A NOTE ON CHROMO-NATURAL INFLATION

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CONTENTS

Review

- An inflationary scenario not only resolves the issues of standard big-bang model but also puts forward explanation for origin of CBM anisitropy and large scale structure of the Universe.
- But there exists no conclusive model of inflation based on particle physics.
- Main difficulty : inflaton potential needs to be fine tuned to realize slow roll inflation.
- But it is difficult to protect these fine tunings against radiative corrections.Mass of the scalar field is sensitive to quantum loop corrections.
- In models where the smallness of coupling of inflaton field is protected by a symmetry of the action , the quantum correction to the potential in under control.

Inflation with axionic field

- In a string motivated inflationary model, the flatness of the inflaton potential is in general not guaranteed - a necessary condition for slow-roll inflation.
- Treating the inflaton as a Nambu-Goldstone boson endowed with shift symmetry, an inflationary model has been devised in which the flatness of the potential becomes natural ('Natural inflation')getting away with fine-tuning problem. Ref:K Freese et al,Phy Rev D 70 (2004) 083512.
- Non-perturbative quantum effect slightly braks the symmetry generating appropriate periodic potential.
- Natural inflation seemed to be observationally viable (a la' PLANCK).**But** requires an axionic decay constant of the order of or larger than Planck mass.
- Appears difficult to realize in the realm of string theory / any other fundamental theory.
- To resolve we need models with effective axion decay constant subplankian (though real one is super-plankian)
- Leads to multifield generalization of 'Natural Inflation'.

- Simplest axionic inflationary model with sub-Planckian axionic decay constant has been proposed by Adshead et al (Phy.Rev.Lett.108(2012)261302, [hep-th/1202.2366] ; Phys.Rev. D86 (2012) 043530 [hep-th/12032264])

- There a collection of non-abelian gauge fields in addition to the axionic field. An SU(2) gauge field is assumed to initiate inflation.

- Slow-roll inflation $\rightarrow$ by transfer of axionic energy into classical gauge fields.

- This energy exchange is mediated by the coupling between the axion and Chern-Simon term of the non-abelian gauge field.

- Since the Chern-Simon term is a total derivative term, it respects the axionic shift symmetry. But this leads to absence of any other axion-gauge field interaction.

- The higher order corrections to the gauge fields are small and thus the coefficient of the Chern-Simon term could be tuned to make the potential sufficiently flat.

- This model is referred as **Chromo-natural Inflation** in literature.

Chromo-natural inflation

- The structure of gauge coupling generates an additional damping effect for the axion which further slows down its motion (E. Martinec et al , JHEP 1302 (2013) 027).
- However, when the linear perturbation of gauge fields are taken into account, the theory involving both the gauge field and the axion field develops a perturbative instability.
- The instability arises due to classical gauge field background which violates parity.
- Neverthless, this instability disappears when the mass of the vector field fluctuation is much greater than the Hubble scale.
- Dimastrogiovanni et al (JCAP 1302 (2013) 046) demonstrated that when this condition is satisfied, it is possible to integrate out the massive gauge field fluctuations maintaining the modifications introduced by the gauge fields in the dynamics of the axion field.
- In this limit, the *Chromo-natural Inflation* is exactly equivalent to a single scalar field effective theory with a non-minimal kinetic term.

Aim of Present Study

- For Chromo Inflation, the standard form of coupling of the axion field with a Chern-Simon term is linear.
- The Chern-Simon term being odd under parity, in principle, any parity-odd function of the axion field can couple to it.
- The linear term can be thought of as the minimal coupling.
- We consider generalizing it to any odd function of the axion field. For simplicity, we consider the function to be a sine function.
- $\sin\left(\frac{\chi}{f}\right)$ is taken as the Chern-Simon term coupling.
- Dimastrogiovanni et al (JCAP 1302 (2013) 046) investigated the inflationary scenario considering linear coupling between gauge and axionic field assuming that the cubic term equation of motion of inflaton field dominates over the linear term .
- With such an assumption, the obsevables spectral index n_s and tensor-scalar ratio r were reported in [0.951, 0.975] and [0.01, 0.03] respectively for $\frac{\chi}{f}$ in [0.1, 2.3]. Running of spectral index in the chosen range of field variable and other model parameters though were not reported but range of r is consistent with the observation of Planck but is disfavoured by the recent BICEP2 results.

