# Quintessence and D3-Brane/Anti-brane Universe

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( With: Abhishek K. Singh, Sunita Singh and Supriya Kar)

Motivation:

- Brane-anti brane pair formation at cosmological horizon
- An accelerated expanding universe in the cosmology

$\Longrightarrow$ a growth in extra dimension

## Overview

1. Quintessence
2. String-brane model
   $\rightarrow$ Geometric torsion
   $\rightarrow$ Gravitational $(3\bar{3})$-brane
   $\rightarrow$ Torsion curvature
   $\rightarrow$ Irreducible curvature scalar $\mathcal{K}$
3. Resulting geometries
4. Outcomes

## Quintessence:

- Accelerated cosmic expansion.

(A.G. Riess et al. Astron.J.116 (1998))

- Friedmann equation: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$
  Equation of state: $\omega = \frac{P}{\rho}$

  for $\ddot{a} > 0 \quad \Rightarrow \quad \omega < -\frac{1}{3}$
- Cosmological observation $\Rightarrow -1 < \omega < \frac{-1}{3}$
- Negative pressure and variable energy density
- The presence of repulsive gravity called dark energy
- Quintessence is one of the candidate for dark energy
- Quintessence $\longrightarrow$ apparently fifth hidden fundamental force
  $\longrightarrow$ described by dynamical scalar field
  $\longrightarrow$ scalar field because of minimal coupling

# String-brane model

JHEP **05** (2013) 033; Phys.Rev.**D 88** (2013) 066001 ; Nucl.Phys.**B 879** (2014) 216; Int.J.Mod.Phys.**A 29** (2014) 1450164

( A.K. Singh, S. Singh, S. Kar and KPP)

- Consider the non-linear $U(1)$ dynamics with a background $G^{(NS)}_{\mu\nu}$
- Poincare dual $\mathcal{F}_{\mu\nu} \rightarrow H_{\mu\nu\lambda}$

$\Longrightarrow$ Dynamics of Kalb- Rammond field on $D_4$-brane.

$$S = -T_{D_4} \int d^5x \sqrt{-G^{(NS)}}\; H_{\mu\nu\lambda} H^{\mu\nu\lambda} .$$

- Where $\quad G^{NS}{}_{\mu\nu} = \left( g_{\mu\nu} - B^{(NS)}_{\mu\lambda} g^{\lambda\rho} B^{(NS)}_{\rho\nu} \right)$

( Seiberg and Witten; JHEP (1999) )

- Equation of motion is

$$\partial_\lambda H^{\lambda\mu\nu} + \frac{1}{2} G^{\alpha\beta}_{(NS)} \partial_\lambda \; G^{(NS)}_{\alpha\beta} \; H^{\lambda\mu\nu} = 0 \; .$$

## Geometric torsion

- Generically $\mathcal{D}_\lambda$ absorbs $H_3$ and may be defined as:
  $\mathcal{D}_\lambda \hat{B}_{\mu\nu} = \nabla_\lambda \hat{B}_{\mu\nu} + \frac{1}{2} H_{\lambda\mu}{}^{\rho} \hat{B}_{\rho\nu} - \frac{1}{2} H_{\lambda\nu}{}^{\rho} \hat{B}_{\rho\mu}$

$$\begin{aligned} \Longrightarrow \mathcal{H}_{\mu\nu\lambda} &= \mathcal{D}_\mu \hat{B}_{\nu\lambda} + \text{cyclic in } (\mu, \nu, \lambda) \\ &= H_{\mu\nu\lambda} + \left( H_{\mu\nu\alpha} \hat{B}^{\alpha}_{\lambda} + \text{cyclic} \right) + H_{\mu\nu\beta} \hat{B}^{\beta}_{\alpha} \hat{B}^{\alpha}_{\lambda} + \ldots \end{aligned}$$

- However for a constant NS two form: $\nabla_\mu B^{NS}_{\nu\lambda} = 0$
  $\mathcal{D}_\lambda B^{NS}_{\mu\nu} = \frac{1}{2} H_{\lambda\mu}{}^{\rho} B^{NS}_{\rho\nu} - \frac{1}{2} H_{\lambda\nu}{}^{\rho} B^{NS}_{\rho\mu}$

$$\mathcal{H}_{\mu\nu\lambda} = \left( H_{\mu\nu\alpha} B^{(NS)\alpha}{}_{\lambda} + \text{cyclic} \right) + H_{\mu\nu\beta} B^{(NS)\beta}{}_{\alpha} B^{(NS)\alpha}{}_{\lambda} + \ldots$$

$\Longrightarrow B_2^{(NS)}$ (couples to $H_3$) defines a Geometric Torsion: $\mathcal{H}_3$

