The effect of non-Gaussianity on error predictions for the EoR 21-cm power spectrum

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The effect of non-Gaussianity on error predictions for the Epoch of Reionization (EoR) 21-cm power spectrum

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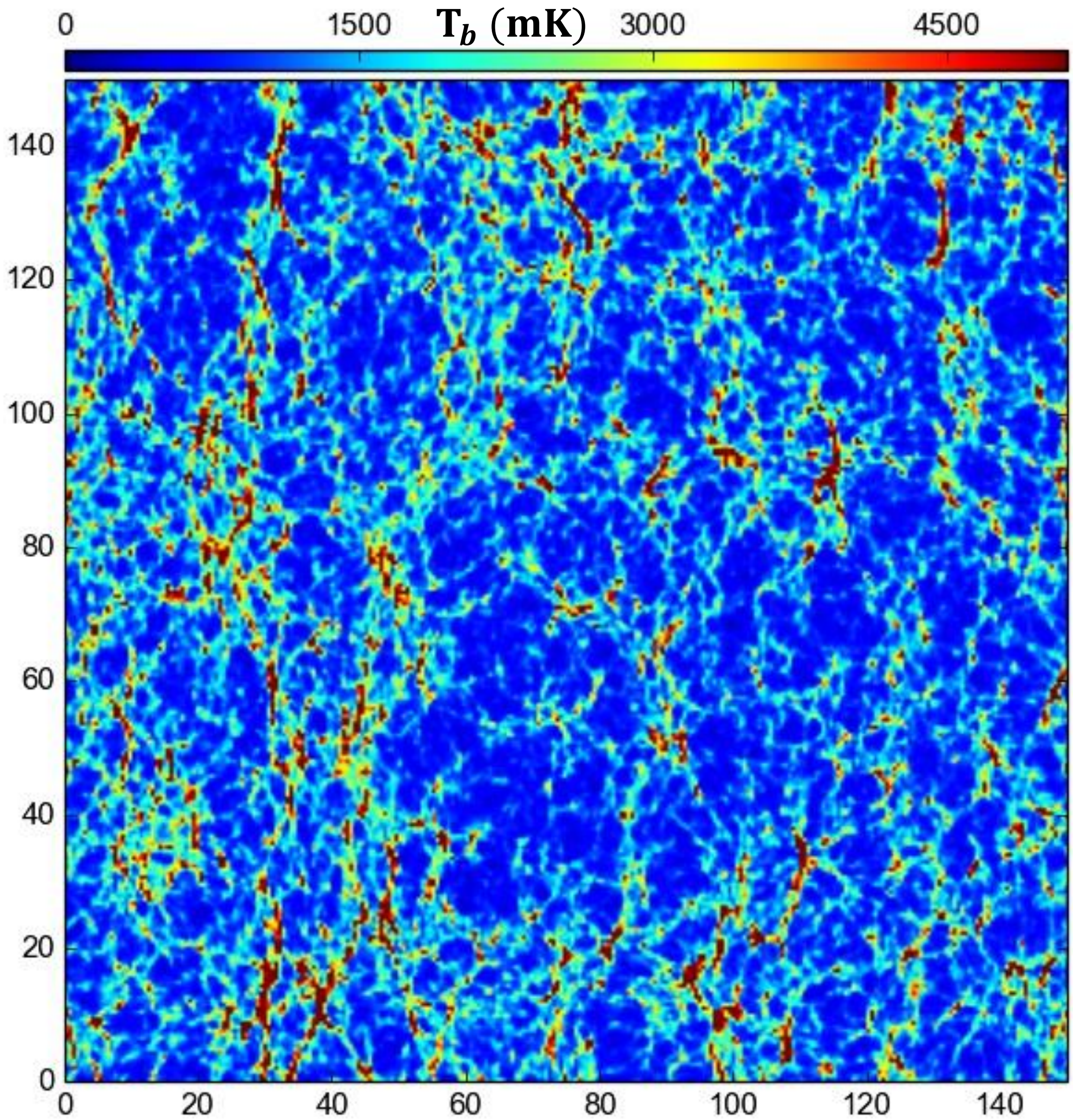

T_b (mK)
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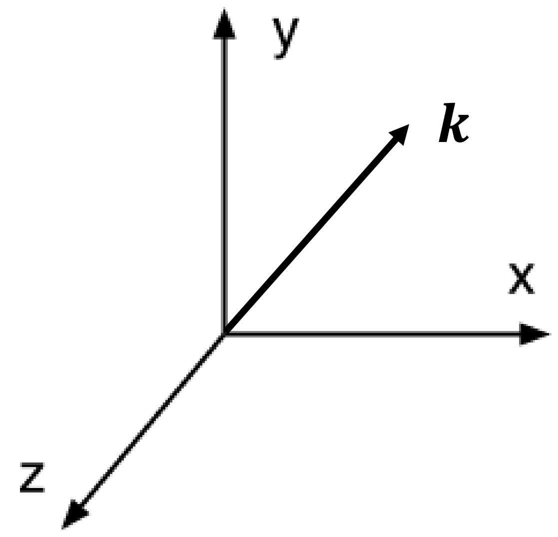
Simple example

- Power spectrum estimator

$$\hat{P}(k) = \frac{\Delta(\boldsymbol{k})\Delta(-\boldsymbol{k})}{V}$$



- Mean power spectrum

$$\langle \hat{P}(k) \rangle = P(k)$$

- S.d. of power spectrum

$$\delta\hat{P}(k) = P(k)$$

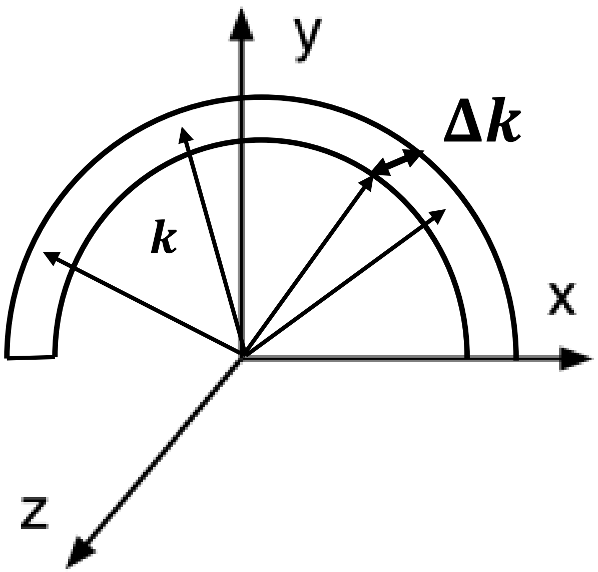
Very uncertain!

Power spectrum estimator

- Binned power spectrum estimator

$$\hat{P}(k) = (N_k V)^{-1} \sum \Delta(\boldsymbol{k})\Delta(-\boldsymbol{k})$$

averaged over k-modes ($\boldsymbol{k}$ to $\boldsymbol{k} + \Delta\boldsymbol{k}$)



- Bin averaged power spectrum

$$\langle \hat{P}(k) \rangle = \bar{P}(k)$$

- And for a Gaussian random field the s.d.

