Effect of phantom dark energy on Gravitational Lensing

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Outline of the presentation

- **Introduction and Motivation**
- **Phantom dark energy**
- **Gravitational lensing**
- **Results**
- **Conclusion**

Introduction

- ΛCDM model is presently considered as standard model of Cosmology
- However, the current cosmological observations indicates that the equation of state parameter w is slightly less than -1 though considering the present uncertainties on observed w, it also admits w=-1 (cosmological constant).
- $w<-1$ implies that dark energy can be phantom in nature.

Motivation

- An important question is whether phantom dark energy affects the local gravitational phenomena such as gravitational bending of light .
- And if so, whether phantom dark energy can be discriminated from cosmological constant from a local gravitational phenomenon, at least in principle.

Phantom dark energy

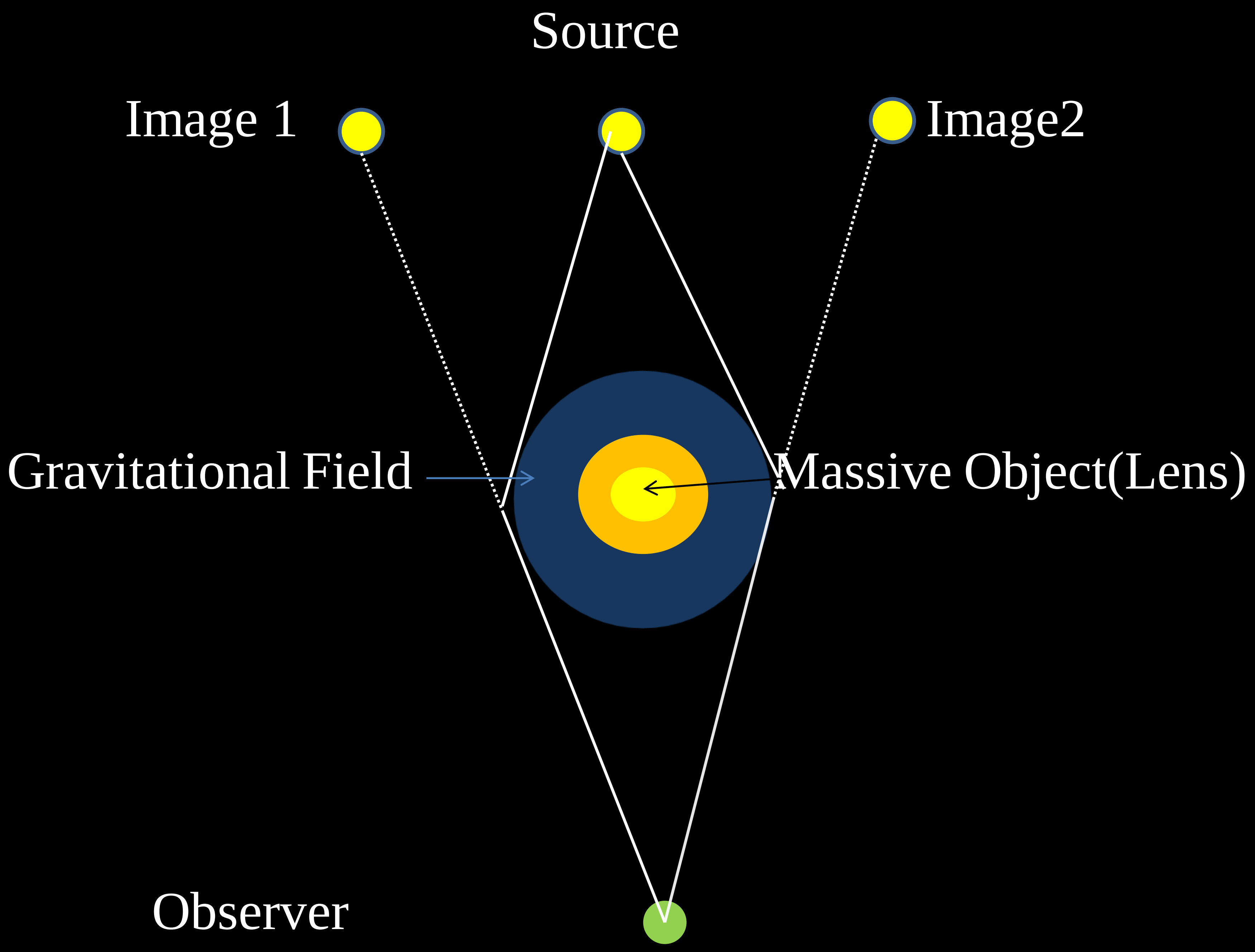
- Dark energy: Dark Energy can be modeled by a self interacting scalar field with a potential.

