# INFLUENCE OF GLOBAL EXPANSION ON THE LOCAL KINEMATICS IN GALAXIES

**Arunava Bhadra**
**High Energy & Cosmic Ray Research Ctr.**
**University of North Bengal**

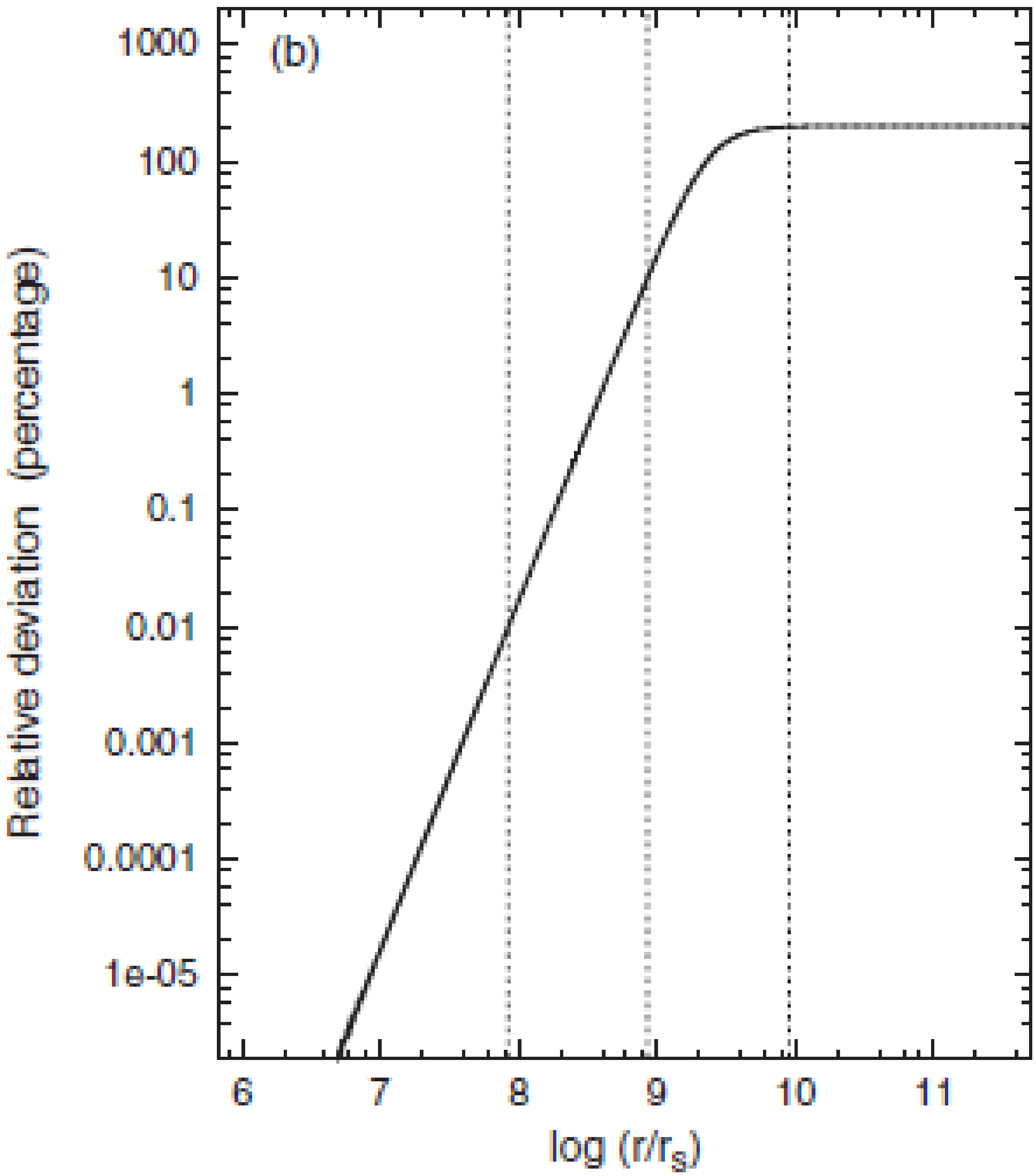
## COSMOLOGICAL CONSTANT AND LOCAL GRAVITY:

- The simplest and most attractive general relativistic model that explains the late time accelerated expansion of the Universe is the ΛCDM model
  - Λ denotes the positive cosmological constant with a value of nearly $10^{-56}$ cm$^{-2}$
  - CDM refers to cold dark matter.
- In the presence of a repulsive cosmological constant (positive "Λ" ) the space-time geometry exterior to a static spherically symmetric metric is Schwarzschild–de Sitter (SDS), rather than Schwarzschild metric.

