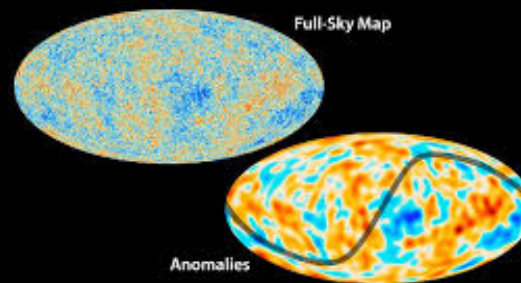
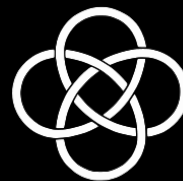


# Imprints of Isotropy Violated Gravitational Wave Background in CMB



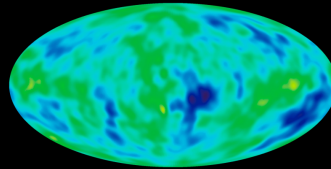
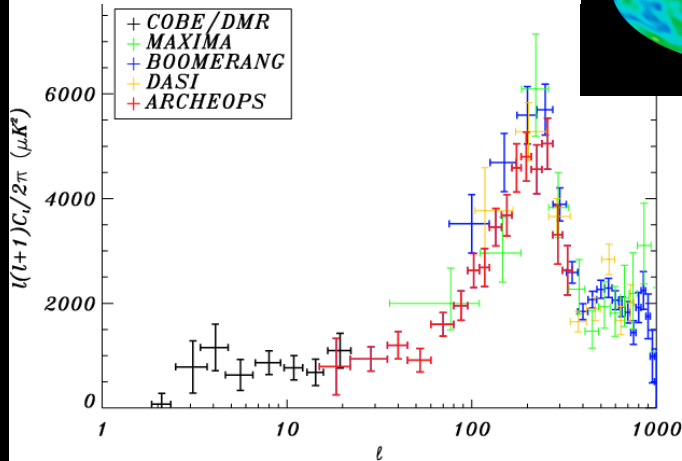
Suvodip Mukherjee



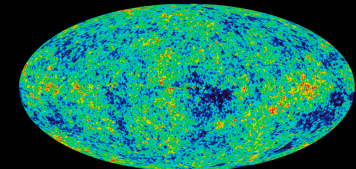
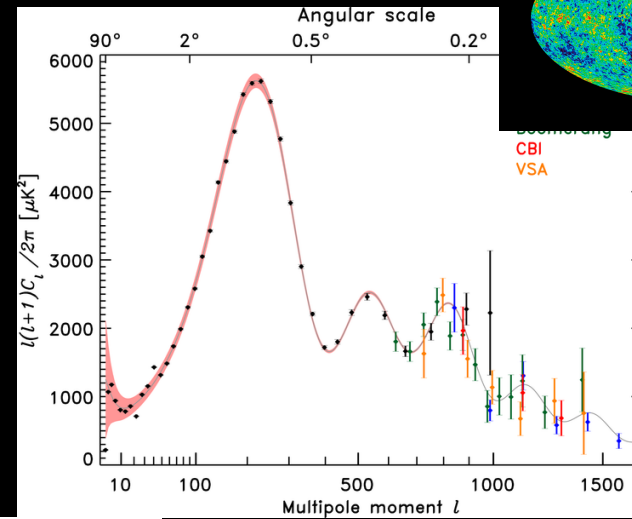
Supervisor  
Prof. Tarun Souradeep

# Journey so far

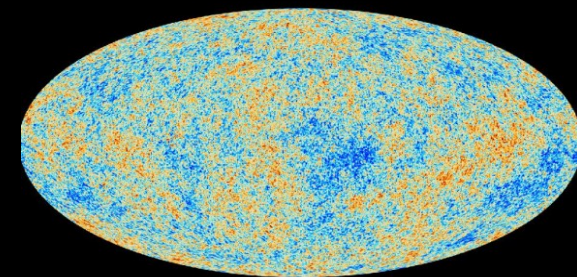
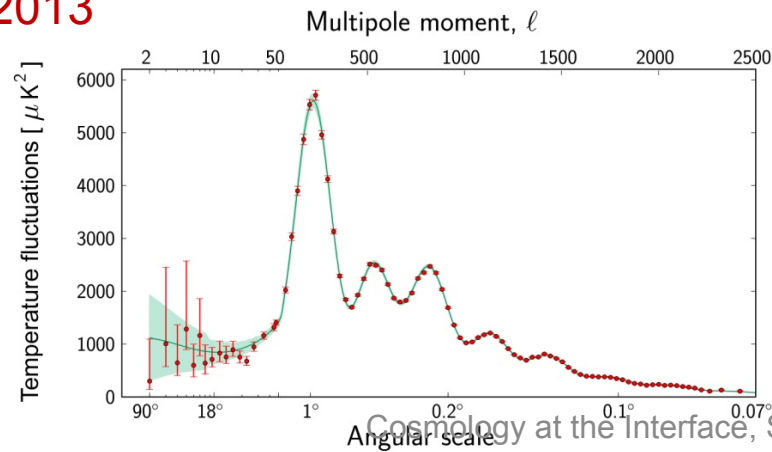
By COBE in 1992



By WMAP in 2003

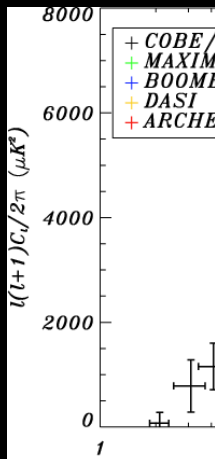


By Planck in 2013



# Journey so far

COBE in



By Planck

Planck Collaboration: Cosmological parameters

arXiv: 1303.5076

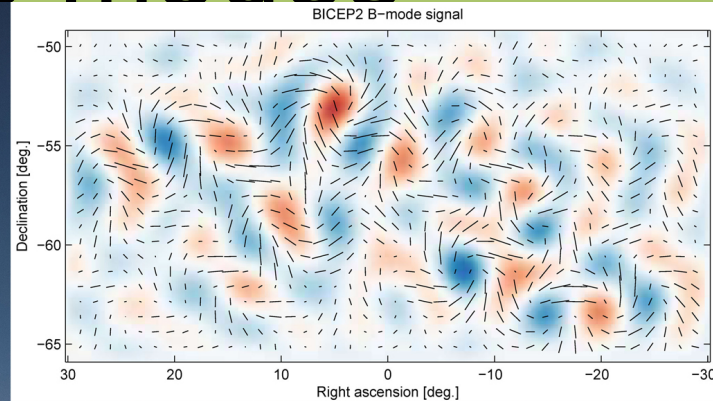
Parameter	Planck+WP		Planck+WP+highL		Planck+lensing+WP+highL		Planck+WP+highL+BAO	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022032	$0.02205 \pm 0.00028$	0.022069	$0.02207 \pm 0.00027$	0.022199	$0.02218 \pm 0.00026$	0.022161	$0.02214 \pm 0.00024$
$\Omega_c h^2$	0.12038	$0.1199 \pm 0.0027$	0.12025	$0.1198 \pm 0.0026$	0.11847	$0.1186 \pm 0.0022$	0.11889	$0.1187 \pm 0.0017$
$100\theta_{MC}$	1.04119	$1.04131 \pm 0.00063$	1.04130	$1.04132 \pm 0.00063$	1.04146	$1.04144 \pm 0.00061$	1.04148	$1.04147 \pm 0.00056$
$\tau$	0.0925	$0.089^{+0.012}_{-0.014}$	0.0927	$0.091^{+0.013}_{-0.014}$	0.0943	$0.090^{+0.013}_{-0.014}$	0.0952	$0.092 \pm 0.013$
$n_s$	0.9619	$0.9603 \pm 0.0073$	0.9582	$0.9585 \pm 0.0070$	0.9624	$0.9614 \pm 0.0063$	0.9611	$0.9608 \pm 0.0054$
$\ln(10^{10} A_s)$	3.0980	$3.089^{+0.024}_{-0.027}$	3.0959	$3.090 \pm 0.025$	3.0947	$3.087 \pm 0.024$	3.0973	$3.091 \pm 0.025$
$A_{100}^{PS}$	152	$171 \pm 60$	209	$212 \pm 50$	204	$213 \pm 50$	204	$212 \pm 50$
$A_{143}^{PS}$	63.3	$54 \pm 10$	72.6	$73 \pm 8$	72.2	$72 \pm 8$	71.8	$72.4 \pm 8.0$
$A_{217}^{PS}$	117.0	$107^{+20}_{-10}$	59.5	$59 \pm 10$	60.2	$58 \pm 10$	59.4	$59 \pm 10$
$A_{143}^{CIB}$	0.0	$< 10.7$	3.57	$3.24 \pm 0.83$	3.25	$3.24 \pm 0.83$	3.30	$3.25 \pm 0.83$
$A_{217}^{CIB}$	27.2	$29^{+6}_{-9}$	53.9	$49.6 \pm 5.0$	52.3	$50.0 \pm 4.9$	53.0	$49.7 \pm 5.0$
$A_{143}^{ISZ}$	6.80	...	5.17	$2.54^{+1.1}_{-1.9}$	4.64	$2.51^{+1.2}_{-1.8}$	4.86	$2.54^{+1.2}_{-1.8}$
$r_{143 \times 217}^{PS}$	0.916	$> 0.850$	0.825	$0.823^{+0.069}_{-0.077}$	0.814	$0.825 \pm 0.071$	0.824	$0.823 \pm 0.070$
$r_{143 \times 217}^{CIB}$	0.406	$0.42 \pm 0.22$	1.0000	$> 0.930$	1.0000	$> 0.928$	1.0000	$> 0.930$

