Neutrinos and Cosmology

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In Nov. 2014, we(Graciela Gelnini, Danny Marfatia and I) hosted a workshop on neutrinos at KITP, UCSB. Among others, we had invited Gary Steigman, Georg Raffelt and Petr Vogel to give talks. In the next few slides I summarise a few topics covered by them. I feel that the contents may be of some interest.

Contents:

- Constraints on "sterile neutrinos" or other relativistic degrees of freedom from BBN and CMB("Equivalent neutrinos")
- Constraints on the sum of neutrino masses from CMB, BAO,SDSS,Ly-Alpha, SNI-a+WL…..("weighing neutrinos with the universe")
- Attempts to detect Cosmic Neutrino Background and a new proposal

The "Effective Number Of Neutrinos" & Counting "Equivalent Neutrinos"

In the early Universe the energy density is dominated by the contributions from ER (extremely relativistic) particles. The early Universe is "Radiation Dominated" (R).

When $T \ll m_e$, the only ER standard model (SM) particles are the photons and neutrinos.

$$\rho \approx \rho_R = \rho_\gamma + 3\,\rho_\nu \gg \rho_B$$

where, $\rho_\nu / \rho_\gamma = 7/8\,(T_\nu / T_\gamma)^4$

The SM neutrinos decouple when $T_\gamma = T_\nu \approx 2 - 3$ MeV, before (barely) $e^\pm$ annihilation.

IF neutrino decoupling were instantaneous, and, IF $T_{\nu d} \gg m_e$, then after the $e^\pm$ pairs have annihilated, $(T_\nu / T_\gamma)^3 = 4/11$.

With these assumptions and, in this regime,

$$\rho / \rho_\gamma = 1 + 3\,[\,7/8\,(4/11)^{4/3}\,]$$

N_{eff}, the <u>"Effective Number of Neutrinos"</u>,

is defined by : $\rho / \rho_\gamma \equiv 1 + N_{eff} [7/8 (4/11)^{4/3}]$

or, $N_{eff} \equiv 3 [11/4 (T_\nu / T_\gamma)^3]^{4/3}$ (when $T_\gamma << m_e$).

If neutrino decoupling were instantaneous

and, if electrons were massless, $N_{eff} = 3$.

Since $T_{\nu d}$ is <u>not</u> >> m_e, $N_{eff} \approx 3.02$.

Since neutrino decoupling is <u>not</u>

instantaneous, $N_{eff} \approx 3.05$.

An "Equivalent Neutrino", ξ, is a very light ($m_\xi << m_e$) particle that may, or may not, be a Majorana fermion ("neutrino").

If ξ is populated in the early Universe, either thermally or via mixing with the SM neutrinos, $\rho_R \rightarrow \rho_R + \rho_\xi \equiv \rho_R + \Delta N_\nu \rho_\nu$.

$\Delta N_\nu = \rho_\xi / \rho_\nu$ is the number of equivalent neutrinos (a measure of dark radiation).

If ξ is a Majorana fermion ("neutrino") and if ξ is fully populated / mixed, $\Delta N_\nu = 1$ (sterile ν).

<u>But</u>, if ξ is a fully populated / mixed, real scalar, $\Delta N_\nu = 4/7$. In general, $\Delta N_\nu \leq 1$ (Dark Radiation).

N_{eff} and ΔN_ν are related by :

$$N_{eff} = N^0_{eff}\,(1 + \Delta N_\nu / 3)\,, \quad N^0_{eff} = 3\,[(11/4)^{1/3}(T_\nu / T_\gamma)_0]^4$$

The expansion rate, the Hubble parameter (H), depends on the mass / energy density : $H \;\alpha\; \rho^{1/2}$

BBN Predicted Primordial Abundances Depend On Two Physical / Cosmological Parameters (ignoring any lepton (neutrino) asymmetry).

Baryon Density (Nucleon Asymmetry) Parameter

- $\eta_B \equiv n_N / n_\gamma$; $\eta_{10} \equiv 10^{10}\,\eta_B = 274\,\Omega_B h^2$

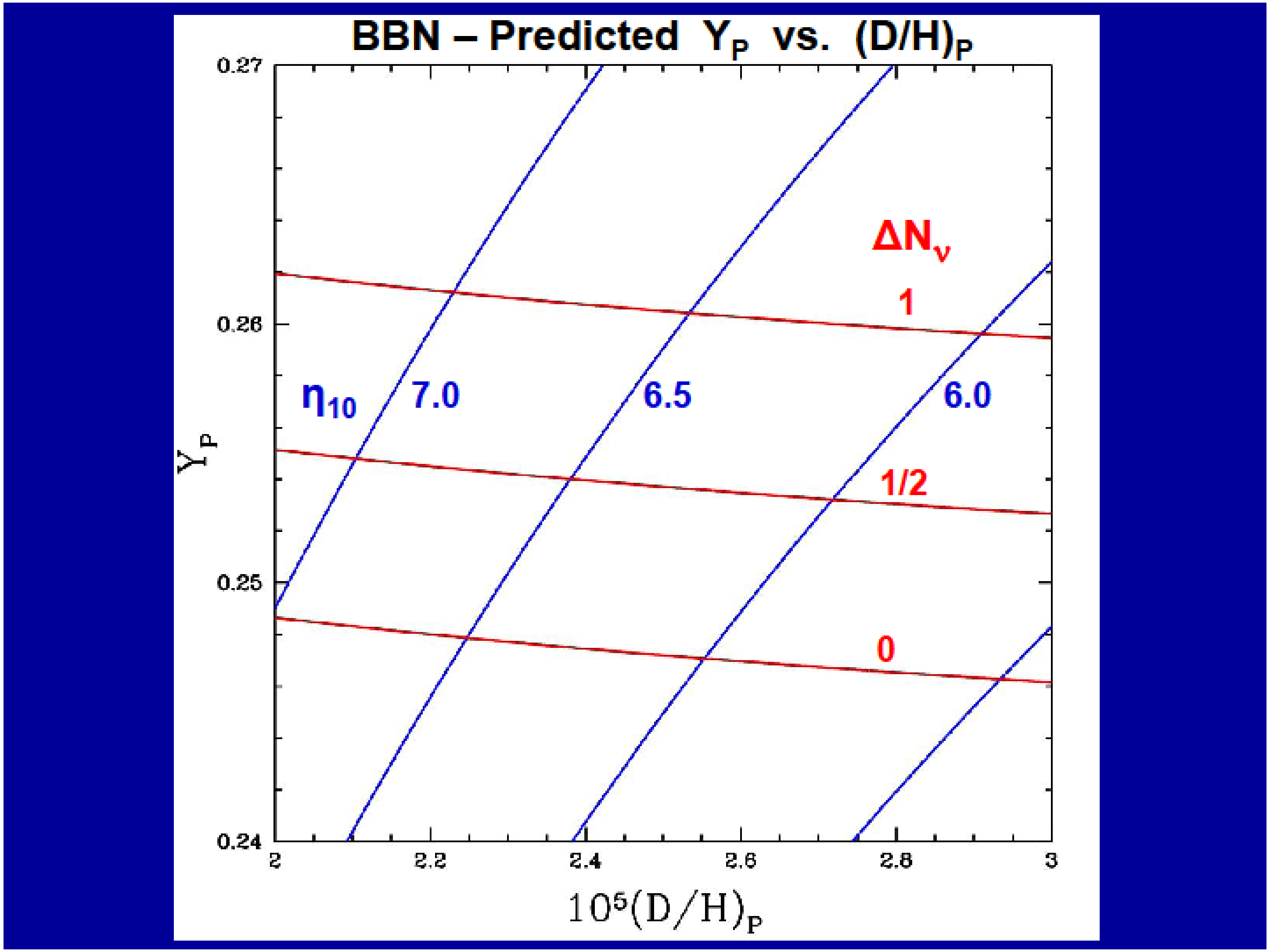
Expansion Rate (Dark Radiation) Parameter

- $S^2 = (H'/H)^2 = \rho'/\rho$; S depends on ΔN_ν (N_{eff})
- **SBBN** : $\Delta N_\nu = 0$ ($S = 1$)

- η_B Probes "Standard" Cosmology / Physics
- D ($y_{DP} = 10^5 (D/H)_P$) is sensitive to η_B
- $\Delta N_\nu \neq 0$ Probes Non - Standard Physics
- ^{4}He (Y_P) is sensitive to ΔN_ν

  * Two parameters (η_B and ΔN_ν)

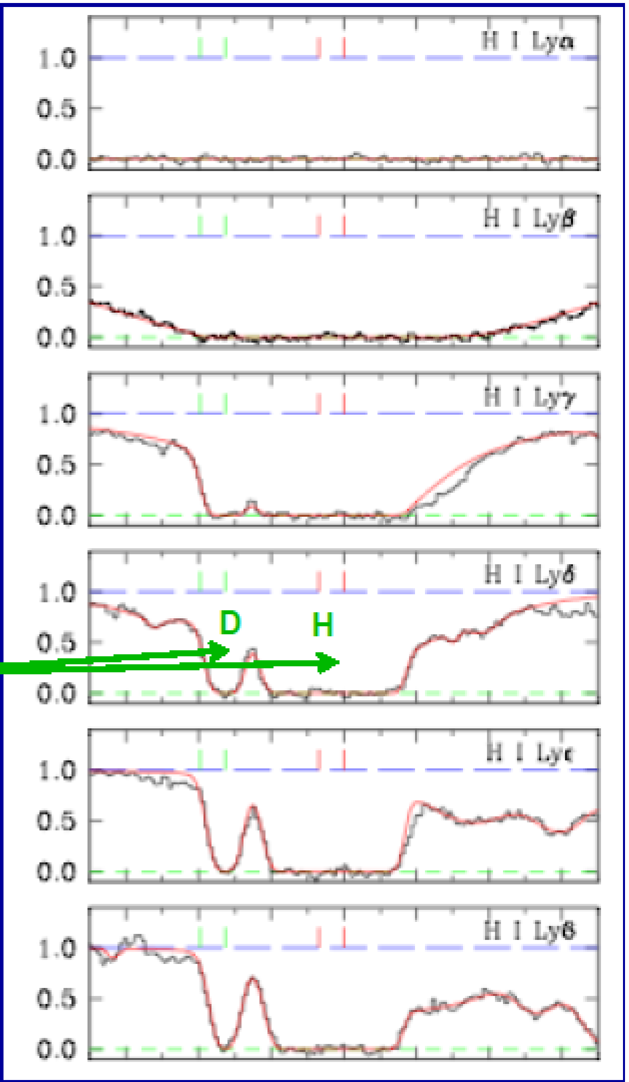
    Two observables (y_{DP} and Y_P)


BBN – Predicted Y_P vs. $(D/H)_P$
0.27
0.26
0.25
0.24
Y_P
2
2.2
2.4
2.6
2.8
3
$10^5(D/H)_P$
ΔN_ν
1
1/2
0
η_{10}
7.0
6.5
6.0

Primordial (nearly) D

Finding D at low - Z in the Ly - α Forest

D and H absorption spectra are identical, <u>except</u> for an isotope shift of ~ 80 km/s