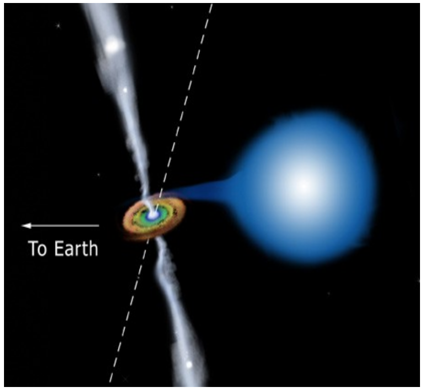
- The cosmological expansion thereby may affect any local gravitational phenomenon like perihelion shift of the orbits of gravitationally bound systems [Kagramanova et al, PLB 06], geodetic precession, gravitational bending of light [Bhadra et al PRD 2010] etc.,
- General perception is that owing to its tiny value cosmological constant does not lead to any significant observable effects in a local gravitational phenomenon.
- In the Solar system the influence of cosmological constant is known to be maximum in the case of perihelion shift of mercury orbit where the $\Lambda$ contribution is about $10^{-15}$ of the total shift [Kagramanova et al, PLB 06]

- The contribution of repulsive $\Lambda$ could be significant (larger than the second order term) when kiloparsecs to megaparsecs-scale distances are involved, such as the gravitational bending of light by cluster of galaxies [Ishak et al, MNRAS 2008].

- The trajectories of both small and large magellanic clouds in the gravitational field of the Milky Way are affected significantly (~10% level or higher) by $\Lambda$ [Stuchlík, JCAP 2010]


(b)
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-05
Relative deviation (percentage)
6
7
8
9
10
11
log (r/r_s)

- A plausible local scenario where cosmological constant may contribute significantly is the relativistic accretion around massive BHs which involve distance scale of the order of hundreds of parsecs or even more.




- A few studies have been carried out so far to investigate the effect of $\Lambda$ in astrophysical jet/accretion flow paradigm

- So far the findings include:
  - $\Lambda$ can have strong collimation effect on jets [Stuchlík et al AA, 2000]

- Λ introduces an outflow of matter through the outer cusp of accretion disk causing a considerable impact on the runway instability [Rezzolla, Zanotti, and Font, AA, 2003].

- Characteristic peaks in the iron line profile generated in the BH accretion disk in a SDS background are less noticeable [Slobodov, Schleich, and Witt, 2004]

- In Bondi-type accretion flow around a nonrotating BH Λ suppresses the mass accretion rate of the flow and dramatically impacts the transonic nature of the accretion flow [Karkowski and Malec, PRD 2013, Mach, et al, PRD, 2013].

- All the works on the effect of $\Lambda$ on accreting systems are carried out under some restricted conditions/situations. This is because the study of accreting BH systems involves solving general relativistic (GR) magnetohydrodynamic (MHD) equations in a strong gravitational field regime.

- Owing to the complex and nonlinear character of the underlying equations in GR regime, analytical/ quasinumerical treatment of the problem is virtually discarded.

- Even numerical simulation is complicated by several issues such as different characteristic time scales for propagating modes of general relativity and relativistic hydrodynamics.

- Early works on the accretion related phenomena were based on pure Newtonian gravity.

- A few GR effects were incorporated ad hoc manner.

- After the seminal work of Paczyński and Witta [AA, 1980], most of the authors treated accretion and its related processes around BHs using hydrodynamical/MHD equations in the Newtonian framework by using some pseudo-Newtonian potentials (PNPs) which are essentially modification of Newtonian gravitational potential developed with the objective to reproduce (certain) features of relativistic gravitation.

- Adopting PNPs, one can comprehensively construct more realistic accretion flow models in simple Newtonian paradigm, while the corresponding PNP would capture the essential GR effects in the vicinity of the compact objects.

- Several PNPs exist in the astrophysical literature which are mostly prescribed in an ad hoc manner

- PNPs are mostly developed for Schwarzschild and Kerr BHs [See Ghosh, Sarkar and Bhadra, MNRAS, 2014 and references therein].

- If the effects of cosmological constant need to be properly revealed in different astrophysical phenomena in local galactic scales circumventing the complex GR treatment, it is desirable to have a correct Newtonian analogous-like potential corresponding to SDS geometry that will reproduce all the salient features of the SDS metric with reasonable accuracy and extensively mimic a wide spectrum of GR behavior.

