Black Hole Superradiance, Alternative Theories and Floating Orbits

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Outline:

Introduction and Motivation

Alternatives to GR

BH dynamics : stable and unstable situations

Quasinormal modes vs Superradiance

Extreme mass ratio inspirals

Floating orbits and resonances

Conclusion

Why do we study black holes?

Black holes as theoretical laboratories

Probe gravity in the extreme regime

Very simple objects – Only characterized by 3 parameters M, Q and J.
"Hydrogen atom" of General relativity

Bridge between classical and quantum gravity

In reality, it is impossible to get isolated black holes in equilibrium : complex distribution of matter around them. Always in perturbed state.

Important question to ask :

"Can black holes be perturbed?"

"How can they be perturbed?"

"If at all they oscillate, then are they stable ?"

Golden Era in Black Hole Physics

(i) our capacity to observationally scrutinize the region close to the horizon within a few Schwarzschild radii, with radio and deep infrared interferometry (Doeleman et al '08, Fish et al. '11)

(ii) the ability to measure black hole spins more accurately than ever before using X-ray Spectroscopy (McKlintock et al, '11, Risaliti et al, 1302.7002)

(iii) huge technological progress in gravitational-wave observatories, gathering data at design sensitivities for several years and are now being upgraded to sensitivities one order of magnitude higher (black hole binaries are thought to be among the first objects to ever be detected in the gravitational-wave spectrum);

(iv) ability to numerically evolve black hole binaries at the full nonlinear level and its immediate importance for gravitational-wave searches and high-energy physics (Pretorius, '05, Campanellis et al, '06, Cardoso et al, '12)

(v) improvement of perturbative schemes, either by an understanding to handle the self-force, or by faster and more powerful methods (Poisson et al, '11, Pani et al,'11)

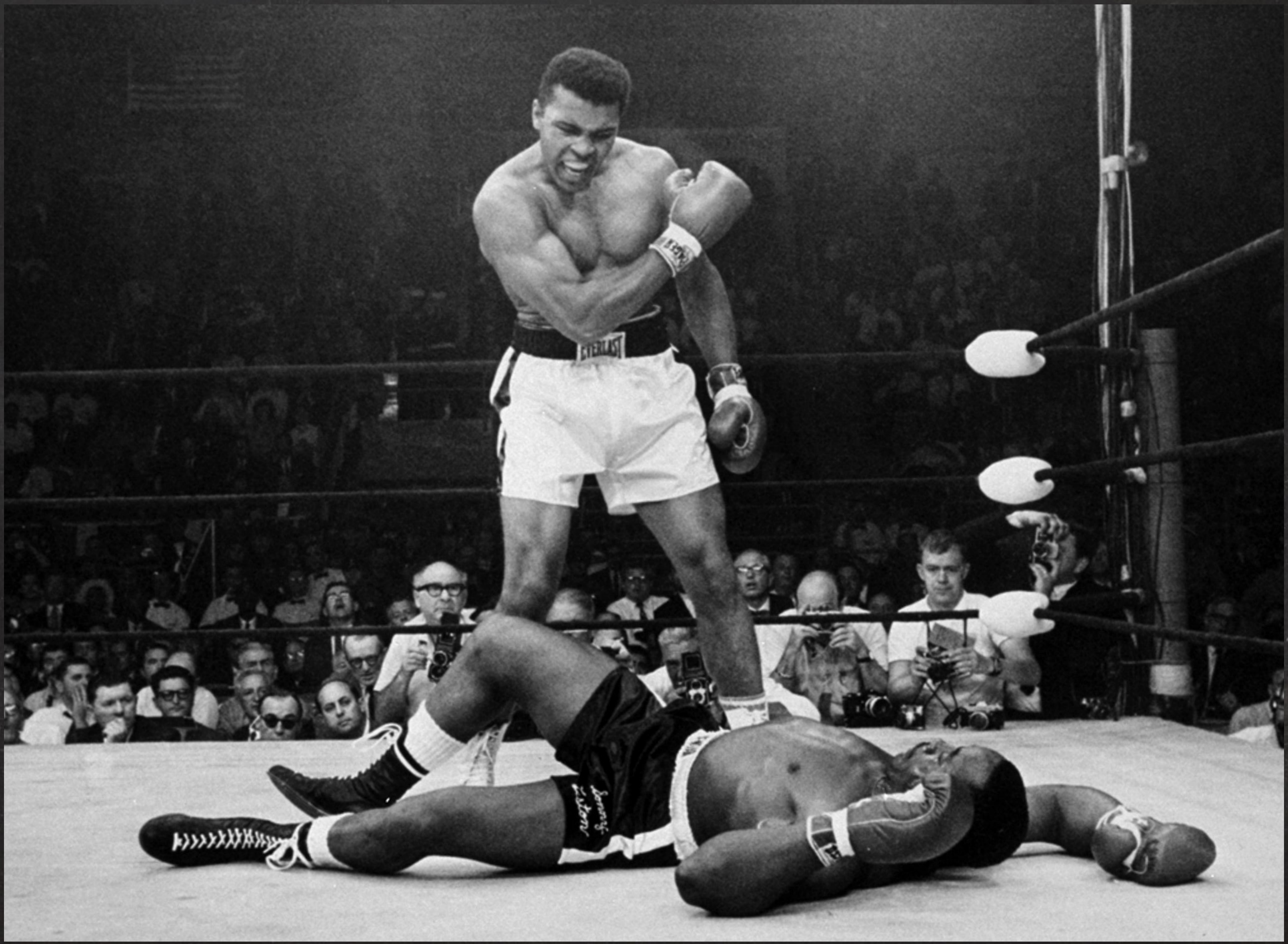
Golden Era in Black Hole Physics

(i) the gauge/gravity duality relating field theories to gravitational physics in anti-de Sitter spacetimes via holography. Gauge/gravity duality opens up a whole new framework to understand traditionally very complicated phenomena through black hole physics

(ii) extensions of the Standard Model to encompass fundamental ultra-light scalar fields, either minimally coupled or coupled generically to curvature terms. These theories include, for instance, generalized scalar-tensor theories and the "axiverse scenario". Ultralight scalars lead to interesting new phenomenology with possible smoking gun effects in black hole physics, and are a healthy and "natural" extension of GR (Arvanitaki et al, '10)

(iii) The formulation of TeV-scale gravity scenarios, either with warped or flat extra-dimensions, most of which predict black hole formation from particle collisions at scales well below the "traditional" Planck scale (Cardoso et al, NR-HEP Roadmap, 2012)

Testing General Relativity – Against what?



Slide courtesy: Emanuele Berti

Foundations of GR : very well tested in the regime of weak gravitational field, small space time curvature. However: conceptually disjoint from QFT, Singularities, Dark Energy, Dark Matter...

GR requires extension/modification at strong gravitational field → Introduce addition DoF

GR is compatible with all observational tests in weak gravity conditions, a major goal of present and future experiments is to proble astrophysical systems where gravity is strong

1. Strong gravitational field $\varphi \sim M/r \longrightarrow v \sim \sqrt{\frac{M}{r}}$

2. Strong curvature (a quantitative measure is the tidal force) $R^{r}_{0r0} \sim M/r^3$

♣ Can probe strong gravity by observations of weak fields : Spontaneous scalarization

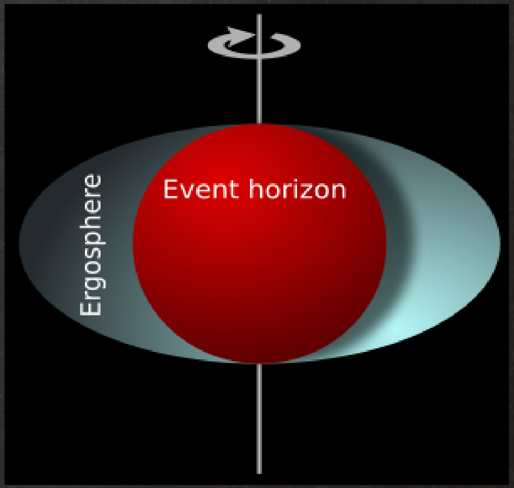
Damour, Esposito-Farese

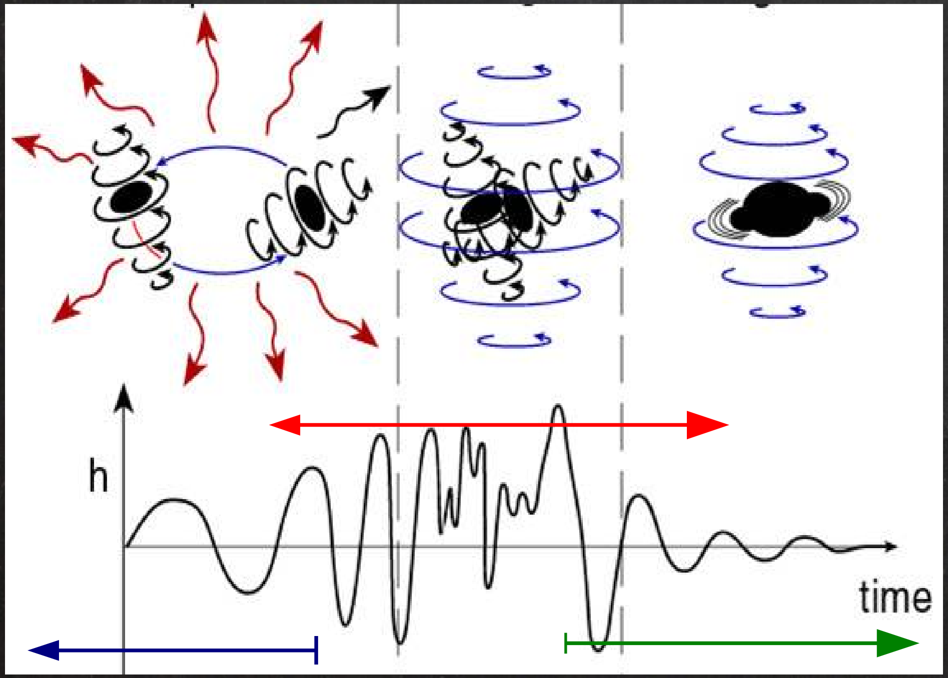
♣ measurements of particle dynamics around strong field regimes are not necessarily "smoking guns" of hypothetical modifications to general relativity!!