- Phantom Dark Energy: Phantom is dark energy of a very large negative pressure.

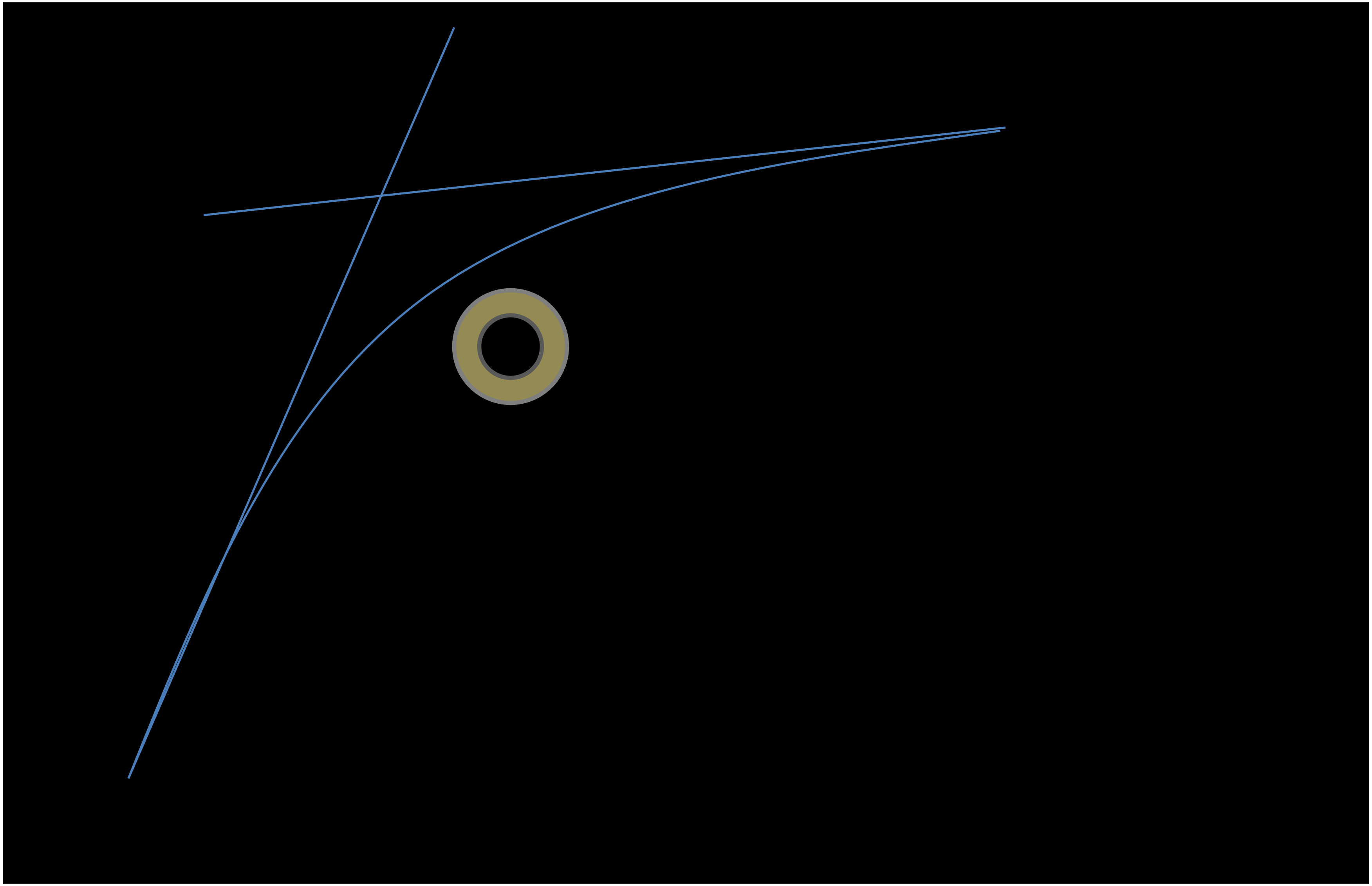
$$w < -1$$

Gravitational lensing

- Source
- Lens
- Observer
- Images
- Deflection Angle


Source
Image 1
Image2
Gravitational Field
Massive Object(Lens)
Observer

Gravitational lensing



Gravitational Weak Lensing in SDS spacetime

- Schwarzschild-de Sitter space time

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

$$\alpha = \psi - \phi$$

$$= \frac{2m}{r_0} - \frac{m r_0}{r^2} - \frac{\Lambda r_0 r}{6} + \frac{\Lambda {r_0}^3}{6r}$$

- The Λ has been found to reduce the deflection angle (Bhadra, Biswas, Sarkar, PRD, 2010)

- ## Strong Field deflection angle

$$\alpha(\theta) = -\bar{a} log\left(\frac{\theta D_{OL}}{u_m} - 1\right) + \bar{b} + o(u_0 - u_m)$$

$$\bar{a} = 1 - \frac{81}{4}\Lambda^2 M^4$$

$$\bar{b} = -\pi + b_R + \left(1 - \frac{81}{4}\Lambda^2 M^4\right) log2 \frac{\left(1 + 36\Lambda M^5\right)^2}{\left(\frac{1}{3} - 3\Lambda M^2\right) M(1 - 72\Lambda M^5)^3}(1 - M)$$

$$u_m \simeq 3\sqrt{3}M\left(1 + \frac{9}{2}\wedge M^2\right)$$

Influence of dark Energy on Gravitational Lensing