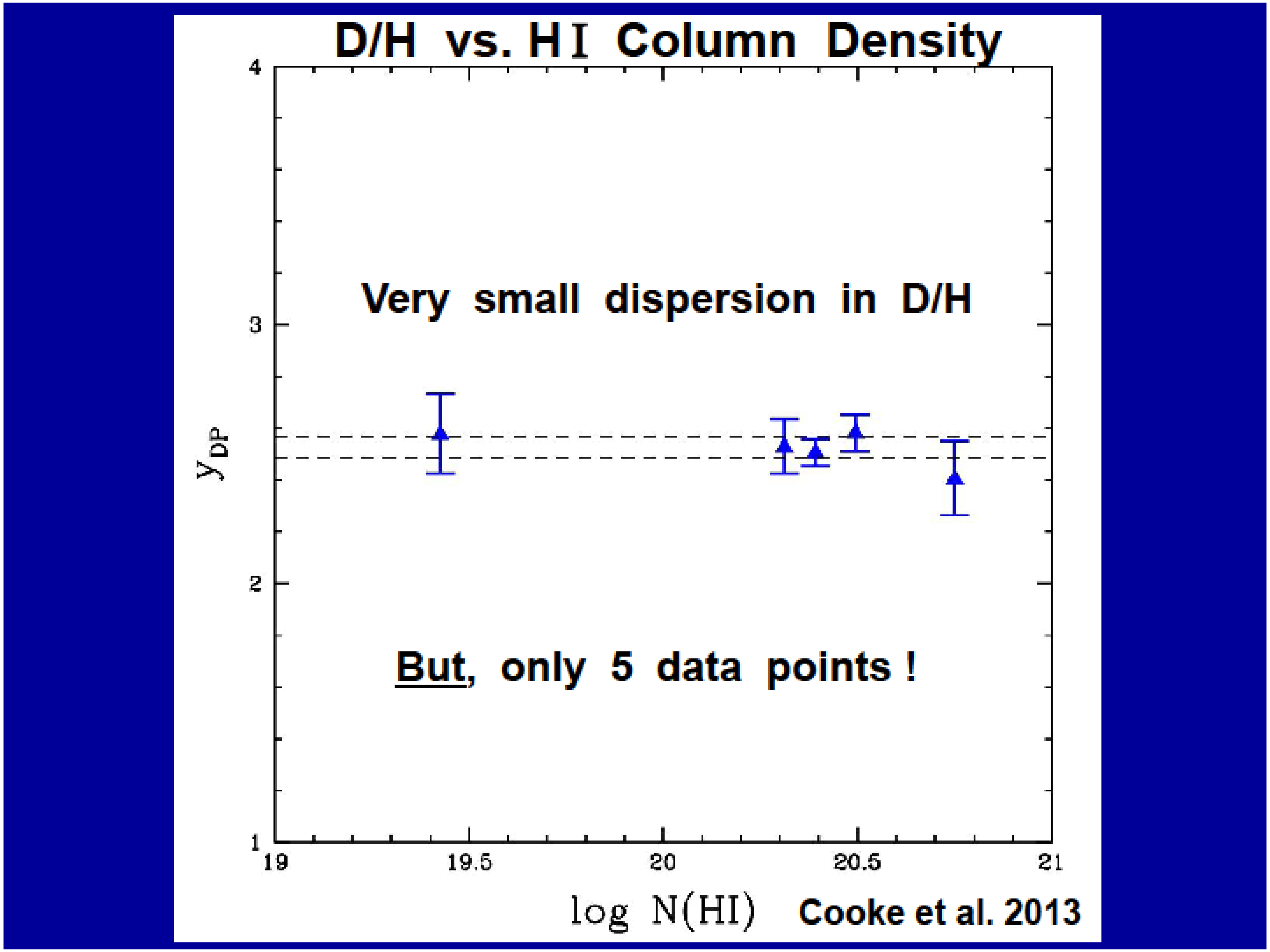


Cooke et al. 2013

Recent Results For Nearly Primordial Deuterium

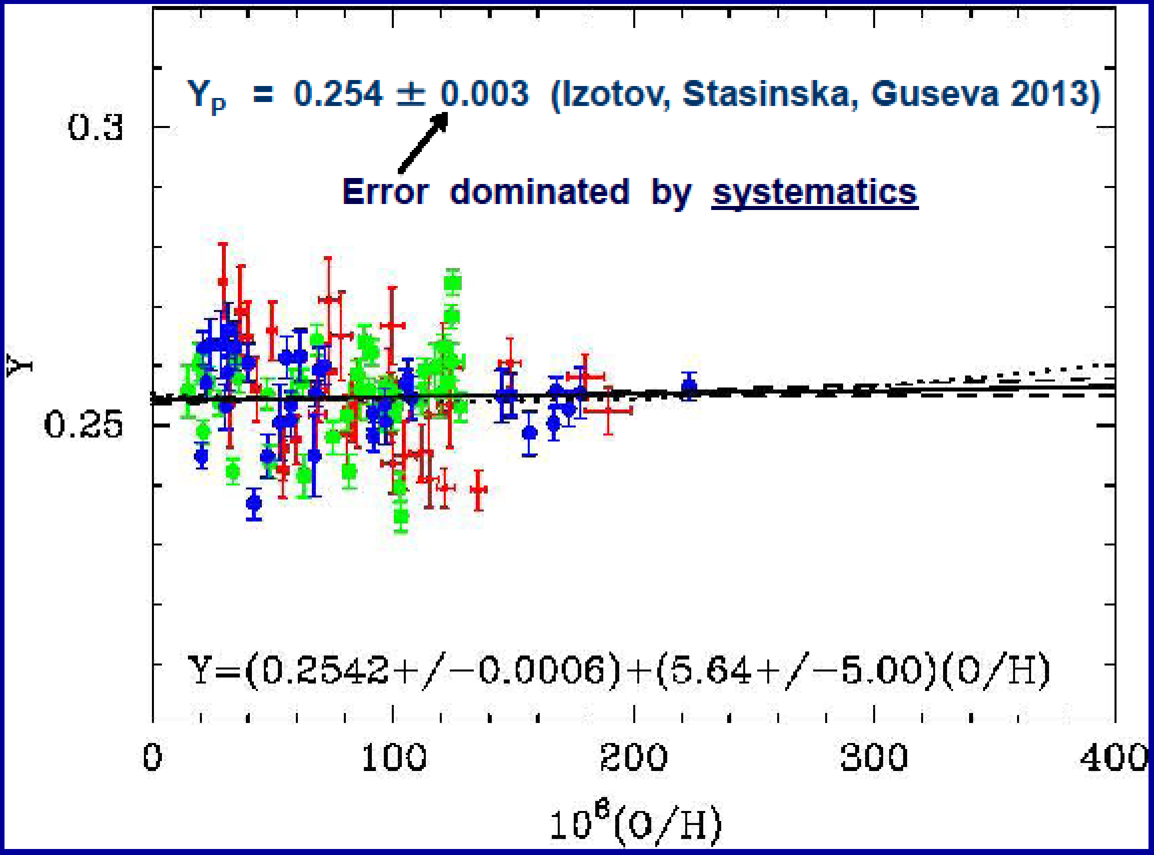
Previous D observations had large dispersion among the D/H determinations.

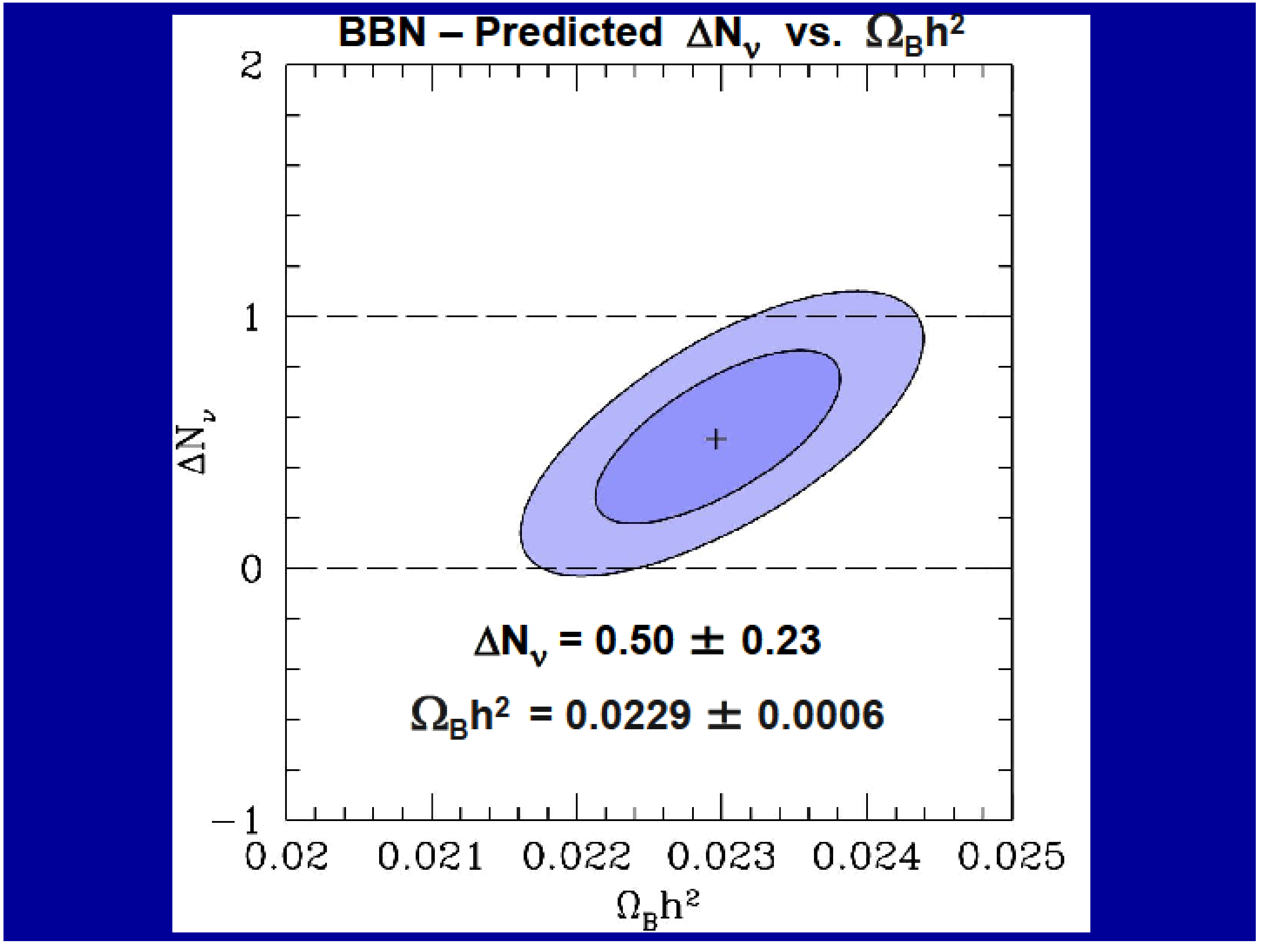
Cooke et al. 2013 restricted their analysis to DLAs (log N(H I) > 19), allowing them access to many lines in the Lyman series, helping to reduce some sources of systematic errors.



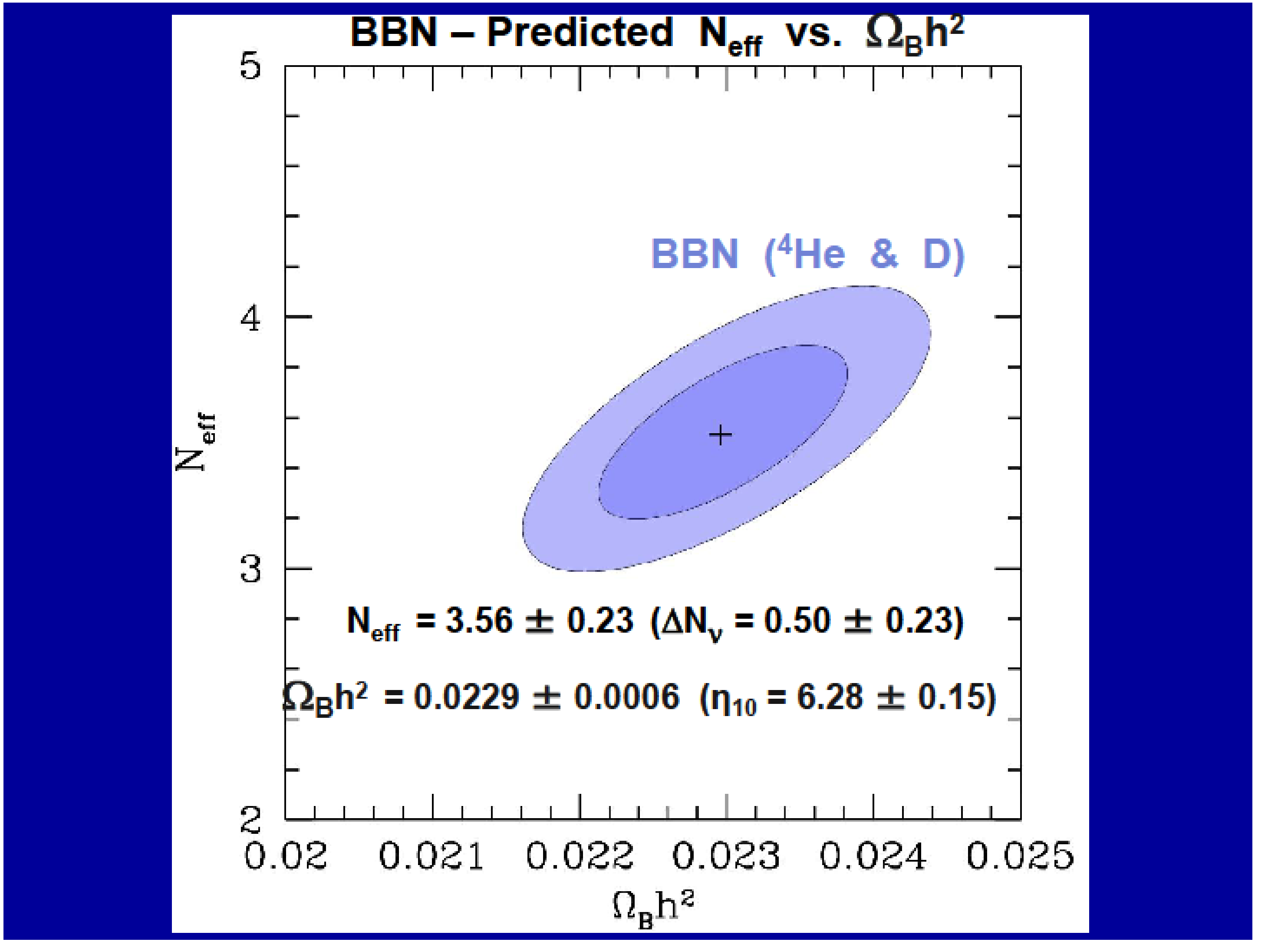
D/H vs. H I Column Density
Very small dispersion in D/H
But, only 5 data points !
4
3
2
1
19
19.5
20
20.5
21
y_DP
log N(HI)
Cooke et al. 2013

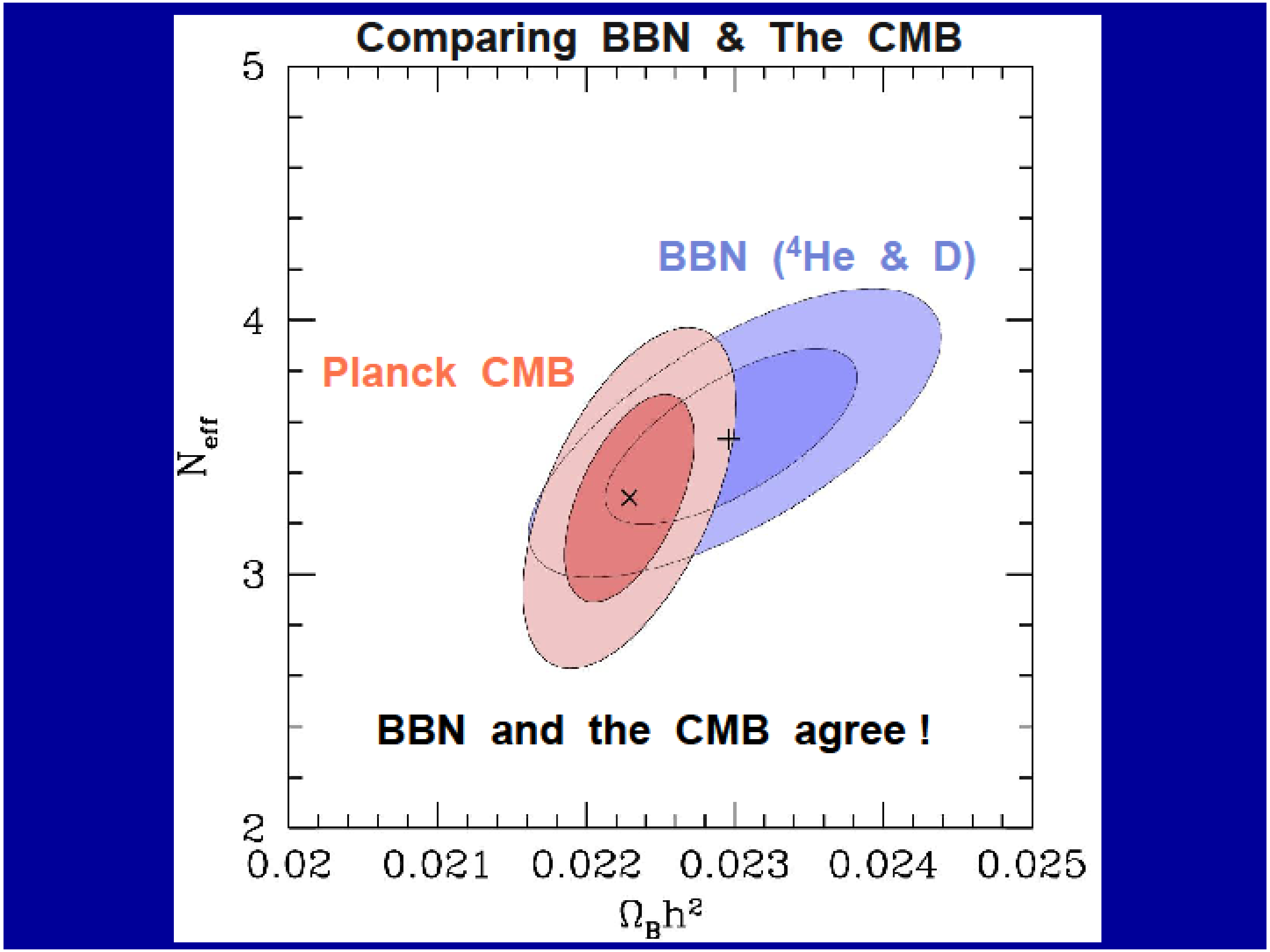
$^4He/H$ is inferred from H and He recombinations observed in Low – Z, Extragalactic H II regions.