$$\delta \hat{P}(k) = \sqrt{\frac{\overline{P^2}(k)}{N_k}}$$

- So, the error comes down as $1/\sqrt{N_k}$

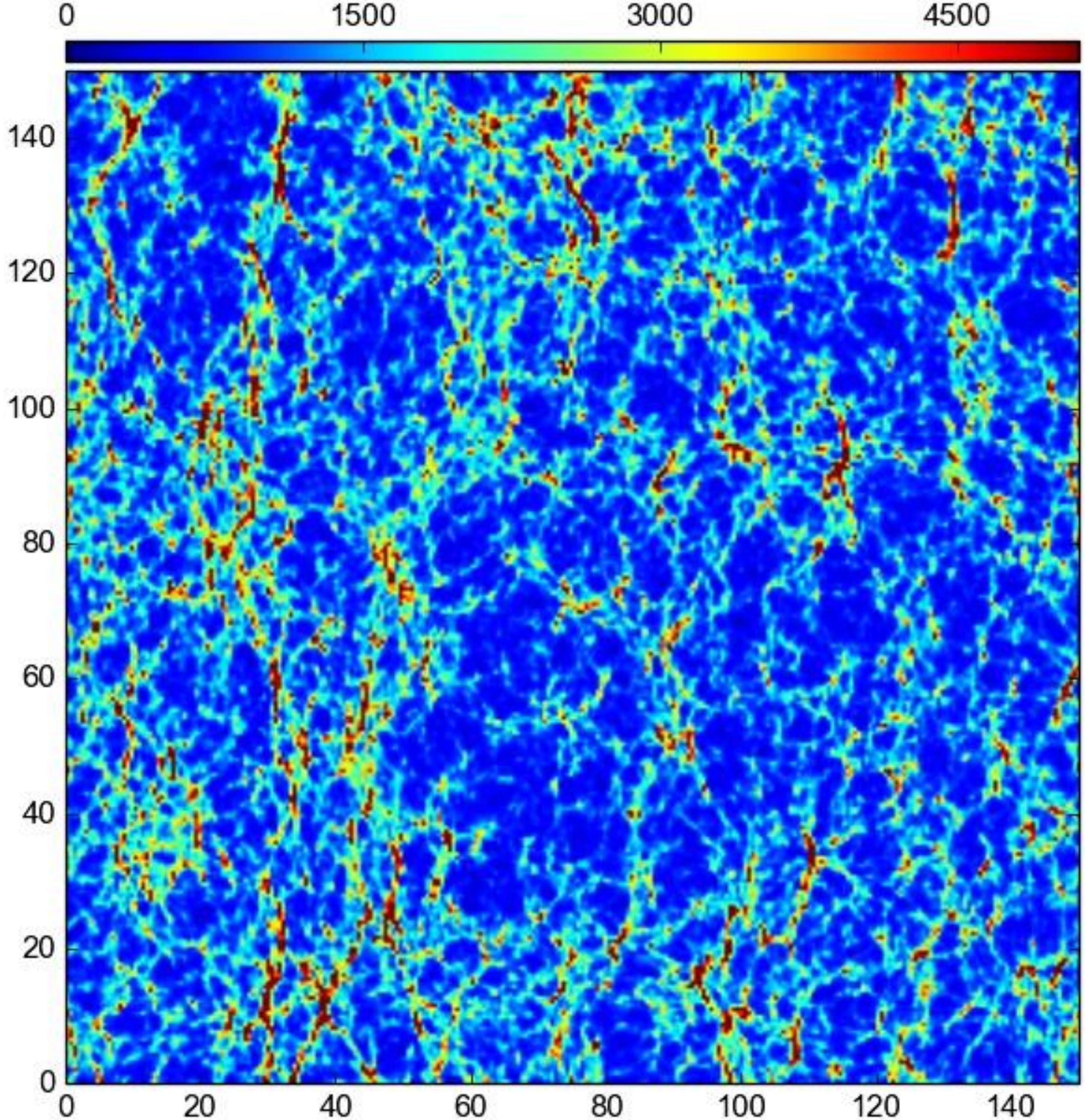
Motivations

- It is commonly assumed, as in all the sensitivity estimates (e.g. Morales 2005, McQuinn et al. 2006, Beardsley et al. 2013, Jensen et al. 2013, Pober et al. 2014 etc.),

  that the EoR 21-cm signal is purely Gaussian random variable

- How good is this assumption?