**LambdaCDM Model is the Best fit Cosmological Model**

$\Omega_\Lambda$	0.6817	$0.685^{+0.018}_{-0.016}$	0.6830	$0.685^{+0.017}_{-0.016}$	0.6939	$0.693 \pm 0.013$	0.6914	$0.692 \pm 0.010$
$\sigma_8$	0.8347	$0.829 \pm 0.012$	0.8322	$0.828 \pm 0.012$	0.8271	$0.8233 \pm 0.0097$	0.8288	$0.826 \pm 0.012$
$z_{re}$	11.37	$11.1 \pm 1.1$	11.38	$11.1 \pm 1.1$	11.42	$11.1 \pm 1.1$	11.52	$11.3 \pm 1.1$
$H_0$	67.04	$67.3 \pm 1.2$	67.15	$67.3 \pm 1.2$	67.94	$67.9 \pm 1.0$	67.77	$67.80 \pm 0.77$
Age/Gyr	13.8242	$13.817 \pm 0.048$	13.8170	$13.813 \pm 0.047$	13.7914	$13.794 \pm 0.044$	13.7965	$13.798 \pm 0.037$
$100\theta$	1.04136	$1.04147 \pm 0.00062$	1.04146	$1.04148 \pm 0.00062$	1.04161	$1.04159 \pm 0.00060$	1.04163	$1.04162 \pm 0.00056$
$r_{drag}$	147.36	$147.49 \pm 0.59$	147.35	$147.47 \pm 0.59$	147.68	$147.67 \pm 0.50$	147.611	$147.68 \pm 0.45$

**Table 5.** Best-fit values and 68% confidence limits for the base  $\Lambda$ CDM model. Beam and calibration parameters, and additional nuisance parameters for “highL” data sets are not listed for brevity but may be found in the Explanatory Supplement (Planck Collaboration ES 2013).

# Recent Measurement of B modes



Detection of primordial gravitational waves with tensor to scalar ratio ( $r$ ) of 0.2

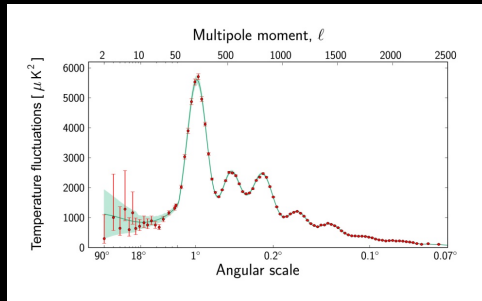
??

Possible origin from dust  
(Planck arXiv: 1409.5738, (2014))

or

Something else ??(This talk)

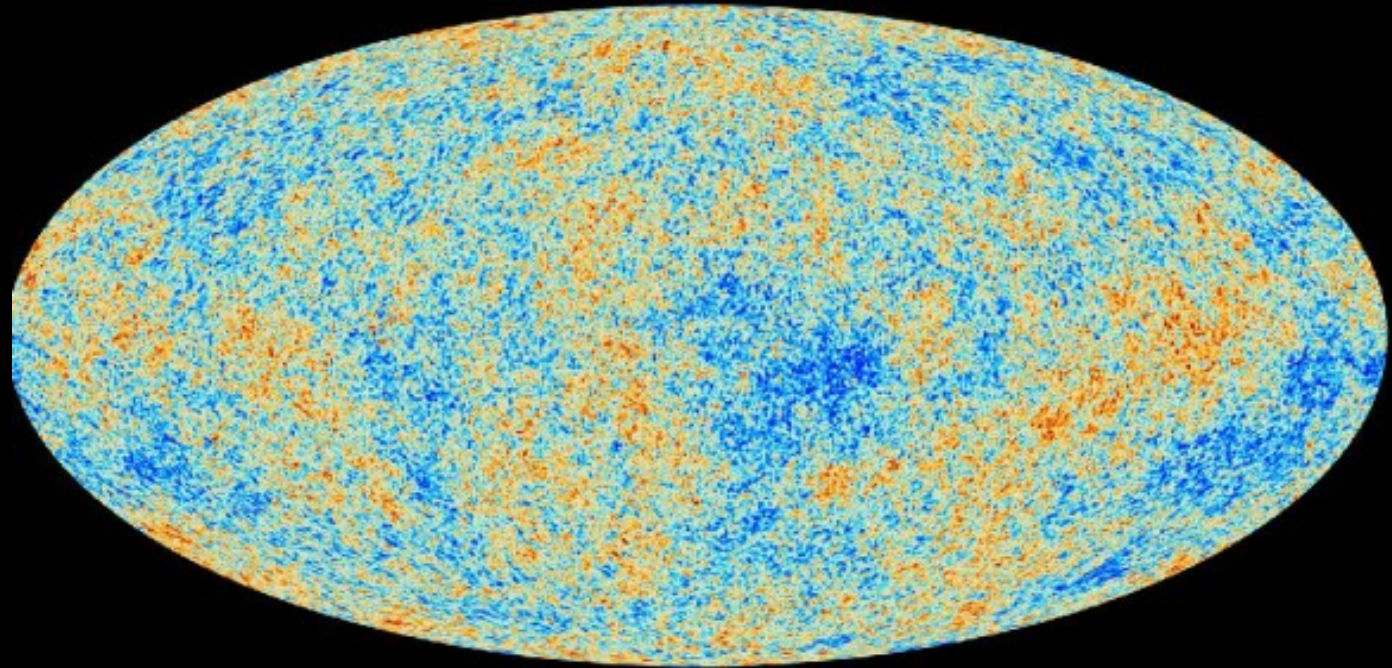
**Table 5.** Best-fit values and 68% confidence limits for the base  $\Lambda$ CDM model. Beam and calibration parameters, and additional nuisance parameters for “highL” data sets are not listed for brevity but may be found in the Explanatory Supplement (Planck Collaboration ES 2013).