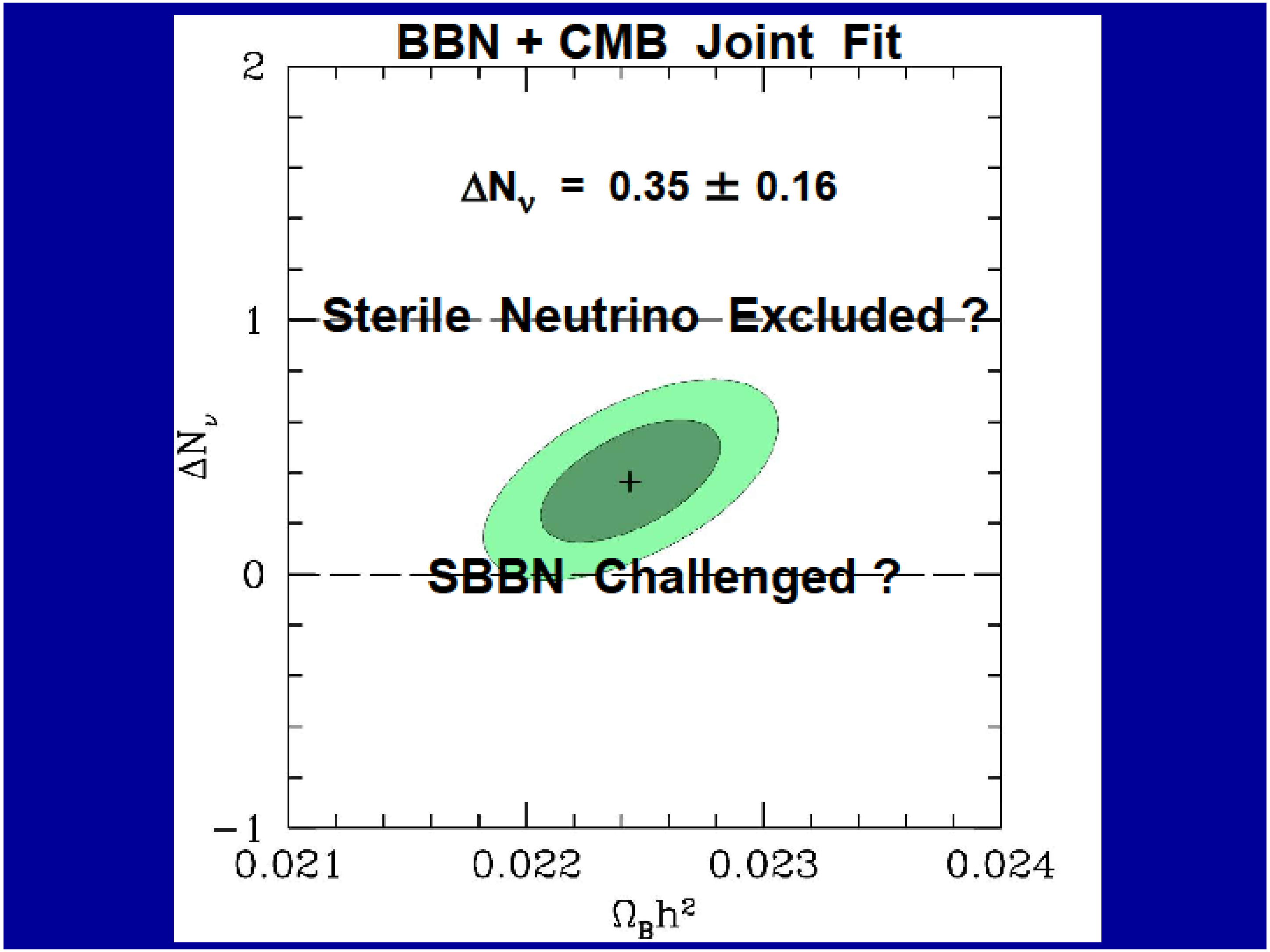




BBN – Predicted ΔN_ν vs. $\Omega_B h^2$
2
1
0
−1
ΔN_ν
0.02
0.021
0.022
0.023
0.024
0.025
$\Omega_B h^2$
$\Delta N_\nu = 0.50 \pm 0.23$
$\Omega_B h^2 = 0.0229 \pm 0.0006$


BBN – Predicted N_{eff} vs. $\Omega_B h^2$
BBN (^{4}He & D)
N_{eff} = 3.56 ± 0.23 (ΔN_ν = 0.50 ± 0.23)
$\Omega_B h^2$ = 0.0229 ± 0.0006 (η_{10} = 6.28 ± 0.15)
N_{eff}
5
4
3
2
0.02
0.021
0.022
0.023
0.024
0.025
$\Omega_B h^2$


Comparing BBN & The CMB
BBN (^{4}He & D)
Planck CMB
BBN and the CMB agree !
5
4
3
2
0.02
0.021
0.022
0.023
0.024
0.025
N_{eff}
$\Omega_B h^2$



BBN + CMB Joint Fit
ΔN_ν = 0.35 ± 0.16
Sterile Neutrino Excluded ?
SBBN Challenged ?
2
1
0
−1
ΔN_ν
0.021
0.022
0.023
0.024
Ω_B h²

Lepton Asymmetry

An Excess of Neutrinos vs. Antineutrinos (or, vice - versa).

Neutrino Mixing (Oscillations) Ensures the SAME asymmetry for all SM Neutrinos.

Lepton Asymmetry is measured by the degeneracy parameter ξ, related to the chemical potential μ, by $\xi = \mu / kT$

($\xi \geq 0$ for more ν than anti - ν).

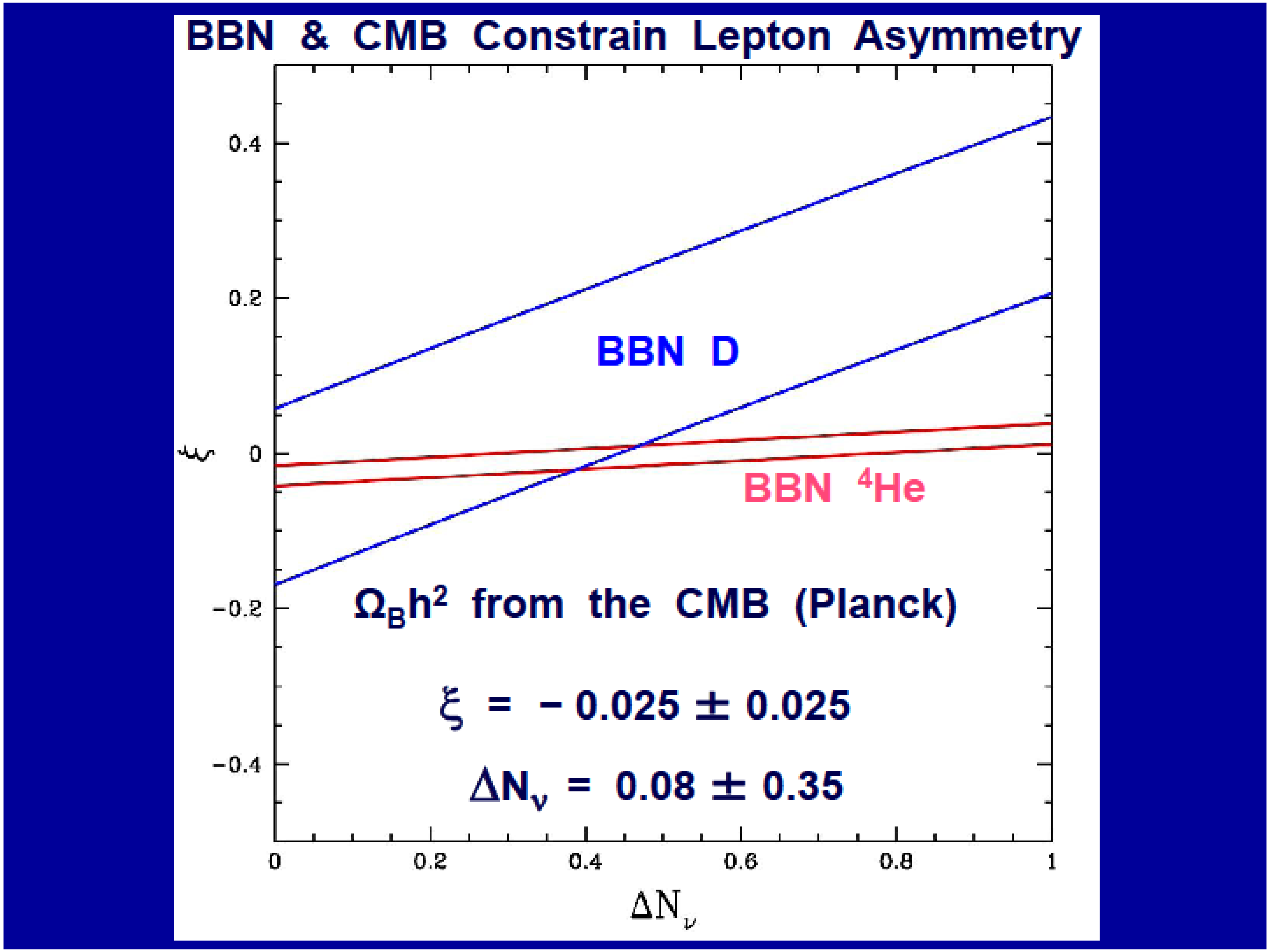
Electron Neutrinos and Antineutrinos play key roles in regulating the neutron - to - proton ratio.

For BBN there are (now) three parameters but, only two observables.

Unless is $|\xi|$ “large”, Lepton Asymmetry is invisible to the CMB.

Use the CMB to constrain $\Omega_B h^2$ (η_{10}).

Use BBN (D & ^{4}He) to constrain ΔN_ν and ξ.


BBN & CMB Constrain Lepton Asymmetry
0.4
0.2
0
−0.2
−0.4
ξ
0
0.2
0.4
0.6
0.8
1
ΔNν
BBN D
BBN 4He
ΩBh2 from the CMB (Planck)
ξ = − 0.025 ± 0.025
ΔNν = 0.08 ± 0.35

How do BBN and the CMB change in the presence of a light WIMP?

BBN & The CMB With A Light WIMP

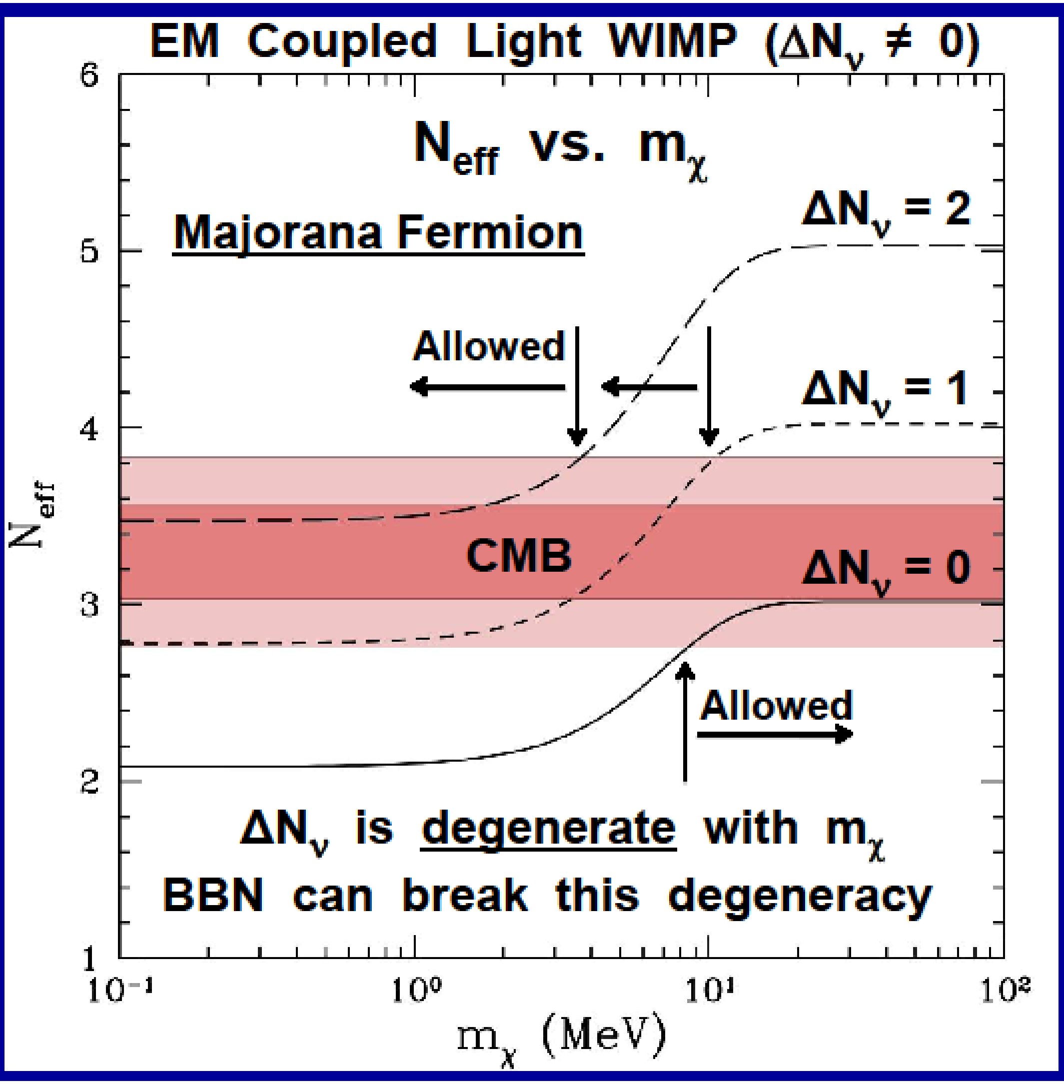
Very light WIMPs, thermal relics, annihilate late in the early Universe, changing the energy and photon densities at BBN and at recombination.

The CMB Confronts A Light WIMP

In the presence of an electromagnetically coupled light WIMP ($m_\chi \leq 30$ MeV), the effective number of neutrinos is: $N_{eff} = N^0_{eff}(1 + \Delta N_\nu / 3)$, where N^0_{eff} now depends on the WIMP mass.

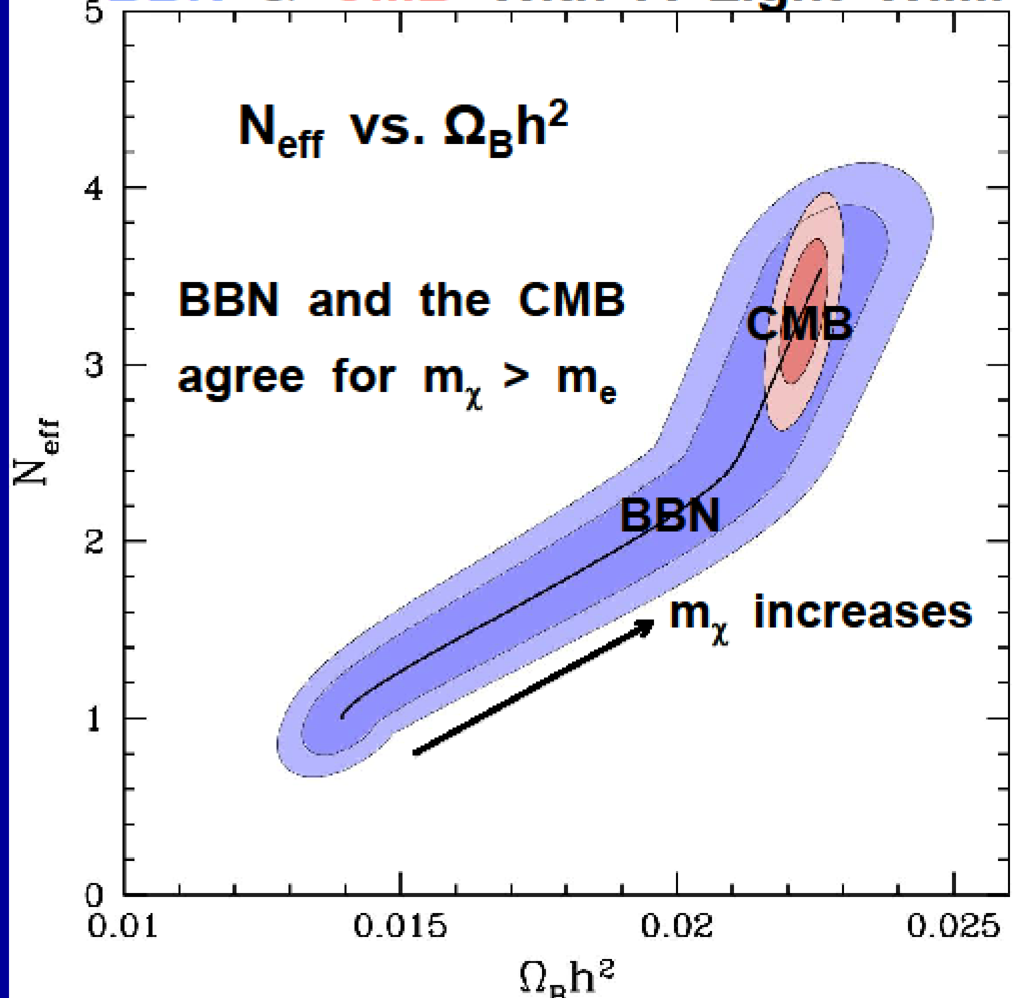
The annihilation of an EM coupled, light WIMP heats the photons relative to the neutrinos:

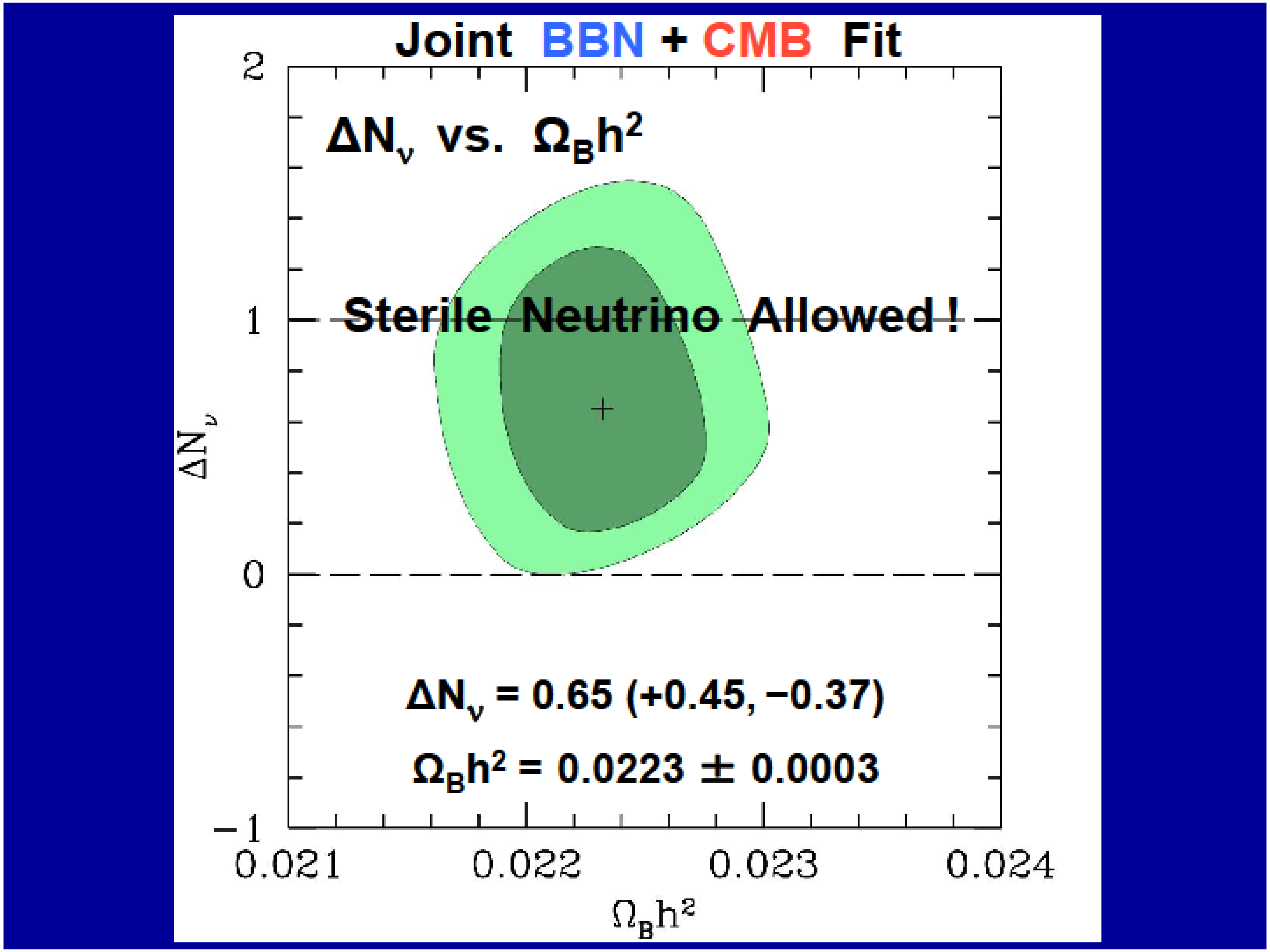
$$(T_\nu / T_\gamma)_0 \leq (4/11)^{1/3} \Rightarrow N^0_{eff} \leq 3 \;;\; N_{eff} \leq 3 + \Delta N_\nu$$



EM Coupled Light WIMP (ΔNν ≠ 0)
Neff vs. mχ
Majorana Fermion
ΔNν = 2
ΔNν = 1
ΔNν = 0
Allowed
CMB
Allowed
ΔNν is degenerate with mχ
BBN can break this degeneracy
Neff
mχ (MeV)
6
5
4
3
2
1
10⁻¹
10⁰
10¹
10²

BBN & CMB With A Light WIMP




Joint BBN + CMB Fit
ΔN_ν vs. Ω_B h^2
Sterile Neutrino Allowed !
ΔN_ν = 0.65 (+0.45, −0.37)
Ω_B h^2 = 0.0223 ± 0.0003
2
1
0
−1
0.021
0.022
0.023
0.024
Ω_B h^2
ΔN_ν

SUMMARY

BBN & CMB are consistent, constraining light WIMPs and the number of Equivalent Neutrinos.

In the absence of a light WIMP ($m_\chi > 30$ MeV) BBN & CMB are consistent, provided that $\Delta N_\nu \approx 0.35$ ($N_{eff} \approx 3.4$).

But, SBBN ($\Delta N_\nu = 0$) and a sterile neutrino ($\Delta N_\nu = 1$) are both disfavored.

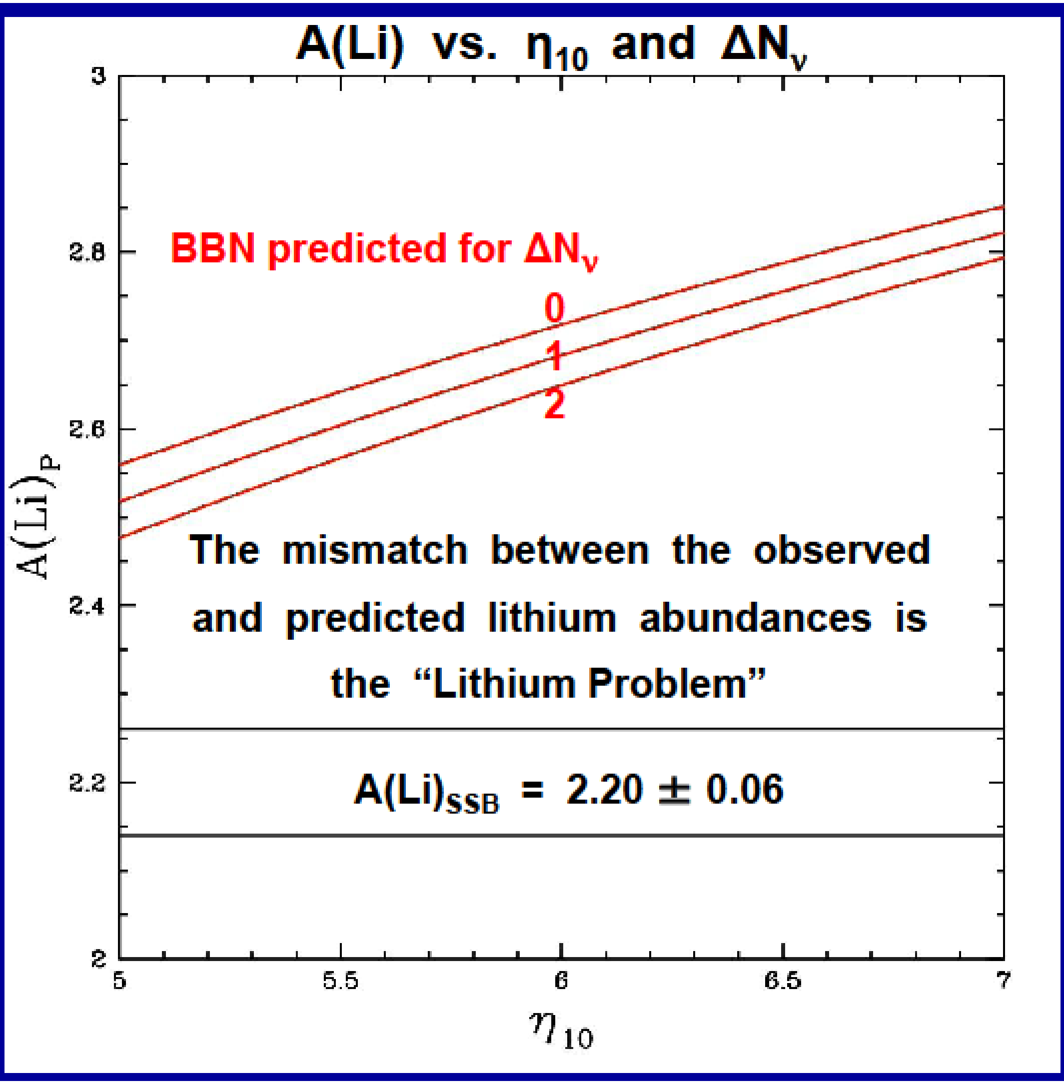
SUMMARY

BBN & CMB exclude an EM Coupled WIMP with $m_\chi \le 1 - 2$ MeV.

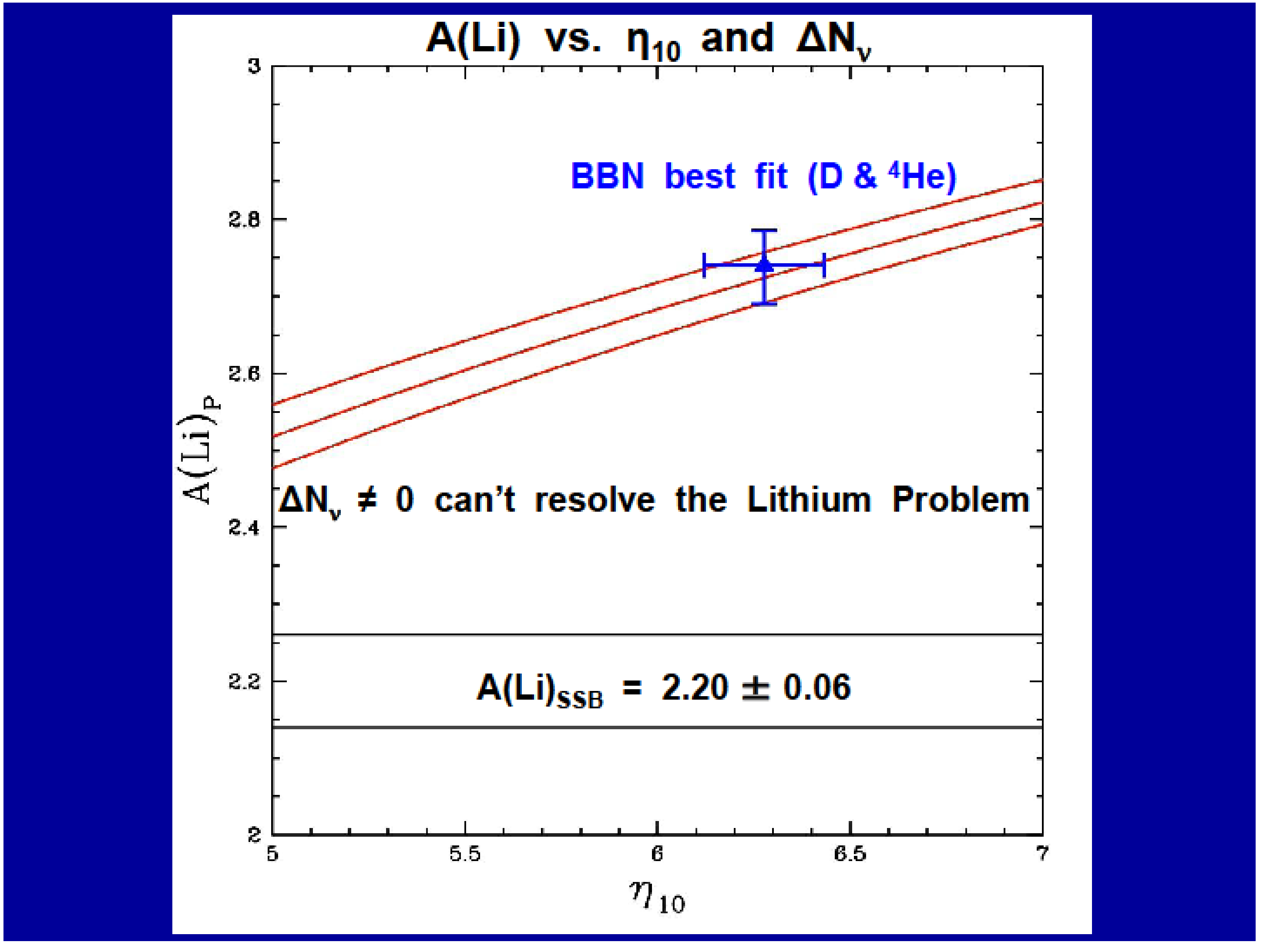
BBN & CMB favor an EM Coupled WIMP with $m_\chi \approx 5 - 10$ MeV, allowing for a sterile neutrino.

With or without an EM Coupled Light WIMP there is a lithium problem.

There is a so=called “Lithium problem” in BBN which we ignore here!