- Ionized bubbles introduce non-Gaussianity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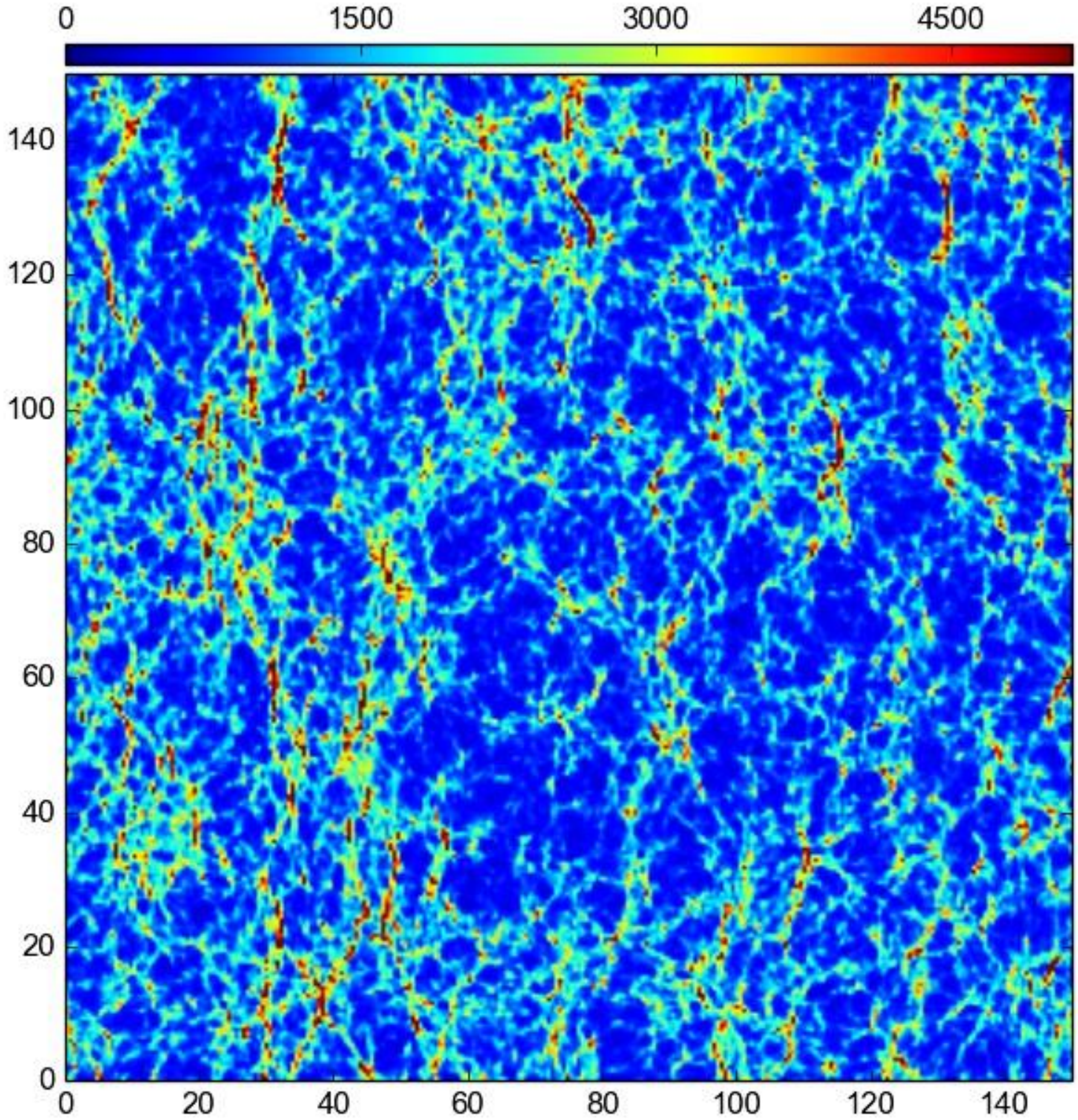
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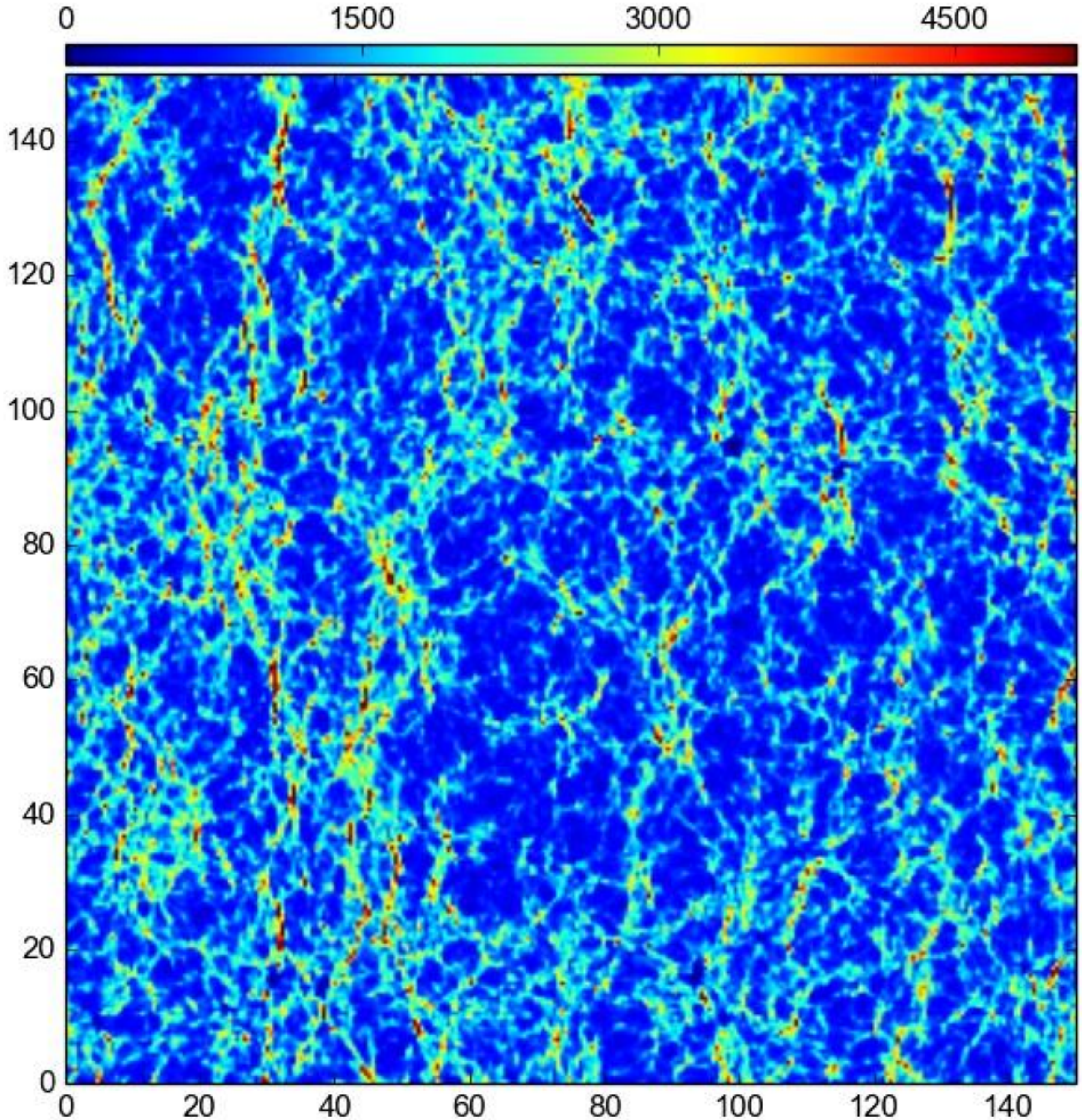