Classic theorems in Brans-Dicke theory, recently extended to generic scalar tensor theories and f(R) theories shows : the solutions of the field equations in vacuum always include the Kerr metric as a special case. (Thorne, Hawking, Bekenstein, Psaltis, Sotiriou...)

Kerr solution is so ubiquitous that probes of the Kerr metric alone will not tell us whether the correct theory of gravity is indeed GR.

Measurement of the metric around BH spacetime will not be sufficient to probe GR.





However, the dynamics of BHs (as manifested in their behaviour when they merge or are perturbed by external agents) will be very different in GR and in alternative theories.

Dynamical measurements of Binary inspiral and merger will be sensitive to the dynamics of the theory. Gravitational radiation (which bears the imprint of dynamics of the grav field) has the potential to tell GR from its alternatives

Finding a contender

A "serious contender"should at least be

(i) well defined in a mathematical sense, e.g. by having a well posed initial value problem

(ii) phenomenologically, the theory must be simple enough to make physical predictions that can be validated by experiments

Scalar field DoF

Additional matter fields

$$\mathcal{L} = f_0(\phi)R - \omega(\phi)g^{ab}\partial_a\phi\partial_b\phi - M(\phi) + \mathcal{L}_{\rm mat}[\Psi, A^2(\phi)g_{ab}] + f_1(\phi)\mathcal{R}^2_{\rm GB} + f_2(\phi)R^*_{abcd}R^{abcd}$$

(See Clifton, Ferreira, Padilla, Skordis, Phys. Rept. '11 For a review)

Bergmann-Wagoner theory: $f_1(\phi) = f_2(\phi) = 0$

Brans Dicke Theory: $A(\phi) = 1;\ f_0(\phi) = \phi;\ \omega(\phi) = \omega_{BD}/\phi;$
$f_1(\phi) = f_2(\phi) = M(\phi) = 0$

BW type theories with a massive scalar field gives rise to interesting effects in BH physics and binary dynamics.

Black hole dynamics and Superradiance

Measurements based on Kerr metric alone does not necessarily differentiate between GR and its alternatives.

BHs are ideal astrophysical laboratories for strong field gravity

Recent results in Numerical Relativity confirmed that the dynamics of BHs can be approximated surprizingly well using linear perturbation theory. (Buonano et al,'07, Berti et al,'07)

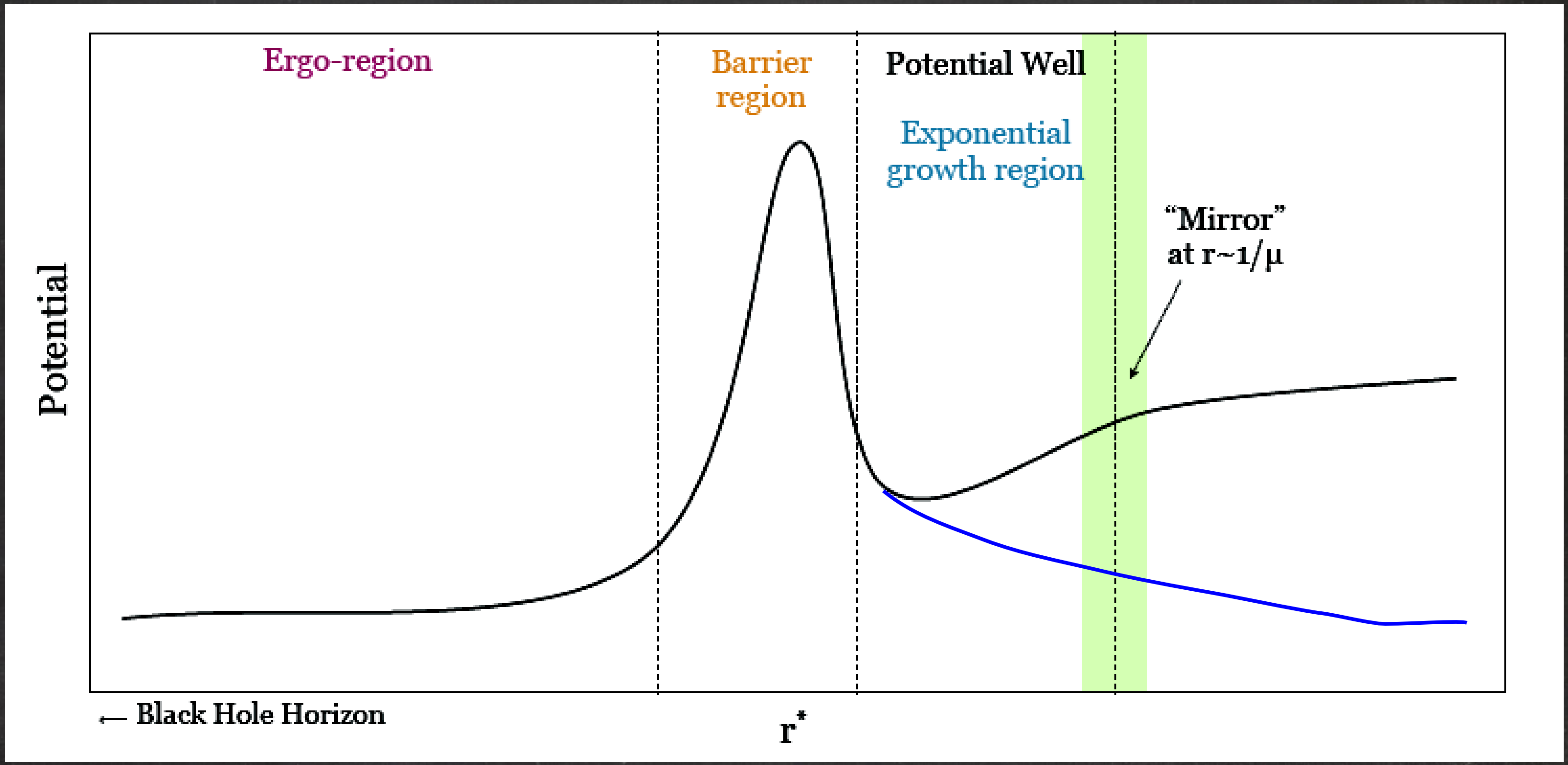
In perturbation theory, the behaviour of test field of any spin (s=0,1,2 for scalar, electromagnetic and gravitational fields) can be described in terms of an effective potential.

It is the shape of the potential which determines the stable or unstable nature of the BH perturbations

↓

Stable dynamics : Quasinormal modes

Superradiant instability: amplification of perturbation modes



Quasinormal modes

For massless scalar perturbation

$$V \to 0 \text{ as } r^* \to \pm\infty$$

Ingoing waves at the horizon, outgoing waves at infinity

Discrete "ringdown" spectrum

Massive scalar: Superradiance

Scalar field mass creates a non zero potential barrier such that at infinity $V \to \mu^2$

Superradiance: black hole bomb when

$$0 < \omega < m\Omega_H$$

Stable/unstable nature is governed by the shape of the potential

Fig: Arvanataki et al' 09

Stable dynamics: QNM: Characteristic modes of vibrations

Characteristic modes of vibrations play crucial role in physics
– Spectroscopy, seismology, atmospheric science, civil engineering,...

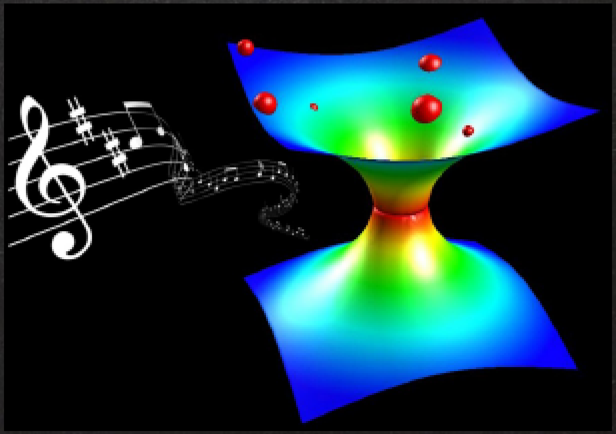
A star can oscillate due to perturbations ! Oscillations carried by fluid making up the star.

Black hole does not possess any material to sustain such oscillations.

In fact Black hole is not a material object, it is a singularity hidden by a horizon. How can it possibly oscillate?

Oscillations involve spacetime metric outside the horizon

BH perturbations carry the characteristic imprint of gravitational interactions



"Hearing the shape" of spacetime itself

How do we perturb a black hole

"Stability analysis consists in finding out whether a system breaks apart if an ant sneezed in its vicinity" - E. Salpeter

Theoretically this ant's sneeze (perturbation) can be performed in two ways:

By adding field to the BH background

Reduces to propagation of the field in the BH background in linear approximation (when the field does not backreact on the background)

$$(\nabla^\mu \nabla_\mu - \mu_s^2)\phi = 0$$

$$\{\gamma^a e_a^\mu (\partial_\mu + \Gamma_\mu) + \mu_d\}\Psi = 0$$

$$\nabla^\mu F_{\nu\mu} - \mu_p^2 A_\mu = 0$$

By perturbing the black hole metric (the background) itself

Gravitational perturbations

Most relevant for astrophysical purposes

$$g_{\mu\nu} = g_{\mu\nu}{}^{BG} + h_{\mu\nu}$$

Background **Perturbation**

$$\delta R_{\mu\nu} = 0$$

Can we simplify them?

SC, K.S. Gupta' 06, SC,08, SC, '09

Why this is hard?