- Here, we would derive a Newtonian-like potential from the conserved Hamiltonian of the test particle motion which would then be an approximate relativistic potential analogue corresponding to the SDS metric.

# PNP FOR THE SDS GEOMETRY

- For a general class of static spherically symmetric spacetimes of the form (in the standard coordinates system)

$$ds^2 = -f(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2,$$

- the Lagrangian density reads

$$2\mathcal{L} = -f(r)c^2\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{f(r)}\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\Omega}{d\tau}\right)^2.$$

- *The two constants of motion are given by*

$$\mathcal{P}_t = \frac{\partial\mathcal{L}}{\partial\dot{t}} = -c^2 f(r)\frac{dt}{d\tau} = \text{constant} = -\epsilon$$

- and

$$\mathcal{P}_\Omega = \frac{\partial\mathcal{L}}{\partial\dot{\Omega}} = r^2\frac{d\Omega}{d\tau} = \text{constant} = \lambda,$$

- For massive particles

$$2\mathcal{L} = g_{\alpha\beta} p^\alpha p^\beta = -m^2 c^2$$

- Thus we get

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{\epsilon^2}{c^2} - c^2\right) - c^2[f(r) - 1] - f(r)\frac{\lambda^2}{r^2}.$$

- From geodesic Eqs. $\quad \dfrac{dt}{d\tau} = \dfrac{\epsilon}{c^2}\dfrac{1}{f(r)}.$

- Thus one gets $\quad \dfrac{dr}{dt} = \dfrac{c^2}{\epsilon} f(r)\sqrt{2E_{GN} - c^2[f(r) - 1] - f(r)\dot{\Omega}^2 \dfrac{r^2}{f(r)}},$

Overdot denotes derivative w.r.t. t, $E_{GN} = (\epsilon^2 - c^4)/2c^2$

- In the low energy limit

$$E_{GN} = \frac{1}{2}\left(\frac{dr}{dt}\right)^2 \frac{1}{f(r)^2} + \frac{r^2\dot{\Omega}^2}{2f(r)} + \frac{c^2}{2}[f(r) - 1].$$

- The Hamiltonian in the Newtonian regime with the generalized analogous potential in spherical polar geometry

$$E_{GN} \equiv \frac{1}{2}(\dot{r}^2 + r^2\dot{\Omega}^2) + V_{GN} - \dot{r}\frac{\partial V_{GN}}{\partial \dot{r}} - \dot{\Omega}\frac{\partial V_{GN}}{\partial \dot{\Omega}},$$