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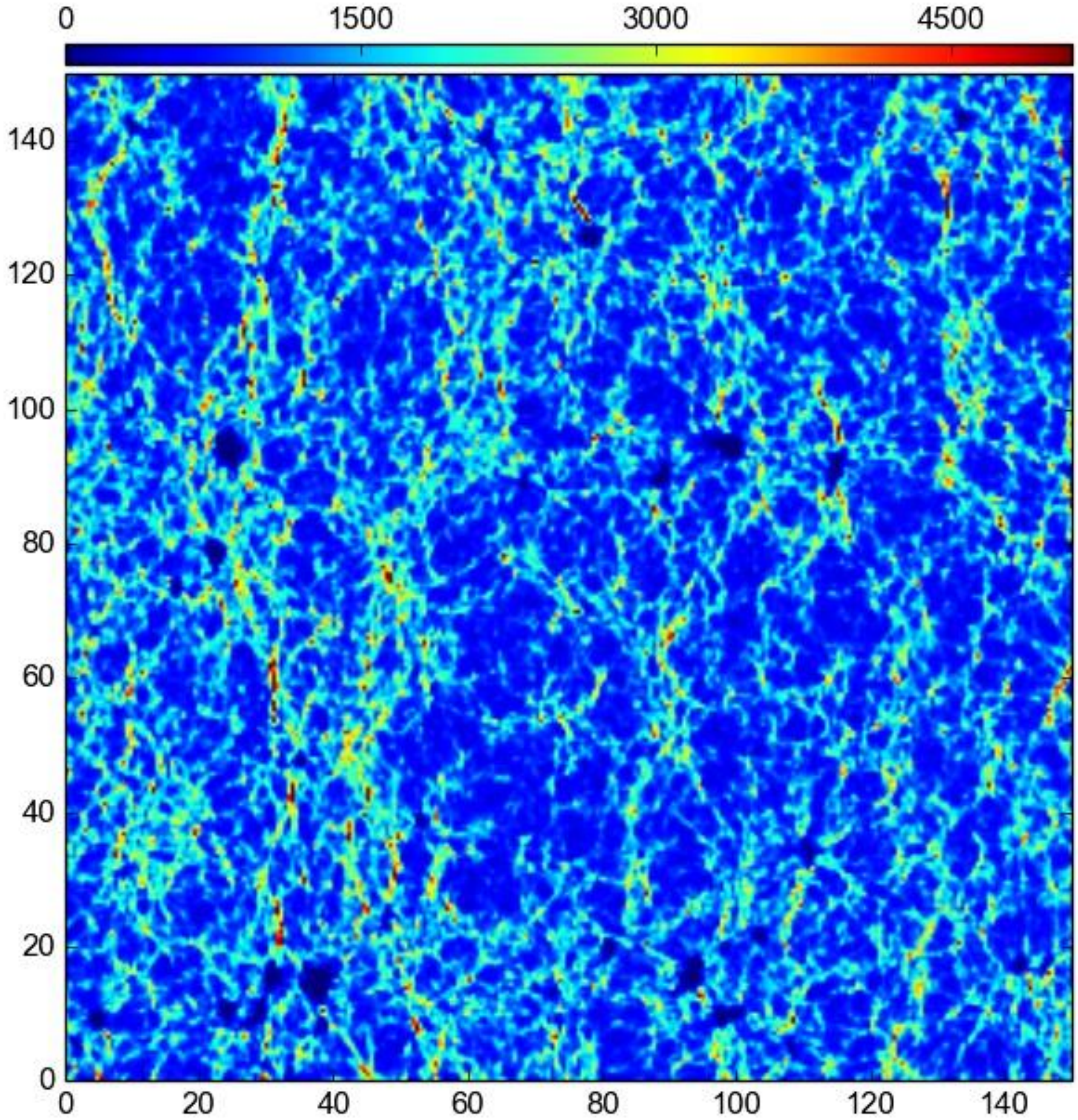
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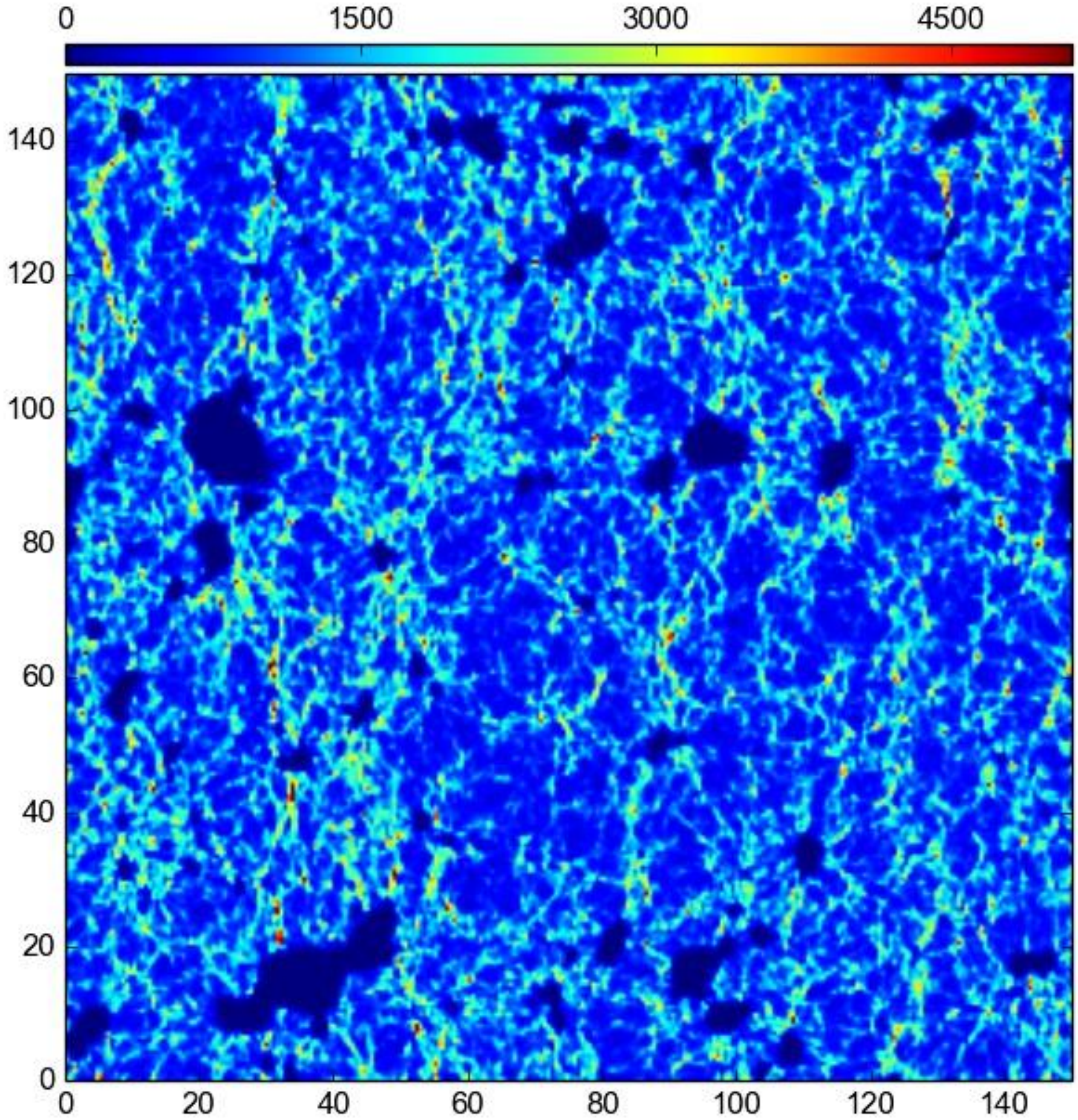
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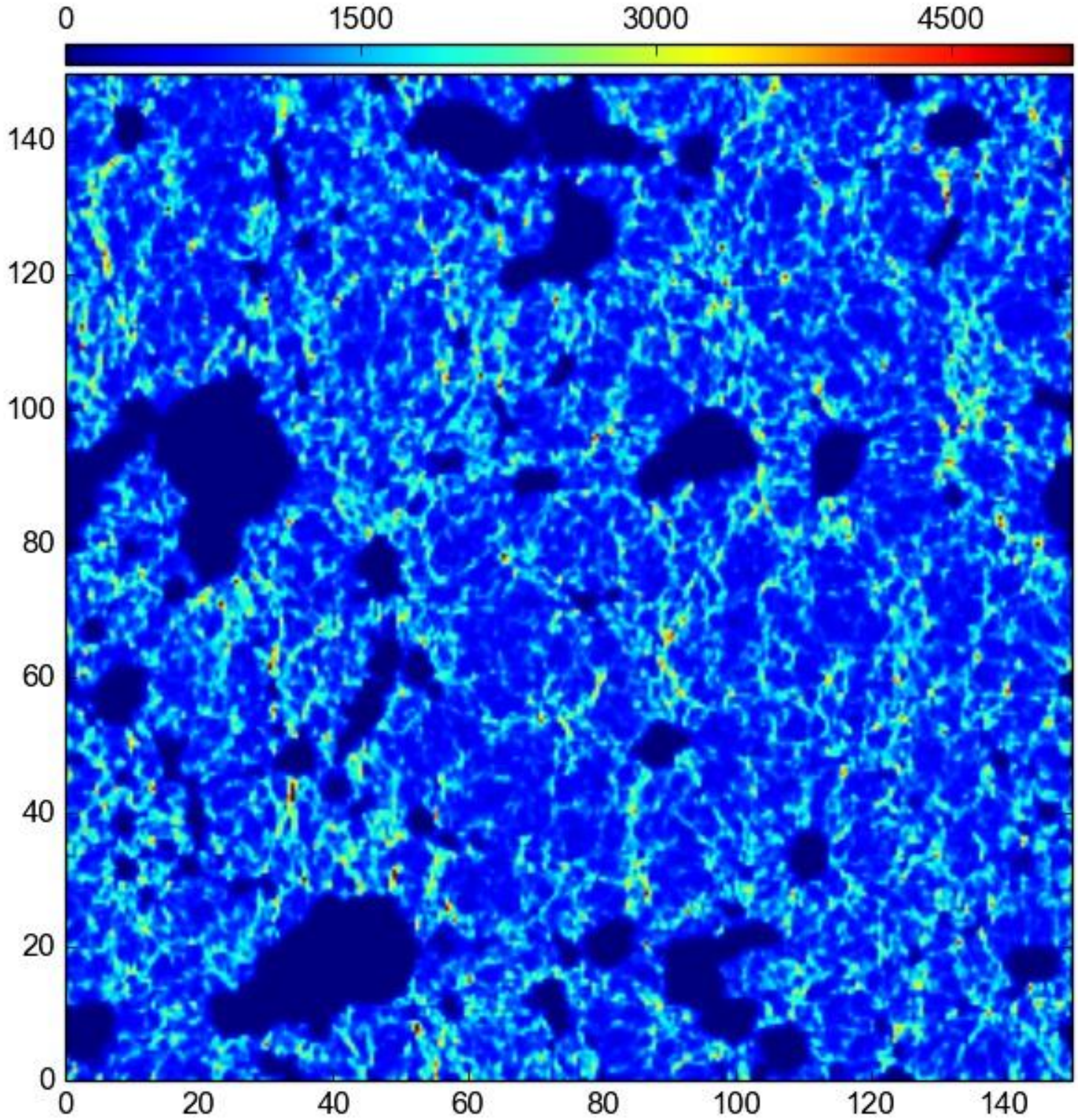
