Geometry of The Universe Described by Wet Dark Fluid in f(R, T) Theory of Gravity

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Abstract:

The Bianchi type-III cosmological model in $f(R, T)$ gravity is investigated with the equation of state for wet dark fluid i. e. $p_{WDF} = \omega(\rho_{WDF} - \rho^*)$. Using exponential and power law expansion, we obtained the exact solution of the field equations. The various cosmologiacl parameters has been discussed.

Introduction

One of the biggest challenging and surprising discoveries in modern cosmology is the observation of an accelerated expansion of the universe. The significant discovery in modern cosmology is the fact that our universe is currently undergoing an accelerated expansion phase. Various cosmological observations like distance measurements from type Ia Supernovae (Riess et al. [1], Perlmutter et al. [2], Tonry et al. [3], Knop et al [4], Riess et al. [5]), measurements of angular diameter distance using stadard ruler like acoustic oscillations in in Cosmic Microwave Background Radiation (CMBR)(Komactu et al. [6]) as well as Baryon Acoustic Oscillations (BAO) (Blake et al [7], Percival et al. [8]) in matter power spectra and measurements of gravitational clustering (Eisenstein et al. [9]) suggest that two third of the total energy density of our universe is contributed an exotic component with negative pressure (konwn as dark energy) which results this late time acceeration of the universe.

Introduction Continued

The other important role for the late time cosmic acceleration is the modification of gravity. There has been significant development in the construction of dark energy models by modifying the geometrical part of the Einstein-Hilbert action. This phenomenological approach is called modified gravity, which can successfully explain the rotation curves of galaxies, the motion of galaxy clusters, the bullet cluster and cosmological observations without the use of dark matter or Einstein's cosmological constant (Nojiri and Odintsov [10], Elizalde et al. [11], Jamil et al. [12], Capozziello et al. [13, 14]). There are several modified gravity theories like $f(R)$ garvity, $f(G)$ gravity, $f(T)$ gravity and so on (Copeland et al. [15], Frieman et al [16], Setare and Jamil [17]). One of the recent modified gravity theories is the $f(R,T)$ gravity (Harko et al. [18]).

- **Wet dark fluid**

Here we are motivated to use the wet dark fluid (WDF) as a model for dark energy which stems from an empirical equation of state proposed by Tait [19] and Hayword [20] to treat water and aqueous solution. The equation of state for wet dark fluid is

$$p_{WDF} = \omega(\rho_{WDF} - \rho^{*}) \tag{1}$$

and is motivated by the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressure possible. One of the virtues of this model is that the square of the sound speed, c_s^2, which depends on $\frac{\partial p}{\partial \rho}$. We treat (1) as a phenomenological equation (Chiba et al. [21]). The parameter ω and ρ^{*} are taken to be positive and restrict ourselves to $0 \leq \omega \leq 1$.

Introduction Continued

To find the WDF energy density, we use the energy conservation equation

$$\rho'_{WDF} + 3H(p_{EDF} + \rho_{WDF}) = 0 \tag{2}$$

From equation of state (1) and using $3H = \frac{V'}{V}$ in the above equation, we get

$$\rho_{WDF} = \frac{\omega}{1+\omega}\rho^* + \frac{c}{V^{(1+\omega)}} \tag{3}$$

where c is the constant of integration and V is the volume expansion. Wet dark fluid naturally includes the components: a piece that behaves as a cosmological constnat as well as a standard fluid with an equation of state $p = \omega\rho$. We can show that if we take $c \geq 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$.

$$p_{WDF} + \rho_{WDF} = (1+\omega)\rho_{WDF} - \omega\rho^* = (1+\omega)\frac{c}{V^{(1+\omega)}} \tag{4}$$

In this section we consider the Bianchi type-III metric as

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 dz^2 \quad (5)$$

where a_1, a_2, a_3 are cosmic scale factors and m is a positive constnat. The energy momentum tensor of the matter source is given by

$$T_\mu^\nu = (p_{WDF} + \rho_{WDF}) u_\mu u^\nu - p_{WDF} \delta_\mu^\nu \quad (6)$$

where u^μ is the flow vector satisfying

$$g_{\mu\nu} u^\mu u^\nu = 1 \quad (7)$$

From (6) we obtained

$$T_0^0 = \rho_{WDF}, \, T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \quad (8)$$

The field equations for $f(R,T)$ gravity model are given as (Harko et al. [18])

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T)$$

$$= 8\pi T_{\mu\nu} - f_T(R,T)(T_{\mu\nu} - \Theta_{\mu\nu})$$
(9)

where

$$T_{\mu\nu} = \frac{-2\delta(\sqrt{-g})}{\sqrt{-g}\delta g^{\mu\nu}}L_m, \Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu},$$

$$\mathrm{f}_R(R,T) = \frac{\partial f(R,T)}{\partial R} and f_T(R,T) = \frac{\partial f(R,T)}{\partial T} (10)$$

Basic Field Equations Continued

Here $f(R, T)$ ia an arbitrary function of Ricci scalar R and the trace T of the stress energy tensor of matter $T_{\mu\nu}$ and L_m is the matter Lagrangian density and in the present study we have assumed that the stress energy tensor of matter as

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu - pg_{\mu\nu} \tag{11}$$

We take the function as (Harko et al. [18])

$$f(R, T) = R + 2f(T) \tag{12}$$

Using (10) and (11), the field equation (9) takes the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2f'(T)T_{\mu\nu} + (2pf'(T) + f(T))g_{\mu\nu} \tag{13}$$

where the overhead prime indicates differentiation with respect to the argument.