- that will then be equivalent to $E_{GN}$ obtained earlier from GR

- The generalized analogous potential would then be given by

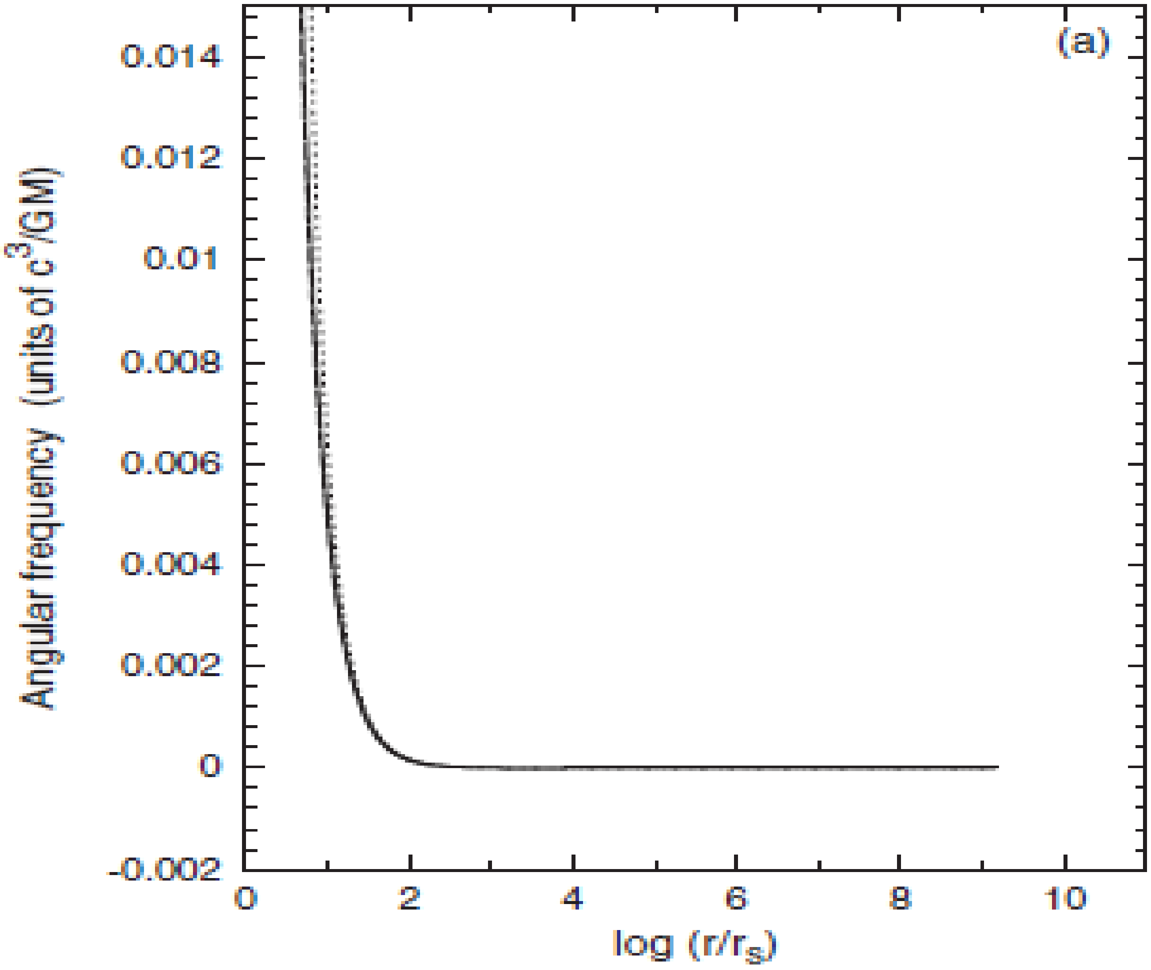
$$V_{GN} = \frac{c^2[f(r)-1]}{2} - \frac{[1-f(r)]}{2f(r)}\left[\frac{1+f(r)}{f(r)}\dot{r}^2 + r^2\dot{\Omega}^2\right].$$