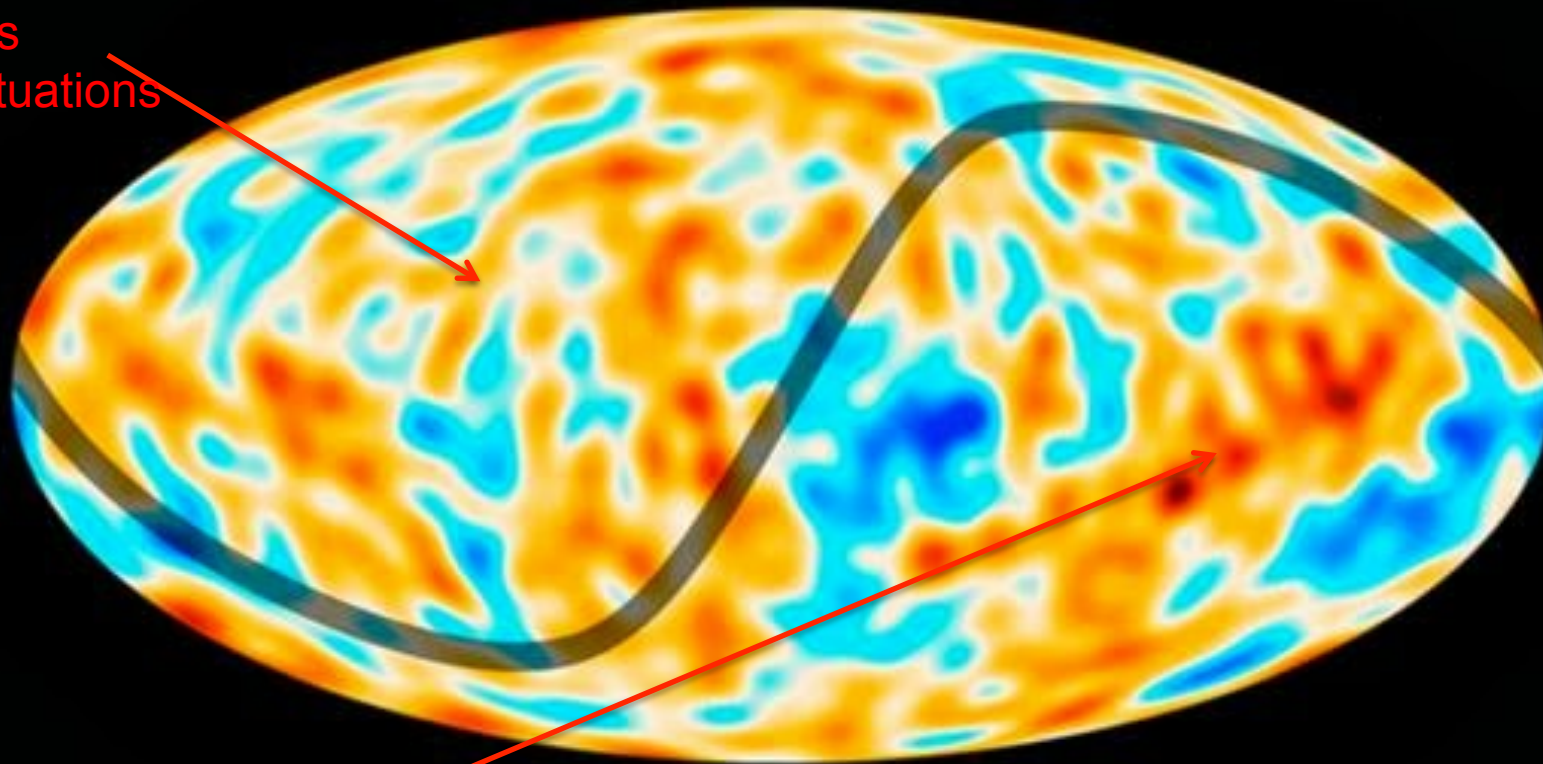
Planck Collaborations: Cosmological parameters

Parameter	Planck-MAP	Planck-MAP+lowℓ	Planck-MAP+lowℓ+SU(2) <sub>CMB</sub>	Planck-MAP+lowℓ+SU(2) <sub>CMB</sub> +ΛCDM
$H_0$	68.0 ± 0.9	68.0 ± 0.9	68.0 ± 0.9	68.0 ± 0.9
$\Omega_b h^2$	0.02204 ± 0.00018	0.02204 ± 0.00018	0.02204 ± 0.00018	0.02204 ± 0.00018
$\Omega_c h^2$	0.1188 ± 0.0007	0.1188 ± 0.0007	0.1188 ± 0.0007	0.1188 ± 0.0007
$\Omega_m h^2$	0.315 ± 0.008	0.315 ± 0.008	0.315 ± 0.008	0.315 ± 0.008
$n_s$	0.963 ± 0.006	0.963 ± 0.006	0.963 ± 0.006	0.963 ± 0.006
$\ln 10^{10} A_s$	3.090 ± 0.027	3.090 ± 0.027	3.090 ± 0.027	3.090 ± 0.027
$\tau$	0.086 ± 0.007	0.086 ± 0.007	0.086 ± 0.007	0.086 ± 0.007
$\sigma_8$	0.80 ± 0.02	0.80 ± 0.02	0.80 ± 0.02	0.80 ± 0.02
$\Omega_{DE}$	0.685 ± 0.007	0.685 ± 0.007	0.685 ± 0.007	0.685 ± 0.007
$w$	-0.95 ± 0.07	-0.95 ± 0.07	-0.95 ± 0.07	-0.95 ± 0.07
$\Omega_{DE} h^2$	0.28 ± 0.01	0.28 ± 0.01	0.28 ± 0.01	0.28 ± 0.01
$\Omega_{DE} h^2 w$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^2$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^3$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^4$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^5$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^6$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^7$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^8$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^9$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{10}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{11}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{12}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{13}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{14}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{15}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{16}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{17}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{18}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{19}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01
$\Omega_{DE} h^2 w^{20}$	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01	0.27 ± 0.01

Table S. Best-fit values and 68% confidence limits for the best- $\Lambda$ CDM model. Best-fit values and additional nuisance parameters for "full" data sets are not listed for brevity but may be found in the Supplementary Appendix (Planck Collaboration IX 2015).



