fully exclusive calculations implemented in independent (parton level) Monte Carlo numerical codes DYNNLO/HNNLO.

- Vector boson prod. (DY): Most "classical" process in hadron collisions (constrain for PDFs fits, measure of M_W, beyond the SM analysis). NNLO total cross section [Hamberg, Van Neerven, Matsuura('91)], [Harlander, Kilgore('02)] and rapidity distribution [Anastasiou, Dixon, Melnikov, Petriello('03)] known since long time.
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  Large QCD corrections: $\sim +100\%$ at NLO, $\sim +25\%$ at NNLO.

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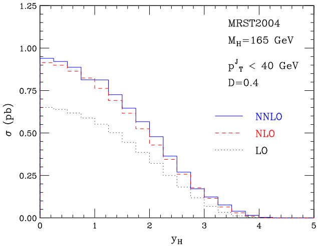

Higgs boson rapidity spectrum ($M_H = 165$ GeV). Final-state jets required to have $p_T < 40$ GeV.

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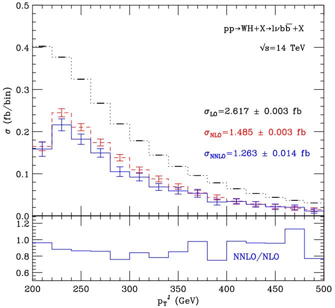
Associated VH production



Fat jet p_T spectrum ($m_H = 120 GeV$). Cuts: $p_T^l > 30\ GeV, |\eta^l| < 2.5$, $p_T^\nu > 30\ GeV, p_T^W > 200\ GeV$. Jets: C/A alg. (R=1.2). Jet veto: No jets with $p_T > 20 GeV$ & $|\eta| < 5$.

- Important LHC channel with boosted analysis: large-p_T Higgs boson through a collimated $b\bar{b}$ pair decay (fat jet).
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- Total cross section: $gg \to HZ$ top-loop $\sim g^2\lambda_t^2\alpha_S^2$ (non DY-like) (+5% at the LHC) [Kniehl('90)] [Brein,Harlander,Djouadi('00)]→vh@nnlo, $\sim g^2\lambda_t^2\alpha_S^3$ [Altenkamp et al.('12)] and $\sim g^3\lambda_t\alpha_S^2$ to WH and ZH ($\sim$ 1-2% at the LHC) [Brein,Harlander,Wiesemann, Zirke('11)].

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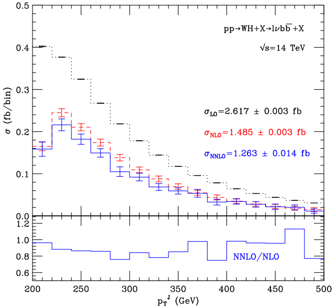

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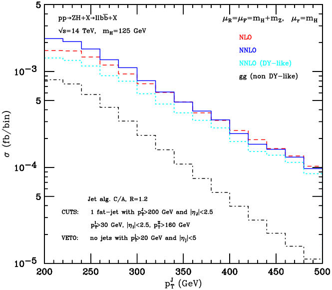

p_T spectrum of the b-jet pair at the LHC 14 TeV.

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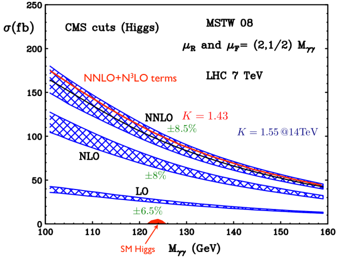
Diphoton production



$M_{\gamma\gamma}$ spectrum at the LHC
$\sqrt{s} = 7\ TeV$
Scales:
$M_{\gamma\gamma}/2 < \mu_R = \mu_F < 2M_{\gamma\gamma}$
Cuts:
$p_T^{\gamma,hard} > 40\,GeV$,
$p_T^{\gamma,soft} > 30\,GeV$,
$|\eta^\gamma| < 2.5$ (excl. $1.44 < |\eta^\gamma| < 1.57$),
$100 < M_{\gamma\gamma} < 160\,GeV$.