- For the SDS geometry

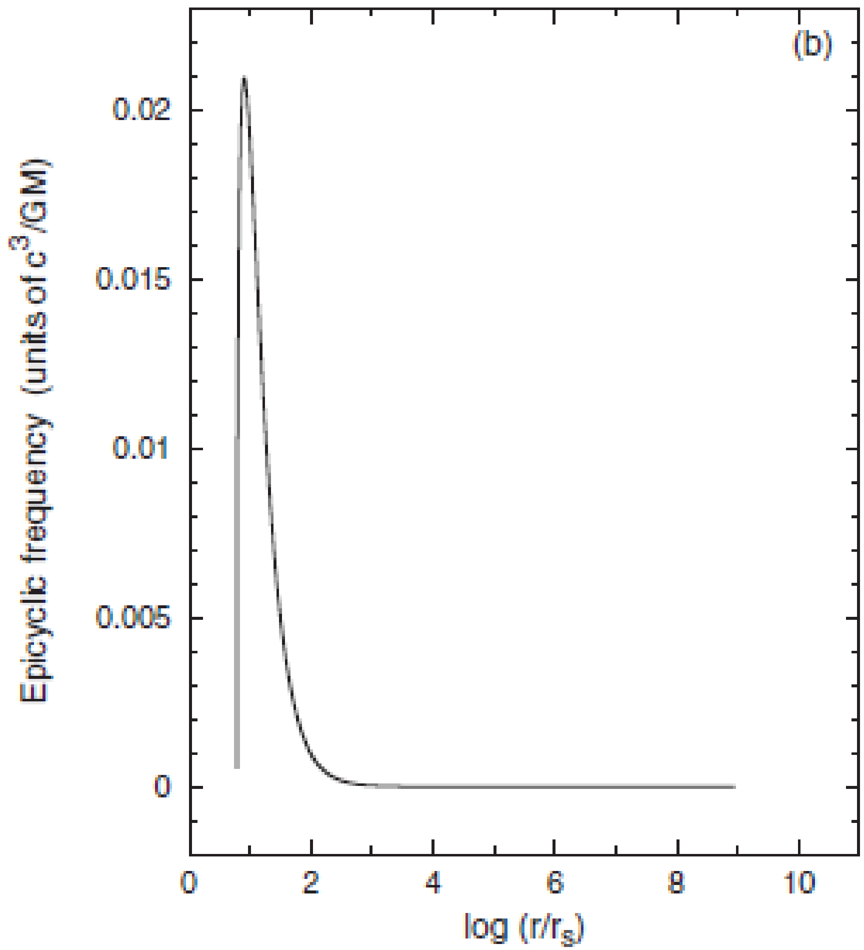
$$f(r) = 1 - 2r_s/r - \Lambda r^2/3$$

- Therefore , PNP for the SDS metric

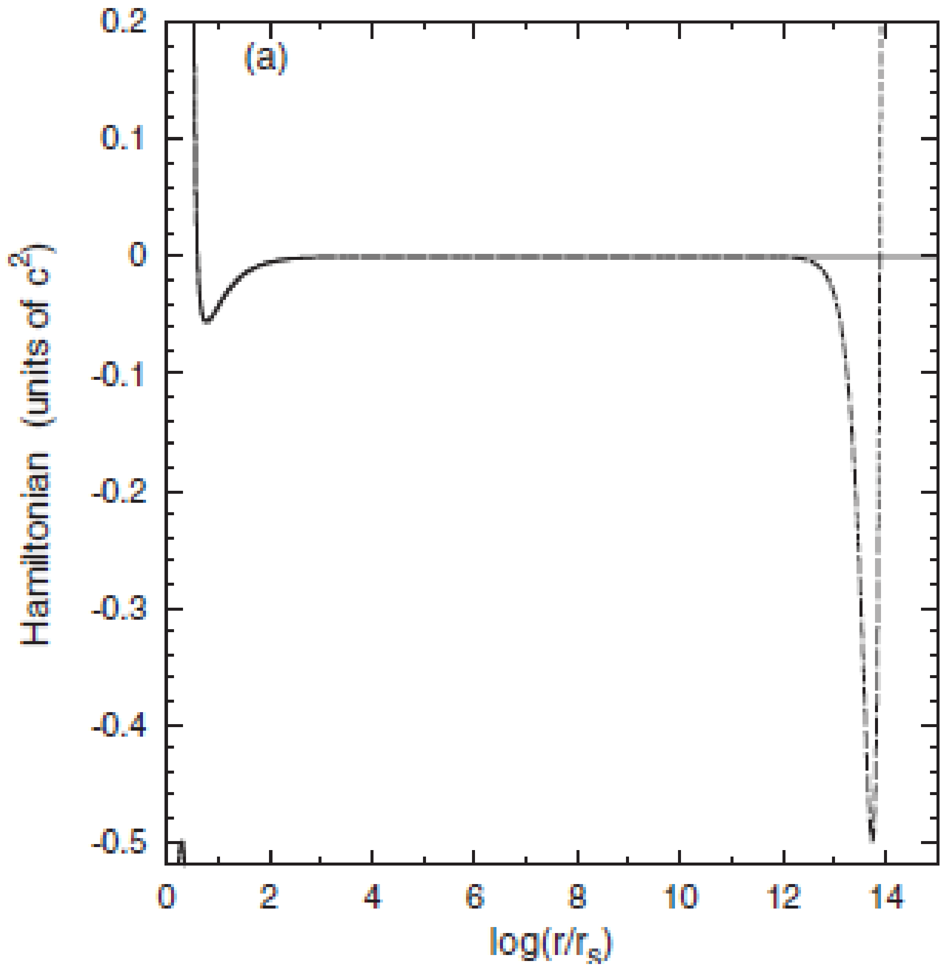
$$V_{\mathrm{DS/ADS}} = -\left(\frac{GM}{r} + \frac{\Lambda c^2 r^2}{6}\right) - \left(\frac{2r_s + \frac{\Lambda r^3}{3}}{r - 2r_s - \frac{\Lambda r^3}{3}}\right) \times \left(\frac{r - r_s - \frac{\Lambda r^3}{6}}{r - 2r_s - \frac{\Lambda r^3}{3}}\dot{r}^2 + \frac{r^2\dot{\Omega}^2}{2}\right),$$