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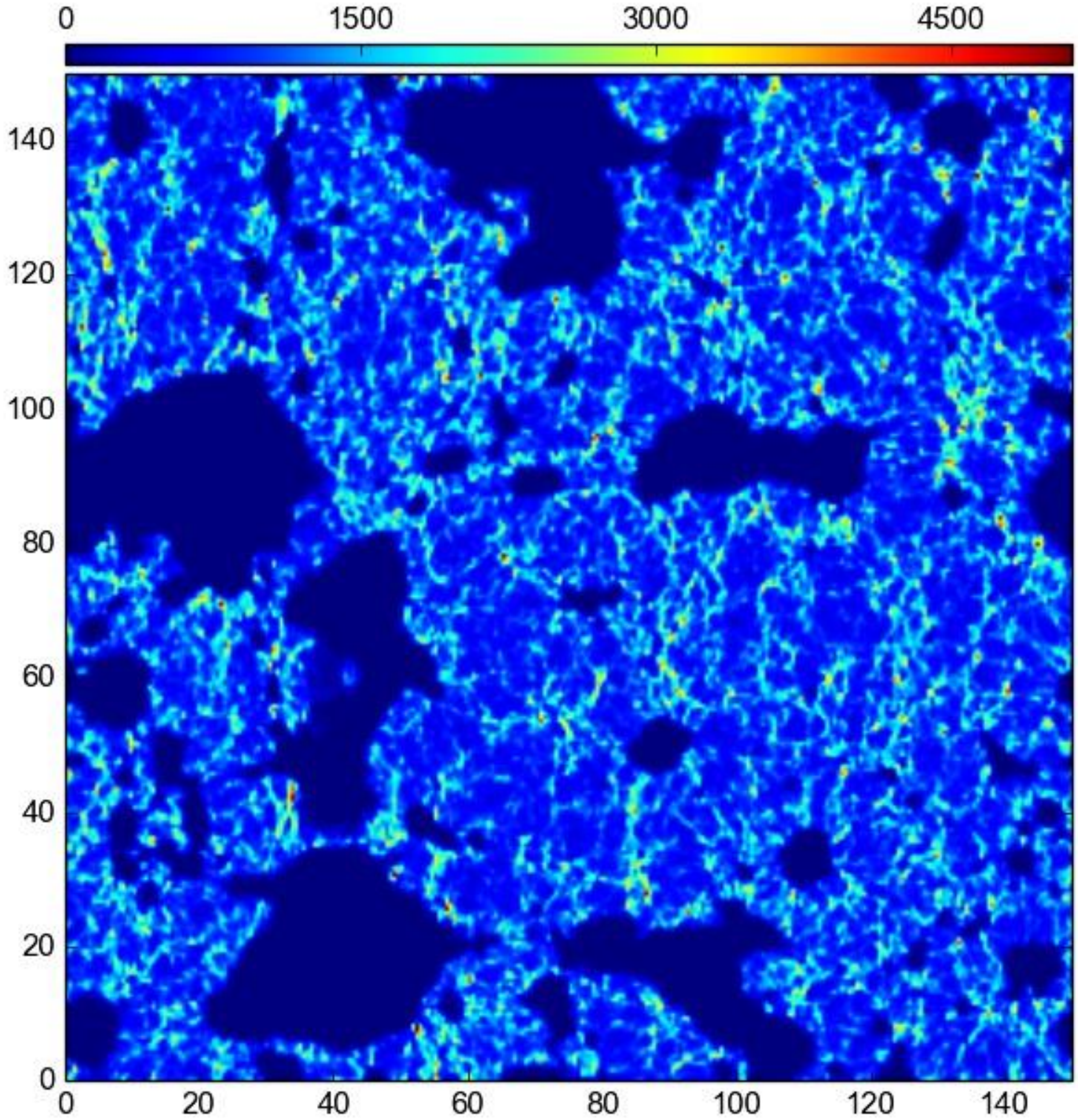
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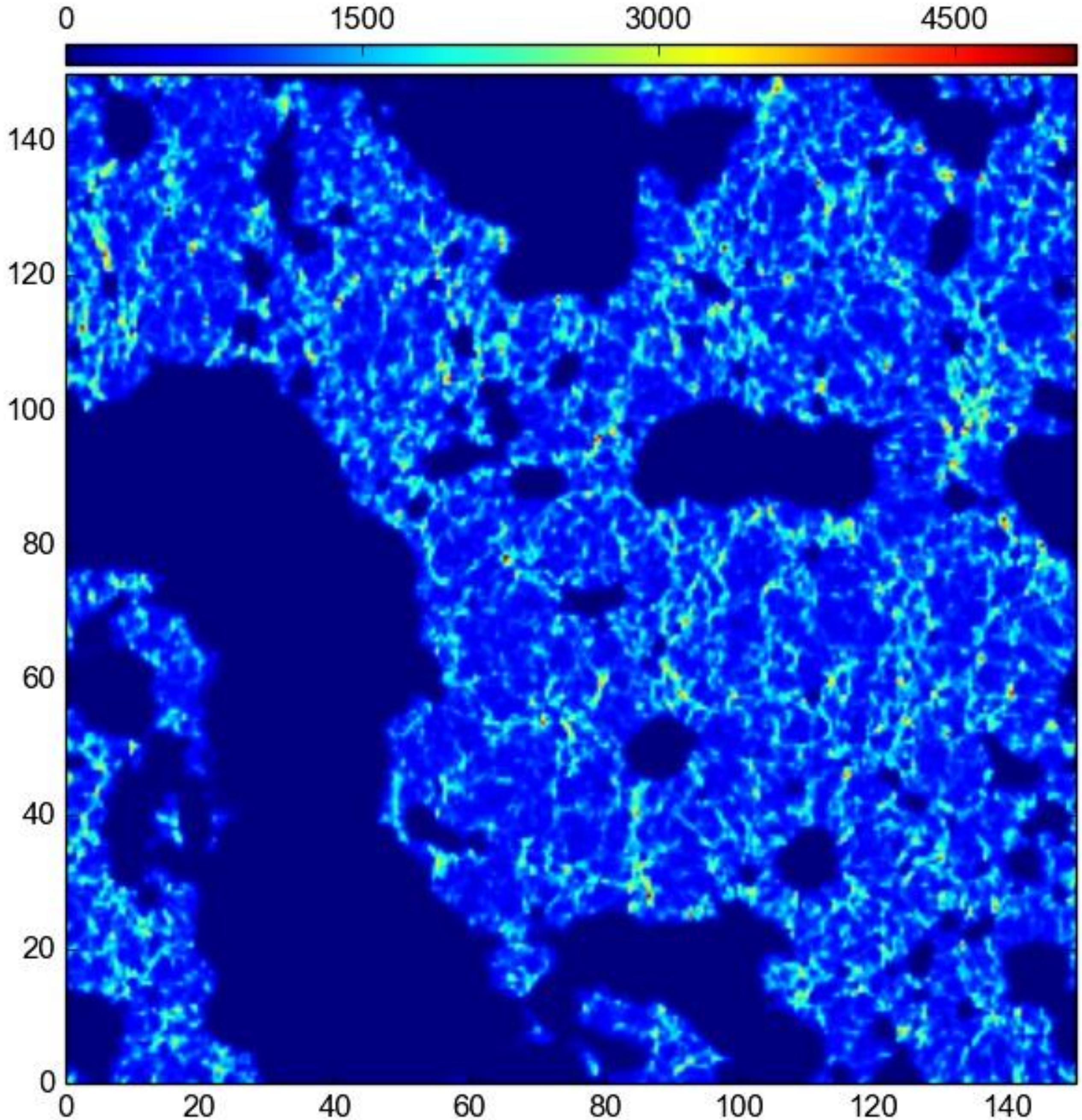
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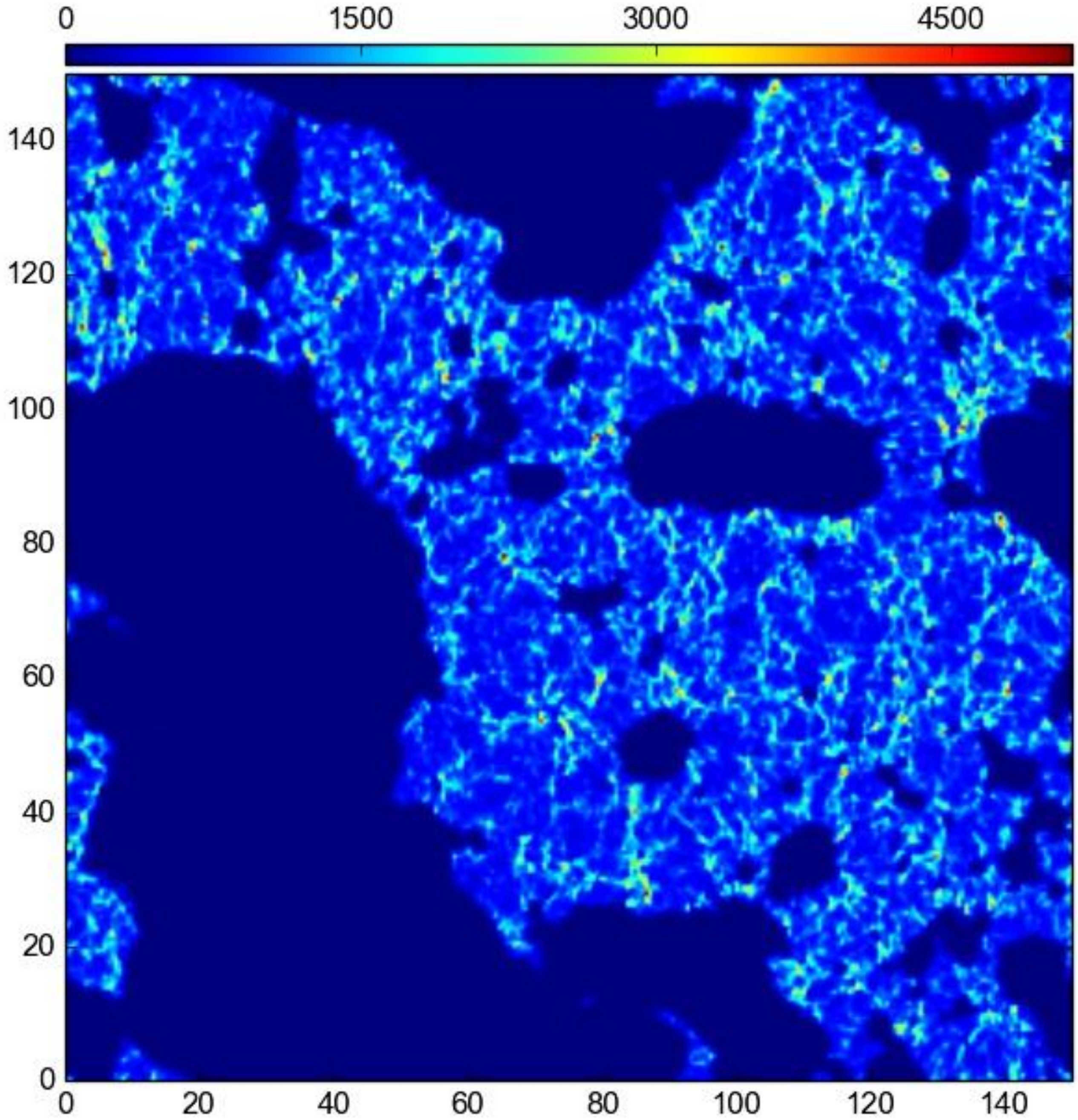