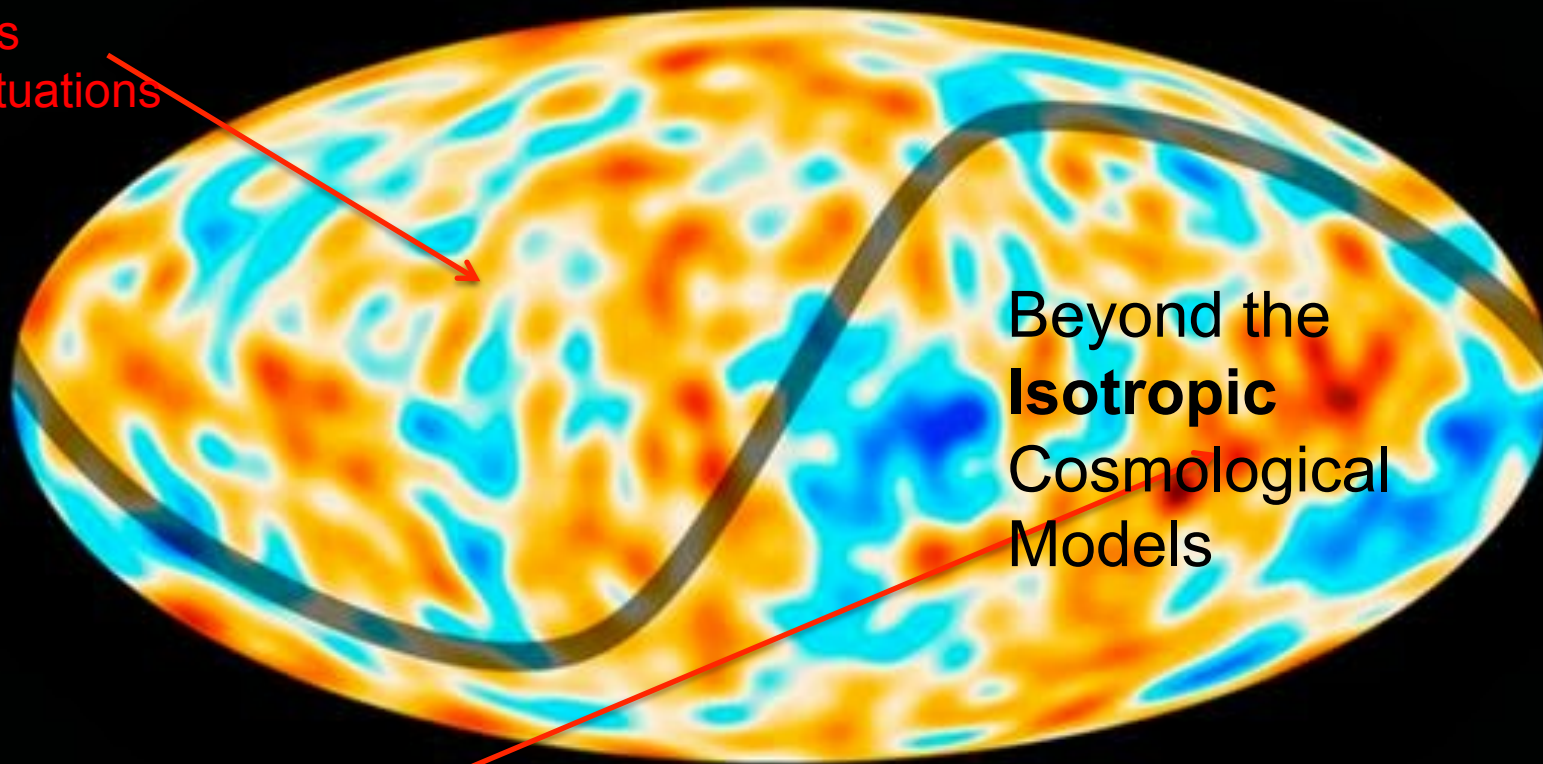
Less  
fluctuations



More  
fluctuations

At large angular scales signature of  
isotropy violated CMB field

Less  
fluctuations

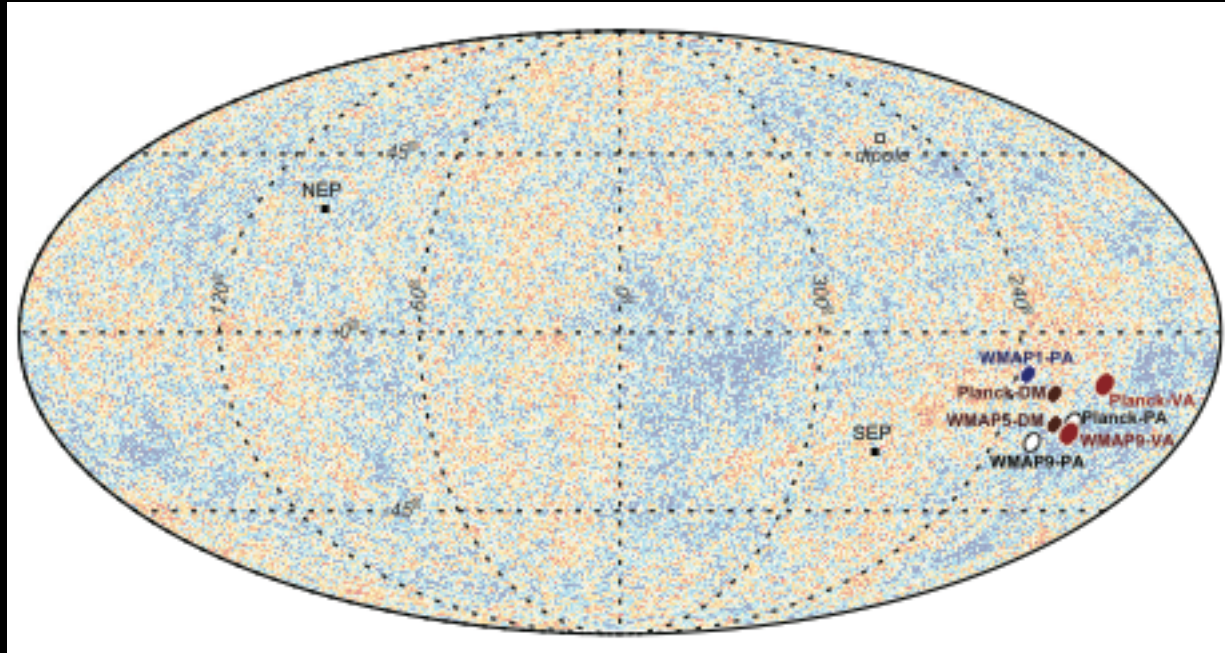


Beyond the  
**Isotropic**  
Cosmological  
Models

More  
fluctuations

At large angular scales signature of  
isotropy violated CMB field

# Signature observed in WMAP and Planck



F. K. Hansen, A. J. Banday and K. M. Gorski,  
M. N. R. A. S. 354, 641, (2004).

H. K. Eriksen et al., Astrophys. J., 605,14,  
(2004).

Y. Akrami et al., Astrophys. J., 784, L42,  
(2014)



# Language to quantify isotropy violation : **BipoSH coefficients**

Hajian and Souradeep  
 Astrophys. J. 597, L5, (2003).

$$\delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}).$$

$$Q(\hat{n}) \pm iU(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{+l} (E_{lm} \pm iB_{lm})_{\pm 2} Y_{lm}(\hat{n})$$

$$C^{XX'}(\hat{n}_1, \hat{n}_2) = \sum_{JNl_1l_2} A_{l_1l_2|XX'}^{JN} \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{JN}$$

**Bipolar Spherical Harmonics (BipoSH) coefficients**

# Isotropic Universe

## BipoSH coefficients are zero

Under the assumption of  
Statistical Isotropy

$$\left\langle a_{lm}^* a_{l'm'} \right\rangle = C_l^{TT} \delta_{ll'} \delta_{mm'},$$

$$\left\langle E_{lm}^* E_{l'm'} \right\rangle = C_l^{EE} \delta_{ll'} \delta_{mm'},$$

$$\left\langle B_{lm}^* B_{l'm'} \right\rangle = C_l^{BB} \delta_{ll'} \delta_{mm'},$$

$$\left\langle E_{l'm'}^* a_{lm} \right\rangle = C_l^{TE} \delta_{ll'} \delta_{mm'}.$$

# Isotropy violated Universe, BipoSH coefficients are **non-zero**

Under ~~the~~ assumption of  
Statistical Isotropy

$$\langle a_{lm}^* a_{l'm'} \rangle = C_l^{TT} \delta_{ll'} \delta_{mm'},$$

$$\langle E_{lm}^* E_{l'm'} \rangle = C_l^{EE} \delta_{ll'} \delta_{mm'},$$

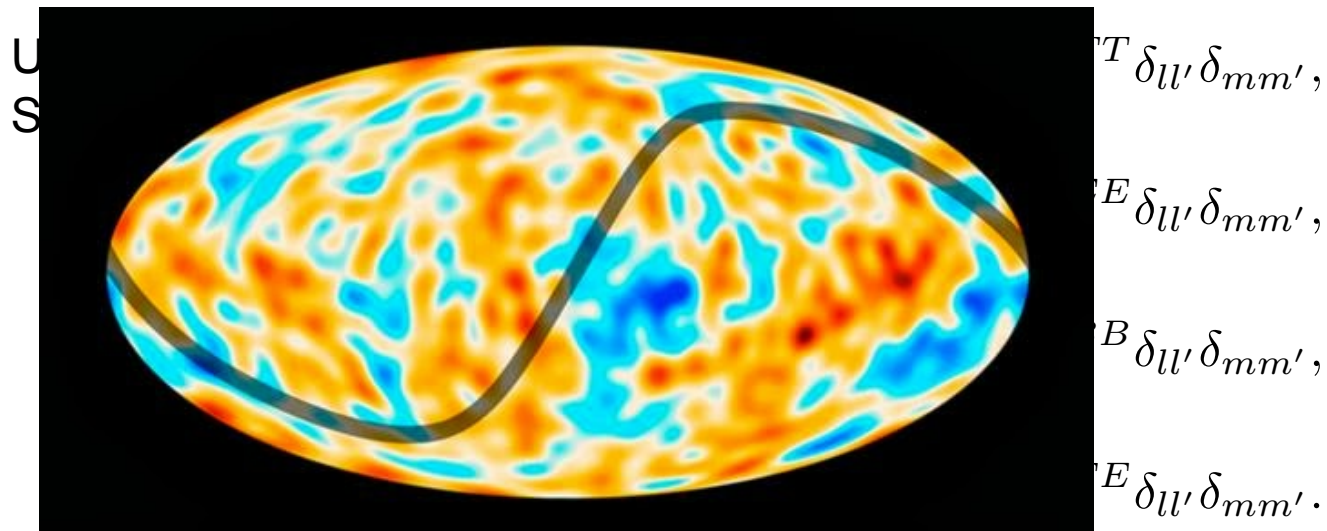
$$\langle B_{lm}^* B_{l'm'} \rangle = C_l^{BB} \delta_{ll'} \delta_{mm'},$$

$$\langle E_{l'm'}^* a_{lm} \rangle = C_l^{TE} \delta_{ll'} \delta_{mm'}.$$

$$\langle X_{l_1 m_1} X_{l_2 m_2}'^* \rangle = \sum_{LM} A_{l_1 l_2}^{LM} |X X'| (-1)^{m_2} C_{l_1 m_1 l_2 - m_2}^{LM}, \quad X = E, B, T$$