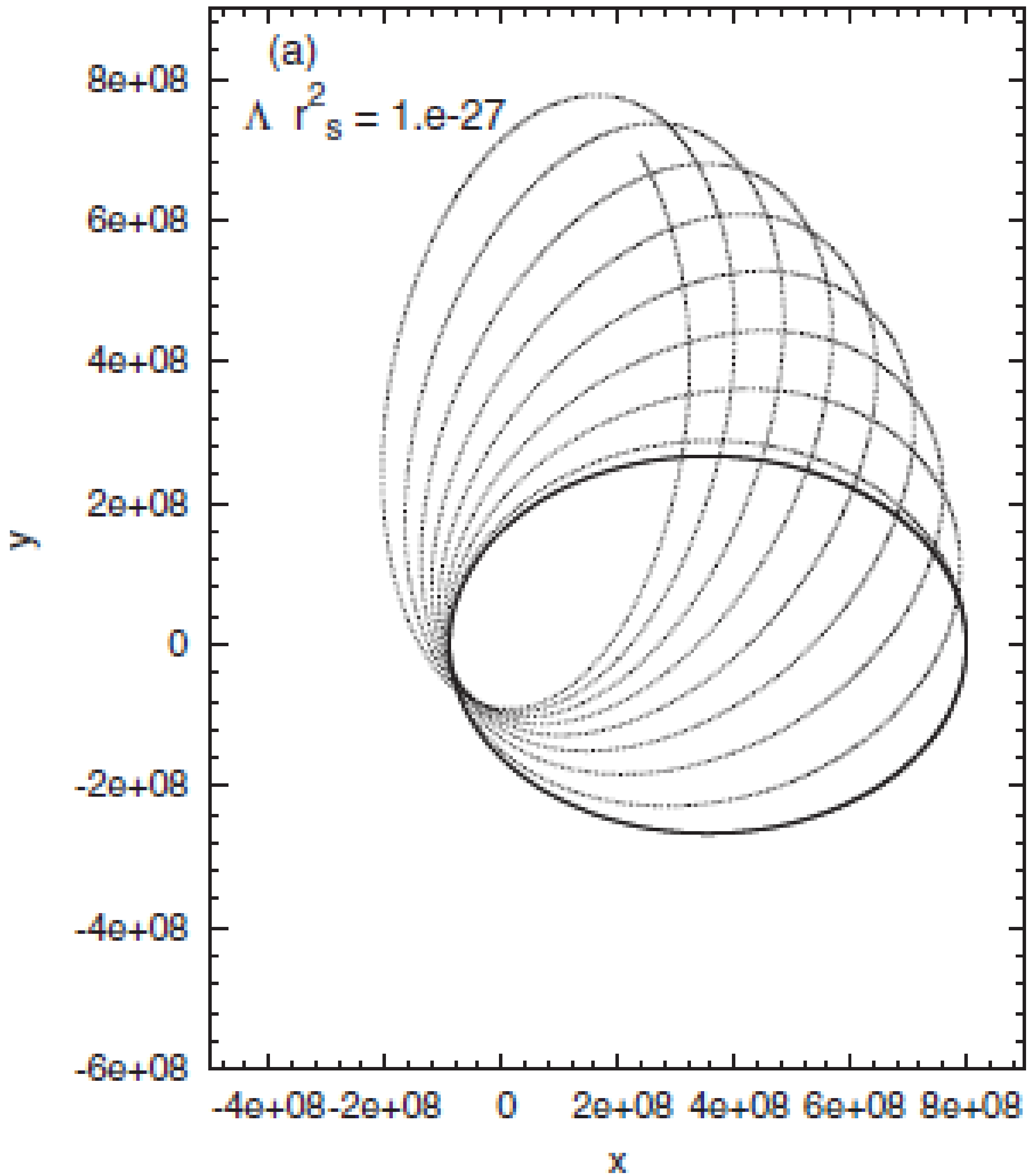



The variation of the angular frequency with r for the test particle in circular orbit. Solid, and long dashed Curves correspond to PNP and GR respectively.




The variation of the epicyclic frequency with r


(a)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Hamiltonian (units of $c^2$)
0
2
4
6
8
10
12
14
$\log(r/r_s)$


(a)
$\Lambda\ r_s^2 = 1.e\text{-}27$
8e+08
6e+08
4e+08
2e+08
0
-2e+08
-4e+08
-6e+08
y
-4e+08
-2e+08
0
2e+08
4e+08
6e+08
8e+08
x

# EFFECT OF Λ ON BONDI ACCRETION RATE

- Bondi accretion is a spherically symmetric steady gaseous accretion onto a compact star/Galaxy

- Dynamically Bondi solution is transonic in nature with two critical radii or sonic radii at which the radial velocity exactly equals to sound speed a in the limits $r \to \infty$ and $r \to 0$ respectively. The limit $r \to \infty$ in astrophysical sense represents the location of the ambient medium (ISM/IGM).