Chromo Natural Inflation with Sinusoidal coupling

We start with the action:

$$\begin{aligned} S_{chromo} &= \int d^4x\,\sqrt{-g}\,[-\frac{M_P^2}{2}R - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - \frac{1}{2}(\partial\chi)^2 - \mu^4\left[1+\cos\left(\frac{\chi}{f}\right)\right] \quad (1) \\ &+ \frac{\lambda}{8}\sin\left(\frac{\chi}{f}\right)\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}], \end{aligned}$$

where, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \bar{g}f^{abc}A^b_\mu A^c_\nu$, $\chi \rightarrow$ axion field and $A^a_\mu \rightarrow$ the $SU(2)$ gauge fields ,$\bar{g} \rightarrow$ gauge coupling constant, $f \rightarrow$ the axion decay constant,$\mu \rightarrow$ mass scale of the theory,$\lambda \rightarrow$ gauge parameter, $f^{abc} \rightarrow$ structure constant of the gauge group (with normalization condition $f^{123} = 1$) .
$\epsilon^{\mu\nu\rho\sigma}$ is chosen as $\epsilon^{0123} = \frac{1}{\sqrt{-g}}$ with standard FRW metric
$ds^2 = -N^2dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, N being the Laps function .

Using the $SU(2)$ gauge freedom which allows for an isotropic configuration for $A_i^a = \delta_i^a a(t)\psi(t)$ and $A_0^a = 0$, where $a(t)$ is the scale factor, one can rewrite the preceding action as an action involving two scalar fields (χ and ψ) coupled to gravity as:

$$S = \int d^4x \sqrt{-g}\, \left[-\frac{M_P^2}{2} R - \frac{1}{2} (\partial\chi)^2 + \frac{1}{4\Lambda^4} (\partial\chi)^4 - \mu^4 \left(1 + \cos\left(\frac{\chi}{f}\right)\right)\right]. \quad (2)$$

The effect of the gauge field in the single field effective theory is captured by the term $\frac{1}{4\Lambda^4} (\partial\chi)^4$ where $\Lambda^4 = \frac{8 f^4 \bar{g}^2}{\lambda^4}$.
This important observation is the key factor which distinguishes Chromo-natural inflation from other axion inflationary models.

This effective action is of the form :

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_P^2 R}{2} + P(X,\chi)\right], \tag{3}$$

with $X = -\frac{1}{2}(\partial\chi)^2$ and the matter Lagrangian density

$$P(X,\chi) = X + \frac{X^2}{\Lambda^4} - V(\chi) \tag{4}$$

where, $V(\chi) = \mu^4\left(1+\cos\left(\frac{\chi}{f}\right)\right)$ is the potential of the axionic field χ.

Now, the basic dynamical equations defining the inflationary scenario i.e, the Friedmann equation and the equation of motion of the inflaton field are

$$H^2 = \frac{\rho}{3M_P^2}, \tag{5}$$

$$\dot{\rho} = -3H(\rho + P), \tag{6}$$

where, $\rho = 2XP_{,X} - P$ is the energy density of the scalar field, P is the pressure term which is same as the matter Lagrangian density in this case and $(P_{,X})$ denotes derivative of P with respect to X.