**Bipolar Spherical Harmonics (BipoSH) coefficients**

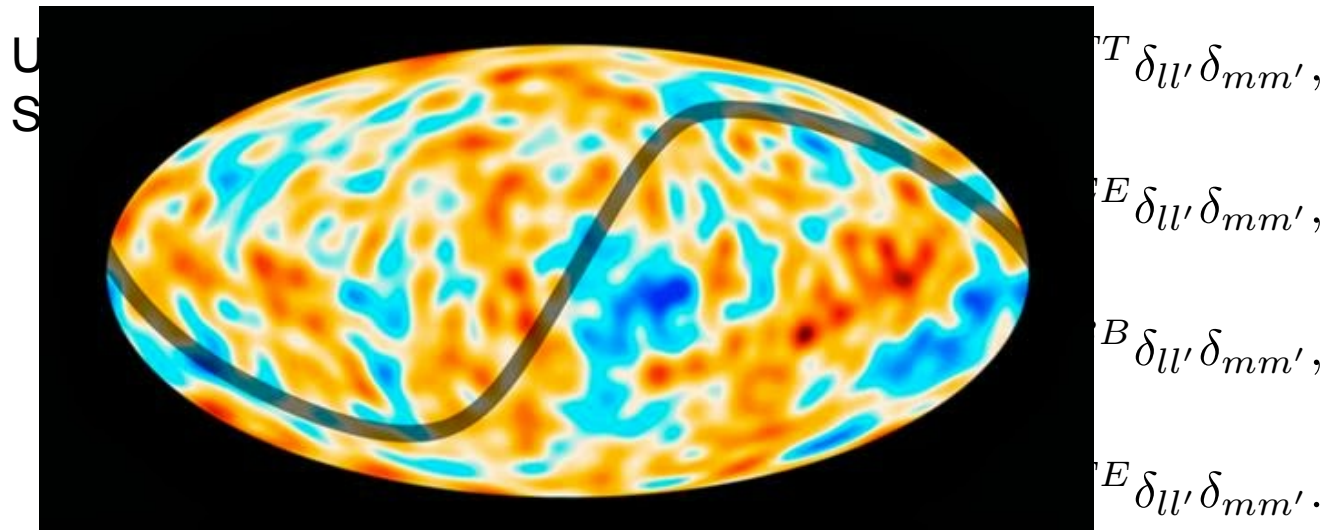
# Isotropy violated Universe, BipoSH coefficients are **non-zero**



$$\langle X_{lm_1} X'_{l+1m_2} \rangle = \sum_N A_{ll+1|XX'}^{1N} (-1)^{-m_2} C_{lm_1 l+1-m_2}^{1N}$$

**Dipolar  
Asymmetry**

# Isotropy violated Universe, BipoSH coefficients are **non-zero**



$$\langle X_{lm_1} X'_{l+1m_2} \rangle = \sum_N A_{ll+1|XX'}^{1N} (-1)^{-m_2} C_{lm_1 l+1-m_2}^{1N}$$

**Signal in Planck ??**

**Dipolar  
Asymmetry**

# Results from Planck-2013

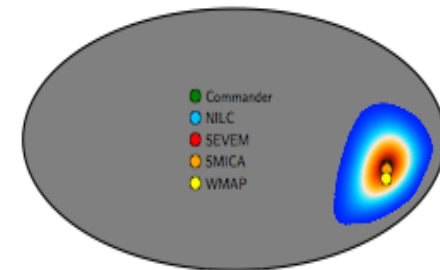
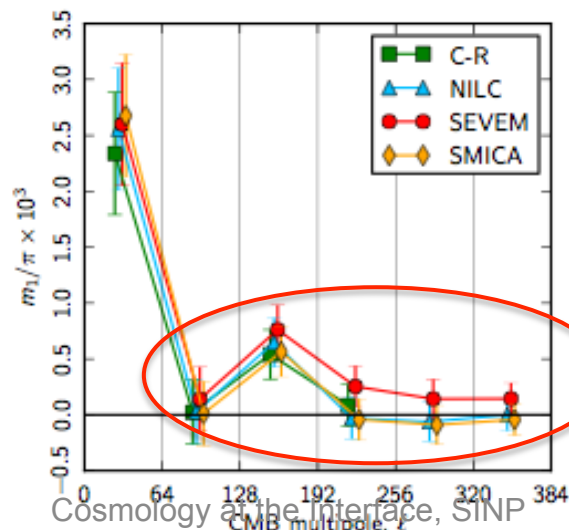
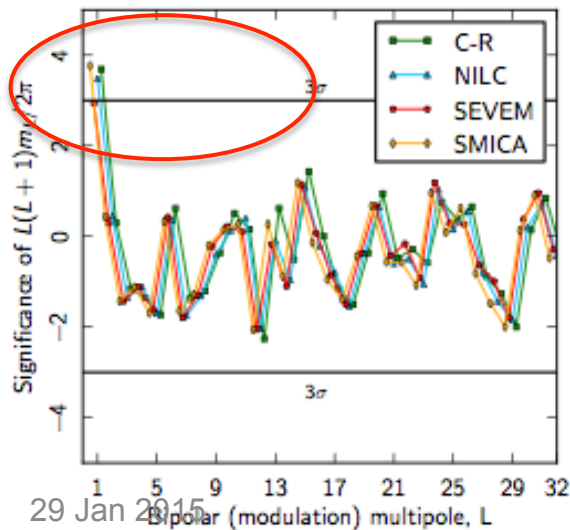
Souradeep et al.  
Planck XXIII  
A&A 571, A23, 2014

$$\tilde{T}(\hat{n}) = [1 + M(\hat{n})]T(\hat{n})$$

$$\tilde{T}(\hat{n}) = [1 + \sum_{LM} m_{LM} Y_{LM}(\hat{n})]T(\hat{n})$$

Only Dipolar (L=1)

( $l = 217.5^\circ, b = -20.2^\circ$ )



No signal at  
**small scale**

# Salient Features of isotropy violation

- Dipolar in nature. All higher L terms are consistent with zero.
- Scale dependent feature. Signal decays beyond ( $l > 60$ )

No Known model yet to explain these two features

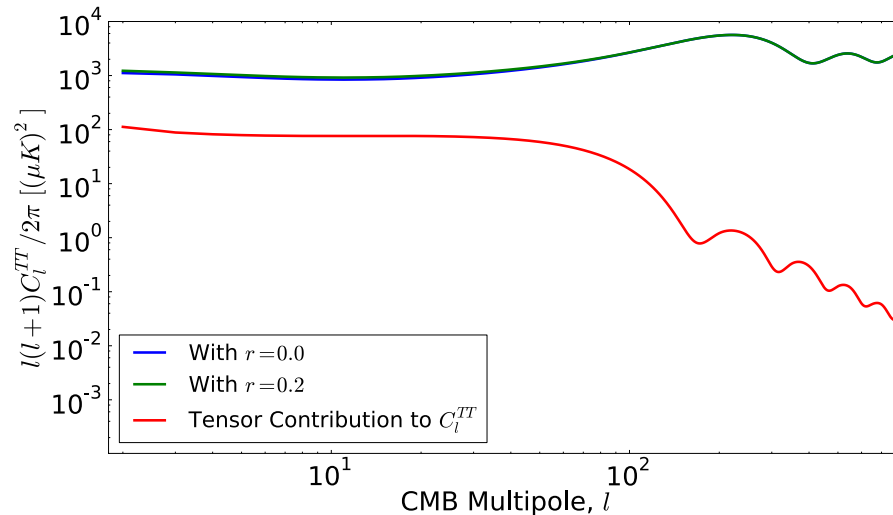
# Salient Features of isotropy violation

- Dipolar in nature. All higher L terms are consistent with zero.
- Scale dependent feature. Signal decays beyond ( $l > 60$ )

Physical Mechanisms which decays at large angular scale.