- The masss flow is considered as usual to be locally adiabatic, with $P \propto \rho^{\gamma}$ so that

$$a = \sqrt{\gamma P/\rho}.$$

- The mass accretion rate here is then defined by the relation $$|\dot{M}| = 4\pi r^2 \rho v_r.$$

- Using PNP for the SDS geometry, instead of usual Newtonian potential, the Bondi Eq. at critical point $r_c$ lead to

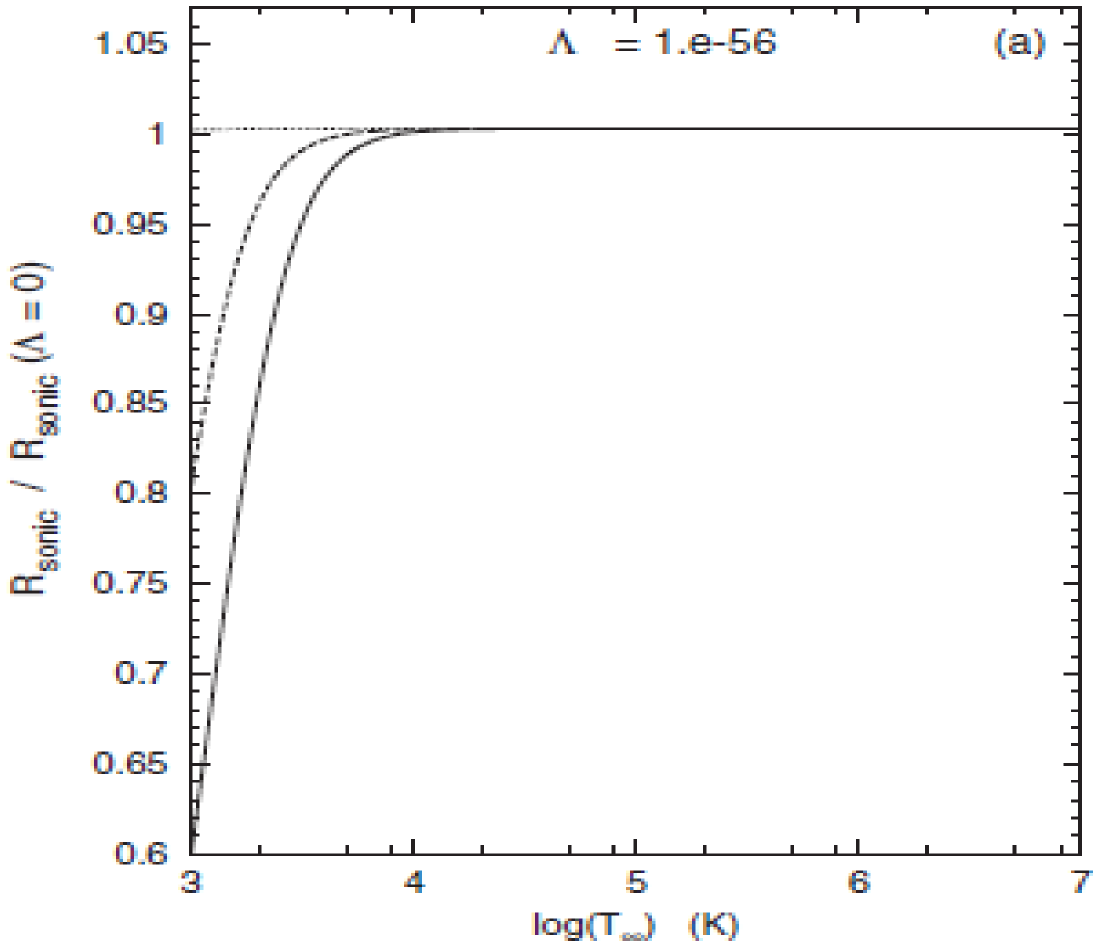
$$\frac{v_r^2|_c}{2} + \frac{a_c^2}{\gamma - 1} + V_{\mathrm{DS}}|_c = \frac{a_\infty^2}{\gamma - 1}$$

- and $$\frac{2a_c^2}{r_c} - \left.\frac{dV_{\mathrm{DS/ADS}}}{dr}\right|_c = 0,$$