Newton vs. **Einstein**

Equations are much more complex

There are many sources of gravity

Gravity is a source of gravity (non-linearlity)

1 eq, 1 variable, simple differential operator

$$\nabla^2\Phi(x,t) = 4\pi\rho(x,t)$$

Linear differential operator

Only mass density

Eqns and independent variables>1, complicated differential operator

$$G_{\mu\nu}[g_{\alpha\beta}(x^\nu)] = 8\pi T_{\mu\nu}[\cdots]$$

Highly non-linear differential operator

$$G_{\mu\nu}[g_{\alpha\beta}] \sim \mathcal{O}[g] + \mathcal{O}[g^2] + \cdots$$

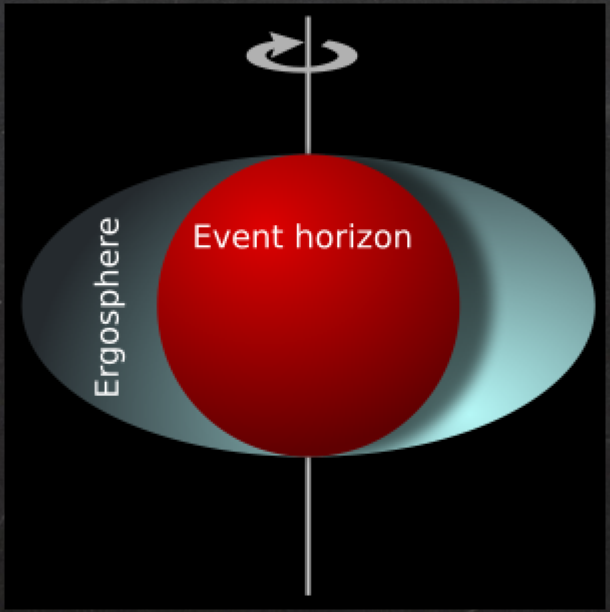
Density, velocity, pressure, kinetic energy, EM fields.....

Kerr metric:

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta)\sin^2\theta}{\Sigma} dt d\phi$$
$$- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2 \cos\theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating black hole with mass M and angular momentum J=aM



Teukolsky equations for Kerr perturbations:

Using Newmann-Penrose formalism it is possible to reduce complicated equations describing Kerr perturbations to a wave equation

(Teukolsky, 1972, '73)

Introduce a tetrad of null vectors at each point in spacetime and project all tensorial quantities on them. The NP equations are relations linking the tetrad vectors, spin coefficients, Weyl tensor, Ricci tensor and scalar curvature.

By Fourier transforming a spin-s field and expanding into spin weighted spheroidal harmonics

$$\psi_s(t,r,\theta,\phi) = \frac{1}{2\pi}\int e^{-i\omega t} \sum_{\ell=|s|}^{\infty} \sum_{m=-\ell}^{\ell} e^{im\phi} {}_sS_{\ell m}(\theta) R_{\ell m}(r) d\omega$$

One can find the separated ODEs for S and R

The radial equation:

$$\Delta R_{\ell m,rr} + 2(s+1)(r-M)R_{\ell m,r} + V(\omega,r)R_{\ell m} = 0$$

The angular equation

$$\left[\frac{\partial}{\partial u}(1-u^2)\frac{\partial}{\partial u}\right]{}_sS_{\ell m} + \left[a^2\omega^2u^2 - 2a\omega s u + s + A_{\ell m} - \frac{(m+su)^2}{1-u^2}\right]{}_sS_{\ell m} = 0$$

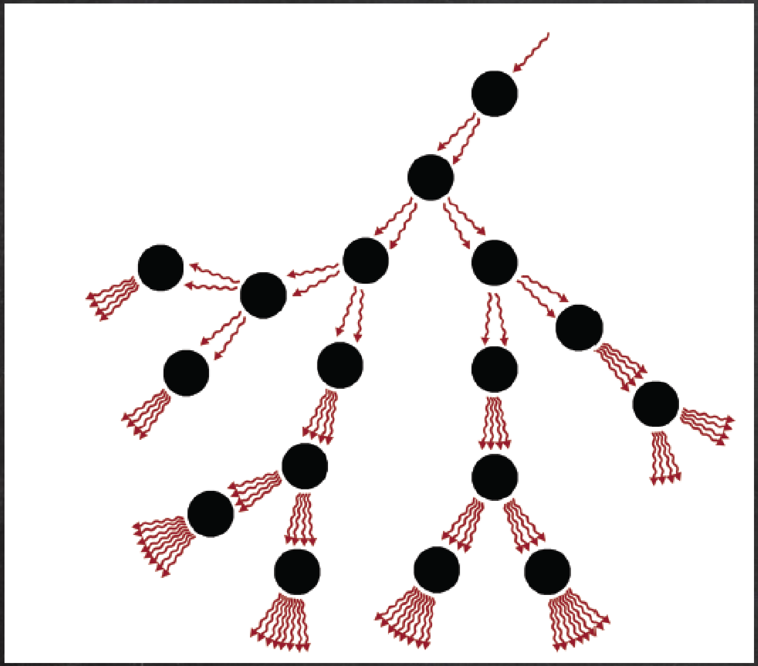
$$u = \cos\theta$$

The solutions of the angular equation are known as spin weighted spheroidal harmonics and the determination of angular separation constant in general is a difficult task to perform: need to take help of numerics

Superradiance: $0 < \omega < m\Omega_H$

(1) Black hole fission

(Press, Teukolsky' 72)



Hypothetical chain reaction in a cluster of rotating black holes. The incident arrow denotes an incident wave on the rotating black hole, which is then amplified and exits with larger amplitude, before interacting with other black holes. The super-radiantly scattered wave interacts with other black holes, in an exponential cascade.

Should work if $\ell_{\mathrm{mean-free-path}} < L$

Size of the cluster

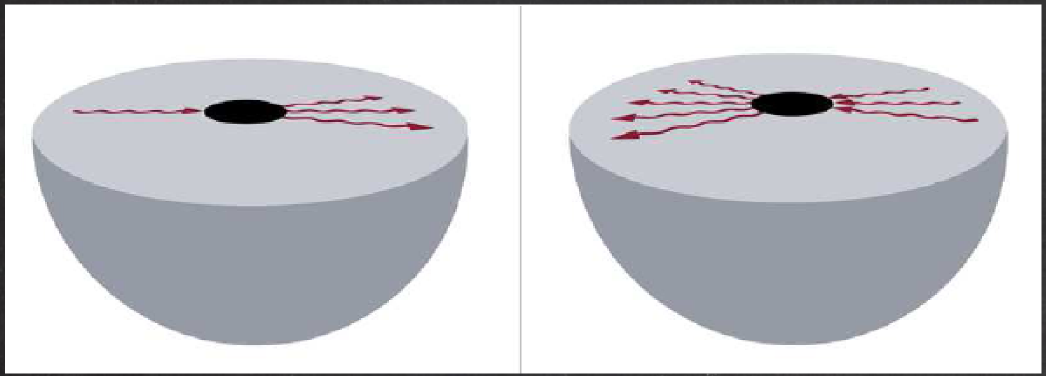
DOES NOT WORK: cluster lies within its own Schwarzschild radius, making the fission process impossible

Figure: A. Sousa (DyBHO)

Superradiance:

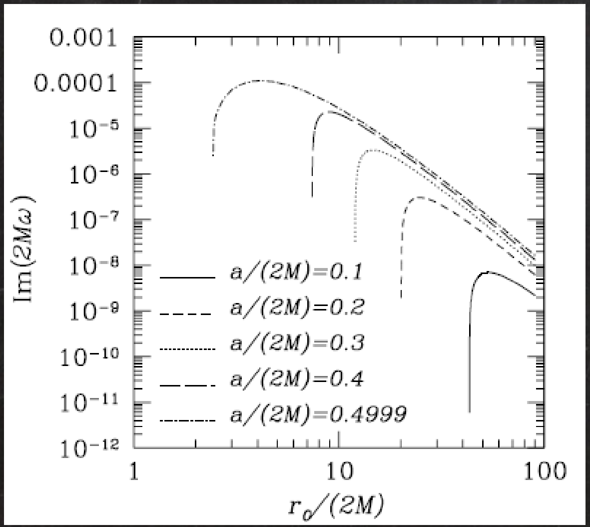
(2) Black hole bombs

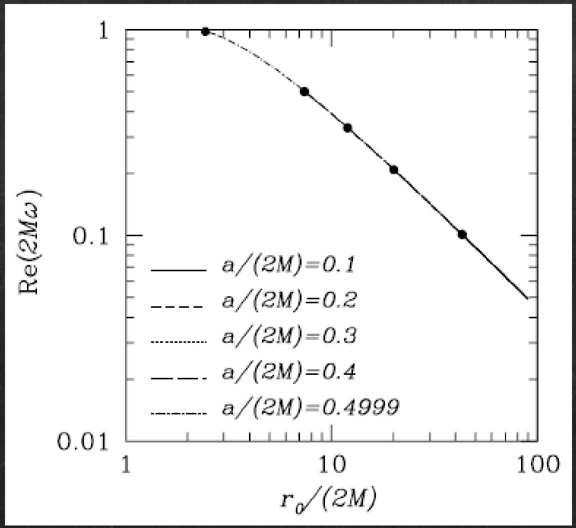
(Press, Teukolsky'72, Cardoso et al ;04, Rosa'09, Dolan '12)



simple way to tap the hole's rotation energy via superradiance is to enclose the rotating black hole inside a perfectly reflecting cavity

Any initial perturbation will get successively amplified near the black hole and reflected back at the mirror, thus creating an instability





Cardoso, Dias, Lemos, Yoshida

Instability time scale is large

$$\tau = 1/\mathrm{Im}(\omega) = \mathrm{r}_0^{2(\ell+1)}$$

bombs can become very efficient, if instead of rotation one considers charged black holes and charged scalar fields

Herdeiro et al'13

Superradiance:

(3) Black hole bombs in AdS:

AdS space times are natural realizations of BH bomb instability as their timelike boundary is perfectly suited to play the role of reflecting cavity

Kodama'07, Uchikata et al' 09, Cardoso et al '08

(4) Nature provides its own mirror: Massive scalar fields

Instability is regulated by the parameter $M\mu$ → Strongest when $M\mu \sim 1$

i.e. When the Compton wavelength of the perturbing field is of the order of the size of the BH