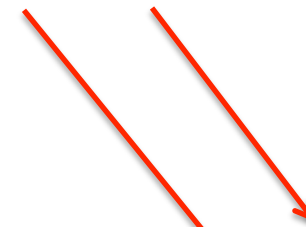


# Mixed Modulation Model



Mukherjee,  
arXiv:1412.2491

Need to determine



$$\tilde{X}(\hat{n}) = [1 + (1 - \alpha)M_s(\hat{n})]X^s(\hat{n}) + [1 + \alpha M_t(\hat{n})]X^t(\hat{n})$$

Unequal contribution from  
scalar and tensor

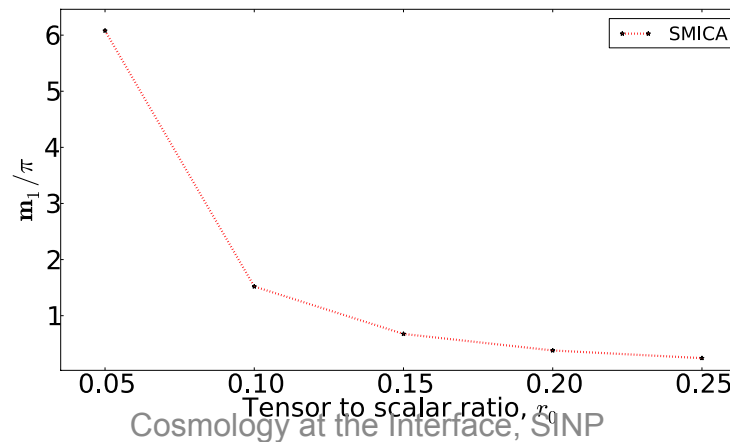
Solves the scale  
dependent problem.

# Determining the value of $M_t$ (A)

Mukherjee,  
arXiv:1412.2491

$${}^t A_{ll+1|TT}^{1N} = M_t^{1N} \frac{\Pi_{ll+1}}{\Pi_1} \frac{[{}^t C_l^{TT} + {}^t C_{l+1}^{TT}]}{\sqrt{4\pi}} C_{l0l+10}^{10}$$

$$\Pi_{l_1 l_2 \dots l_n} = \sqrt{(2l_1 + 1)(2l_2 + 1) \dots (2l_n + 1)}$$



# Mixed Modulation Model

Mukherjee,  
arXiv:1412.2491

$$\tilde{X}(\hat{n}) = [1 + (1 - \alpha)M_s(\hat{n})]X^s(\hat{n}) + [1 + \alpha M_t(\hat{n})]X^t(\hat{n})$$

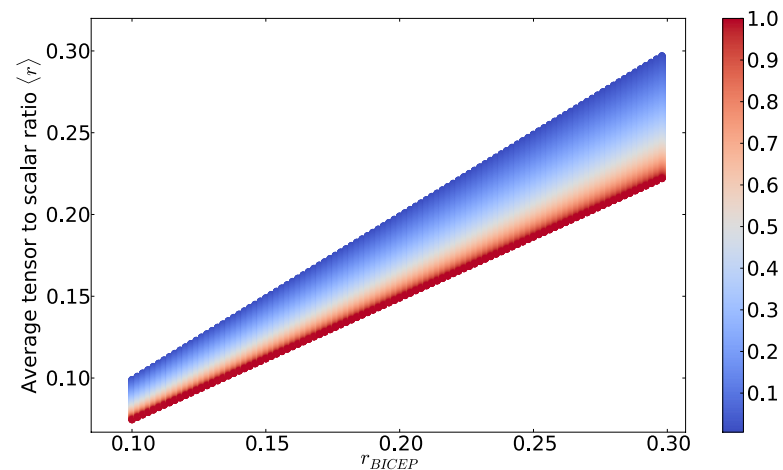
**Implication:**

**Direction dependent tensor to scalar ratio**

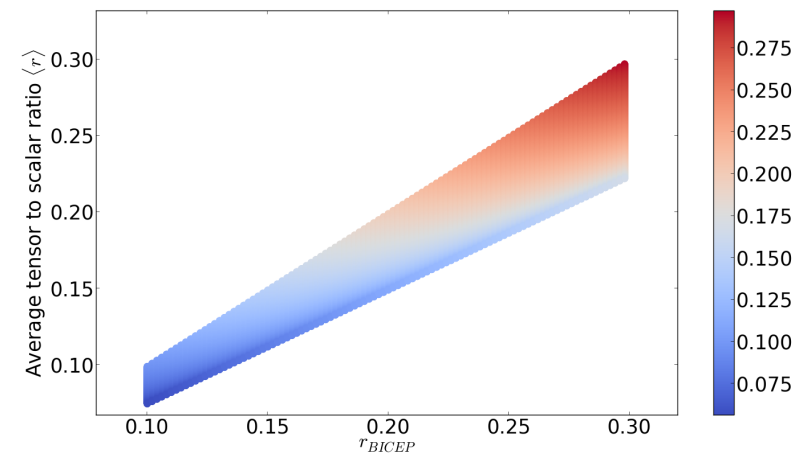
$$r(\hat{n}) = r_o [1 + \alpha A \hat{p} \cdot \hat{n}]^2$$
$$\langle r(\hat{n}) \rangle = r_o (1 + (\alpha A)^2 / 3)$$
$$A = 1.5 \sqrt{\frac{m_1}{\pi}}$$

# Determining the value of $\alpha$ and $r_0$

Mukherjee,  
arXiv:1412.2491



$\alpha$



$r_0$

Different values of tensor to scalar ratio from  
Planck and BICEP

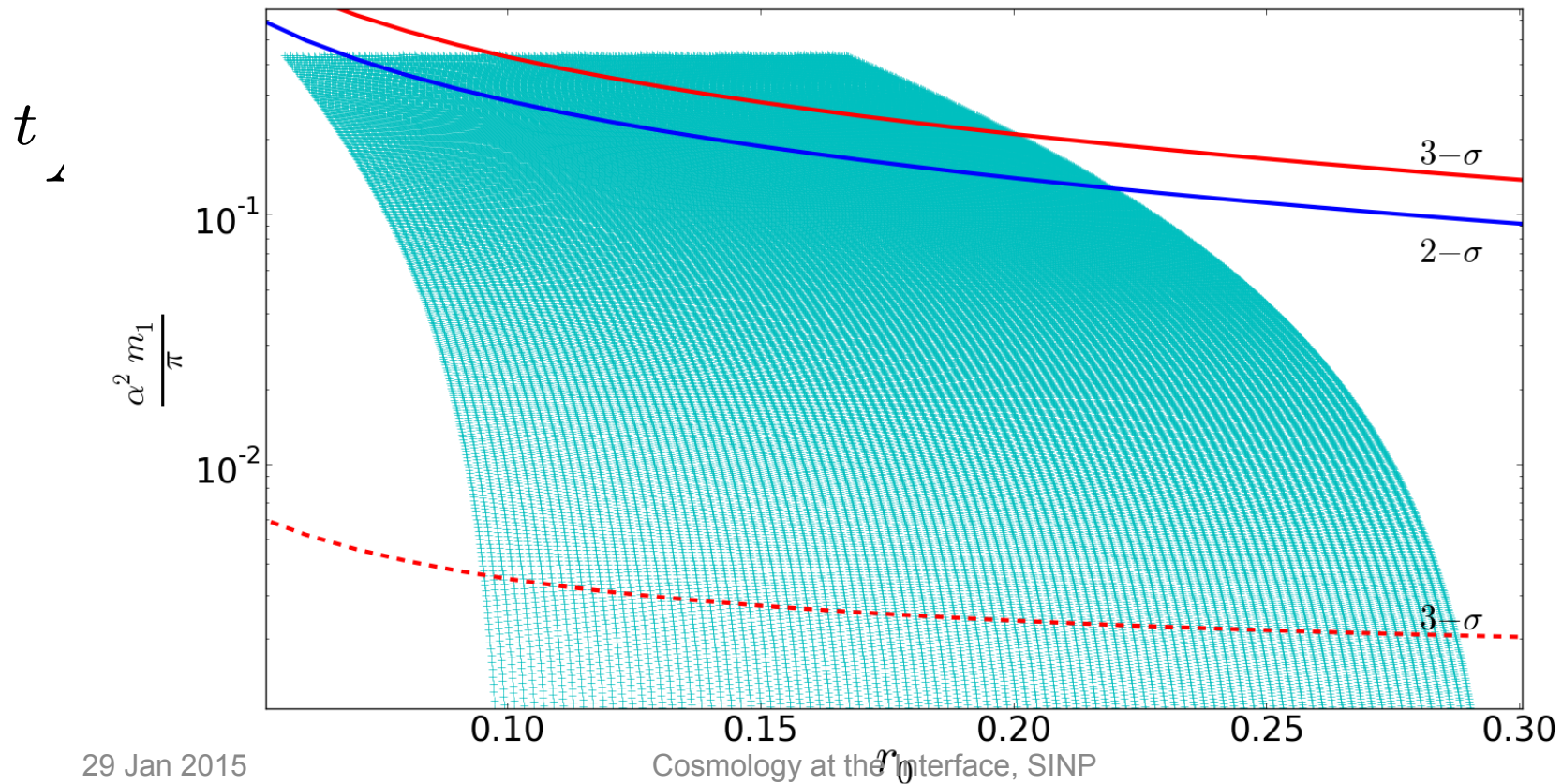
# Prediction of Mixed Modulation model: Measurable SI violated B modes