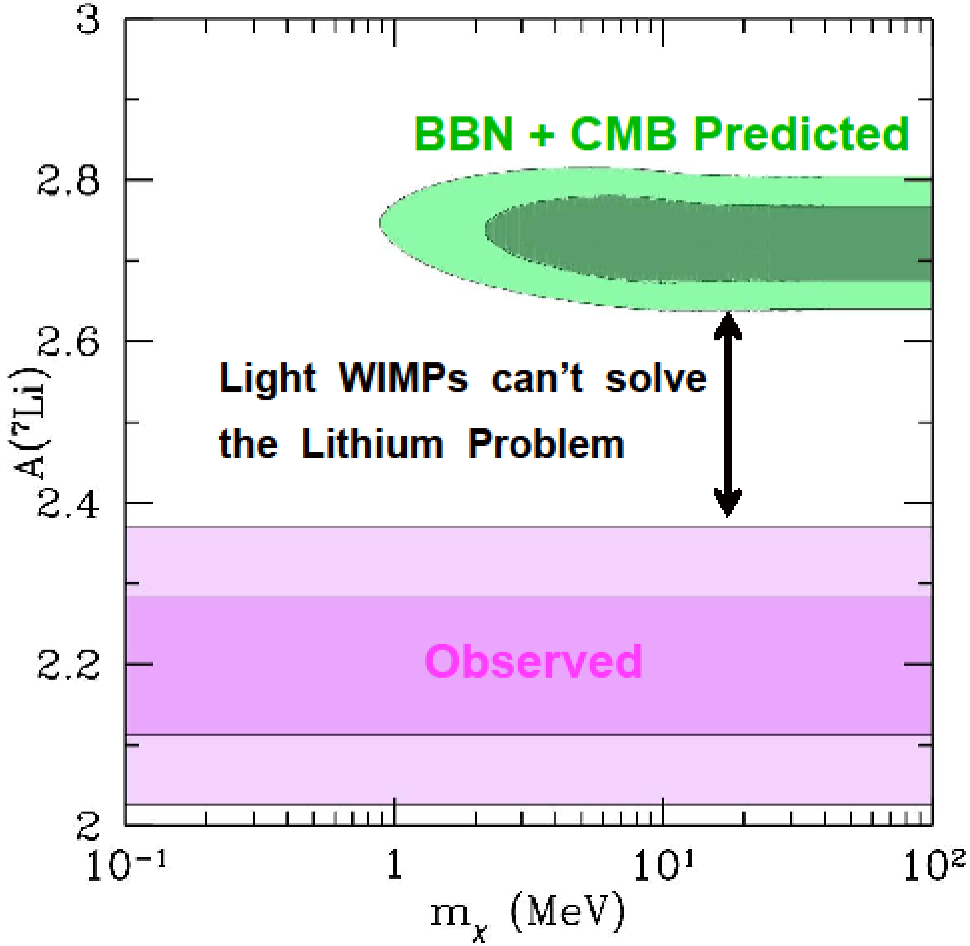


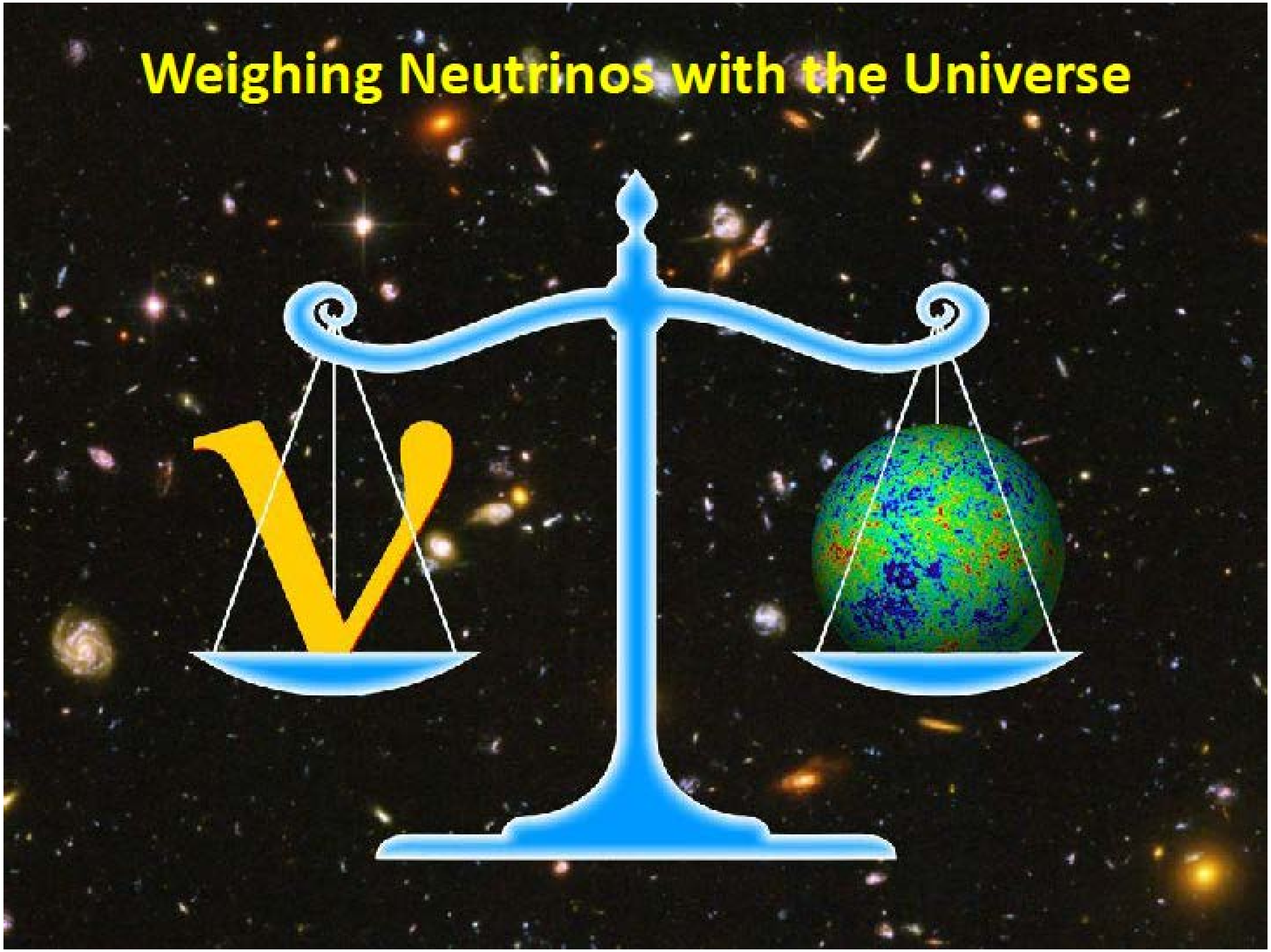
A(Li) vs. η10 and ΔNν
BBN predicted for ΔNν
0
1
2
The mismatch between the observed and predicted lithium abundances is the "Lithium Problem"
A(Li)SSB = 2.20 ± 0.06
A(Li)P
η10
3
2.8
2.6
2.4
2.2
2
5
5.5
6
6.5
7



A(Li) vs. η10 and ΔNν
BBN best fit (D & 4He)
ΔNν ≠ 0 can't resolve the Lithium Problem
A(Li)SSB = 2.20 ± 0.06
A(Li)P
η10
3
2.8
2.6
2.4
2.2
2
5
5.5
6
6.5
7

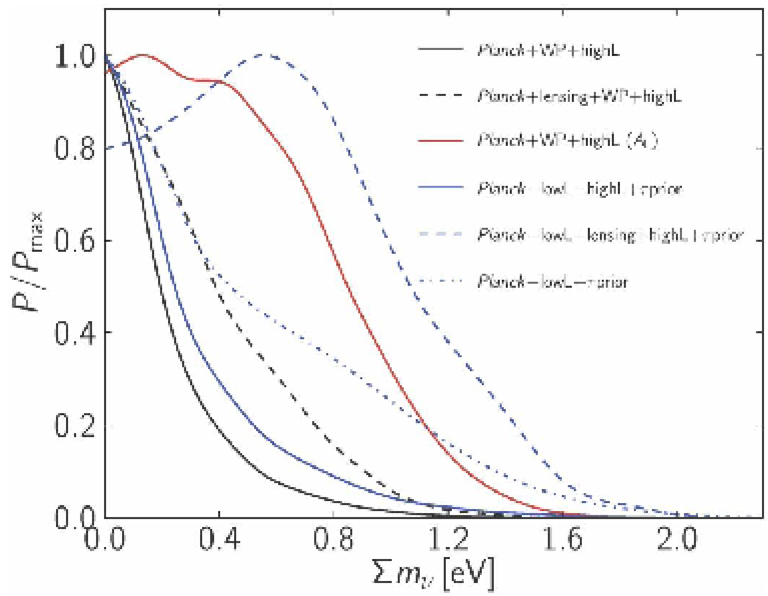
Lithium Predicted vs. Observed




Weighing Neutrinos with the Universe

Neutrino Mass Limits Post Planck (2013)

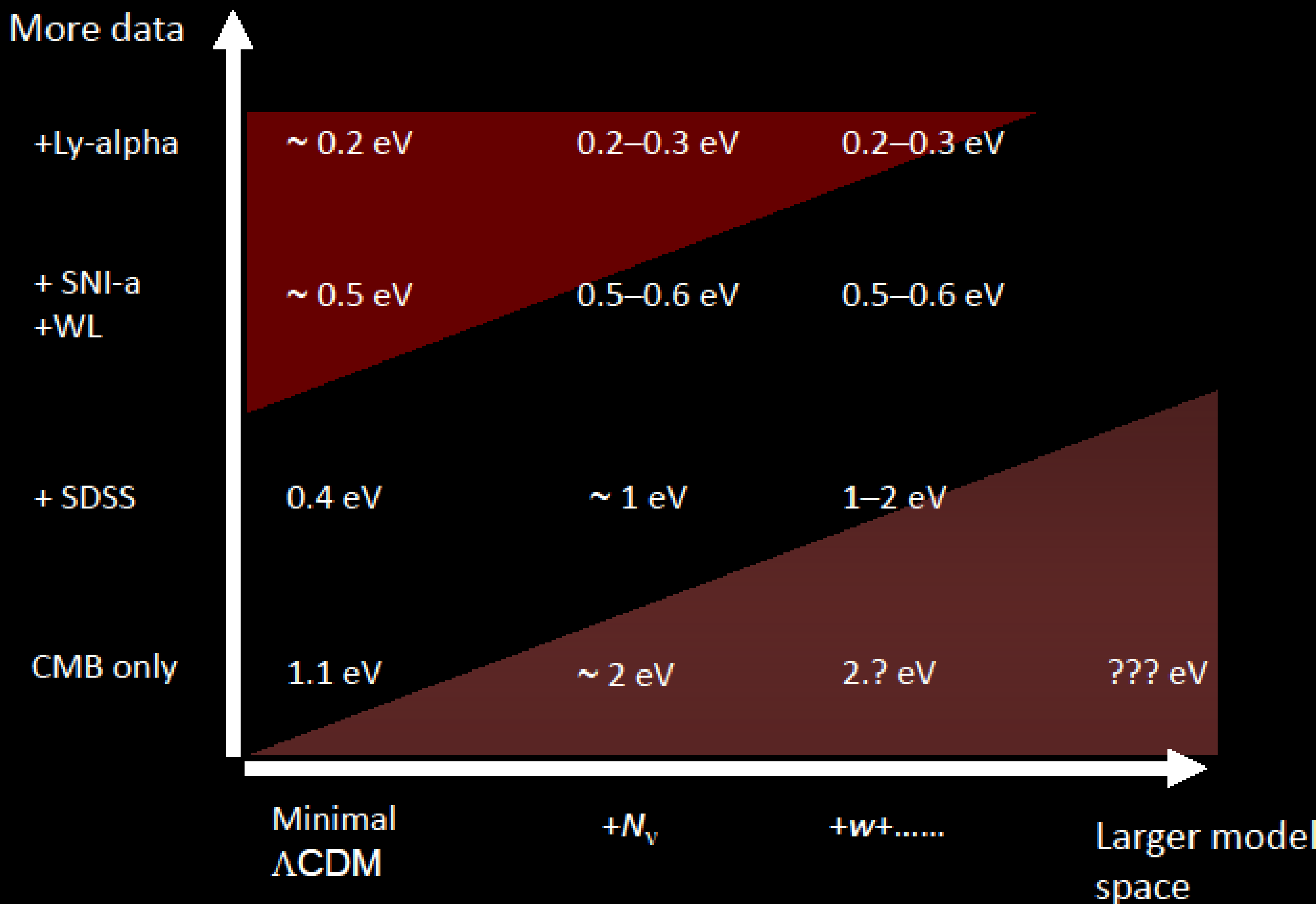
Depends on used data sets
Many different analyses in the literature



Planck alone: Σm_ν < 1.08 eV (95% CL)
CMB + BAO limit: Σm_ν < 0.23 eV (95% CL)

Ade et al. (Planck Collaboration), arXiv:1303.5076

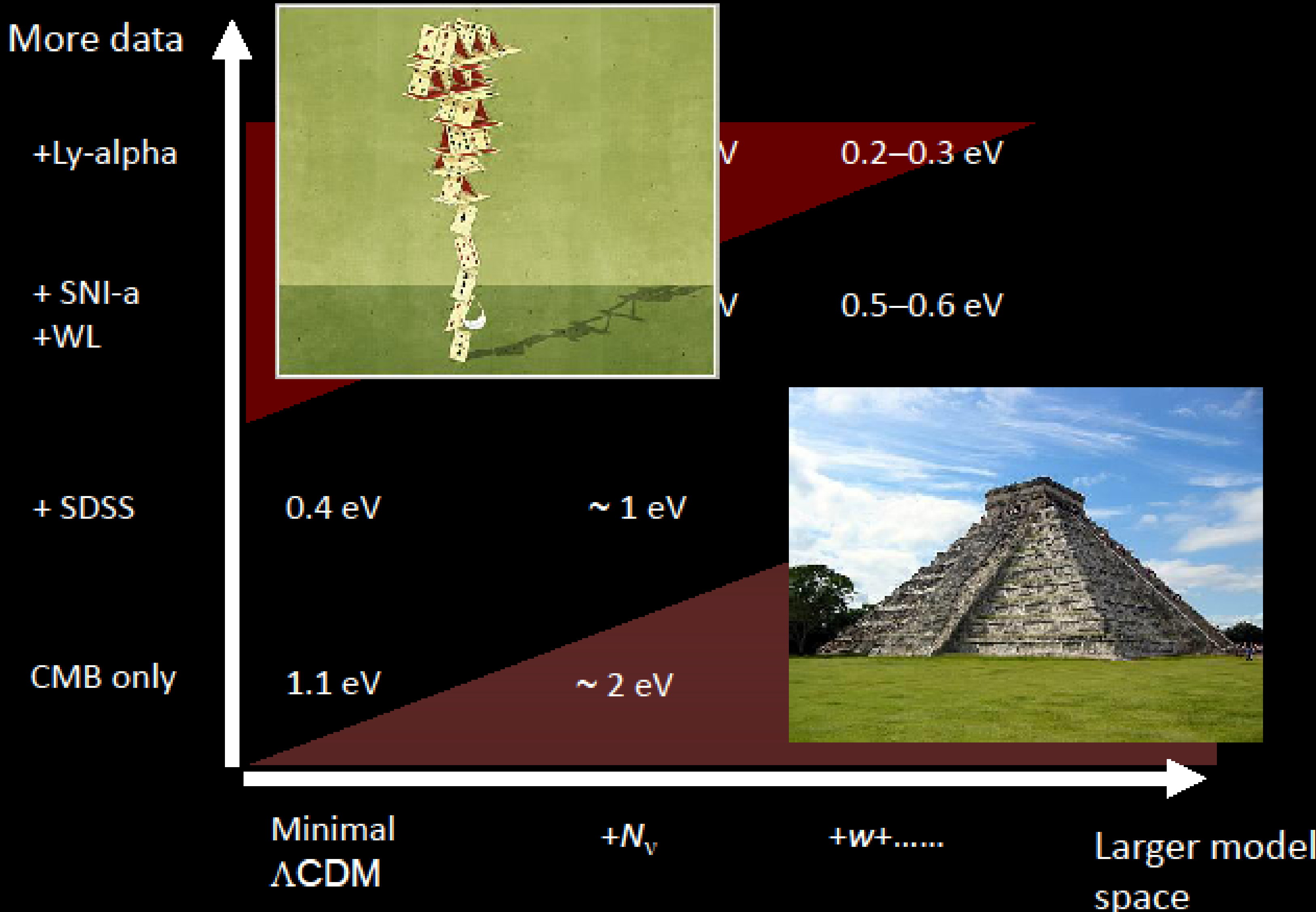
Neutrino Mass from Cosmology Plot (Hannestad)