- $\mathcal{H}_3$ is described by a dynamical $B_2^{(NS)}$ in the frame-work

- The gauge invariance of $\mathcal{H}_3^2$ is achieved with a notion of metric

$$f_{\mu\nu} \;=\; C\; \mathcal{H}_{\mu\lambda\rho}\bar{\mathcal{H}}^{\lambda\rho}{}_{\nu}\ .$$

- The stringy (geometric torsion) correction modifies the metric:

$$\begin{aligned} G_{\mu\nu} \;&=\; G^{(NS)}_{\mu\nu} \;+\; f_{\mu\nu} \\ &=\; \left(g_{\mu\nu} - B^{(NS)}_{\mu\lambda} B^{(NS)\lambda}_{\nu} \;+\; C\; \bar{\mathcal{H}}_{\mu\lambda\rho}\; \mathcal{H}^{\lambda\rho}{}_{\nu}\right) \end{aligned}$$

- $\mathcal{H}_3 \Rightarrow$ a gravitational pair of $(3\bar{3})$-brane by $B_2$ on $D_4$-brane
- Schwinger Mechanism: $A_\mu \xrightarrow{vacuum}$ particle + anti-particle
- Hawking Radiation : $A_\mu \xrightarrow{black\;\;hole\;\;horizon}$ particle + anti-particle

Hawking; CMP (1975)

## Gravitational $(3\bar{3})$-brane

- Since a two form quanta is explored to produce a pair in $U(1)$
  $\Longrightarrow$ Stringy (not fundamental) quantum gravity effects
- A pair is created across a (cosmological) horizon
  $\Longrightarrow$ They cannot annhiliate each other
- They move in opposite directions
  $\Longrightarrow$ An extra fifth (transverse) dimension in between
- The extra dimension is hidden to the 3-brane and $\bar{3}$-brane universes $\Longrightarrow$ Significance of closed string modes in the scenario
- Vacuum created 3-brane $\longrightarrow$ a gravitational 3-brane vacuum
  $\Longrightarrow$ (Low energy) Type IIA on $S^1$ + BPS $D_3$ -brane

## Torsion curvature

- Commutators of the covariant derivative on a $D_4$ -brane:
  (i) on a scalar field:

$$\Big[\mathcal{D}_\mu\ ,\ \mathcal{D}_\nu\Big]\phi\ =\ \mathcal{H}_{\mu\nu}{}^{\lambda}\ \partial_\lambda\phi$$

  $\Rightarrow$ Nontrivial curvature even with a linear $\phi(x)$ $\rightarrow$ Quintessence !

  (ii) on a gauge field:

$$\Big[\mathcal{D}_\mu\ ,\ \mathcal{D}_\nu\Big]A_\lambda =\ \mathcal{K}_{\mu\nu\lambda}{}^{\rho}A_\rho$$

  Where $\mathcal{K}_{\mu\nu\lambda}{}^{\rho}\ \equiv\ \partial_\mu\mathcal{H}_{\nu\lambda}{}^{\rho} - \partial_\nu\mathcal{H}_{\mu\lambda}{}^{\rho} + \mathcal{H}_{\mu\lambda}{}^{\sigma}\mathcal{H}_{\nu\sigma}{}^{\rho} - \mathcal{H}_{\nu\lambda}{}^{\sigma}\mathcal{H}_{\mu\sigma}{}^{\rho}$

- For a non propagating torsion: $\mathcal{K}_{\mu\nu\lambda\rho} \rightarrow R_{\mu\nu\lambda\rho}$
  $\longrightarrow$ Riemannian (Einstein vacuum)
  $\Longrightarrow$ D-brane world volume correction becomes insignificant

## Irreducible curvature scalar $\mathcal{K}$

$$4\mathcal{K}_{\mu\nu} = -\left(2\partial_\lambda \mathcal{H}^\lambda{}_{\mu\nu} + \mathcal{H}_{\mu\rho}{}^\lambda \mathcal{H}_{\lambda\nu}{}^\rho\right)$$
$$\text{and} \quad \mathcal{K} = -\tfrac{1}{4}\mathcal{H}_{\mu\nu\lambda}\mathcal{H}^{\mu\nu\lambda}$$

- Geometric torsion dynamics on a gravitational pair of $3\bar{3}$-brane

$$S_{3-\bar{3}} = \frac{1}{3C_2^2}\int d^5x\sqrt{-G^{(NS)}}\ \mathcal{K}^{(5)}$$

- Extra dimension between the brane/anti-brane $\Longrightarrow \mathcal{K}^5$ on $\mathrm{S}^1$

$$S_{3-\bar{3}} = \frac{1}{3C^2}\int_{\bar{3}b} d^4x\sqrt{-G^{(NS)}}\ \mathcal{K}\ -\ \frac{1}{4}\int_{3b} d^4x\sqrt{-G^{(NS)}}\bar{\mathcal{F}}_{\mu\nu}\mathcal{F}^{\mu\nu}$$
$$\text{where} \qquad \bar{\mathcal{F}}_{\mu\nu} = (2\pi\alpha')\left(F_{\mu\nu} + \mathcal{H}^\lambda_{\mu\nu}\mathcal{A}_\lambda\right)$$