Transformation into a single field effective theory

For this purpose, it is convenient to scale the gauge fields and the coupling λ as

$$A^a_\mu = \frac{1}{\sqrt{\bar{g}}}\tilde{A^a_\mu},\ \lambda = \sqrt{\bar{g}}\tilde{\lambda}. \tag{7}$$

In the SRHS limit, i.e when the mass of the gauge field is considered much bigger than the Hubble parameter H (in other words implementing the condition $\bar{g} \rightarrow \infty$, keeping $\tilde{A^a_\mu}$ and $\tilde{\lambda}$ fixed), the action (21) takes the form :

$$\begin{aligned} S_{SRHS} &= \int d^4x\,\sqrt{-g}\,[-\frac{M_P^2}{2}R - \frac{1}{4}f^{abc}\tilde{A}^b_\mu\tilde{A}^c_\nu f^{ade}\tilde{A}^\mu_d\tilde{A}^\nu_e - \frac{1}{2}(\partial\chi)^2 & (8)\\ &- \mu^4[1+\cos\left(\frac{\chi}{f}\right)] - \frac{\tilde{\lambda}}{2}\sin(\frac{\chi}{f})\epsilon^{\mu\nu\rho\sigma}\partial_\mu\tilde{A}^a_\nu f^{abc}\tilde{A}^b_\rho\tilde{A}^c_\sigma] \\ &= \int d^4x\,\sqrt{-g}\,[-\frac{M_P^2}{2}R - \frac{1}{4}\left(\tilde{A}^a_\mu\tilde{A}^\mu_a\right)^2 + \frac{1}{4}\left(\tilde{A}^a_\mu\tilde{A}^a_\nu\right)\left(\tilde{A}^\mu_b\tilde{A}^\nu_b\right) - \frac{1}{2}(\partial\chi)^2 \\ &- \mu^4[1+\cos\left(\frac{\chi}{f}\right)] + \frac{\tilde{\lambda}}{6}\partial_\mu[\sin\left(\frac{\chi}{f}\right)]\epsilon^{\mu\nu\rho\sigma}f^{abc}\tilde{A}^a_\nu\tilde{A}^b_\rho\tilde{A}^c_\sigma] & (9) \end{aligned}$$

- For the last term in the above equation we have carried out a partial integration. In the above action, gauge fields only appear algebraically and as the action is quadratic in $\tilde{A}_0^a$, the resulting equation of motion can be easily solved for $\tilde{A}_0^a$. But the solution for $\tilde{A}_i^a$ can not easily be found out as the action is not quadratic in $\tilde{A}_i^a$, rather have a general algebraic form.

- Therefore, it is solved at a single space time point, x_p, in a locally inertial rest frame.

- In fact, we can write a time-like four-vector $\partial_\mu\left[\sin\left(\frac{\chi}{f}\right)\right]$ at the point x_p as a Lorentz boost of a vector pointing only in the time direction and having the same invariant form as follows:

$$\partial_\mu[\sin\left(\frac{\chi}{f}\right)](x_p) = \Lambda_\mu^\nu(x_p)\delta_{\nu 0}\sqrt{-\left[\partial[\sin\left(\frac{\chi}{f}\right)](x_p)\right]^2}, \tag{10}$$

where, $\Lambda_a^A(x_p)\Lambda_b^B(x_p)\eta_{AB} = \eta_{ab}$. Similarly, for the gauge field we can write

$$\tilde{A}_\mu^a(x_p) = \Lambda_\mu^\nu(x_p)\bar{A}_\nu^a(x_p). \tag{11}$$

- Local Lorentz invariance of the action can be used to write the gauge-field part of the Lagrangian as :

$$\begin{aligned} L_A(x_p) &= -\frac{1}{4}\left(\bar{A}^a_\mu \bar{A}^\mu_a\right)^2 + \frac{1}{4}\left(\bar{A}^a_\mu \bar{A}^a_\nu\right)\left(\bar{A}^\mu_b \bar{A}^\nu_b\right) \\ &+ \frac{\bar{\lambda}}{6}\sqrt{-(\partial(\sin(\frac{\chi}{f}))(x_p))^2}\delta_{\nu 0}\epsilon^{\mu\nu\rho\sigma} f^{abc} \bar{A}^a_\mu \bar{A}^b_\rho \bar{A}^c_\sigma \end{aligned} \tag{12}$$

$$\begin{aligned} &= -\frac{1}{2}\left(\bar{A}^a_0\right)^2\left(\bar{A}^b_i\right)^2 + \frac{1}{2}\left(\bar{A}^a_0 \bar{A}^a_i\right)\left(\bar{A}^0_b \bar{A}^i_b\right) - \frac{1}{4}\left(\bar{A}^a_i \bar{A}^i_a\right)^2 \\ &+ \frac{1}{4}\left(\bar{A}^a_i \bar{A}^a_j\right)\left(\bar{A}^i_b \bar{A}^j_b\right) + \frac{\tilde{\lambda}}{6}\sqrt{-(\partial(\sin(\frac{\chi}{f}))(x_p))^2}\epsilon^{ijk} f^{abc} \bar{A}^a_i \bar{A}^b_j \bar{A}^c_k. \end{aligned} \tag{13}$$