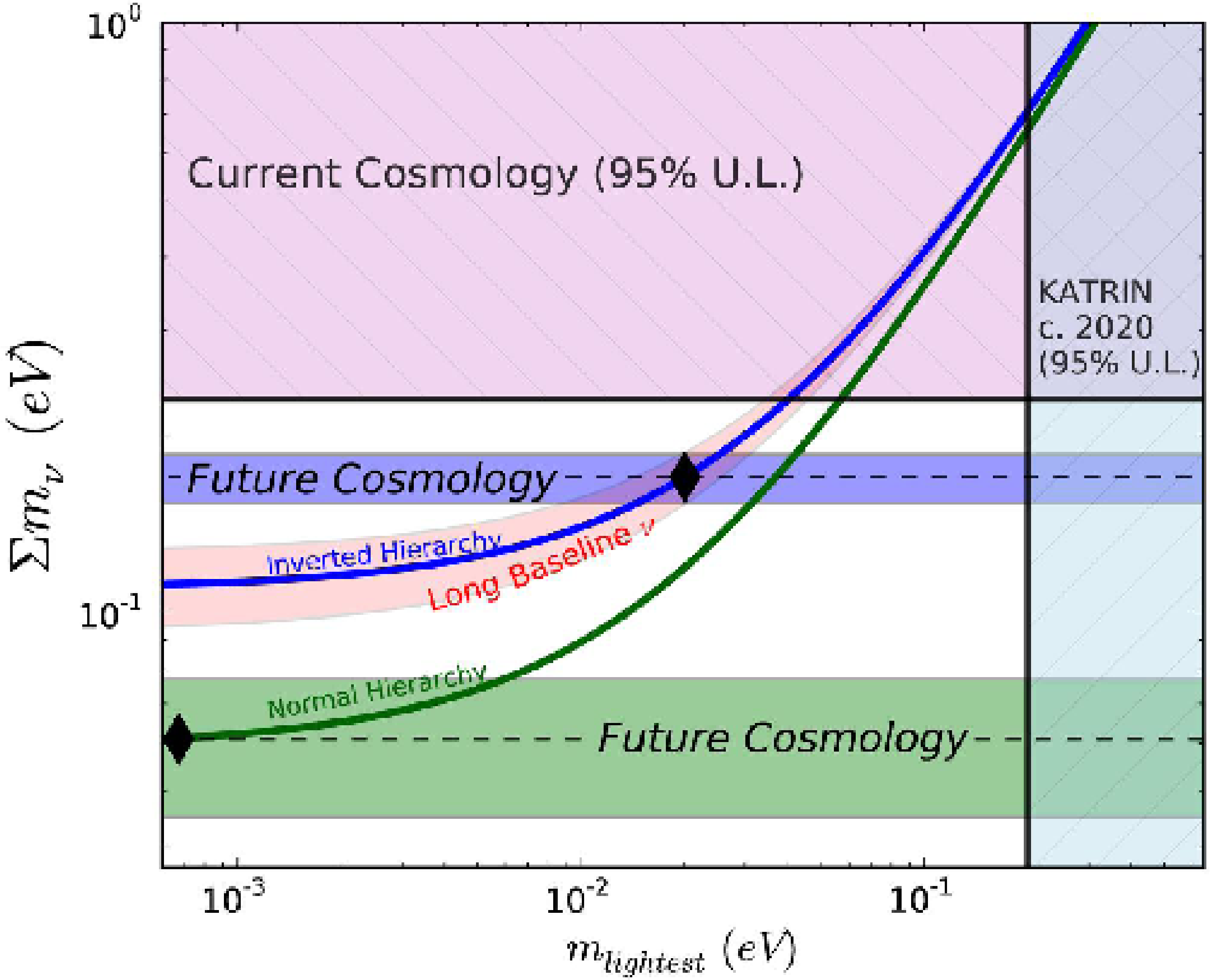
Neutrino Mass from Cosmology Plot (Hannestad)

More data	Minimal ΛCDM	$+N_\nu$	$+w+$......	Larger model space
+Ly-alpha		V	0.2–0.3 eV	
+ SNI-a +WL		V	0.5–0.6 eV	
+ SDSS	0.4 eV	~ 1 eV	1–2 eV	
CMB only	1.1 eV	~ 2 eV	2.? eV	??? eV

Neutrino Mass from Cosmology Plot (Hannestad)

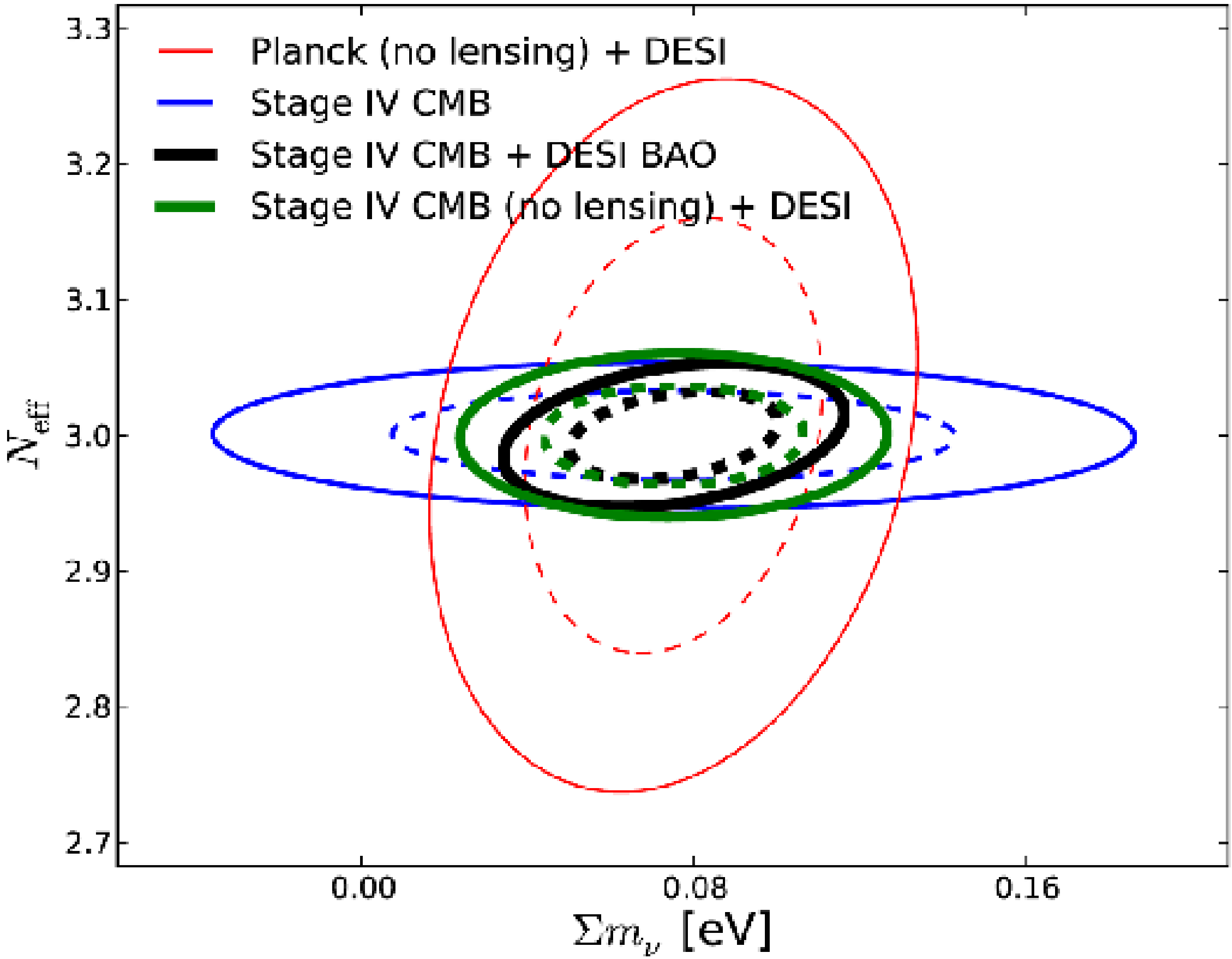


Neutrino-Mass Sensitivity Forecast



Community Planning Study: Snowmass 2013, arXiv:1309.5383

Nu-Mass and N-eff Sensitivity Forecast



Community Planning Study: Snowmass 2013, arXiv:1309.5383

These are then firm predictions of the Hot Big-Bang Cosmology:

Neutrino number density = 112 neutrinos/cm^3 for each flavor, i.e.,
56 neutrinos and 56 antineutrinos of each flavor

Neutrino temperature = 1.94 K = 1.67×10^{-4} eV

If one could confirm (or find deviations) from these predictions, one would test the theory at $t \sim 1$ sec, $T \sim 1$ MeV, and redshift $z \sim 10^{10}$, much earlier and hotter than the tests based on BBN and CMB.

There is, therefore, strong motivation to try to detect these CνB.

Clustering neutrino density enhancement

Massive particles become nonrelativistic when their mass exceeds the temperature of the Universe. From than on they can become bound, i.e., concentrate in structures of various sizes. Their densities in these structures can far exceed the average density derived from cosmological measurements and arguments.

The overall energy density (critical density for $\Omega = 1$) of the Universe is

$$\rho_c = 1.05\text{x}10\ h_{100}\ \text{eV/cm} \sim 5\ \text{keV/cm} \quad (\text{since } h_{100} \sim 0.73)$$

component	average ρ(keV/cm^3)	Structure	Enhancement
baryons	0.2	galaxy(disk)	$\sim 5\text{x}10^6$
dark matter	1.0	galaxy(halo)	$\sim 3\text{x}10^5$
Neutrinos	**112(Σm_ν/keV)**	**clusters**	**~1 - 100**

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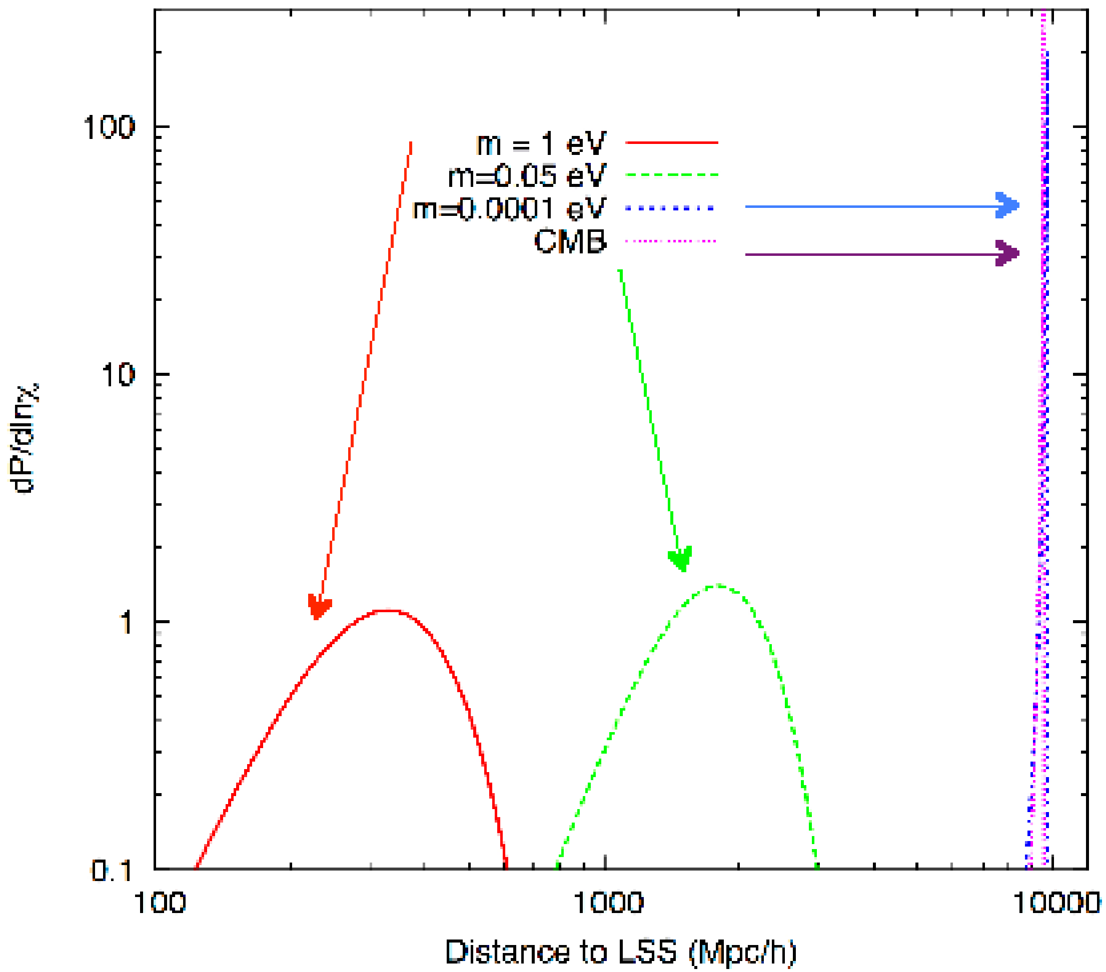
Neutrinos	**112(Σm_ν/keV)**	**clusters**	**~1 - 100**

An interesting and contraintuitive consequence of finite nuclear mass, and thus the fact that neutrino are nonrelativistic now, is the fact that the last scattering surface for them is much closer that for the CMB photons even though they decoupled earlier.



The probability that a neutrino of mass *m* last scatters at a given comoving distance from us. The large spread is the consequence of the momentum distribution of the neutrinos.

From Dodelson & Vesterinen, PRL103, 2009

How do we detect Cosmic Neutrino Background (CNB)?

de Broglie wavelength $\lambda_\nu = h/p_\nu \sim 2.4$ mm (for $p_\nu \sim 3T_\nu$)

A sphere with $d = \lambda_\nu$ contains ~ 10^{21} nucleons. If neutrinos interact coherently with all of them, it should help a lot.

The first idea, from ~1980 when people believed that $m_\nu \sim 30$ eV, was to use the coherent scattering on macroscopic objects.

To describe the reflection or refraction on a thin foil, it was proposed to use the concept of index of refraction

$$n = 1 + N \lambda_\nu^2 f(0)/2\pi,$$

where N is the number of density of target atoms and f(0) is the forward scattering amplitude.

Deviation of index of refraction from unity is obtained the same way as in the treatment of the MSW effect for matter neutrino oscillations

$$n-1 = \pm [G_F N (3Z - A)]/(2^{3/2} T_\nu) \quad \text{for } \nu_e \ (\nu_e)$$

$$n-1 = \pm [G_F N (Z - A))]/(2^{3/2} T_\nu) \quad \text{for } \nu_\mu, \nu_\tau \ (\nu_\mu, \nu_\tau)$$

T_ν is the kinetic energy of the nonrelativistic neutrinos.

For ν_μ on gold $1-n \approx 10^{-7}$ (eV/m_ν) for v_ν = 500 km/s
and the critical scattering angle $\theta_c = [2(1-n)]^{1/2} \approx 1.5$ **arcmin**

Consider neutrinos with flux density **j (neutrinos/sr cm^2 sec)**.
Collision rate for area of 1 cm^2 with angles less than θ_c is
$2\pi j\ \theta_c$ and the momentum transfer is $p_\nu\,\theta_c$

The **pressure** of the `neutrino wind' is then
$dp/dt = 4\pi\ \rho_\nu N\ G_F (A-Z) / 2^{1/2}$
linear in G_F and independent of v_ν (Opher,74,82; Lewis,80)

Unfortunately, this derivation is wrong !!!