- Main irreducible background of Higgs production via $gg \to H + X \to \gamma\gamma + X$
- The q_T-subtraction formalism cannot deal with IR divergences in the final state: rely on Frixione smooth cone isolation (no *fragmentation component*) Fully exclusive NNLO corrections for *direct component*→2γNNLO. [Catani,Cieri,de Florian,G.F.,Grazzini('11)]
- Naive LO and NLO scale variation bad estimate of perturbative uncertainty.
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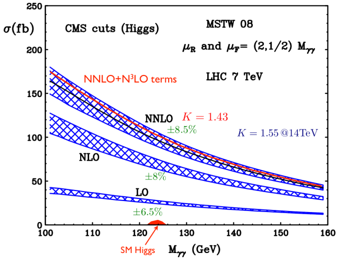

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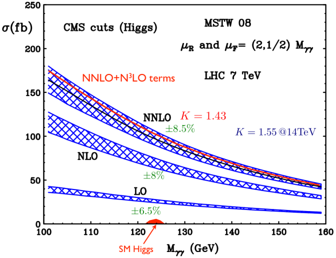

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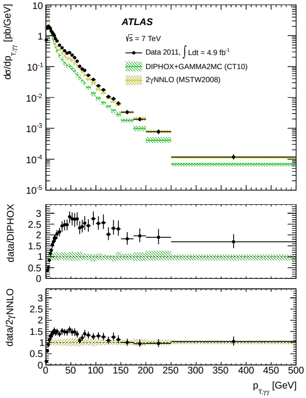

$p_{T,\gamma\gamma}$ spectrum, NLO and NNLO QCD corrections compared with ATLAS data.

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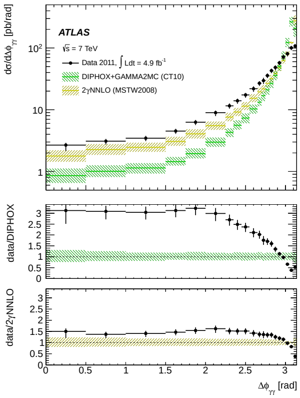

Azimuthal angle $\Delta\phi_{\gamma\gamma}$ spectrum, NLO and NNLO QCD corrections compared with ATLAS data.

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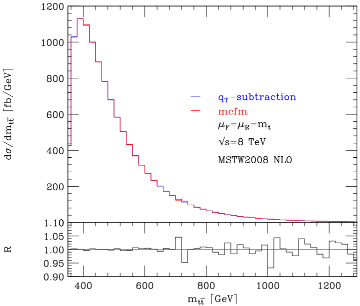
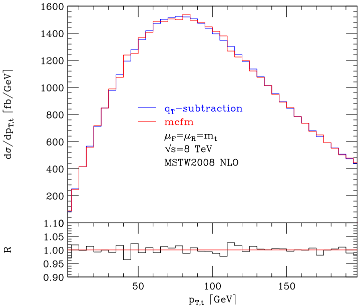
Top quark pair production

By exploiting the q_T resummation for $t\bar{t}$ production possible to set up a (N)NLO fixed-order calculation within the q_T subtraction formalism [Bonciani,Catani,Grazzini,Sargsyan,Torre('15)].



Invariant mass distribution of the $t\bar{t}$ pair at the LHC ($\sqrt{s} = 8$ TeV) at NLO accuracy. Comparison of q_T-subtraction with the MCFM (dipole subtr.) results.



p_T distribution of the top at the LHC ($\sqrt{s} = 8$ TeV) at NLO accuracy. Comparison of q_T-subtraction with the MCFM (dipole subtr.) results.

Conclusions

- **Overview on q_T resummation formalism:** difference between $q\bar{q}$ annihilation and gluon fusion process, hard-collinear factors and universality.
- NNLL(NNLO)+NLO q_T-resummation for Drell-Yan and Higgs production in gluon fusion.

  Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results for Drell-Yan consistent with the experimental data in a wide region of q_T.
- Added full kinematical dependence on the vector/Higgs boson and on the final state leptons/photons.
- **Overview on q_T subtraction formalism at NNLO:** illustrative phenomenology result on several differential distribution for vector boson, Higgs boson, associated VH, diphoton production and top quark pair production.

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Thank you!