- Now, varying the action with respect to $\bar{A}^a_0$ and remembering the fact that $\bar{A}^a_i$ is having non-zero value and the action is quadratic in $\bar{A}^a_0$, one gets

$$\bar{A}^a_0(x_p) = 0. \tag{14}$$

With this value, $L_A(x_p)$ becomes:

$$L_A(x_p) = -\frac{1}{4}\left(\bar{A}_i^a \bar{A}_a^i\right)^2 + \frac{1}{4}\left(\bar{A}_i^a \bar{A}_j^a\right)\left(\bar{A}_b^i \bar{A}_b^j\right) + \frac{\bar{\lambda}}{6}\sqrt{-(\partial(\sin(\frac{\chi}{f}))(x_p))^2}\epsilon^{ijk} f^{abc} \bar{A}_i^a \bar{A}_j^b \bar{A}_k^c, \quad (15)$$

yielding a cubic equation after variation with respect to $\bar{A}_i^a$.
It is found that $\bar{A}_i^a = \delta_i^a \frac{\bar{\lambda}}{2}\sqrt{-(\partial(\sin(\frac{\chi}{f}))(x_p))^2}$ satisfies the cubic equation and can be taken as the background solution of the chromo natural inflation. It is interesting to note that this solution is of similar form as was originally chosen as the isotropic background solution.
Finally, one gets

$$L_A(x_p) = \frac{1}{4\bar{\Lambda}^4}\left[\partial(\sin(\frac{\chi}{f}))(x_p)\right]^4, \quad (16)$$

where, $\bar{\Lambda}^4 = \frac{8\bar{g}^2}{\lambda^4}$.

As this is evaluated in a local inertial frame, using the principle of covariance, one can take the general form of the Lagrangian as $L = \frac{1}{4\bar{\Lambda}^4}\left[\partial(\sin(\frac{\chi}{f}))\right]^4$.
Finally, including this part, the effective action becomes

$$\begin{aligned} S_{\chi} &= \int d^4x\,\sqrt{-g}\,[-\frac{M_P^2}{2}R - \frac{1}{2}\left(\partial\chi\right)^2 + \frac{1}{4\bar{\Lambda}^4 f^4}\cos^4\left(\frac{\chi}{f}\right)\left(\partial\chi\right)^4 \\ &- \mu^4\left(1+\cos\left(\frac{\chi}{f}\right)\right)]. \end{aligned} \quad (17)$$

Thus, it is evident that, within the SRHS limit, the chromo inflationary scenario with a more general form of Chern-Simon term can be transformed in to a single field effective theory where, the effect of the gauge field is captured by the non-minimal kinetic term for the axionic scalar field in the single field action.

Chromo inflationary dynamics in single field effective theory

It is clear from equation (17) that this single field action for the chromo inflation is a single field theory with the matter Lagrangian density $P(X, \chi)$ given by

$$P(X, \chi) = X + \frac{\cos^4\left(\frac{\chi}{f}\right) X^2}{\Lambda^4} - V(\chi) \tag{18}$$

where, $\Lambda^4 = \bar{\Lambda}^4 f^4 = \frac{8f^4\bar{g}^2}{\lambda^4}$ and X is same as in section II. The form of the energy density is found to be

$$\rho = X + \frac{3\cos^4\left(\frac{\chi}{f}\right) X^2}{\Lambda^4} + V(\chi). \tag{19}$$