Light primordial BHs

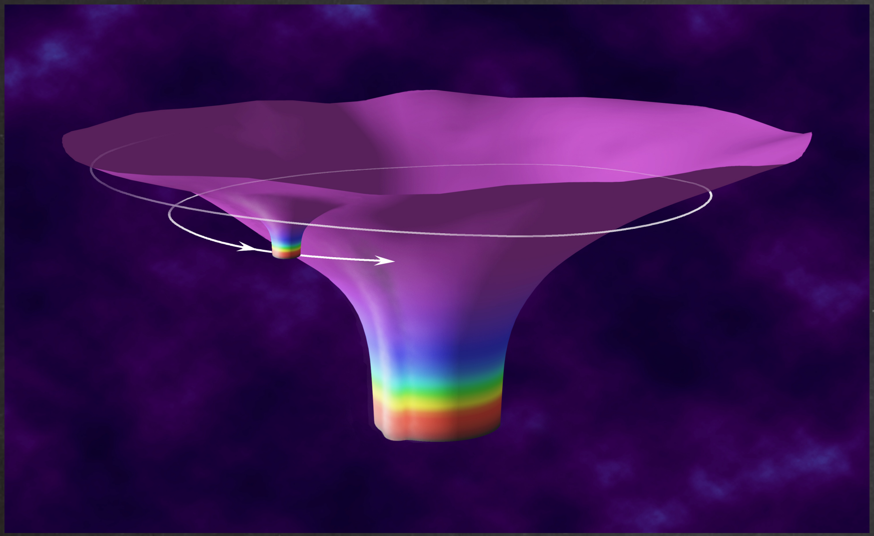
Ultralight exotic particles

Ex: "String axiverse" scenario:

$$10^{-33}\ \mathrm{eV} < \mathrm{m} < 10^{-18}\ \mathrm{eV}$$

Superradiance can put stringent constraints on the mass of the perturbing field.

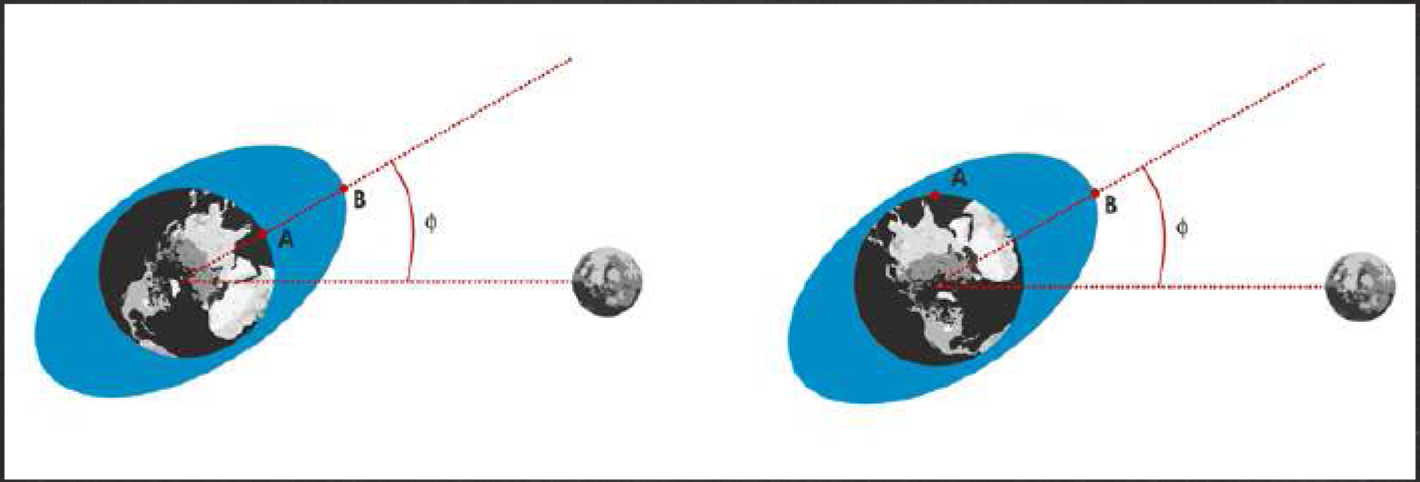
(Arvanitaki et al, '09, Arvanitaki et al '11, Pani et al' 12)



Extreme mass ratio inspirals

Galactic cannibalism: Capture of small black hole/neutron star by a supermaasive black hole

Tide in Earth Moon system



Gravitational pull of moon on earth produces tides and because earth rotates, there are two tides a day. Tidal effects are responsible for constant drift of the moon's orbit (**tidal acceleration**), and for its synchronous rotation with earth (**tidal locking**). Tides are caused by differential forces on the oceans and they raise tidal bulges on the earth

Since $\Omega_{\text{earth}} > \Omega_{\text{moon}}$, the bulges lead the earth-moon direction by a constant angle, which would be zero if there is no friction. Friction between earth's crust and the ocean slows down the earth's rotation period (0.002 s/century). Conservation of angular momentum then lifts the moon to a higher orbit (4cm/yr) with a longer period and larger semi major axis.

Cardoso, Pani, CQG, 30 (2013) 045011

Tides on black holes and its ``moon''

A more cleaner system in the context of tides : Black holes : extremely ``simple'', much lesser parameters.

Consider a ``moon'' of mass m_p orbiting with angular velocity Ω around a rotating BH of mass M and angular velocity Ω_H at a distance r_0, dissipates energy (through tidal heating) at the event horizon at a rate

$$\dot{E}_H \sim \frac{G^7}{c^{13}} \frac{M^6 m_p^2}{r_0^6} \Omega(\Omega - \Omega_H)$$

(Hartle, '73,'74; Poisson, '09)

$\Omega < \Omega_H$, energy flowing out of the BH. BH spun down by the moon

Not the end of the story ...

BH's are general relativistic objects. Pure space-time fabric. Any tidal distortion carries energy (in the form of gravitational wave) away to infinity:

$$\dot{E}_\infty \sim \frac{32}{5} \frac{G^4}{c^5} \frac{M^3 m_p^2}{r_0^5}$$

Tidal acceleration is only possible if $|\dot{E}_H / \dot{E}_\infty| > 1$

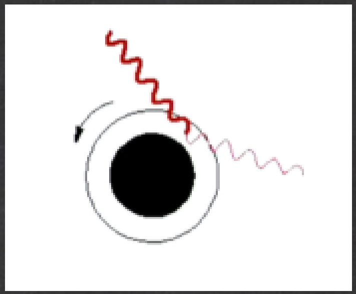
$$\frac{\dot{E}_H}{\dot{E}_\infty} \sim \left(\frac{v}{c}\right)^5 > 1$$

Tidal acceleration not possible in GR

Another take on tidal dissipation: Superradiance

Equivalent but complimentary approach ⇒ use a wave-like perspective by considering small moon as the time dependent disturbance in a stationary rotating space-time

A wave scattered off a rotating BH is superradiantly amplified



$$\phi \sim e^{-i\omega t}$$

$$\omega < m\Omega_H$$

A massless field in the vicinity of the rotating BH

$$\frac{d^2\Psi}{dr_*^2} - V_{\rm eff}\Psi = 0$$

In a scattering experiment of a wave with frequency **ω** and azimuthal and time dependence, $e^{-i\omega t + im\phi}$ the above equation has the asymptotic behaviour

$$\Psi \sim \mathcal{T}(r - r_+)^{-i\chi(\omega - m\Omega_H)} + \mathcal{O}(r - r_+)^{i\chi(\omega - m\Omega_H)}, \text{as} \quad r \to r_+$$

$$\sim \mathcal{R}e^{i\omega r} + e^{-i\omega r}, \text{as} \quad r \to \infty$$

$$\chi = 2r_+(r_+^2 + a^2)/(r_+^2 - a^2); a/M = cJ/(GM^2)$$

The BC's: incident wave of unit amplitude from spatial infinity giving rise to a reflected wave of amplitude **R** and a transmitted wave of amplitude **T** at the horizon. The **O** term describes a putative outgoing flux at the horizon.

$$\Psi \sim \mathcal{T}(r-r_+)^{-i\chi(\omega - m\Omega_H)} + \mathcal{O}(r-r_+)^{i\chi(\omega - m\Omega_H)}, \text{as} \quad r \to r_+$$

$$\sim \mathcal{R}e^{i\omega r} + e^{-i\omega r}, \text{as} \quad r \to \infty$$

Two linearly independent solutions

$$W_{\rm NH} = -2i(\omega - m\Omega_H)(|\mathcal{T}|^2 - |\mathcal{O}|^2)$$

$$W_\infty = 2i\omega(|\mathcal{R}|^2 - 1)$$

Wronskian W must be constant

Evaluate and equate the W near the horizon and near infinity

$$|\mathcal{R}|^2 = 1 - \frac{\omega - m\Omega_H}{\omega}(|\mathcal{T}|^2 - |\mathcal{O}|^2)$$

< 1 in general

If < 0, then $|R|^2>1$

= 0 at the horizon, "nothing can come out of horizon"

Superradiance **(Excess energy comes from BH's rotational energy)**

(Misner '72, Zel'dovich '71)

Dissipation is crucial. Interesting effects such as Black Hole Bomb!

Without the ingoing BC at horizon, there is no superradiance.

evidence of a correspondence between the two perspectives : tidal absorption and heating at the horizon with the wave absorption and superradiance.

Superradiance requires dissipation, the role of which is played by the horizon (BHs are perfect absorbers) and as we already have some idea that tidal acceleration also need dissipation.

Still the energy dissipated through tidal effects at the horizon of a rotating BH in pure GR is much more smaller than the energy emitted in gravitational waves at infinity.

However, this effect can be enormously amplified when coupling to light scalar degrees of freedom is allowed in the theory.

Two body problem in GR

Newtonian case: Solved exactly by reducing to one body problem

GR : Difficult to solve exactly due to non-linearity of GR

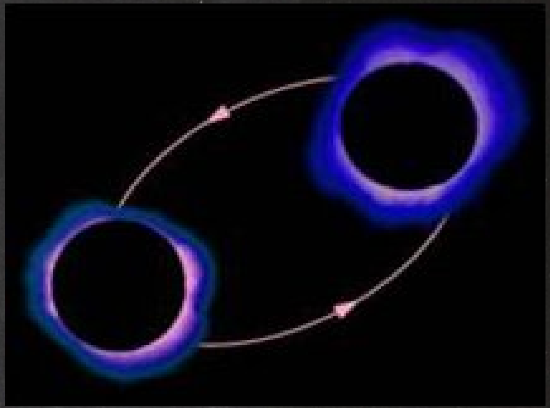
2 body problem is well motivated from the viewpoint of studying gravitational waves

Understanding of orbital evolution and accurate prediction of wave forms are necessary for the detection of GWs.