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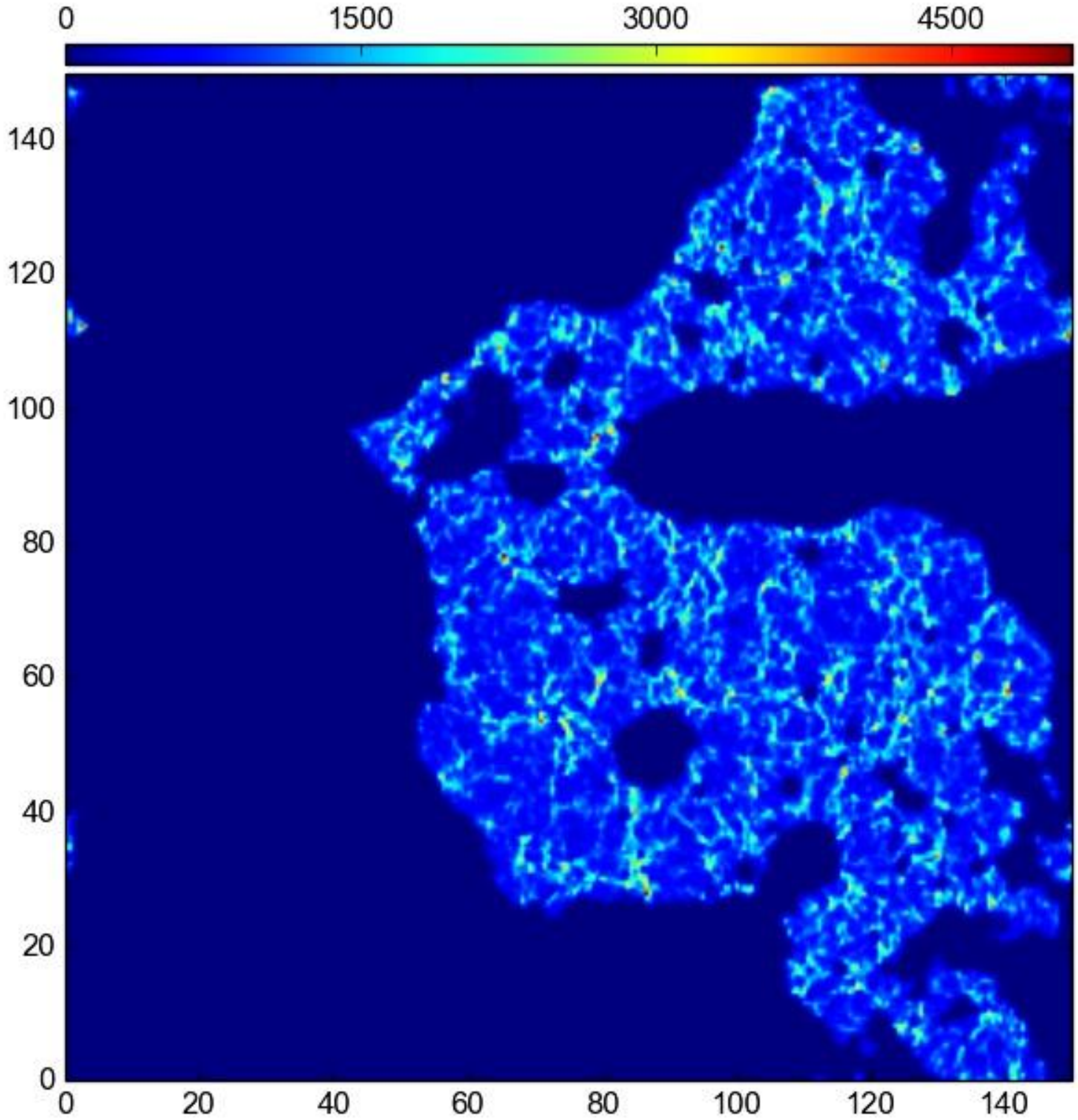
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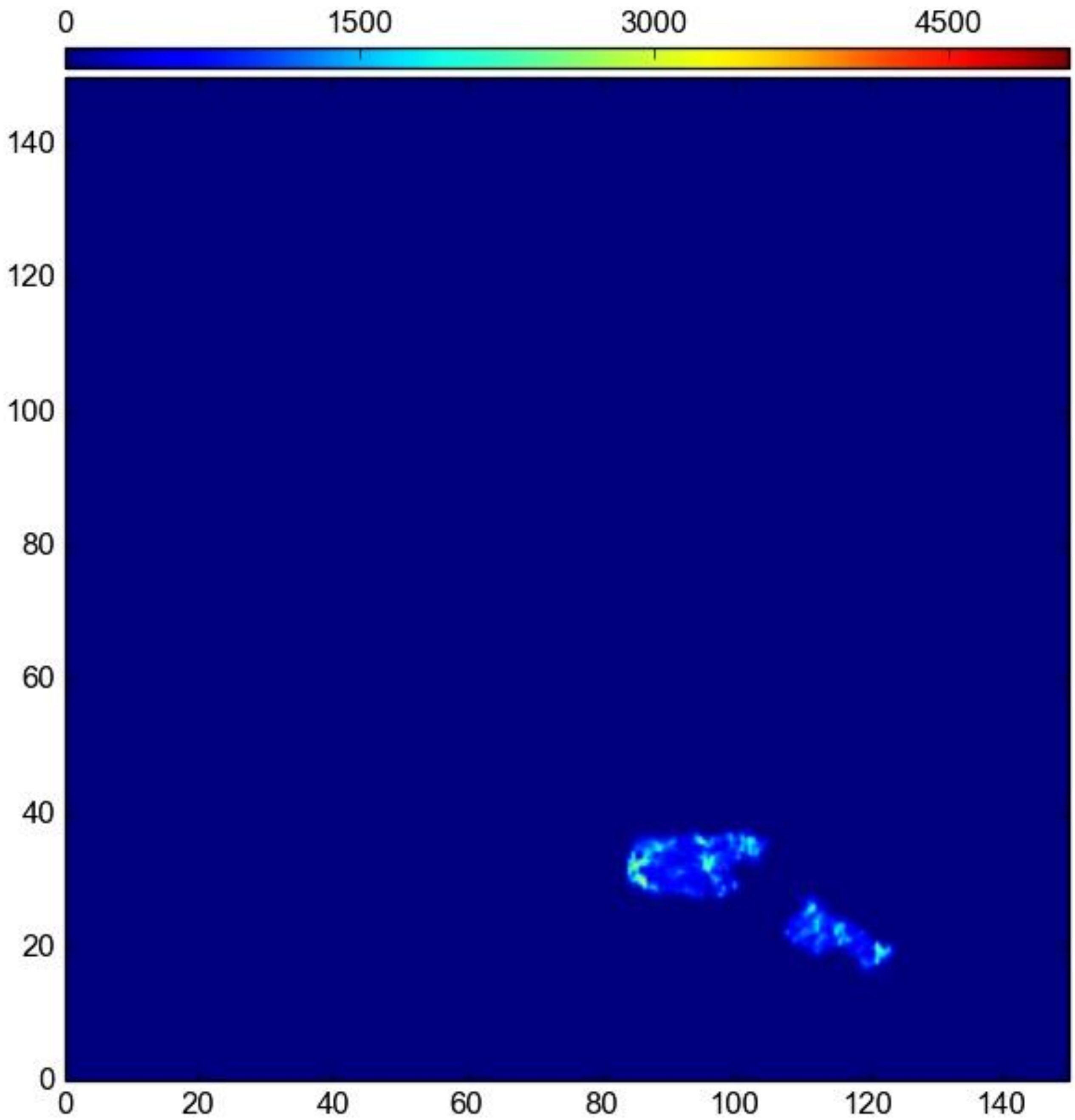
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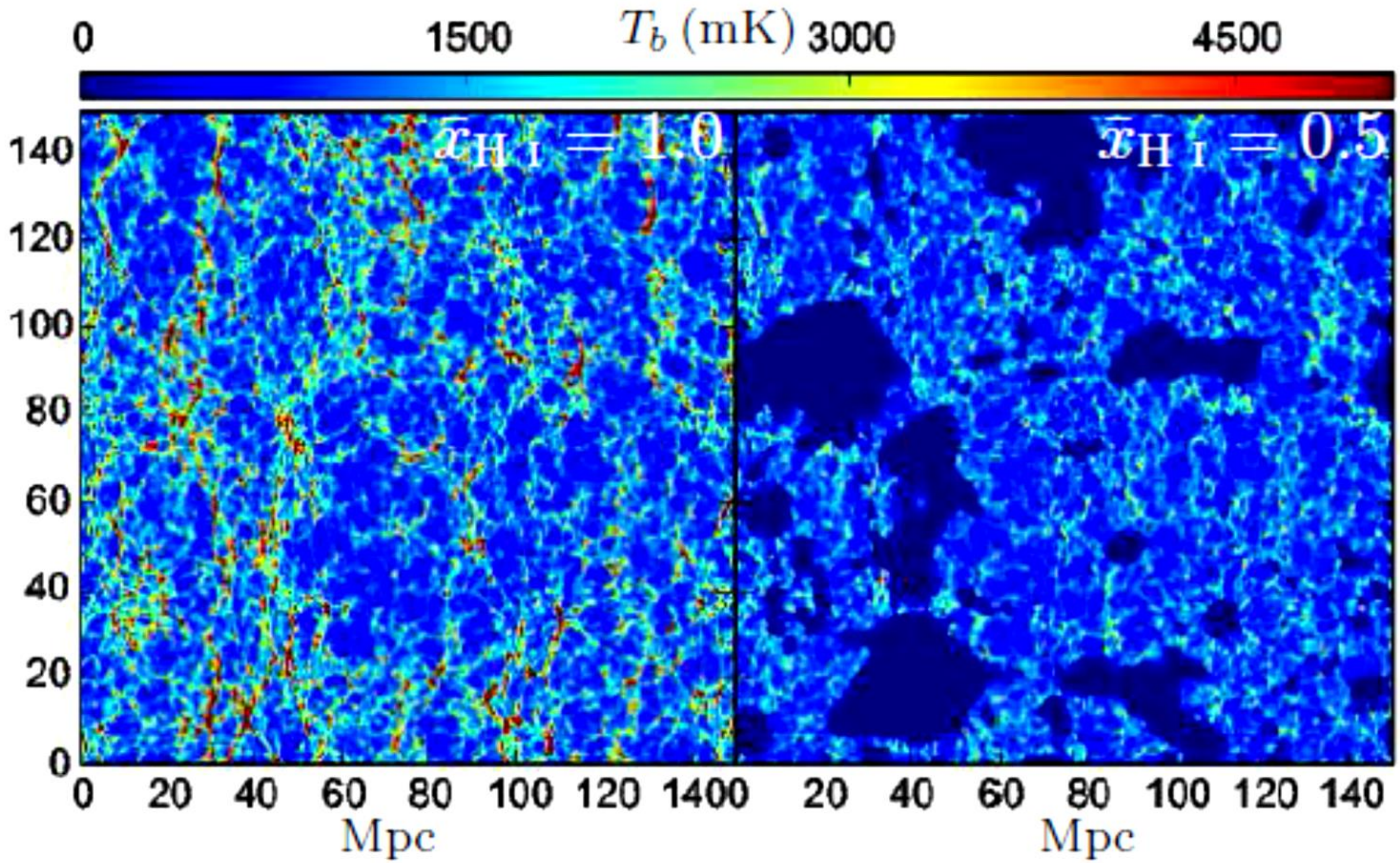