Now, from this action, the Friedmann equation and the background equation of motion of the axionic scalar field in slow roll regime can be expressed as :

$$H^2 = \frac{V(\chi)}{3M_P^2}, \tag{20}$$

$$3H\dot{\chi}\left(1+\frac{1}{\Lambda^4}\cos^4(\frac{\chi}{f})\dot{\chi}^2\right) = \frac{\mu^4}{f}\sin(\frac{\chi}{f}). \tag{21}$$

From this cubic equation, one can obtain the expression for $\dot{\chi}$ and use this to evaluate all the observables in terms of parameters of this model.

the power spectrum for the scalar and the tensor perturbations can be expressed respectively as

$$P_k^\zeta = \frac{\sqrt{3}\mu^8(1+\cos(\frac{\chi}{f}))^2\Lambda^4}{36\pi^2 M_P^4(\dot{\chi})^4\cos^4(\frac{\chi}{f})}, \tag{22}$$

$$P_k^h = \frac{2\mu^4}{3\pi^2 M_P^4}(1+\cos(\frac{\chi}{f})). \tag{23}$$

In the following tables we display these values for some choice of parameters μ, Λ and f so that the power spectrum satisfies the COBE normalization constraint ie, $P_k^\zeta \sim 2\times 10^{-9}$. It may also be noted than , in this case the end of inflation is marked by $\eta = 1$ which runs faster than ε, contrary to the previous section.

μ	f	Λ	n_s	r	α_s
0.01	0.01	0.000001	0.950553	0.110265	-0.0012196
0.01	0.1	0.00001	0.950554	0.110261	-0.00121958
0.01	1	0.0001	0.950538	0.110016	-0.00122026
0.01	5	0.0005	0.950477	0.103476	-0.00122068
0.01	9	0.0009	0.950254	0.0894433	-0.000915847

Table : Few allowed model parameters for $N = 40$ (When Chern-Simon term contains linear function of χ)

μ	f	Λ	n_s	r	α_s
0.01	0.01	0.00000165	0.973024	0.188191	-0.00482996
0.01	0.1	0.000016	0.967254	0.177693	-0.00350042
0.01	1	0.00018	0.960173	0.160723	-0.00220467
0.01	5	0.0006	0.95029	0.119164	-0.00122968
0.01	10	0.0008	0.949265	0.0964403	-0.00119981

Table : Few allowed model parameters for $N = 40$ (When Chern-Simon term contains sine function of χ)

μ	f	Λ	n_s	r	α_s
0.01	0.01	0.000001	0.966876	0.0725007	-0.000545603
0.01	0.1	0.00001	0.966872	0.0725081	-0.000545736
0.01	1	0.0001	0.966864	0.0723371	-0.000545925
0.01	5	0.0005	0.966811	0.0674989	-0.000545901
0.01	10	0.0007	0.966788	0.0630496	-0.000547662

Table : Few allowed model parameters for $N = 60$ (When Chern-Simon term contains linear function of χ)

μ	f	Λ	n_s	r	α_s
0.01	0.01	0.0000012	0.979481	0.121725	-0.00177654
0.01	0.1	0.000012	0.979352	0.121661	-0.00176474
0.01	1	0.00012	0.978734	0.120114	-0.00169073
0.01	5	0.0006	0.967059	0.0901083	-0.000507499
0.01	10	0.0008	0.965114	0.0652003	-0.000516045

Table : Few allowed model parameters for $N = 60$ (When Chern-Simon term contains sine function of χ)

Conclusions

- when the linear coupling is generalized to sinusoidal coupling of scalar field to the Chern-Simon term in the action, we found that the model can successfully achieve large tensor to scalar ratio as predicted by BICEP2 as well as spectral index consistent with PLANCK observations. These results are best obtained for sub-Planckian values of the axion decay constant ($f \sim 0.1 M_P$).
- With linear scalar field coupling to Chern-Simon term it is not possible to achieve BICEP2 result. The model is also not consistent with the combined analysis of BICEP2 and PLANCK if polarized dust power spectra is taken into account in the sense that tensor-to -scalar ratio can not reach the upper bound 0.1 since this model has a running spectral index.

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