- The strong field expression for bending involves cosmological constant. This effect mainly occurs through the expression of impact parameter. [ICRC2011/1046, K. Sarkar and A. Bhadra]

- If the dark energy is phantom in nature!

Lensing by phantom dark energy

- Action

$$S = \int \sqrt{g}d^4x\left[R + \varepsilon g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)\right]$$

- R is scalar curvature
- $\varepsilon = -1$ corresponds to a phantom field

$$V(\phi) = -\frac{c}{b^2}(3 - 2cos^2\psi) - \frac{r_0}{b^3}\{3sin\psi cos\psi + \psi(3 - 2cos^2\psi)\}$$

$$r_0 = 3m$$

- General Static Spherically Symmetric Metric

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

- Solution to the Einstein-scalar equation

$$A(r) = 1 + \frac{r_0 r}{b^2} + (r^2 + b^2)\left[\frac{c}{b^2} + \frac{r_0}{b^3} tan^{-1}\left(\frac{r}{b}\right)\right]$$

$$A(r) = B(r)^{-1}$$

$$b = constant > 0$$

K. A. Bronnikov and J.C. Fabris, Phys. Rev. Lett. 96. 251101 (2006)

- In general Bronnikov-Fabris solution describes **regular black hole with a de Sitter asymptotic behavior far beyond the horizon**

- Eiroa et al [PRD, 2013] estimated gravitational lensing in both weak and strong field regime by phantom dark energy considering the solution is asymptotically flat.