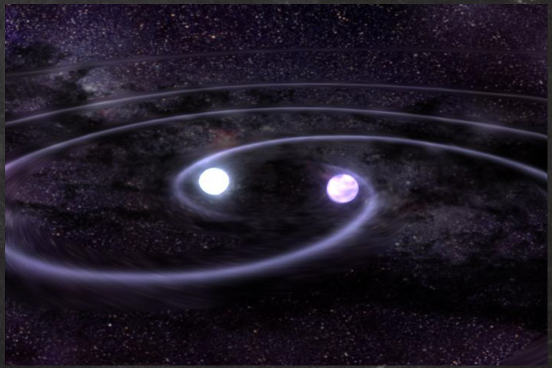
Approximate methods work very well in multiscale problems, for example in the two body problems in which one of the two scales is much larger than the other

Approximate methods are there everywhere in physics : in GR as well:

- Post Newtonian expansion: weak field approximation of Einstein's field equations (expansions in a small parameter, which is the ratio of the velocity of matter, forming the gravitational field, to the speed of light)
- Perturbation theory: stability, oscillations, GW emission ...



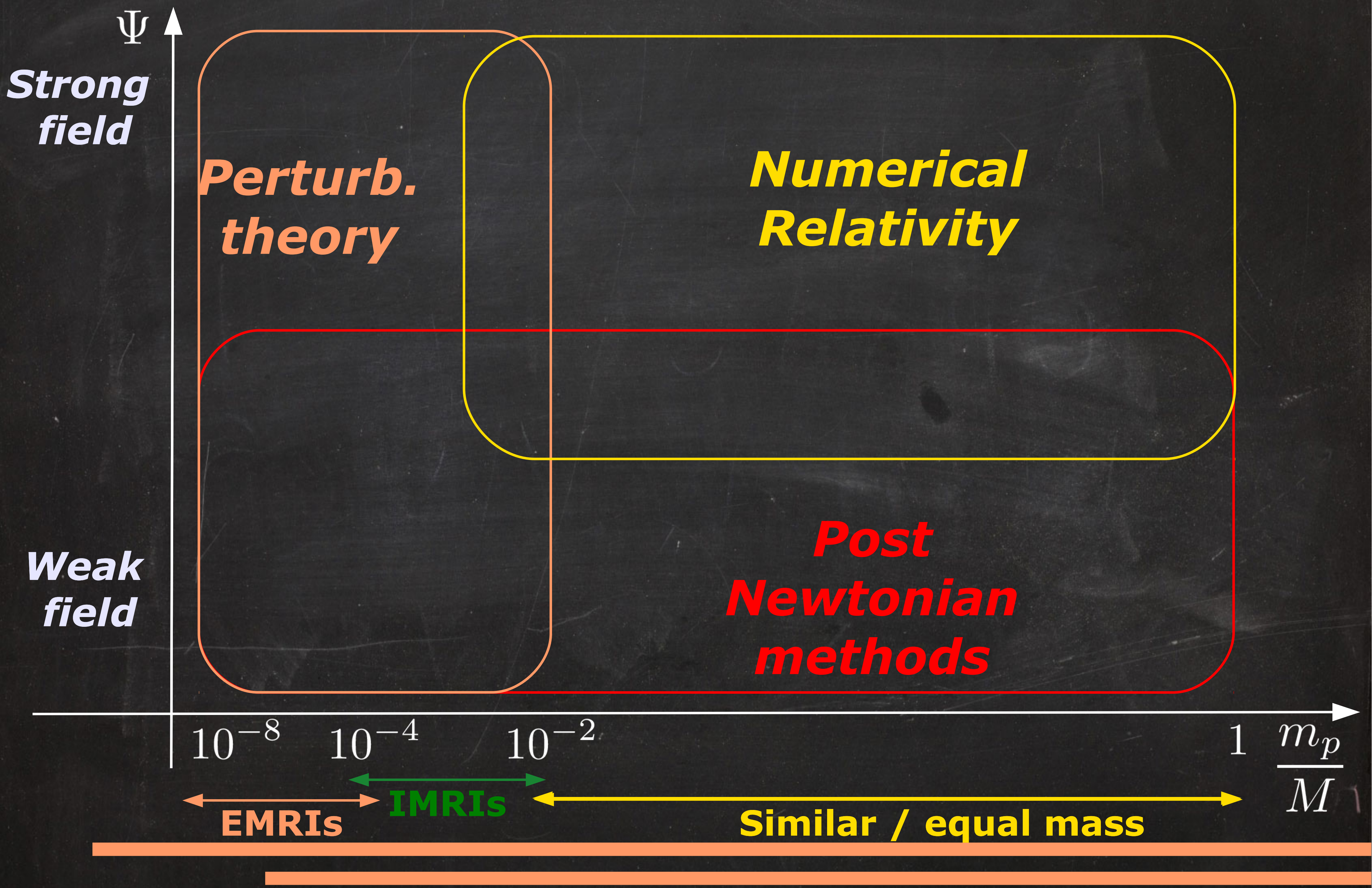
Black hole binaries



Compact object binaries

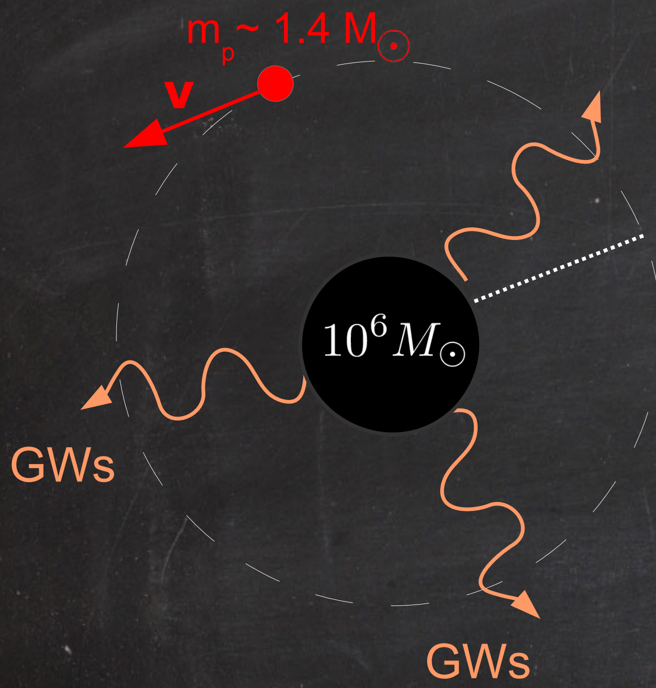
Two body problem in GR

(Hinderer, Flanagan, 2008)



EMRIs: Extreme Mass Ratio Inspirals

Emission due to "point-like" particles:



- Evidences of supermassive BHs in galactic centers
- $M \sim 10^6 - 10^9\ M_\odot$
- Two scales: M and m_p
- Mass ratio: $10^{-8} - 10^{-4} \rightarrow$ Not a regime for NR
- v~c $\rightarrow$ Not a regime for PN either!
- Perfect regime for perturbation theory

Source for GW space based detectors

~ 10^5 cycles during last year probing really the strong curvature regime

EMRIs: how does the system evolve?

- **Geodesic motion (just based on energy conservation)** (Tanaka, Cutler, Poisson,Hughes,...)

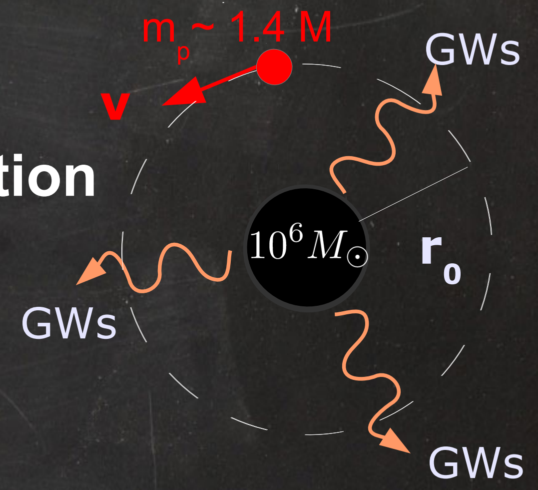
  - **At first order approx. treat the smaller object as point particle orbiting around Kerr BH along the Geodesics**
  - **Stress energy tensor of Einstein equation is given by the point particle**
  - **These are source term in Teukolsky equation**
  - **Solve Teukolsky equation with source to get the energy and angular momentum flux assuming adiabatic evolution**
  - **Difficult for generic orbits in Kerr**
  - **Neglects the self force**

Self force : (Barack, Sago, Norichika ...)

“gravitational self-force” corrections to geodesic motion, analogous to “radiation reaction forces” in electrodynamics.

important to calculate these self-force corrections in order to be able to determine accurate inspiral motion in the extreme mass ratio limit.