- In a low energy limit, i .e. for a large 5th dimension $\mathcal{F}_{\mu\nu} \rightarrow F_{\mu\nu}$
- The Poincare dual of $\mathcal{K}$ *i.e.* an axion, on an $\bar{3}$-brane

$\Longrightarrow$ a quintessence in the frame-work

- Two form on $S^1 \Rightarrow$ does not generate a dilaton
- The energy-momentum tensor is computed in a gauge choice,

$$(2\pi\alpha')^2 T_{\mu\nu} = \left( G^{(NS)}_{\mu\nu} + \tilde{C}\ \bar{\mathcal{F}}_{\mu\lambda}\bar{\mathcal{F}}^{\lambda}{}_{\nu} + C\ \bar{\mathcal{H}}_{\mu\lambda\rho}\mathcal{H}^{\lambda\rho}{}_{\nu} \right)$$

$\Longrightarrow$ With $(C = \frac{3}{4},\ \tilde{C} = \frac{3}{2})$ and $(C = -\frac{5}{4},\ \tilde{C} = -\frac{1}{2})$

$$G_{\mu\nu} = \left( G^{(NS)}_{\mu\nu} \pm \bar{\mathcal{F}}_{\mu\lambda}\bar{\mathcal{F}}^{\lambda}{}_{\nu} \pm \bar{\mathcal{H}}_{\mu\lambda\rho}\mathcal{H}^{\lambda\rho}{}_{\nu} \right)$$

- Equation of motion for gauge torsion in four dimension:

$$\partial_\lambda H^{\lambda\mu\nu} + \frac{1}{2}\left( g^{\alpha\beta}\partial_\lambda\ g_{\alpha\beta} \right) H^{\lambda\mu\nu} = 0$$

- Non-linear gauge field equation of motion is

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = 0$$

$$\text{or,}\quad \partial_\mu \mathcal{F}^{\mu\nu} + \frac{1}{2}(g^{\alpha\beta}\partial_\mu g_{\alpha\beta})\mathcal{F}^{\mu\nu} - \frac{1}{2}\mathcal{H}^{\nu}_{\mu\alpha}\mathcal{F}^{\mu\alpha} = 0$$

## Resulting geometries

- Ansatz: $B_{t\theta} = B_{r\theta} = \ b \quad \text{and} \quad B_{t\phi} = \ -M^2 \cos\theta$
  $\mathcal{H}_{t\theta\phi} \to H_{t\theta\phi} = -M^2 \sin\theta \ \text{ and } \ \mathcal{H}_{tr\phi} = \frac{M^2 b}{r^2} \sin\theta$

$$G_{\mu\nu} = \left(g_{\mu\nu} \ + \ B_{\mu\lambda} g^{\lambda\rho} B_{\rho\nu} \ + \ C \ \bar{\mathcal{H}}_{\mu\lambda\rho} g^{\lambda\alpha} g^{\rho\beta} \bar{\mathcal{H}}_{\alpha\beta\nu}\right)$$

- For $C = \pm 1/2$, $r^4 >> b^4$ and $r^8 >> M^8$ on $(3\bar{3})$

$$\begin{aligned} ds^2 \ &= \ - \ \left(1 - \frac{b^2}{r^2} \mp \frac{M^4}{r^4} \pm \frac{M^4 b^2}{r^6}\right) dt^2 \\ &+ \ \left(1 - \frac{b^2}{r^2} \mp \frac{M^4}{r^4} \pm \frac{M^4 b^2}{r^6}\right)^{-1} dr^2 \\ &+ \ r^2 d\theta^2 \ + \ \left(1 \mp \frac{M^4 b^2}{r^6}\right) r^2 \sin^2\theta \ d\phi^2 \end{aligned}$$

- For C= $1/2$ and vanishing geometric torsion term

$$ds^2 = -\left(1-\frac{b^2}{r^2}+\frac{M^4}{r^4}\right)dt^2 + \left(1-\frac{b^2}{r^2}+\frac{M^4}{r^4}\right)^{-1}dr^2 + r^2 d\Omega^2$$

- The torsion modes decouple with a large fifth dimension.
- The decoupling of non-perturbative quantum effects may lead to describe Einstein vacuum in the frame-work
- Under Weyl scaling
- Matrix projection
- For $M < r < b$
- On $(3\bar{3})$