Mukherjee,  
arXiv:1412.2491

$${}^t A_{l+1|BB}^{1N} = \alpha M_t^{1N} \frac{\Pi_{ll+1}}{\Pi_1} \frac{[{}^t C_l^{BB} + {}^t C_{l+1}^{BB}]}{\sqrt{4\pi}} C_{l\ 2l+1-2}^{10}$$

# Prediction of Mixed Modulation model: Measurable SI violated B modes

Mukherjee,  
arXiv:1412.2491



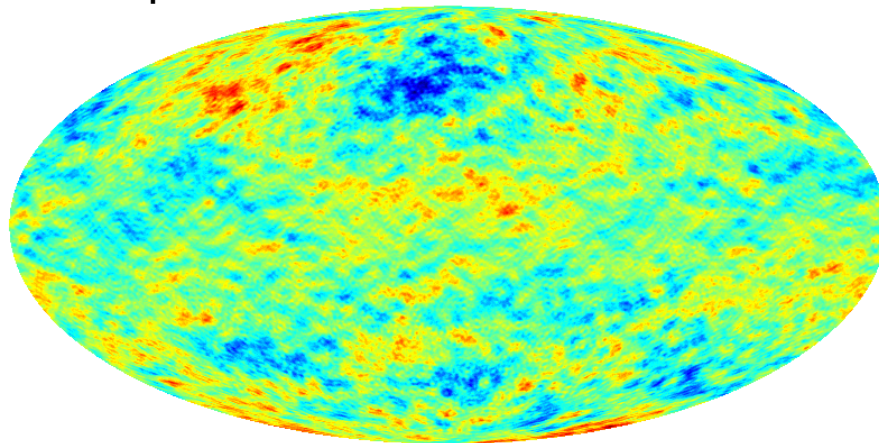
-2


# Comparison of Mixed modulation maps and scale dependent temperature modulation map

- We developed a numerical algorithm **CoNIGs** (**C**ode for **N**on **I**sotropic **G**aussian **s**ky) to produce SI violated Gaussian realization of CMB

Mukherjee and Souradeep Phys. Rev. D 89 063013 (2014)

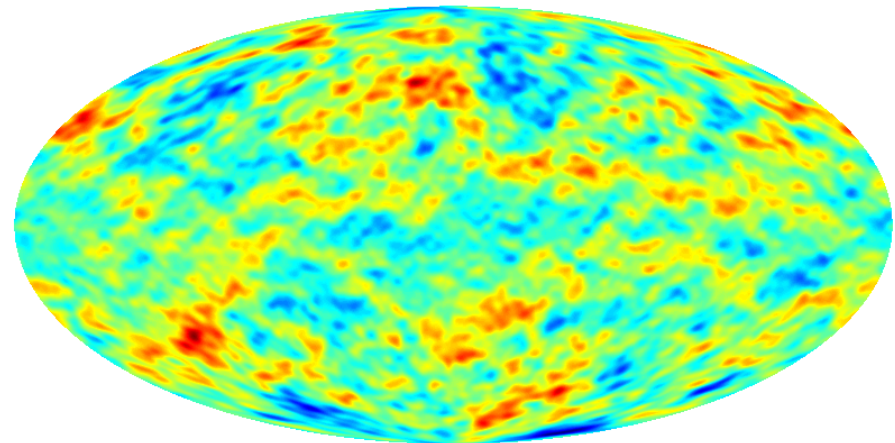
Scale dependent temperature modulation




-0.972  +0.916

29 Jan 2015

Mixed Modulation model



-0.704  +0.638

Cosmology at the Interface, SINP

23

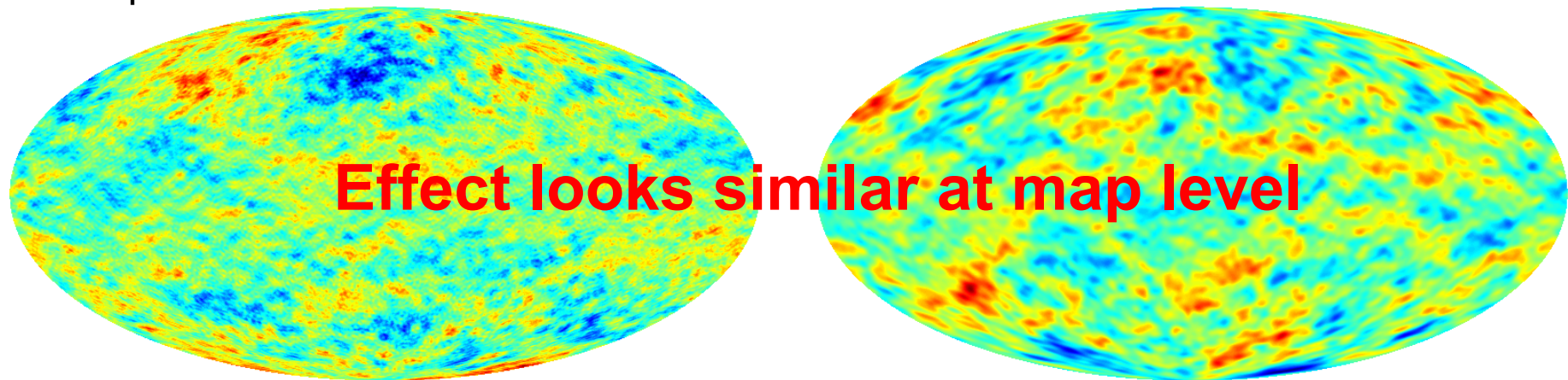
# Comparison of Mixed modulation maps and scale dependent temperature modulation map

- We developed a numerical algorithm **CoNIGs** (**C**ode for **N**on **I**sotropic **G**aussian **s**ky) to produce SI violated Gaussian realization of CMB

Mukherjee and Souradeep Phys. Rev. D 89 063013 (2014)

Scale dependent  
temperature modulation

Mixed Modulation model



-0.972  +0.916

-0.704  +0.638



- Dipolar in nature. All higher  $L$  terms are consistent with zero.
- **Scale dependent feature. Signal decays beyond ( $l > 60$ )**
- **Physical Mechanisms which decays at large angular scale: Stochastic Gravitational Wave Background**