EMRIs: *1-slide computation*



Inhomogeneous Bardeen-Press-Teukolsky (BPT) equation

$$\frac{d^2\Psi_{\ell m}(\omega,r)}{dr_*^2} + V(\omega,r)\Psi_{\ell m}(\omega,r) = \mathcal{T}_{\ell m}(\omega,r)$$

The source term $\mathcal{T}_{\ell m}$ can be calculated from the stress energy tensor of the point particle

Use the Green's function technique

$$\Psi_{\ell m} = \frac{\Psi_\infty}{W}\int_{r_+}^{r} dr'\mathcal{T}_{\ell m}(r')\Psi_{r_+} + \frac{\Psi_{r_+}}{W}\int_{r}^{\infty} dr'\mathcal{T}_{\ell m}(r')\Psi_\infty$$

Energy flux at infinity:

$$\dot{E}_\infty^{\ell m} \sim \Omega_p^2 \left(\frac{m_p}{M}\right)^2 |\Psi_{lm}(r\to\infty)|^2$$

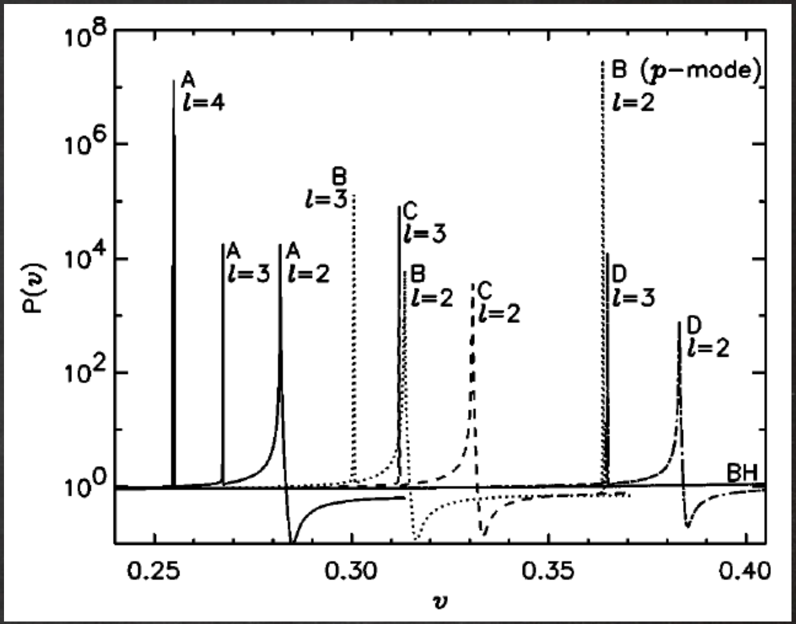
(Teukolsky '73, Detweiler, '74)

Energy flux at the horizon:

$$\dot{E}_{r_+}^{\ell m} \sim \Omega_p\left(\Omega_p - m\Omega_H\right)\left(\frac{m_p}{M}\right)^2 |\Psi_{lm}(r\to r_+)|^2$$

EMRIs: resonances in GW

Non-rotating spherically symmetric Neutron star :
Point like particle in circular orbit.



Pons, Berti, Gualtieri, Minutti, Ferrari, 2002

QNMs of perfect fluid stars can be excited :

$$\Omega_p \sim \omega_{QNM}/m$$

~ Modelled with forced oscillator.

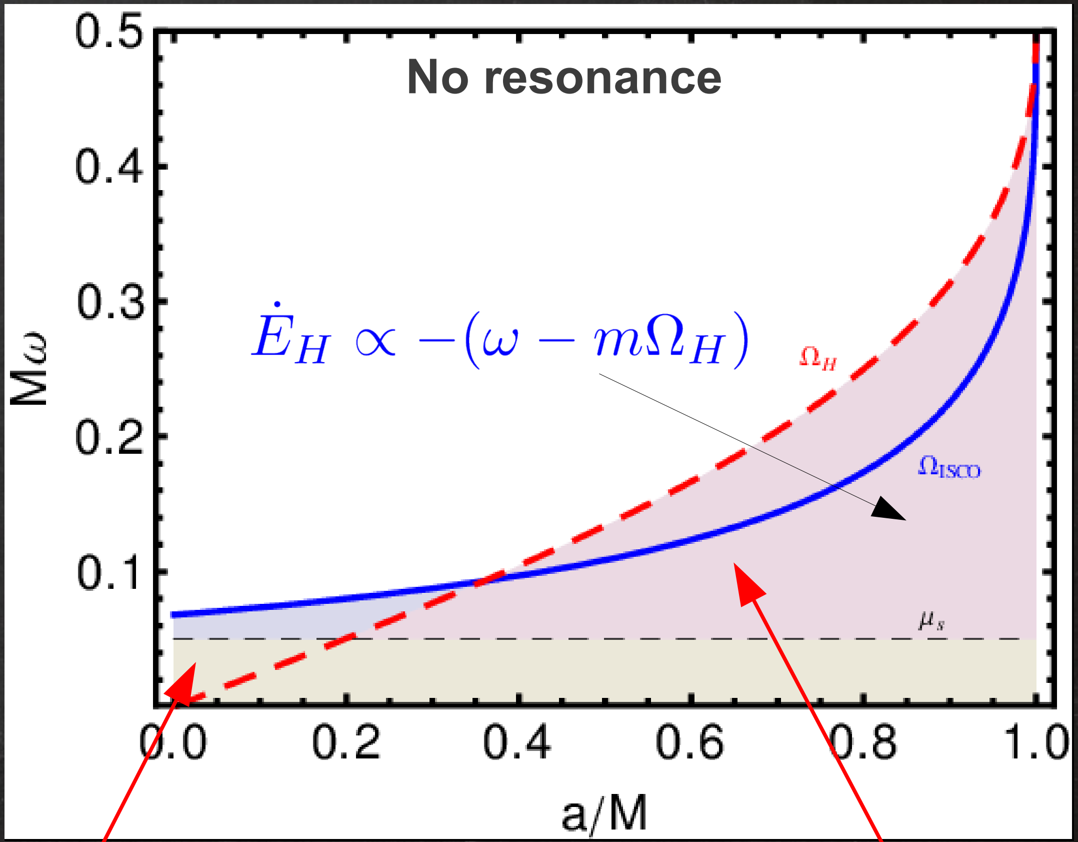
~ Everytime the orbital particle has the frequency same as the QNM of neutron star, there is a resonance.

EMRIs: resonances in GW

BH QNMs can not be excited by orbiting particles $\omega_{QNM} > m\ \Omega_{ISCO}$. In order to excite the QNMs the QN frequency must be below the ISCO frequency.

Situation changes with introduction of light scalar field coupled to matter , it introduces a new scale $\omega_{QNM} \sim \mu_s$

If $\omega < m\ \Omega_H$ → **Superradiance**, the flux at the horizon can be negative. So for a/M > 0.36, one can excite the QNMs as well as superradiance gives a large negative flux at the horizon.



An object orbitting around a BH loses energy in GWs. This follows from energy balance:

$\dot{E}_p + \dot{E}_g + \dot{E}_s = 0$

Usually

$\dot{E}_g + \dot{E}_s > 0 \Rightarrow$ orbit shrinks with time. But, due to superradiance, $\dot{E}_g + \dot{E}_s = 0$

Positive resonance: Sinking Orbit

Negative resonance: Floating orbits

Procedure to follow:

The process we consider is quite general. It occurs in all theories of gravity with Kerr BHs as background solutions and a scalar field of mass μ_s coupled to matter : Brans-Dicke theory with a massive scalar field being an example.

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega_{BD}}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m_s^2(\phi-\phi_0)^2\right) + S_{\text{matter}}$$

Approximate EMRI trajectories as geodesics of test particle in a background of SMBH to leading order in mass ratio

One can then study gravitational and scalar waves emitted and the energy momentum carried away by solving first order perturbation equations of the field equations as a function of the given geodesic.

Any given geodesis is sensitive to background upon which it evolves : we choose Kerr. Most general, stationary, axisymmetric, vacuum spacetime that is also the end point of gravitational collapse in Brans Dicke theory is Kerr metric (Sotiriou, Faraoni, ' 11)

We use adiabatic approximation: The particle is in nearly geodesic motion, allowing to compute, at each time, the emitted energy flux assuming a geodesic orbit which means the radiation reaction timescale is much longer than the orbital timescale

Set up

At first order in perturbation theory, the scalar field equation is $(\Box - \mu_s^2)\varphi = \alpha\mathcal{T}$

Because of the coupling to matter, the orbiting object emits both gravitational and scalar radiation.

Gravitational radiation can be computed using Teukolsky's formalism.

Focus on scalar wave emission. Defining

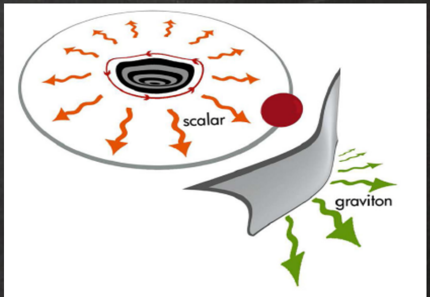
$$\varphi(t,r,\theta,\phi) = \sum_{\ell,m} \int d\omega e^{im\phi - i\omega t} \frac{X_{\ell m}(\omega,r)}{\sqrt{r^2+a^2}} S_{\ell m}(\theta)$$

$$\alpha(r^2 + a^2\cos^2\theta)\mathcal{T} = \sum_{\ell m} \int d\omega e^{im\phi - i\omega t} T_{\ell m\omega} S_{\ell m}(\theta)$$

The non-homogeneous equation for the scalar field

$$\left[\frac{d^2}{dr_*^2} + V\right] X_{\ell m\omega}(r) = \frac{\Delta}{(r^2+a^2)^{3/2}} T_{\ell m\omega}$$

Peak scalar flux for l=m=1, n=0, close to resonance frequency:



$$\dot{E}^{s,\mathrm{peak}}_{r_+} \sim -\frac{3\alpha^2\sqrt{\frac{r_0}{M}}m_p^2 M}{16\pi r_+\left(M^2-a^2\right)\left(M+\sqrt{M^2-a^2}\right)\mathcal{F}}$$

Where, $\mathcal{F} = 1 + 4P^2, P = -2Mr_+(\omega - m\Omega_H)/(r_+ - r_-)$

(Cardoso, SC, Pani, Berti, Gualtieri, PRL'12)

The flux at the horizon grows with r_0

It is negative and large due to superradiance. In fact for generic ℓ ,

$$\dot{E}^{s,\mathrm{peak}}_{r_+} \propto -r_0^{2\ell-3/2}$$

For very small "a" the peak flux at resonance is instead positive, and it can also be very large: for the Schwarzschild geometry,

$$32\pi M^4 \dot{E}^{s,\mathrm{peak}}_{r_+} \sim 3\alpha^2 r_0^2 m_p^2$$

Agrees very well with numerical integration of the Teukolsky equation

Floating orbit

Orbital freq.