Figure 1. A section through one of the simulated redshift space H I brightness temperature maps for $\bar{x}_{\rm H\,I} = 1.0$ (left) which is largely a Gaussian random field, and $\bar{x}_{\rm H\,I} = 0.5$ (right) which has considerable non-Gaussianity due to the discrete ionized bubbles visible in the image. The redshift space distortion is with respect to a distant observer located along the horizontal axis.

Simulating the 21-cm maps

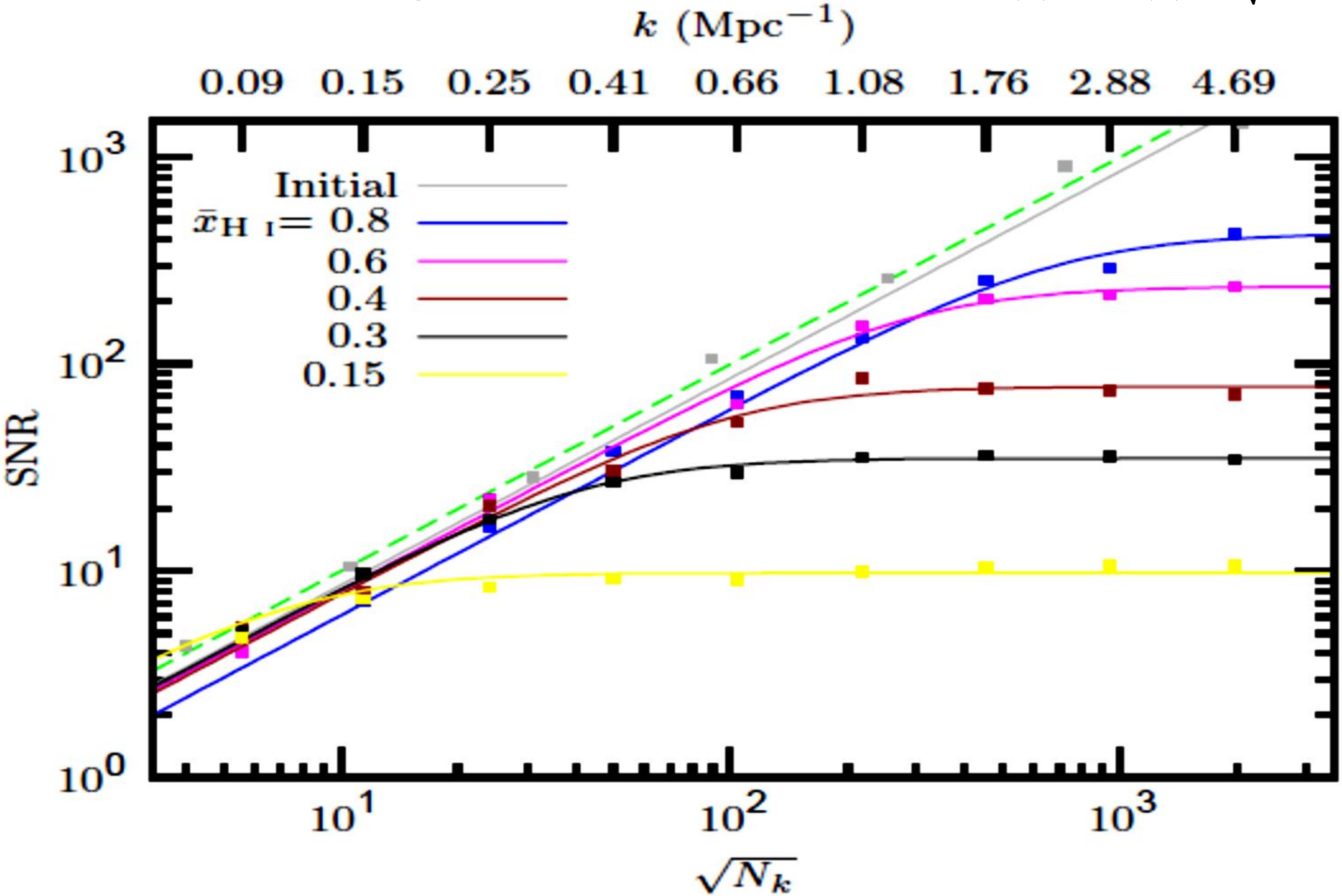
- It is not well established that how mass averaged neutral fraction $\bar{x}_{HI}$ varies with the redshift z?
- We have fixed the redshift $z = 8$ and considered different values of $\bar{x}_{HI}$
- For each value of $\bar{x}_{HI}$, we have simulated 21 statistically independent realizations of the reionization map

- **N-body Simulation:** particle-mesh parallelized code, Box has$(150.08 Mpc)^3$comoving volume, Mass resolution $(M_{part}) = 7.304 \times 10^7 h^{-1} M_{\odot}$

- **Identifying Halos:** Friends-of-Friends (FoF) algorithm, linking length 0.2 times the mean inter-particle separation, require a halo to have at least 10 particles

- **Generating the ionization map:** homogeneous recombination scheme (Choudhury et al. 2009), HI distribution was mapped to redshift space (Majumdar et al. 2013)

Gaussian random field: signal to noise ratio to follow $SNR = P_b(k)/\delta P_b(k) = \sqrt{N_k}$