(Cabibbo & Maiani, 82; Langacker,Leveille & Sheiman, 83)

$$F = -\Delta p_\nu/\Delta t \simeq G_F \int d^3x\ \rho_A(x)\ \nabla\ n_\nu(x)$$

With ρ_A atomic number density of the target, and $\nabla n_\nu(x)$ gradient of the local neutrino density. This gradient vanishes since $n_\nu(x)$ is uniform at the scale of the detector, except for the weak scattering waves that are of order G_F. **Thus the force is $G_F{}^2$.**

Another proposal to use coherence, this time $\sim G_F^2$

(Shvartsman, Braginski, Gershtein, Zeldovich, and Khlopov, 82)

Scatter relic neutrinos on spheres with $r = \lambda$; use the virial motion of Earth with respect to the relic neutrinos, $v \sim 300$ km/s and measure the force on such spheres.

Cross section $\sigma = G_F^2\, m_\nu^2\, k_L^2/\pi$, $k_L = 3Z-A$ (for ν_e), $A-Z$ (for ν_μ, ν_τ)

Force $F = 2 n_\nu v\, m_\nu v\, \sigma\, N_A^2$

(n_ν = density of relic neutrinos, N_A = number of target atoms in each sphere)

Acceleration of each sphere $a = F/m_{sphere}$ is independent of m_ν since $N_A \sim \lambda^3 \sim m_\nu^{-3}$.

$$a_t \approx 2 \times 10^{-28} \left(\frac{n_\nu}{\bar{n}_\nu}\right) \left(\frac{10^{-3}c}{v_{relative}}\right) \left(\frac{\rho_t}{\mathrm{g/cm^3}}\right) \left(\frac{r_t}{\bar{\lambda}}\right)^3 \mathrm{cm/s^2}$$

Take iron spheres, assume clustering $n_\nu/\langle n_\nu \rangle = 100$,

$a \sim 3 \times 10^{-25}$ cm s^{-2}, $F \sim 3 \times 10^{-29}$ dyne

<u>This is ~12 orders of magnitude from the sensitivity of the current Dicke - Eotvos type experiments.</u>

For Majorana ν there is a further $(v_{rel}/c)^2$ suppression.

This force is for Dirac neutrinos, because the forward coherent scattering is from vector interaction, which is zero for Majorana nus. T

here is a very small correction …..

Recent proposals to revive this in some form, e.g. (i)Torsion balance(Hagmann),(ii) heating of superconducting domains(Stodolsky)……….

Using resonance absorption of UHE neutrinos on CνB

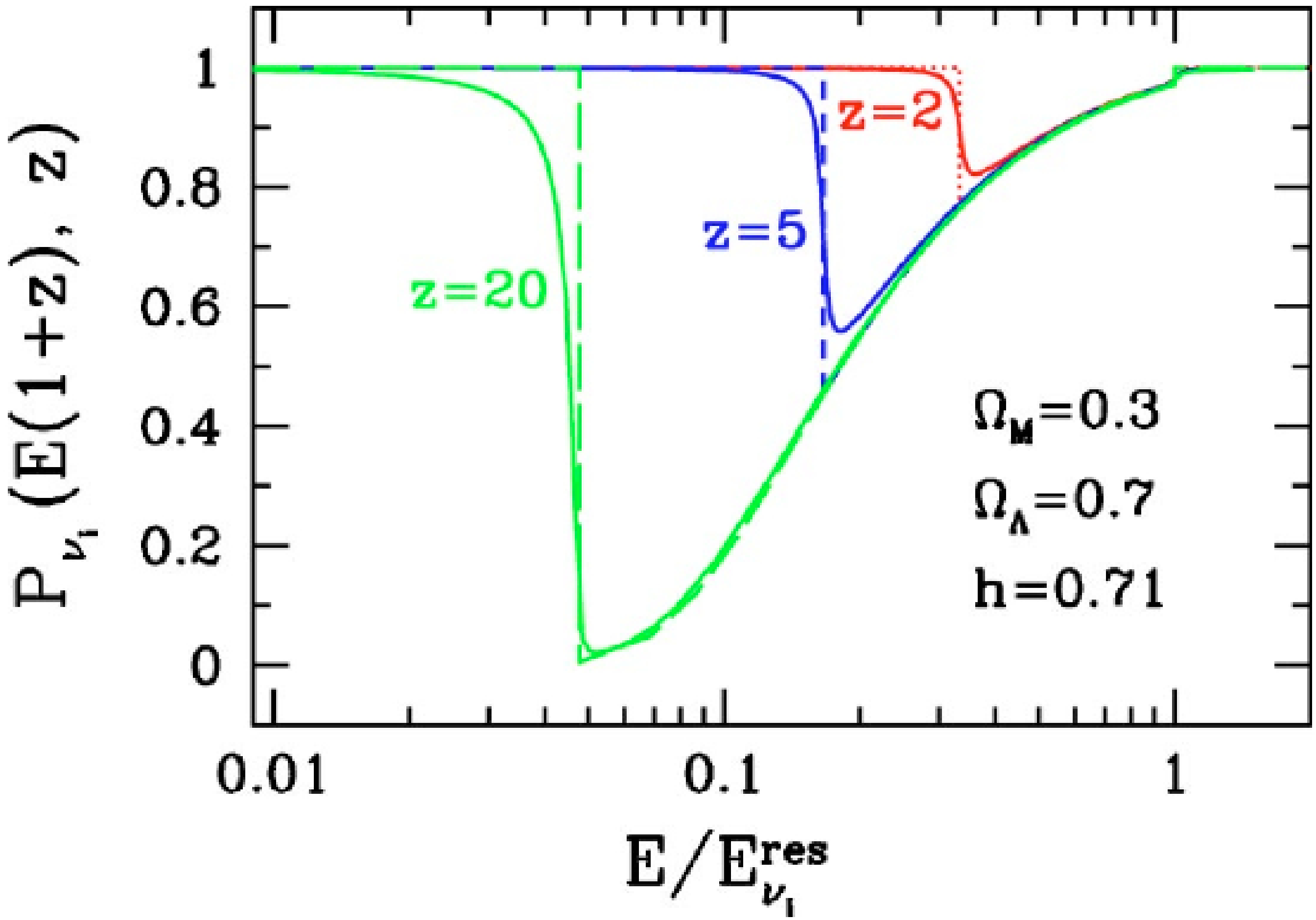
The Universe is transparent to neutrinos with the exception of the resonance annihilation into Z-bosons (Weiler 82).

The resonance energy is $E_\nu^{res} = m_Z^2/2m_\nu = 4.2\times10^{22}$ eV (0.1 eV$/m_\nu$), and the cross section is $\langle\sigma_{\nu\nu}^{ann}\rangle = 2\pi\sqrt{2}G_F = 40.4$ nb.

When the UHE neutrinos are injected at redshift z with energy E_i, They are detected at Earth with $E = E_i/(1+z)$. Thus, the ``dip" in the observed spectrum will be broadened and z dependent.

Clearly, the observable effect will depend on the z and energy Distribution, so far unknown, of the UHE neutrino sources.

Note that the highest energy neutrinos observed so far have energies ~PeV = 10^{15} eV.



Survival probability of a cosmic neutrino injected at redshift *z* with energy E_i, so that at Earth it has energy $E = E_i/(1+z)$, in units of the resonance energy $E_\nu^{res} = m_Z^2/2m_\nu$. Full treatment (full lines) and the narrow width approximation are compared (from Eberle et al, 04)

Since none of these proposals work, by a huge margin, lets consider the usual way of detecting neutrinos, by charged current weak interactions.

The problems to solve:

1) Can one find an appropriate target?
2) How many target atoms can one use in practice?
3) What is the cross section, and is the event rate sufficient?
4) Can one separate the signal from background?

Each of these items is challenging, but it turns out that the needed technological improvements are *only(??!!)* one or few orders of magnitude each, so it is worthwhile to consider them in more detail.

Since the momentum of the CNB $p_\nu \to 0$, we must consider only exothermic reaction, i.e., reactions on unstable targets.
Take the $\nu_e + n \;\to\; p + e^-$ (hypothetical, there are no free neutrons) reaction with $E_e = M_n - M_p + E_\nu$ which remains positive and $E_e \geq m_e$ even when $E_\nu \to 0$?

$$\frac{d\sigma}{d\cos\theta} = \frac{\bar{G}^2}{v_\nu} E_e p_e [(f^2 + 3g^2) + (f^2 - g^2) v_e v_\nu \cos\theta]$$

The cross section now contains $1/v_\nu$, which
means that the rate, σv_ν , remain finite even when $v_\nu \to 0$.
(see Weinberg 62, Cocco,Mangano,Messina 07)

Naturally, the $1/v_\nu$ factor should be there even for the endothermic reactions, but becomes irrelevant since in that case $v_\nu \to c$ (=1 here).
This is a general result for reactions with nonrelativistic projectiles (known long time ago for the case of slow neutrons).

Analogous reactions on unstable nuclear targets A_Z are

$\nu_e + A_Z \to e^- + A_{Z+1}$ or $\bar{\nu}_e + A_Z \to e^+ + A_{Z-1}$

where the allowed $\beta^\pm$ decay of $A_{Z\pm1}$ is characterized by the known nuclear matrix element $|M_{nucl}|^2 \approx 6300/ft_{1/2}$.

The cross section in cm^2 for these exothermic reactions is

$$\sigma = \sigma_0 \times \langle \frac{c}{v_\nu} E_e p_e F(Z, E_e) \rangle \frac{2I' + 1}{2I + 1}$$

with

$$\sigma_0 = \frac{G_F^2 \cos^2\theta_C {m_e}^2}{\pi} |M_{nucl}|^2 = \frac{2.64 \times 10^{-41}}{ft_{1/2}}$$

When $v_\nu \to 0$ the $e^\pm$ energies are monoenergetic $E_e = Q + m_e + m_\nu$

They are separated from the $e^\pm$ β-decay spectrum by $2m_\nu$.

We can consider now the answer to our first question:

Can one find an appropriate target?

Clearly the unstable A_Z target should have halflife $t_{1/2}$ longer than the duration of the measurement, i.e., **$t_{1/2} \geq$ years**.

It could be manmade, or it could exist in nature. However, natural radioactivity has **$t_{1/2} \geq 10^9$ years**.

The target A_Z should also have minimal possible **$ft_{1/2}$** so that the cross section is as large as possible. This means that the superallowed decays, with **$ft_{1/2} \sim 1000$** are preferred.

Now, lets consider the second question:

How many target atoms can one use in practice?

When reviewing possible targets, the tritium (^3H) clearly comes to mind. Its halflife **$t_{1/2}$ = 12.3 years** is just right, and **$ft_{1/2}$ = 1143** is almost as small as the $ft_{1/2}$ for the free neutron decay.

The technology of production is well developed, and using as much as **1 Mcu (2.1x10^{25} tritium atoms)** is very challenging but appears to be technologically possible.

This corresponds to just ~100 g of pure tritium.
(Note, however, that the Karlsruhe facility, handling all tritium for the KATRIN experiment, as well as for ITER, is licensed for maximum only 20 g of tritium.)

What is the cross section, and the event rate?

To estimate the relic neutrino velocity, lets neglect the virial motion and use $v_\nu/c \sim 3T_\nu/m_\nu$, with $T_\nu = 1.9$ K.
With this assumption $\sigma = 1.5\times10^{-41}\,(m_\nu/\text{eV})\ \text{cm}^2$

The CNB capture rate is then independent of m_ν, and v_ν

$$R = \sigma \times v_\nu \times n_\nu \approx 1.8 \times 10^{-32} \times n_\nu/\langle n_\nu\rangle\ \text{s}^{-1}$$

The number of events is
$N_{\nu\,capt} \approx 83\ \text{yr}^{-1}\ \text{Mcu}^{-1}$ for $n_\nu/\langle n_\nu\rangle = 10$
So, the number of events would be reasonably large.

Note that this rate is for Majorana ν, for Dirac ν it is reduced by 0.5 (Long et al. arXiv: 1405.7654.)
Also, there will be a ~1% annual modulation depending on the velocity distribution (Safdi et al., PRD90,043001)

Can we understand that it is possible to have a reasonably large neutrino capture rate with only ~100g of tritium compared with ~500 ton (fiducial) of scintillator in KamLAND?

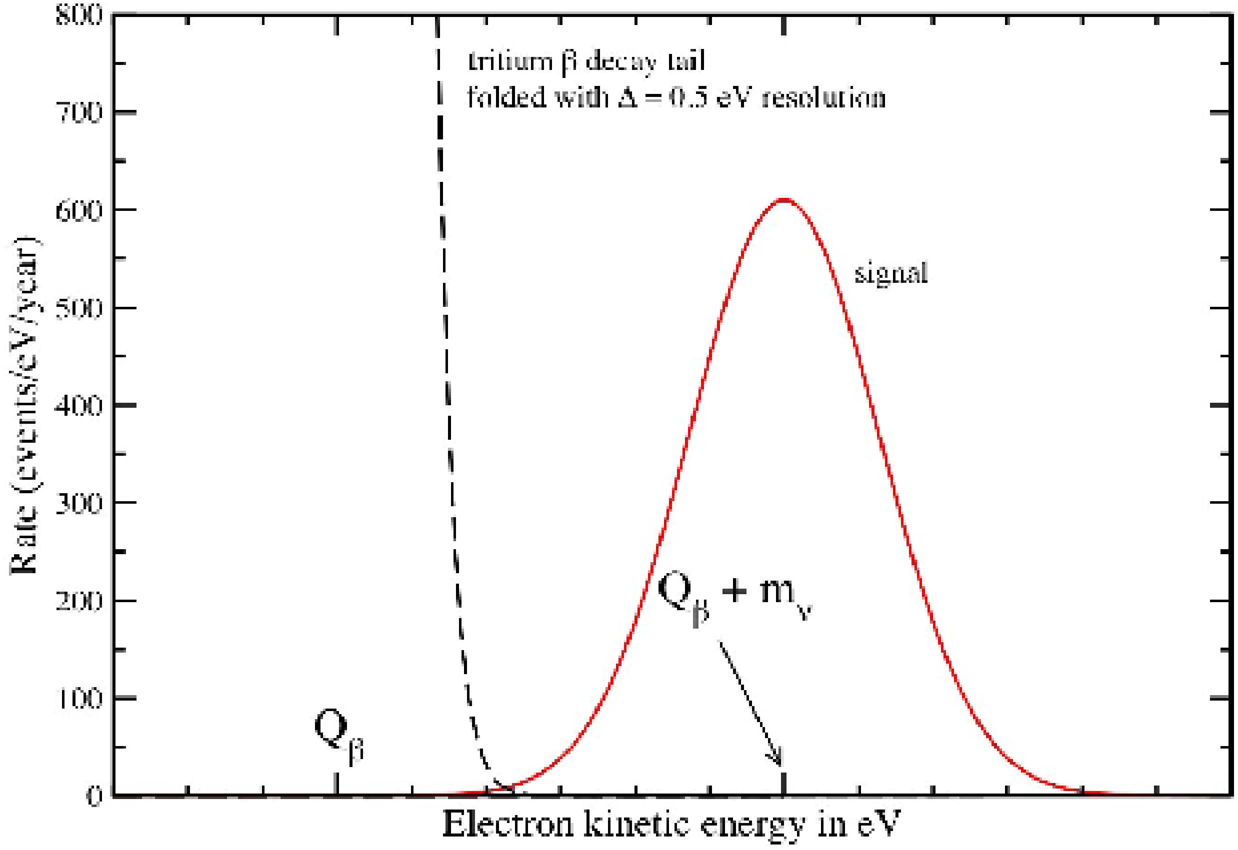
Here are the ratios tritium/KamLAND:

Cross section	~100
Number of targets	$\sim 5 \times 10^{-7}$
Flux	$\sim 10^{4}$
Total	~0.5

Finally, the last and most difficult question:

Can one separate the signal from background?