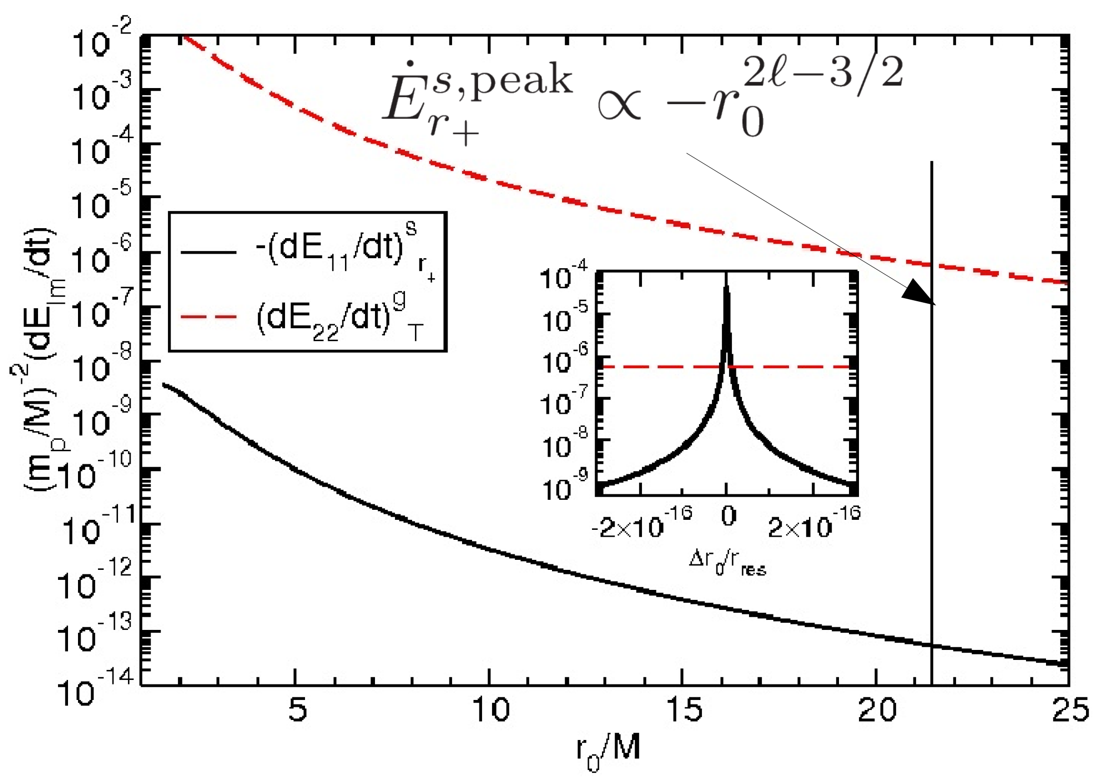
Massive scalar QNM

(Cardoso, SC, Pani, Berti, Gualtieri)

Can have resonances at $\Omega = \omega_R \sim \mu_s$

$$dE_p/dt = -\dot{E}_{\rm total} = -(\dot{E}_S + \dot{E}_G)$$

$$\text{if } \dot{E}_S = -\dot{E}_G \Longrightarrow \dot{E}_p = 0$$



Dominant fluxes of scalar and gravitational energy (l = m = 1 and l = m = 2, respectively) for $\mu_s M = 10^{-2}$, $\alpha = 10^{-2}$ and a = 0.99M.

Delayed inspiral may have observational consequences

Can constrain Brans Dicke parameter

$$\omega_{\rm BD} \leq 10^8, \quad \text{for } a/M = 0.99$$

Compare with present bound: $\omega_{\rm BD} > 4 \times 10^4$

An orbiting body excites superradiant scalar modes close to the BH horizon. This resonances excite the scalar flux at the horizon to (absolute) values which may be larger than the gravitational flux at infinity. The orbiting particle is driven to “floating orbits” for which the total flux is vanishing.

Conclusion

Approximations are not always bad, it tells us important things about black holes, in particular they are extremely helpful in studying GWs, EMRIs, QNMs etc.

An extreme form of energy extraction from a Kerr BH when a massive scalar field is coupled to a point particle in circular orbit around the BH

first example of a phenomenon produced by a resonance between orbital frequencies and proper oscillation frequencies of the BH.

Floating orbits can be instrumental to constrain or prove existence of massive scalars coupled to matter

Current searches for gravitational waves are strongly biased towards general relativity. If light scalar dof couple to matter, binaries may merge in a much more interesting way, and current searches based on matched-filtering techniques mayunderperform.

Still a lot to do: What happens when the companion is of equal mass?
What happens if the companion has spin?

THANK YOU

Back up slides

General perturbation equations:

Most general Einstein Hilbert action:

$$S = \frac{1}{16\pi G}\int d^d x\sqrt{-g}\,(R-2\Lambda) + \int d^d x\sqrt{-g}\,\mathcal{L}_m$$

Einstein equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

This should be supplemented with EOM of matter fields. Together with EE, they form a set of complicated set of non linear partial differential equations describing evolution of all field including the metric. A particular solution of this system forms a set of background fields $g_{\mu\nu}^{\rm BG},\ \Phi^{\rm BG}$

By writing $g_{\mu\nu} = g_{\mu\nu}^{\rm BG} + h_{\mu\nu},\ \Phi = \Phi^{\rm BG} + \phi$ and linearizing full system of equations with respect to the perturbations one obtains a set of linear differential equations satisfied by the perturbations.

Scalar perturbations:

Complex scalar field with conformal coupling

$$\mathcal{L}_m = -\left(\partial_\mu \Phi\right)^\dagger \partial^\mu \Phi - \frac{d-2}{4(d-1)} \gamma\, R \Phi^\dagger \Phi - m^2 \Phi^\dagger \Phi\,.$$

For γ=0 and m=0 one gets usual minimally coupled massless scalar field

Equations of motion
$$\nabla_\mu \nabla^\mu \Phi = \frac{d-2}{4(d-1)} \gamma R \Phi\,, \qquad G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Consider perturbations of the fields $g_{\mu\nu} = g_{\mu\nu}^{\mathrm{BG}} + h_{\mu\nu},\ \Phi = \Phi^{\mathrm{BG}} + \phi$ with $\Phi^{\mathrm{BG}} = 0$

It can be seen that the linearized eom for perturbations decouple and thus the metric fluctuations can be consistently set to zero. The background metric always satisfy $G_{\mu\nu}^{\mathrm{BG}} + \Lambda g_{\mu\nu}^{\mathrm{BG}} = 0$

Scalar perturbation equation
$$\frac{1}{\sqrt{-g_{\mathrm{BG}}}} \partial_\mu \left(\sqrt{-g_{\mathrm{BG}}}\, g_{\mathrm{BG}}^{\mu\nu}\, \partial_\nu \phi \right) = \frac{d(d-2)\gamma}{4L^2}\, \phi\,.$$

Decomposing
$$\phi(t, r, \theta\,) = \sum_{lm} e^{-i\omega t} \frac{\Psi_{s=0}(r)}{r^{(d-2)/2}} Y_{lm}(\theta)\,,$$
We get
$$\frac{d^2 \Psi_s}{dr_*^2} + \left(\omega^2 - V_s\right) \Psi_s = 0$$

$$V_{s=0} = f \left[\frac{l(l+d-3)}{r^2} + \frac{d-2}{4} \left(\frac{(d-4)f}{r^2} + \frac{2f'}{r} + \frac{d\gamma}{L^2} \right) \right]$$

Scalar flux: Calculational details

$$\left[\frac{d^2}{dr_*^2} + V\right] X_{\ell m\omega}(r) = \frac{\Delta}{(r^2+a^2)^{3/2}} T_{\ell m\omega}$$

$$dr/dr_* = \Delta/(r^2+a^2)$$ **Tortoise coordinate in Kerr**

$$V = \left(\omega - \frac{am}{\rho^2}\right)^2 - \frac{\Delta}{\rho^8}\left(\hat{\lambda}\rho^4 + 2Mr^3 + a^2(r^2 - 4Mr + a^2\right) - \frac{\Delta\mu_s^2}{\rho^2}$$

$$\rho^2 = r^2 + a^2, \quad \hat{\lambda} = A_{lm} + a^2\omega^2 - a^2\mu_s^2 - 2am\omega$$

$\hat{\lambda}$ **is determined from the angular eigenfunction**

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS}{d\theta}\right) + \left(a^2(\omega^2 - \mu_s^2)\cos^2\theta - \frac{m^2}{\sin^2\theta} + A_{lm}\right)S = 0$$

The scalar source function has the form

$$T_{\ell m\omega} = \frac{m_p\alpha}{U^t} S^*_{\ell m}(\pi/2)\delta(r - r_0)\delta(m\Omega_p - \omega)$$

Scalar energy flux

$$\left[\frac{d^2}{dr_*^2} + V\right] X_{\ell m\omega}(r) = \frac{\Delta}{(r^2+a^2)^{3/2}} T_{\ell m\omega}$$

Consider two LI solutions of homogeneous equation: $X^{r_+}_{lm\omega}, \quad X^{\infty}_{lm\omega}$

With boundary conditions $X^{\infty, r_+}_{\ell m\omega} \sim e^{ik_{\infty,H} r_*} \quad \text{as} \quad r \to \infty, \, r_+$

$$k_H = \omega - m\Omega_H \quad \text{and} \quad \mathrm{k}_\infty = \sqrt{\omega^2 - \mu_\mathrm{s}^2}.$$

Wronskian $W = X^{r_+}_{\ell m\omega} \frac{dX^{\infty}_{\ell m\omega}}{dr_*} - X^{\infty}_{\ell m\omega} \frac{dX^{r_+}_{\ell m\omega}}{dr_*}$

The scalar energy flux $\dot{E}^s_{r_+,\infty} = \sum_{\ell m} m\Omega_p k_{H,\infty} |Z^{r_+,\infty}_{\ell m\omega}|^2$ (Teukolsky, '73)

$$Z^{r_+,\infty}_{\ell m\omega} \equiv -\alpha \frac{X^{\infty,r_+}_{\ell m\omega}(r_0)}{WU^t} \frac{S^*_{\ell m}(\pi/2)}{\sqrt{r_0^2+a^2}} m_p/M$$

When numerically solving flux equation for a given a/M, we truncate the sum in l when the series evaluated at the ISCO, $r_0=r_{ISCO}$, converges to one part in 10^5 or better. This requires summing up to $l=17$ for $a/M=0.99$, but only up to $l=6$ for $a/M=0$. Such a scheme then implies that our numerical data is accurate to one part in 10^5, which is sufficient for this study.

