A simple solution to the fine tuning problem of the Cosmological constant

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Thursday 29th January, 2015

Outline

- Introduction: The Cosmological constant and fine tuning problem.
- Conformal invariance and fine tuning problem.
- Possible solution: Model with explicitly broken conformal symmetry.
- Extension of mechanism to perturbation theory.
- Conclusions

Introduction: The cosmological constant problem

Why is there a Cosmological Constant problem?

- The cosmological constant Λ appearing in Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Initially introduced to get a static cosmological solution, but later observations shows expanding Universe $\rightarrow \Lambda$ vanish? NO.

- Is empty space really empty? NO.

- Quantum fluctuations exist everywhere$\rightarrow$ vaccum energy.

The cosmological constant problem

- This vacuum energy density is believed to act as a contribution to the cosmological constant.

$$\Lambda_{effective} = \Lambda + 8\pi G < \rho >$$
$$H_0 = 2.134h \times 10^{-42} GeV$$

Using FRW metric the Einstein Eq. gives

$$\Lambda_{effective} \leq H_0^2$$
$$\Rightarrow |\rho_\Lambda|_{obs} \lesssim 10^{-47} GeV^4 \sim 10^{-123} M_{PL}^4$$
$$< \rho >_{th} \sim (10^{-68} \rightarrow 1) \quad \text{(QFT)}$$

- *Problem: Why is the vacuum energy today so small*? Fine tuning of Λ of order of magnitude 60 to 120 is required.

T. Padmanabhan, Phys. Rept. 380, 235 (2003)

Possible solution: Conformal Invariance

- A theory with conformal invariance does not permit a cosmological constant term.

$$
\begin{aligned}
\mathcal{S}_C =& \int d^4x\sqrt{-g}\left[\frac{\beta}{8}\chi^2\tilde{R} + + \frac{1}{2}g^{\mu\nu}D_\mu\chi D_\nu\chi - \frac{\lambda}{4}\chi^4\right] \\
\tilde{R} =& R + 6f^2 g^{\mu\nu}S_\mu S_\nu + 6fg^{\mu\nu}S_{\mu;\nu}\,, \\
D_\mu\chi =& (\partial_\mu - fS_\mu)\chi
\end{aligned}
\tag{1}
$$

- The conformal action displays invariance under the transformation,

$$
g_{\mu\nu} \to \Omega(x)^2 g_{\mu\nu}\,, \chi \to \frac{\chi}{\Omega(x)}\,, S_\mu \to S_\mu - \frac{1}{f}\partial_\mu \ln(\Omega(x))
$$

Cosmological constant problem in conformal model

- All the dimensional parameter in this theory are generated by soft breaking of the conformal symmetry, which require a constant classical solution such that,

$$\chi = \chi_0$$

We choose the value consistent with observations

$$\chi_0 = \frac{M_{PL}}{\sqrt{2\pi\beta}}$$

- Solve the EOM, with $S_\mu = 0$ and find scalar curvature for this solution,

$$R = \frac{2\lambda}{\pi\beta^2} M_{PL}^2$$

- *Problem*: Observations implies a very small value of R and hence fine tuning of λ at each order in perturbation theory is required.

P.Jain et.al, Mod. Phys. Lett. A 22,1651-1661 (2007)
M .E . Shaposhnikov et. al, Phys. Lett. B 671, 162 (2009)

Model with broken conformal symmetry

- We add a symmetry breaking action

$$\mathcal{S}_{SB} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}m^2\chi^2 + \Lambda + \frac{1}{2}m_1^2 S_\mu S^\mu\right] + \ldots \qquad (2)$$

- We make a conformal transformation,

$$g_{\mu\nu} \to \Omega(x)^2 g_{\mu\nu}\,,\, \chi \to \frac{\chi}{\Omega(x)}\,,\, S_\mu \to S_\mu - \frac{1}{f}\partial_\mu \ln(\Omega(x))$$