There are 3.7×10^{16} tritium β decays/s , and hence emitted electrons distributed over the energy interval 0 — Q_β - m_ν and smeared by the detector energy resolution. The fraction of electrons in the energy interval of width Δ just below the endpoint is ~ $(\Delta/Q_\beta)^3$



This is for Δ =0.5 eV m_ν=1 eV and $n_\nu/\langle n_\nu \rangle$ = 50.

There are, thus, two challenging problems:

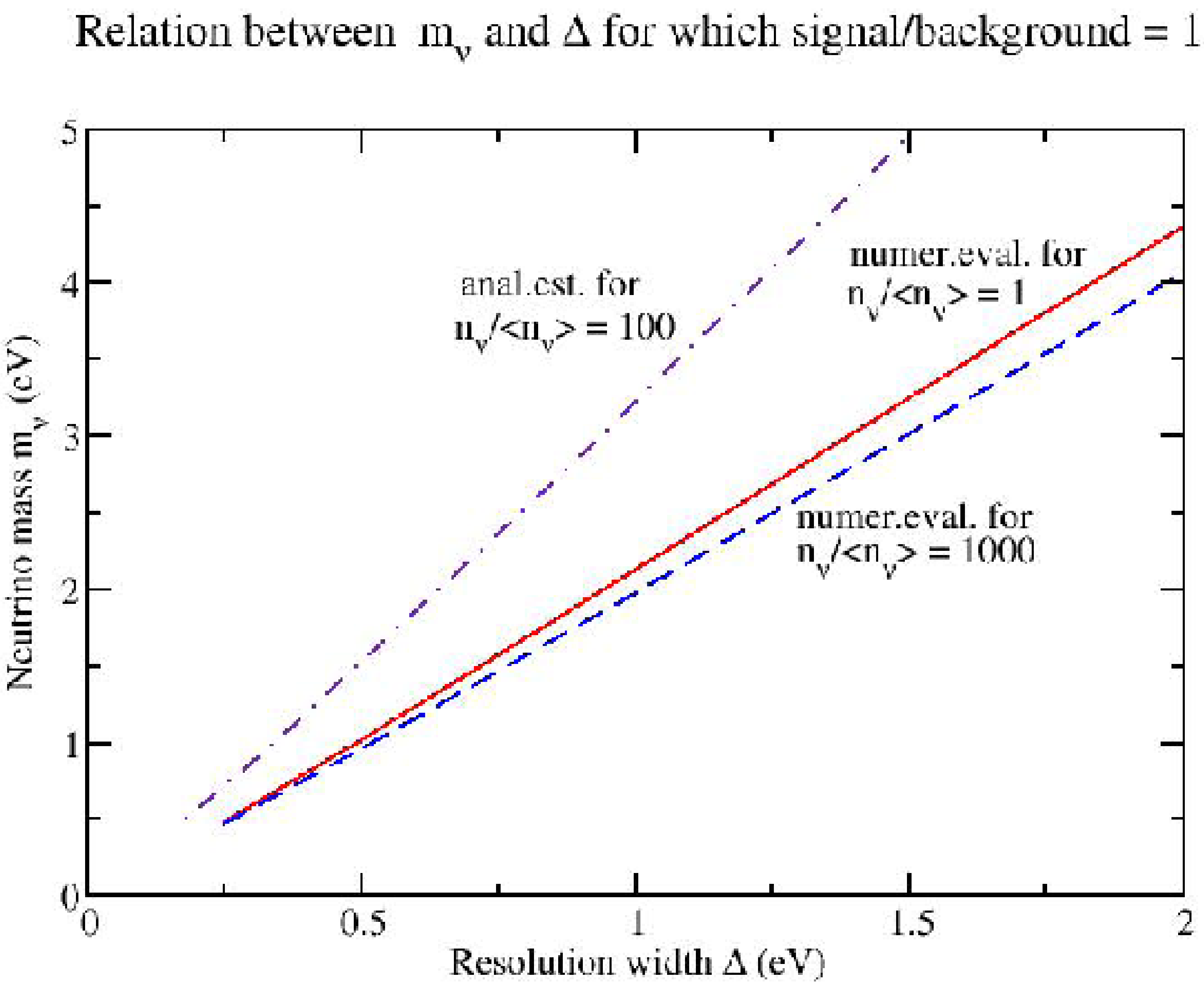
1) Can one filter out up to the ~10^{16} electrons/s that have energies below the endpoint?
In KATRIN design the ratio between electrons in the window of planned 0.2 eV sensitivity and the total decay rate is ~10^{15}. So, the filter used in KATRIN will be essentially capable to reach the required rejection ratio.

2) Can one reach the required energy resolution? And how the signal to background ratio depends on the resolution Δ and on the neutrino mass m_ν?
It turns out one can make an analytic estimate of the ratio

$$\lambda_\nu/\lambda_\beta = 6\pi^2\, n_\nu/\Delta^3 \times (2\pi)^{1/2}\, e^{2z}\,, \quad z = (m_\nu/\Delta)^2$$

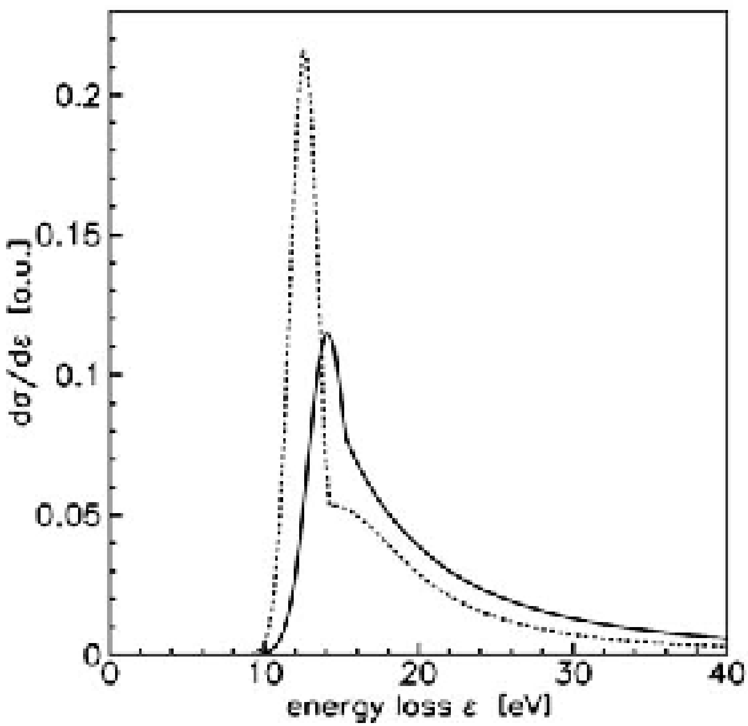
valid reasonably well as long as $m_\nu > \Delta$ (*Cocco et al.*)

The analytic formula suggest that $m_\nu/\Delta \sim 3$ is needed, numerical evaluation gives $m_\nu/\Delta \sim 2$, a somewhat more favorable ratio.



Here are potential killer problems:

1) Past and planned experiments use molecular T_2. The rotational-vibrational states in the final ^3HeT molecule are spread over ~0.36 eV. That essentially limits the achievable resolution. However, using atomic T would be very difficult but obviously necessary.
2) Electrons scatter on T_2 with $\sigma = 3 \times 10^{-18} cm^2$. This limits the source column density and makes sources of 1kCu or more impossible. Totally new arrangement would be needed for stronger sources.



Schematic idea of the `Project 8' of Monreal and Formaggio

Phys. Rev. D80, 051301(2009).

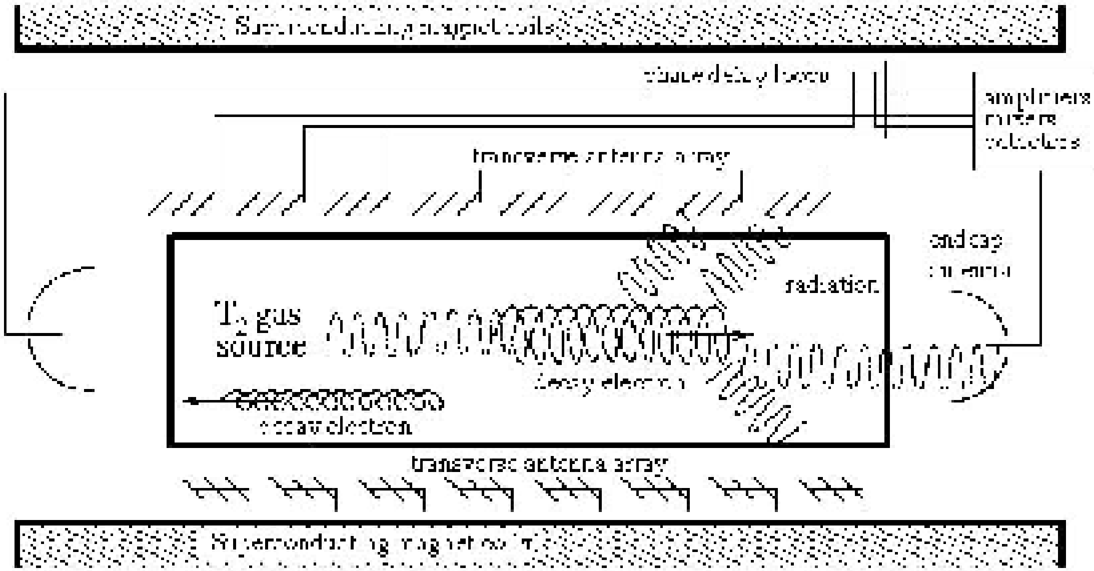


FIG. 1: Schematic of the proposed experiment. A chamber encloses a diffuse gaseous tritium source under a uniform magnetic field. Electrons produced from beta decay undergo cyclotron motion and emit cyclotron radiation, which is detected by an antenna array. See text for more details.

Cyclotron frequency depends on the electron kinetic energy:
$\omega = qB/(m_e + E)$

Each electron emits microwaves at frequency ω and total power

$P(\beta,\theta) =$
$1/4\pi\varepsilon_0 \times 2q^2\omega^2/3c \times \beta^2 \sin^2\theta/(1-\beta^2)$

where β is the electron velocity and θ is the pitch angle

With 100Ci source of atomic tritium the projected sensitivity to neutrino mass of 0.007 eV is estimated.

That the basic idea works as expected was recently demonstrated using a small cell with the gaseous monoenergetic conversion electron source Kr^{83m} (*Asner et al., arXiv: 1408:5362*)

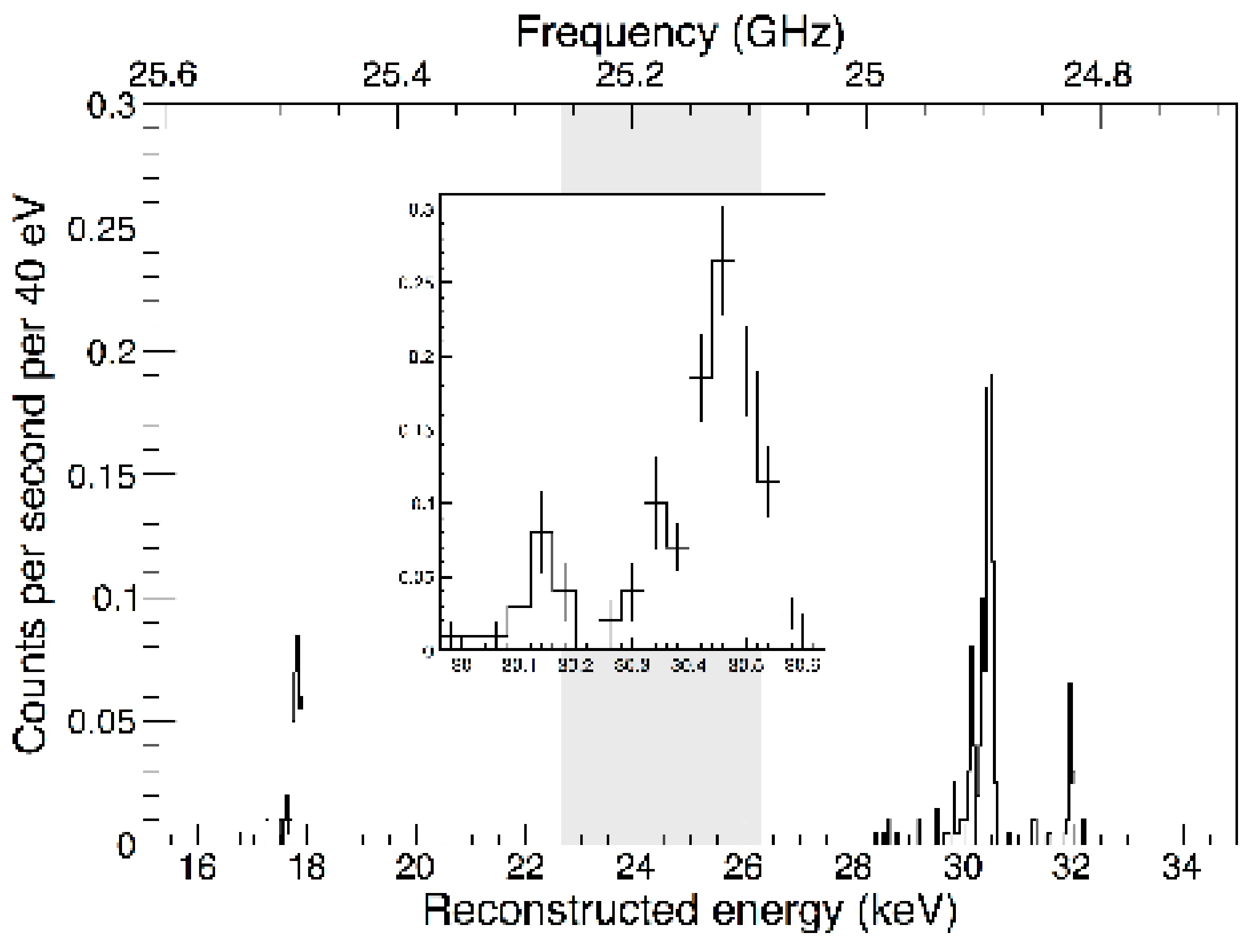


FIG. 3. The kinetic energy distribution of conversion electrons from ^{83m}Kr as determined by CRES. The spectrum shows both the 17 keV, 32 keV and 30 keV complex conversion electron lines. The shaded region indicates the bandwidth where no data were collected. Inset: An expanded view of the 30 keV energy region, where the 30.4 keV conversion electrons can be seen.

Prospects for Relic Neutrino Detection at PTOLEMY: Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield

Plans to use monoatomic tritium source deposited on a graphene substrate and a combination of *MAC-E* filters, cryogenic calorimetry, RF tracking and time-of-flight systems.
(see Betts et al. arXiv: 1307.4738)

Summary

1) We have discussed the challenges and promises of detecting the primordial neutrinos (in particular the ν_e component) using the neutrino capture on radioactive nuclei, with emphasis on tritium as target.
2) Among the various technological challenges of such program, the requirement that the detector resolution is better that the neutrino mass by a factor 2 - 3, appears to be the most difficult one to achieve. It essentially restricts the applicability of the discussed approach.
3) In the next few years a variety of approaches (KATRIN, cosmology & astrophysics, $0\nu\beta\beta$ decay) promise to reach sensitivity to $m_\nu \sim 0.2$ eV or even better. If one or all of these approaches find positive evidence, e.g.. if we can conclude that $m_\nu \geq 0.2$ eV, it would be certainly worthwhile, and perhaps even imperative, to pursue the indicated program vigorously.

Thanks to Gary Steigman, Georg Raffelt and Petr Vogel!