General scalar tensor theories

$$S_{(J)} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[F(\phi)R - Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2U(\phi)\right] + S_{\rm mat}(\Psi_m; g_{\mu\nu})$$

$$S_{\rm mat} = \int d\tau m(\phi)$$

Massive BD when $F(\phi) = \phi, \quad Z(\phi) = \frac{\omega_{\rm BD}}{\phi}, \quad U(\phi) = m_s^2/2(\phi - \phi_0)$

Confromal transformations:

$$\begin{aligned} g^{(E)}_{\mu\nu} &= F(\phi)g_{\mu\nu}\,, \\ \Phi(\phi) &= \frac{1}{\sqrt{4\pi}}\int d\phi\left[\frac{3}{4}\frac{F'(\phi)^2}{F(\phi)^2} + \frac{1}{2}\frac{Z(\phi)}{F(\phi)}\right]^{1/2}, \\ A(\Phi) &= F^{-1/2}(\phi)\,, \\ V(\Phi) &= \frac{2U(\phi)}{F^2(\phi)}\,. \end{aligned}$$

Action in Einstein frame

$$S_{(E)} = \int d^4x\sqrt{-g^{(E)}}\left(\frac{R^{(E)}}{16\pi} - \frac{1}{2}g^{(E)}_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi - \frac{V(\Phi)}{16\pi}\right) - \int d\tau^{(E)}A(\Phi)m(\phi)\,, \quad (1)$$

Modified field equations

$$\begin{aligned} G^{(E)}_{\mu\nu} &= 8\pi\left(T^{(E)}_{\mu\nu} + T^{(\Phi)}_{\mu\nu}\right), \\ \Box^{(E)}\Phi &= \frac{1}{16\pi}\frac{\partial V}{\partial\Phi} + \frac{1}{\sqrt{-g^{(E)}}}\times\int d\tau^{(E)}\frac{\partial\left[A(\Phi)m(\phi)\right]}{\partial\Phi}\delta^{(4)}\left[x^\mu - y^\mu(\tau^{(E)})\right] \end{aligned}$$

stress-energy tensor associated with the matter action $T^{\mu\nu(E)} = 2(-g^{(E)})^{-1/2}\delta S_{\rm mat}/\delta g^{(E)}_{\mu\nu}$

scalar field energy-momentum tensor $T^{(\Phi)}_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g^{(E)}_{\mu\nu}(\partial\Phi)^2 - \frac{1}{16\pi}g^{(E)}_{\mu\nu}V(\Phi)$

The field equations depend only on three generic functions, V(Φ), A(Φ) and m(Φ).

consider scalar perturbations around a constant background scalar field, $\Phi^{(0)}$, and around a metric which is solution of Einstein's equations in an asymptotically-flat spacetime. Hence, we expand about $\Phi - \Phi^{(0)} = \varphi \ll 1$

$$V(\Phi) = \sum_{n=0} V_n \left(\Phi - \Phi^{(0)}\right)^n , \quad A(\Phi) = \sum_{n=0} A_n \left(\Phi - \Phi^{(0)}\right)^n , \quad m(\Phi) = \sum_{n=0} m_n \left(\Phi - \Phi^{(0)}\right)^n ,$$

Expanding the modified field equations in this way, we get:

$$G^{1,(E)}_{\mu\nu} = 8\pi T^{1,(E)}_{\mu\nu} ,$$
$$\left[\Box^{1,(E)} - \mu_s^2\right]\varphi = \alpha\, T^{1,(E)} .$$

$$T^{1,(E)} = A_0 m_0 \int \frac{d\tau^{(E)}}{\sqrt{-g^{0,(E)}}}\delta^{(4)}\left[x^\mu - y^\mu\left(\tau^{(E)}\right)\right] ,$$ is the trace of the stress-energy tensor for a test particle

$$\alpha \equiv \frac{A_1}{A_0} + \frac{m_1}{m_0} , \qquad \mu_s^2 \equiv \frac{V_2}{8\pi} ,$$

Grav. Const : (Damour et. al.)

$$G = \left[1 + \frac{1}{4\pi}\left(\frac{A'(\Phi)}{A(\Phi)}\right)^2\right] A(\Phi)^2 \sim A_0^2 + \frac{A_1^2}{4\pi} \,,$$

Sensitivity: (Will et al.)

$$s \equiv -\frac{d\log m}{d\log G} = -\frac{G(\Phi)}{m(\Phi)}\frac{m'(\Phi)}{G'(\Phi)}$$

$$\sim -\frac{\pi m_1 G}{A_1 m_0}\left(A_2 + 2\pi A_0\right)^{-1} \,.$$

$$\mu_s = \sqrt{\frac{2}{2\omega_{\rm BD}+3}}\, m_s \,.$$

Massless BD

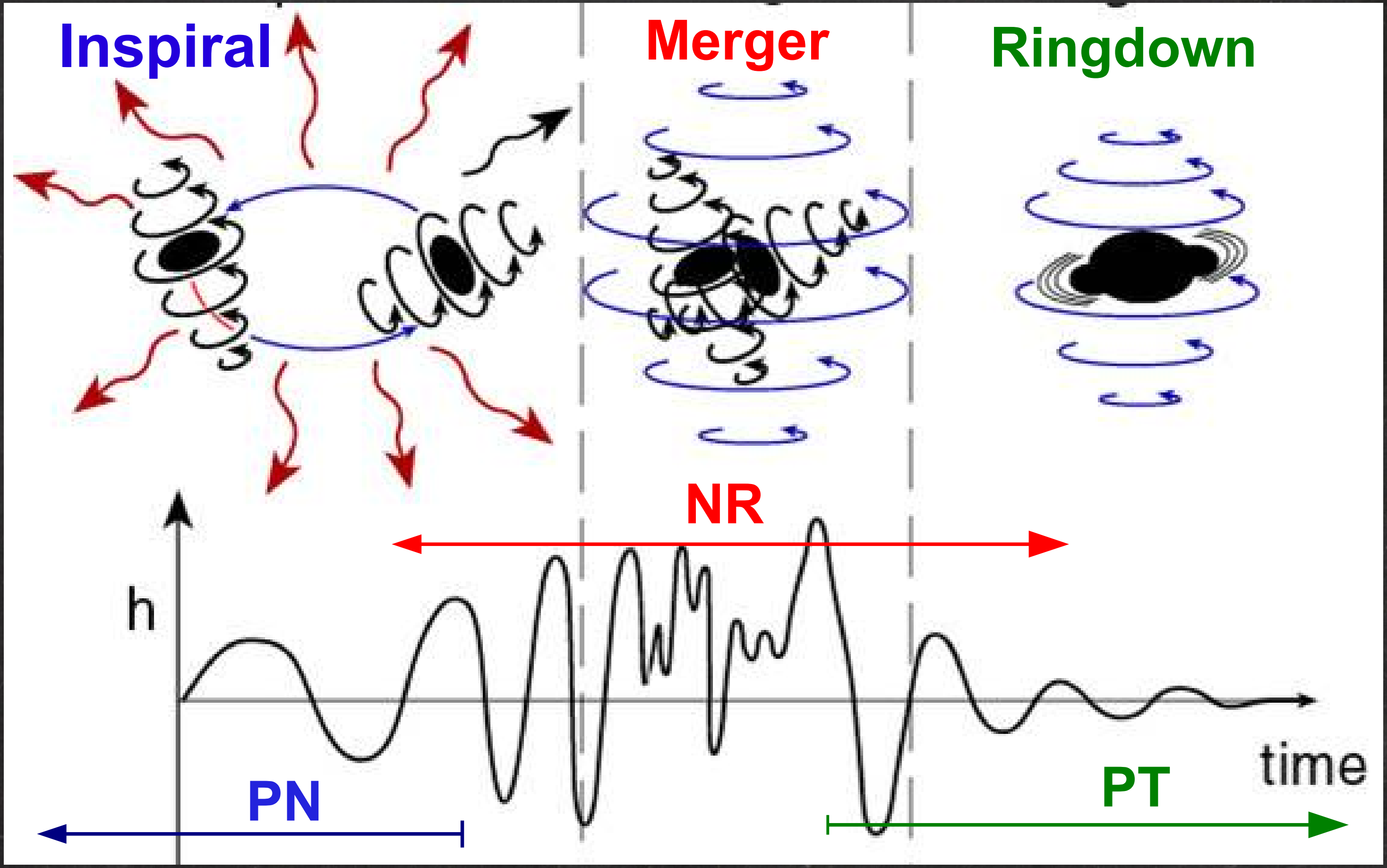
Massive BD

$$F(\phi) = \phi \,, \qquad Z(\phi) = \frac{\omega_{\rm BD}}{\phi} \,, \qquad U(\phi) = 0 \,, \qquad U(\phi) = \frac{m_s^2}{2}(\phi - \phi_0)^2 \,,$$

$$g^{(E)}_{\mu\nu} = \phi\, g_{\mu\nu} \,, \qquad \Phi = \frac{1}{\beta}\ln\phi \,, \quad A_0 = \frac{1}{\sqrt{\phi_0}} \,, \quad A_1 = -\frac{\beta}{2\sqrt{\phi_0}} \quad A_2 = \frac{\beta^2}{8\sqrt{\phi_0}} \,,$$

$$A(\Phi) = e^{-\beta\Phi/2} \,, \qquad T^{(E)}_{\mu\nu} = \frac{1}{\phi} T_{\mu\nu} \,, \quad \beta = \sqrt{16\pi/(2\omega_{\rm BD}+3)} \qquad G = \frac{1}{\phi_0}\frac{2\omega_{\rm BD}+4}{2\omega_{\rm BD}+3} \,.$$

A(n) (astro)-physical picture



(Fig. taken from Kip Thorne's lectures)