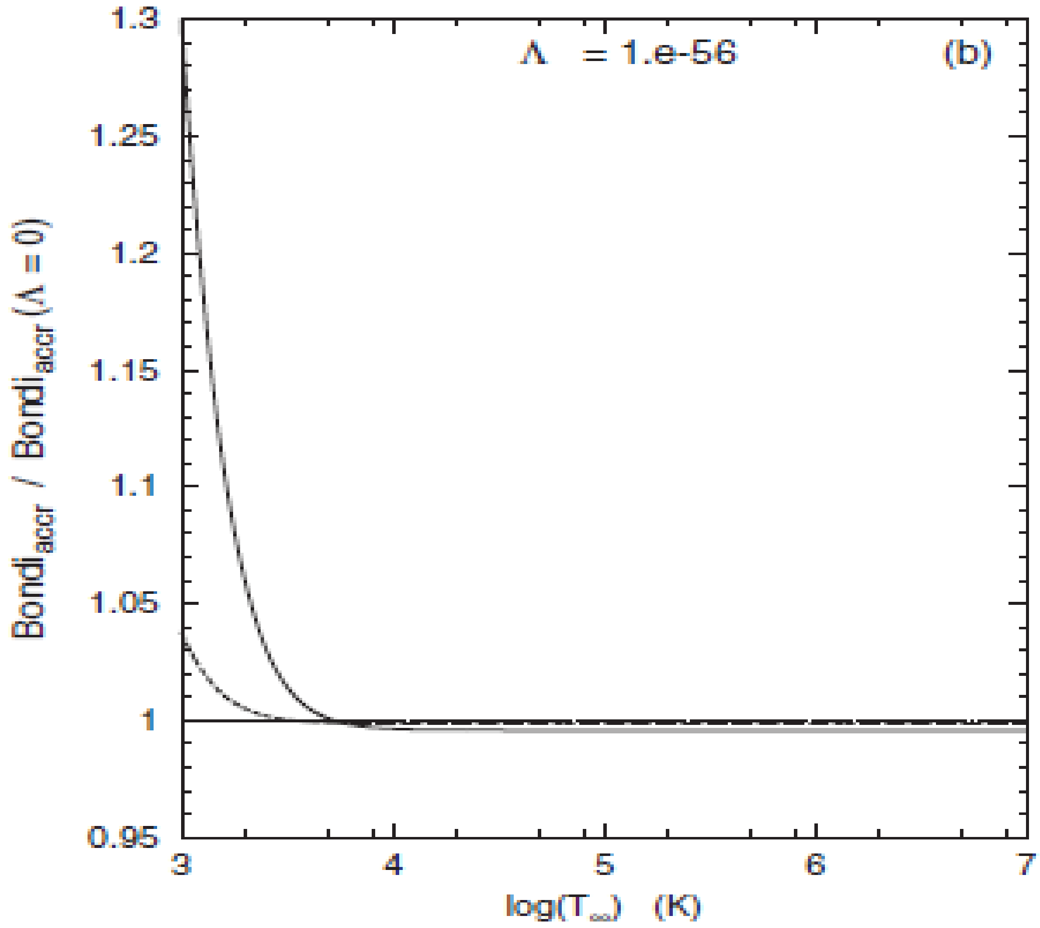
- Solving these two equations we obtain $r_S$ and corresponding $a_C$ in terms of $a_\infty$, where $a_\infty$ is the sound speed at $r \rightarrow \infty$.
- Bondi accretion rate is $$|\dot{M}_t| = 4\pi r_c^2 \rho_c a_c .$$
- Assuming the accreting plasma to be of purely ionized hydrogen, we have

$$a_\infty = \sqrt{\gamma k_B T_\infty / m_p}$$

- and $$\rho_\infty = n_p|_\infty m_p,$$
- $T_\infty$ and $n_p|_\infty$ are the temperature and the number density of the accreting gas at $r \rightarrow \infty$.




Solid, long-dashed and dotted curves are for adiabatic constant ( 4/3, 1.5; 1.66) respectively



Λ = 1.e-56
(b)
Bondi$_{accr}$ / Bondi$_{accr}$ (Λ = 0)
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
3
4
5
6
7
log(T$_{\infty}$) (K)

# CONCLUSIONS:

- The influence of positive cosmological constant is important in understanding the kinematics of the kiloparsecs-scale regions and beyond, in the local galaxies.

- We have obtained modified Newtonian analogous potential corresponding to SDS geometry which reproduces almost all of the GR features with remarkable accuracy.

- The significant effects (at the level of 10% and above over those due to pure Schwarzschild geometry) of $\Lambda$ occur at $r \gtrsim 50$–100 kpc.
- Consequently the local astrophysical kinematics in many massive AGNs/quasars would be strongly influenced by $\Lambda$.
- It is found that the current accepted value of cosmological constant impact, the sonic radius, as well as the Bondi accretion rate moderately when ambient temperature is $T_\infty < 10^4$ K and for smaller values in $\gamma$ ($\lesssim 1.5$)

- For more details, please see the reference

Sarkar, Ghosh and Bhadra, PHYSICAL REVIEW D 90, 063008 (2014)

- Thank you