- Under this transformation, S_C remains unchanged. However S_{SB} becomes,

$$\mathcal{S}_{SB} = \int d^4x\sqrt{-g}[-\frac{1}{2}m^2\Omega^2\chi^2 + \Omega^4\Lambda$$
$$+\frac{m_1^2}{2}\Omega^2 g^{\mu\nu}(S_\mu - \frac{1}{f}\partial_\mu \ln\Omega)(S_\nu - \frac{1}{f}\partial_\nu \ln\Omega)]$$

Jain and Kashyap, arXiv:1408.2620 [hep-ph]
Jain and Kashyap, Mod. Phys. Lett. A, Vol. 29, No. 36 (2014) 1450195

Fine Tuning?

- Choose $\Omega(x)$ such that the term proportional to λ and Λ cancel each other

$$\Omega^4 = \frac{\lambda\chi^4}{4\Lambda} \tag{3}$$

- With this choice the full action again becomes symmetric

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{\beta}{8}\chi^2\tilde{R} - \frac{1}{4}m^2\sqrt{\frac{\lambda}{\Lambda}}\chi^4\right] + \ldots \tag{4}$$

- Effective cosmological constant $\propto m^2\sqrt{\lambda/\Lambda}$

- No fine tuning is required as m do not get contribution from symmetry preserving sector $\rightarrow$ can be chosen arbitrarily small.

Perturbative Expansion

- We maintain conformal invariance at quantum level by using a dynamical mass scale, χ, for regularization.
- If we do the perturbative expansion in the transformed action the fine tuning reappears $\rightarrow$ symmetry breaking terms disappers.
- We propose a generalization of the mechanism by choosing $\Omega(x)$ such that

$$\Omega^2 = \sqrt{\frac{\lambda}{4\Lambda}}\,\chi^2 + \xi\frac{g^{\mu\nu}S_\mu S_\nu}{\chi^2} \tag{5}$$

- With this parameterization the transformed action retains the symmetry breaking terms.

F. Englert et.al, Nucl. Phys. B 117, 407 (1976)
Jain et.al, MPLA, 22,1651(2007)

Perturbative Expansion..

$$\begin{aligned}\mathcal{S}_{SB} &= \int d^4x\sqrt{-g}\Bigg[-\frac{1}{4}m^2\sqrt{\frac{\lambda}{\Lambda}}\chi^4+\frac{\lambda}{4}\chi^4-\frac{1}{2}\xi\left(m^2-2\sqrt{\lambda\Lambda}-\frac{m_1^2}{2}\sqrt{\frac{\lambda}{\Lambda}}\frac{\chi^2}{\xi}\right)S^\mu S_\mu \\ &+ \frac{\xi}{\chi^2}\left(\frac{\xi\Lambda}{\chi^2}+\frac{m_1^2}{2}\right)(S_\mu S^\mu)^2-\frac{m_1^2}{2f}g^{\mu\nu}S_\mu\partial_\nu\Omega^2+\frac{m_1^2}{8f^2}g^{\mu\nu}\frac{1}{\Omega^2}\partial_\mu\Omega^2\partial_\nu\Omega^2\Bigg] \qquad (6)\end{aligned}$$

- Symmetry breaking terms are present in transformed action.
- We obtain a constant classical solution for χ with $S_\mu = 0$.
- While making quantum expansion around classical solution, we need to add counter terms corresponding to cosmological constant and mass term.
- We follow the procedure order by order in perturbation theory.

Conclusions

- A simple mechanism based on conformal invariance.
- By breaking the conformal symmetry explicitly we can solve the fine tuning problem of the cosmological constant problem.
- The symmetry breaking terms do not get contribution from symmetry preserving sector at any order.
- We can choose symmetry breaking parameter m as small as required.
- The cosmic evolution is governed by term $m(\lambda/\Lambda)^{1/4}$.
- Large value of λ and Λ will not affect the evolution.
- We can apply this mechanism perturbatively at each order.

Thank You