- Here we evaluated the deflection angle considering Bronnikov-Fabris solution asymptotically non-flat.

$$For\ \ flat\ space\ time\ A(r) = 1\ at\ r \to \infty$$

$$A(r) = 1 + \frac{r_0 r}{b^2} + (r^2 + b^2)\left[\frac{c}{b^2} + \frac{r_0}{b^3} tan^{-1}\left(\frac{r}{b}\right)\right]$$

As suggested by Bronnikov, $c = -\dfrac{3\pi m}{2b}$,

$A(r) \neq 1$ **at** $r \to \infty$

- The regular black hole with a de Sitter asymptotic behavior far beyond the horizon

Horizon of the Black Hole

$$\boldsymbol{A(r) = 0}$$

$$r_{horizon} = \frac{r_0}{2cb^2}\left[-\left(1+\frac{1}{b^2}\right) \pm \sqrt{\frac{1}{b^2} - \frac{4}{c} - 3}\right]$$

- For asymptotically flat space time such as the Schwarzschild space time the direction of asymptotic light rays is usually evaluated by applying the limit $r \to \infty$ in the orbit equation and the angle between the two asymptotic directions gives the total deflection angle.
- For asymptotically non-flat space-time, such an approach is not valid. Instead, it is important to have the angles that the tangent made with light direction at source and observer's position [Rindler, PRD, 2007].

- The angle that the tangent to the light trajectory made with a coordinate direction is

$$\tan\psi = \left[\frac{A(r_0)r^2}{A(r){r_0}^2} - 1\right]^{-\frac{1}{2}}$$

$$A(r) = 1 + \frac{r_0 r}{b^2} + \left(r^2 + b^2\right)\left[\frac{c}{b^2} + \frac{r_0}{b^3} tan^{-1}\left(\frac{r}{b}\right)\right]$$

$$\psi = \frac{r_0}{\sqrt{r^2 - {r_0}^2}}\left[1 + \frac{1}{b^2} r^3 r_0 \frac{1}{r^2 - {r_0}^2} + \frac{c}{2b^3} r^2\right]$$

- The null geodesic equation for the space time

$$\frac{1}{r^4}\left(\frac{dr}{d\phi}\right)^2 + \frac{A(r)}{r^2} - \frac{1}{h^2} = 0$$

$$h \equiv r^2 \frac{d\phi}{dp}, p \; is \; an \; affine \; parameter$$

- Second order DE for the null geodesic

$$\frac{d^2u}{d\phi^2} + (1+c)u = -\frac{1}{u_0 b^2} \qquad u = \frac{1}{r}$$

$$u = u_0 + u_1$$

u_0: undeflected trajectory

u_1: small perturbation

$$u = \frac{1}{R} sin\left(\sqrt{1+c}\phi + \xi\right) - \frac{1}{u_0 b^2 (1+c)}$$

The bending angle

- The distance of closest approach

$$\frac{1}{r_0} = \frac{1}{R} - \frac{1}{u_0 b^2 (1+c)}$$

$$\frac{1}{r} = \frac{1}{R}\left[\sqrt{1+c}\phi + \xi\right] - \frac{1}{u_0 b^2 (1+c)}$$

$$\phi = \frac{1}{\sqrt{1+c}}\left[1 - \frac{(r-r_0)}{r r_0} u_0 b^2 (1+c) - \frac{1}{r r_0} {u_0}^2 b^4 (1+c)^2\right]$$

- The bending angle $\alpha = \psi - \phi$

Conclusions

- Phantom dark energy influences the gravitational lensing phenomenon.

- If the dark energy is phantom in nature, it appears that it has significantly different effects on Gravitational angle in compare to those by cosmological constant and hence, in principle should be discriminated from local bending observations..

References

- V.Bozza. Phys. Rev. D 66, 103001 (2002)
- Eiroa et. al.,. Phys. Rev. D 66, 024010 (2002)
- A.Bhadra. Phys. Rev. D 67, 103009 (2003)
- K.Sarkar and A.Bhadra, Class.Quan.Grav.23, 22, 61
- Ernesto F. Eiroa and Carlos M. Sendra
  Phys. Rev. D 88, 103007 (2013)
- K. A. Bronnikov and J.C. Fabris, Phys. Rev. Lett. 96. 251101 (2006)
- K.A. Bronnikov , R. A Konoplya and A. Zhidenko, Phys. Rev. D 86. 024028 (2012)
- A. Bhadra, S. Biswas and K. Sarkar, Phys. Rev. D 82, 063003 (2010)
- A. Bhadra, arXiv:1007.1794

THANK YOU