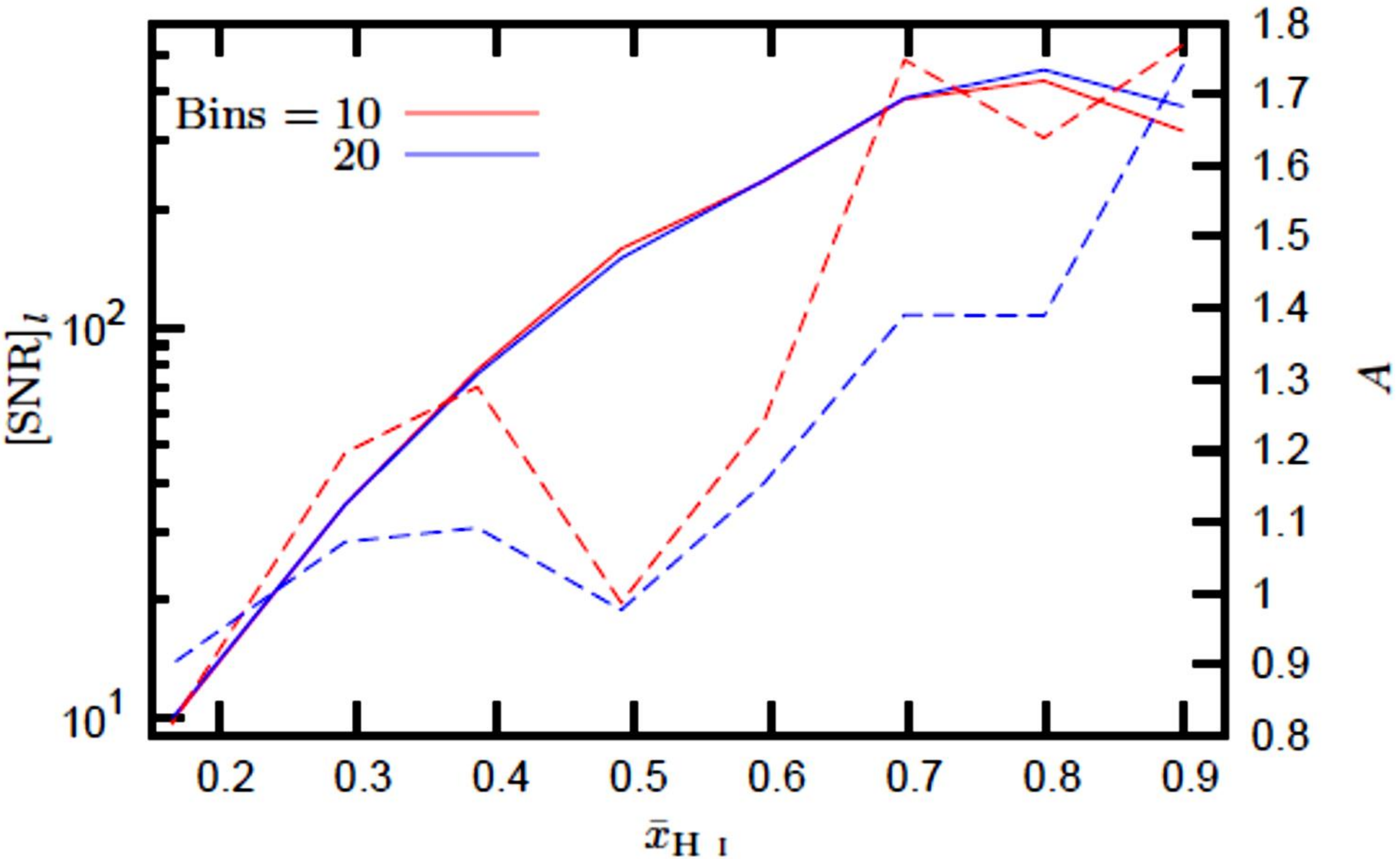
Fitting formula for the SNR

$$\mathrm{SNR} = \frac{\sqrt{N_k}}{A}\left[1 + \frac{N_k}{(A[\mathrm{SNR}]_l)^2}\right]^{-0.5}$$

- Where the parameter A quantifies the deviation from the Gaussian prediction in the low SNR regime

- The deviations from the Gaussian predictions seen at large SNR increase (i.e. $[\mathrm{SNR}]_l$ decrease) as reionization proceeds.

- Used a least-square fit to obtained the best fit A and $[SNR]_l$



Modelling the SNR

- The quantity we are dealing is the binned power spectrum

$$\hat{P}_b(k) = (N_k V)^{-1} \sum_a \tilde{\mathrm{T}}_b(a)\tilde{\mathrm{T}}_b(-a)$$

- The bin averaged power spectrum

$$\langle \hat{P}_b(k) \rangle = \bar{P}_b(k) = (N_k)^{-1} \sum_a P_b(a)$$

and, the variance of power spectrum

$$\langle [\delta \hat{P}_b(k)]^2 \rangle = [\delta P_b(k)]^2 = (N_k)^{-1} \overline{P_b{}^2}(k) + V^{-1} \bar{T}_b(k,k)$$

where $\overline{P_b^2}(k)$ and $\bar{T}_b(k,k)$ are the square of the power spectrum and the trispectrum respectively

Modelling the SNR

- The $\mathrm{SNR} \equiv \bar{P}_b(k)/[\delta P_b(k)]$ can be cast in the from of our fitting formula i.e.

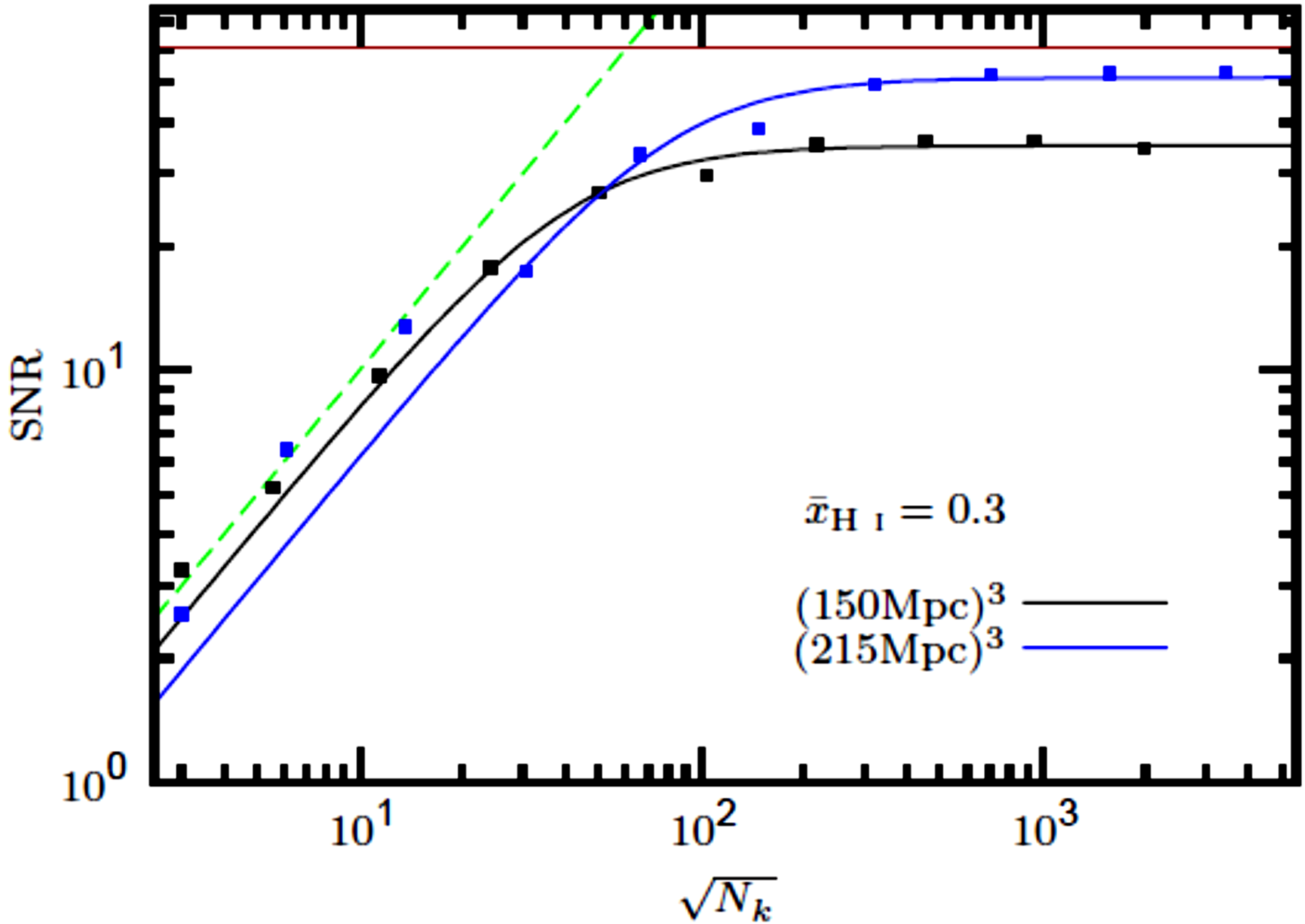
$$\mathrm{SNR} = \frac{\sqrt{N_k}}{A}\left[1 + \frac{N_k}{(A[\mathrm{SNR}]_l)^2}\right]^{-0.5}$$

provided we identify

$$A = \sqrt{\frac{\overline{P_b{}^2}(k)}{[\bar{P}_b(k)]^2}}$$

and

$$[\mathrm{SNR}]_l = \sqrt{\frac{[\bar{P}_b(k)]^2 V}{\bar{T}_b(k,k)}}$$


SNR
10^1
10^0
10^1
10^2
10^3
$\sqrt{N_k}$
$\bar{x}_{\rm H\,I} = 0.3$
$(150\text{Mpc})^3$
$(215\text{Mpc})^3$

Conclusions

- Two components, one a Gaussian random field and another a non-Gaussian component from the discrete ionized bubbles.
- The Gaussian component in different Fourier modes are independent
- The non-Gaussian components however are correlated this can be quantified through bispectrum (Bharadwaj & Panday, 2005), Trispectrum (Mondal et al. 2015) etc.
- The contribution to $\mathrm{SNR} = P_b(k)/\delta P_b(k)$ from the Gaussian component scale as $\sqrt{N_k}$, whereas the non-Gaussian contribution remains fixed even if N_k is increased.

Conclusions

- For a fixed volume V , it is not possible to increase the SNR beyond $[SNR]_l$
- $[SNR]_l$ is proportional to $\sqrt{V}$, and it is possible to achieve a high SNR by increasing the volume.
- The non-Gaussian effect could play an important role in the error predictions for the EoR 21-cm power spectrum

THANK YOU