# Possible Origin from inflation

Mukherjee and Souradeep [in preparation]

$$X = \frac{H_a}{H} \ll 1$$

$$Y = \frac{\dot{H}_a}{\dot{H}} \ll 1$$

A. A. Abolhasani, et al. Phys. Rev. D 89, 063511, (2014).

L. Dai, et al. Phys. Rev. D, 87, 123005, (2013).

$$\tilde{P}_s(k) = P_s(k) [1 + (4X - Y) \hat{p} \cdot \hat{x}]$$

$$\tilde{P}_t(k) = P_t(k) [1 + 2X \hat{p} \cdot \hat{x}]$$

$$\tilde{n}_s - 1 = (n_s - 1) [1 + (Y - 2X) \hat{p} \cdot \hat{x}].$$

# Possible Origin from inflation

Mukherjee and Souradeep [in preparation]

$$X = \frac{H_a}{H} \ll 1$$

$$Y = \frac{\dot{H}_a}{\dot{H}} \ll 1$$

Can be zero

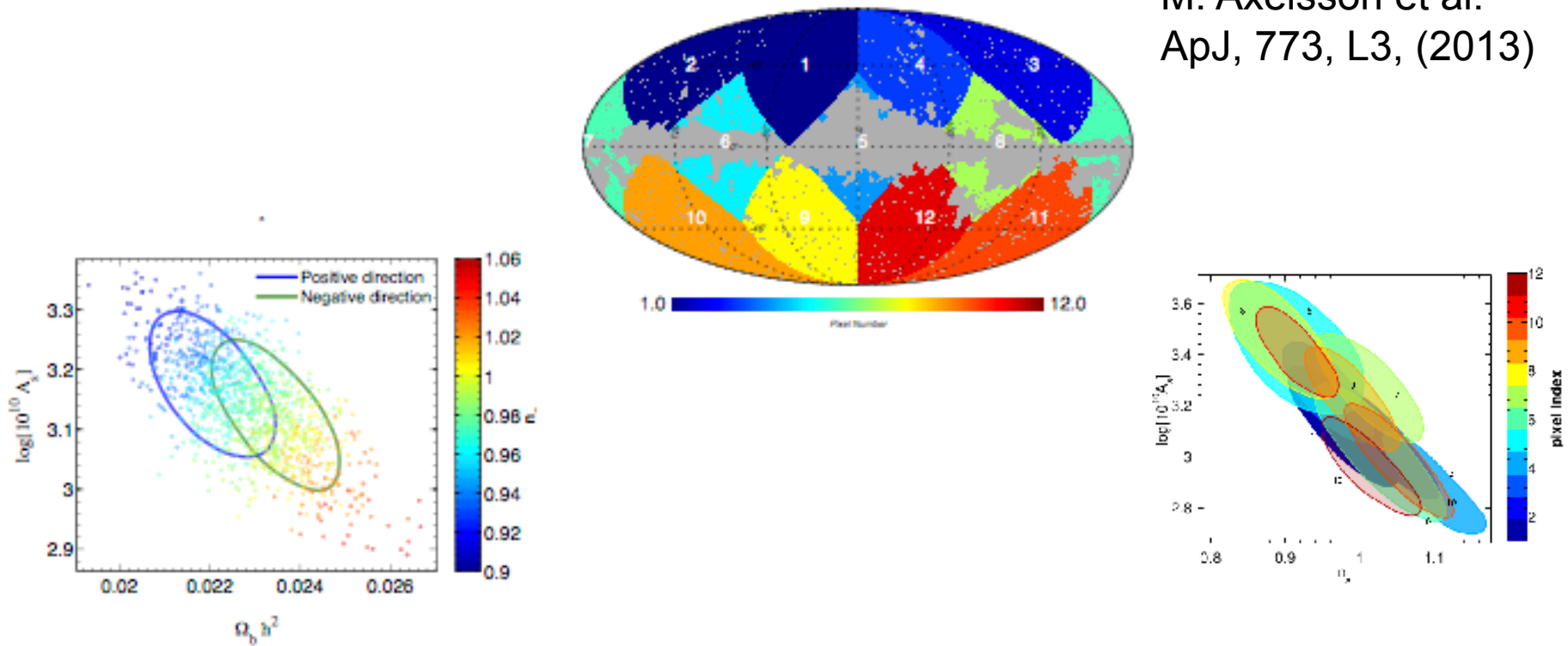
$$\tilde{P}_s(k) = P_s(k) [1 + (4X - Y) \hat{p} \cdot \hat{x}]$$

$$\tilde{P}_t(k) = P_t(k) [1 + 2X \hat{p} \cdot \hat{x}] - \alpha M_t$$

$$\tilde{n}_s - 1 = (n_s - 1) [1 + (Y - 2X) \hat{p} \cdot \hat{x}].$$

# Signature of direction dependent cosmological parameters in Planck

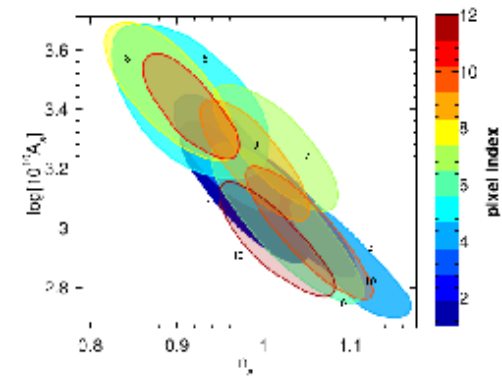
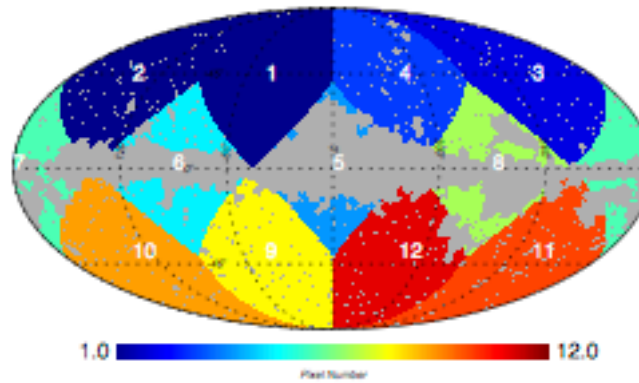
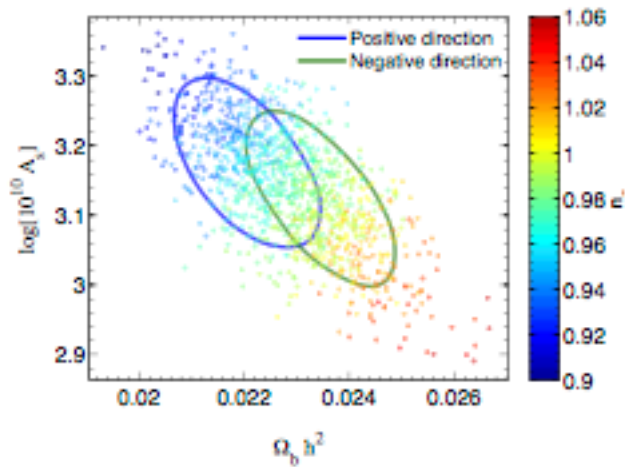
M. Axelsson et al.  
ApJ, 773, L3, (2013)



$$\tilde{n}_s - 1 = (n_s - 1) [1 + (Y - 2X) \hat{p} \cdot \hat{x}].$$

# Signature of direction dependent cosmological parameters in Planck

M. Axelsson et al.  
ApJ, 773, L3, (2013)



Mukherjee and Souradeep [in preparation]

$$4X \geq Y > 2X$$

# Conclusions

- A signature of isotropy violated CMB temperature field is observed in both WMAP and Planck.
- Signature is present only at large angular scales ( $l < 60$ ).
- This can be model by isotropy violated Stochastic Gravitational Wave Background.
- Implies direction dependent tensor to scalar ratio and is measurable from Planck and BICEP-2.
- Observable dipolar BipoSH spectra for B mode polarization are measurable from Planck and PRISM. As a result MM model can be falsified.
- These signatures can be related to the anisotropic Hubble parameter and its derivative.

# Conclusions

- A signature of isotropy violated CMB temperature field is observed in both WMAP and Planck.
- Signature is present only at large angular scales ( $l < 60$ ).
- This can be model by isotropy violated Gravitational Wave Background.
- Implies direction dependent tensor to scalar ratio and is measurable from Planck and BICEP-2.
- Observable dipolar BipoSH spectra for B mode polarization are measurable from Planck and PRISM. As a result MM model can be falsified.
- These signatures can be related to the anisotropic Hubble parameter and its derivative.

Thank you