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{r^2}{b^2}\mp\frac{N^2}{r^2}\pm\frac{M^4}{r^4}\right)dt^2 + \frac{r^4}{b^2}d\Omega^2 \\
&+ \left(1-\frac{r^2}{b^2}\mp\frac{N^2}{r^2}\pm\frac{M^4}{r^4}\right)^{-1}dr^2 \mp \frac{M^4}{r^2}\sin^2\theta\ d\phi^2 \\
&\pm\frac{N^2}{r^2}ds^2_{\rm flat}
\end{aligned}
$$

- It can be rexpressed in terms of renormalized mass $N_{\rm eff}$ as:

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{r^2}{b^2}+\frac{N^2_{\rm eff}}{r^2}\right)dt^2 + \left(1-\frac{r^2}{b^2}+\frac{N^2_{\rm eff}}{r^2}\right)^{-1}dr^2 \\
&+\frac{r^4}{b^2}d\theta^2 + \left(\frac{r^2}{b^2}-\frac{M^4}{r^4}\right)r^2\sin^2\theta \\
&\text{where}\quad N^2_{\rm eff} = N^2\left(\frac{b^2}{r_e^2}-1\right)
\end{aligned}
$$

- It is 4 dimensional TdS black hole in presence of extra dimension in Einstein vacuum
  $\rightarrow$ propagating torsion ensures non-Riemannian geometry
  $\rightarrow$ scalar curvature

$$R = \frac{F(M, b, \theta)}{b^2 r^6 (b^2 M^4 - r^6)}$$

$\rightarrow$ curvature singularities at $r \rightarrow 0$ and at $r \rightarrow (bM^2)^{1/3} \rightarrow$ scalar curvature is sourced by geometric torsion.

$$R \ \rightarrow \ \mathcal{K} = \frac{M^4}{2r^4}\left(1 + \frac{b^2}{r^2}\right)$$

- since $M < r < b$, $r \rightarrow 0$ is not accessible to observation.
- The gravitational repulsion that prevents the formation of singularities in the vacuum created brane/anti-brane

- Similarly can be expressed in terms of renormalised mass $N_{eff}$

$$ds^2 = -\left(1 - \frac{r^2}{b^2} - \frac{N_{\rm eff}^2}{r^2}\right) dt^2 + \left(1 - \frac{r^2}{b^2} - \frac{N_{\rm eff}^2}{r^2}\right)^{-1} dr^2 + \frac{r^4}{b^2} d\theta^2 + \left(\frac{r^2}{b^2} + \frac{M^4}{r^4}\right) r^2 \sin^2\theta d\phi^2 \quad (1)$$

- It is SdS geometry sourced by geometric torsion.
- Curvature singularity is absent at brane regime,
- two horizons:

$$r_c \approx b\left(1 - \frac{N_{\rm eff}^2}{2b^2}\right) \qquad \text{and} \qquad r_e \approx N_{\rm eff}$$

- In low energy limit torsion contribution becomes insignificant
  $\rightarrow$ TdS and SdS approximated by Riemannian curvature.
  $\rightarrow$ spherical symmetry is restored.

$$ds^2 = -\left(1 - \frac{r^2}{b^2} - \frac{N^2}{r^2}\right) dt^2 + \left(1 - \frac{r^2}{b^2} - \frac{N^2}{r^2}\right)^{-1} dr^2 + \frac{r^4}{b^2} d\Omega^2$$

$$ds^2 = -\left(1 - \frac{r^2}{b^2} + \frac{N^2}{r^2}\right) dt^2 + \left(1 - \frac{r^2}{b^2} + \frac{N^2}{r^2}\right)^{-1} dr^2 + \frac{r^4}{b^2} d\Omega^2$$

- The emergent geometries in a low energy limit correspond to classical vacua presumably in Einstein gravity
- It is worked out for a SdS black hole and is given by

$$R = \frac{20}{b^2} + \frac{6N^2}{r^4} + \frac{2(b^2 - r^2)}{b^2 r^4} \left(b^2 - 7r^2\right)$$

- The curvature singularity at $r \rightarrow 0$ is forbidden by the black hole mass $N$ in the brane window.

# Outcomes:

- Investigated geometric torsion dynamics underlying a $D_4$-brane
- KR two form can generate a gravitational pair of $(3\bar{3})$-brane
- The Poincare dual of $\mathcal{K}$ *i.e.* an axion, on an $\bar{3}$-brane
  $\Longrightarrow$ a quintessence in the frame-work
- Its dynamics on influences the effective 3-brane universe
  $\Longrightarrow$through the hidden fifth dimension
- It describe various effective de Sitter quantum geometries