We chose

$$f(T) = \lambda T \tag{14}$$

where λ is constnat With the help of equation (6), (7), (8) and (14), the field equation (13) fro the metric (5) can be written as

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1^2} = -(8\pi + 3\lambda)\rho_{WDF} + \lambda p_{WDF} \tag{15}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF} \tag{16}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF} \tag{17}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = (8\pi + 3\lambda)p_{WDF} - \lambda\rho_{WDF} \tag{18}$$

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = 0 \tag{19}$$

Now performing (15)-(16), (15)-(17), (15)-(18), (20)-(21), (20)-(22) and (21)-(22), we obtained

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{\ddot{a}_2}{a_2}-\frac{\ddot{a}_3}{a_3}-\frac{m^2}{a_1^2}=-(8\pi+2\lambda)(\lambda\rho_{WDF}+p_{WDF}) \quad (20)$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}-\frac{\ddot{a}_1}{a_1}-\frac{\ddot{a}_3}{a_3}-\frac{m^2}{a_1^2}=-(8\pi+2\lambda)(\lambda\rho_{WDF}+p_{WDF}) \quad (21)$$

$$\frac{\dot{a}_1\dot{a}_3}{a_1a_3}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}-\frac{\ddot{a}_2}{a_2}-\frac{\ddot{a}_1}{a_1}-\frac{m^2}{a_1^2}=-(8\pi+2\lambda)(\lambda\rho_{WDF}+p_{WDF}) \quad (22)$$

$$\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\ddot{a}_1}{a_1}-\frac{\ddot{a}_2}{a_2}=0 \quad (23)$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\ddot{a}_1}{a_1}-\frac{\ddot{a}_3}{a_3}-\frac{m^2}{a_1^2}=0 \quad (24)$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{\ddot{a}_2}{a_2}-\frac{\ddot{a}_3}{a_3}-\frac{m^2}{a_1^2}=0 \quad (25)$$

Solutions and Geometry of the Universe Continued

Now eliminating $\frac{m^2}{a_1^2}$ from (24) and (25) we have the equation, which is identical to that of equation (23). So solving equation (23) we obtained $\frac{a_1}{a_3} = m_1 exp\left(l_1 \int \frac{dt}{V}\right)$
This gives

$$a_1 = a_3 m_1 exp\left(l_1 \int \frac{dt}{V}\right) \quad (26)$$

Again from equation (19) we obtained

$$a_1 = a_3 m_2 \quad (27)$$

Here $m_1, m_2 (\neq 0)$ and l_1 are constant of integration and $V = a_1 a_2 a_3$ is the spatial volume of the universe. From equation (26) and (27) we conclude that

$$a_1 = m_2 a_2 = a_3 m_1 exp\left(l_1 \int \frac{dt}{V}\right) \quad (28)$$

In view of $V = a_1 a_2 a_3$, we obtained

$$a_1 = (kV)^{\frac{1}{3}} exp\left(\frac{l_1}{3}\int\frac{dt}{V}\right) \tag{29}$$

$$a_2 = \frac{1}{m_2}(kV)^{\frac{1}{3}} exp\left(\frac{l_1}{3}\int\frac{dt}{V}\right) \tag{30}$$

$$a_3 = \frac{1}{m_1}(kV)^{\frac{1}{3}} exp\left(\frac{-2l_1}{3}\int\frac{dt}{V}\right) \tag{31}$$

To solve we have used two different volumetric expansion laws

$$V = c_1 e^{3lt} \tag{32}$$

$$V = c_1 t^{3n} \tag{33}$$

Now solving the field equations (15) - (19) for the exponential volumetric expansion (32) by considerig (29) - (31), we obtained the scale factor as follows

$$a_1 = Dexp\left(lt - \frac{l_1}{3c_1 l}e^{-3lt}\right) \tag{34}$$

$$a_2 = \frac{D}{m_2}exp\left(lt - \frac{l_1}{3c_1 l}e^{-3lt}\right) \tag{35}$$

$$a_3 = \frac{Dc_2^{\frac{-l_2}{c_1}}}{m_1}exp\left(lt + \frac{2l_1}{3c_1 l}e^{-3lt}\right) \tag{36}$$

where $D = \left(kc_1 c_2^{\frac{l_1}{c_1}}\right)^{\frac{1}{3}}$, c_1 and c_2 are constnats of integration.

The energy density and pressure of the wet dark fluid for this model is obtained as (with the help of equation (1), (3) and (32))

$$\rho_{WDF} = \frac{\omega}{1+\omega}\rho^* + \frac{D_1}{(c_1 c_2^3 e^{3lt})^{1+\omega}} \quad (37)$$

$$p_{WDF} = -\frac{\omega}{1+\omega}\rho^* + \frac{\omega D_1}{(c_1 c_2^3 e^{3lt})^{1+\omega}} \quad (38)$$

The geometry of the universe takes the form

$$ds^2 = dt^2 - \left(D exp\left(lt - \frac{l_1}{3c_1 l}e^{-3lt}\right)\right)^2 dx^2$$

$$-\left(\frac{D}{m_2} exp\left(lt - \frac{l_1}{3c_1 l}e^{-3lt}\right)\right)^2 e^{-2mx} dy^2 - \left(\frac{D c_2^{\frac{-l_1}{c_1}}}{m_1} exp\left(lt + \frac{2l_1}{3c_1 l}e^{-3lt}\right)\right)^2 dz^2 (39)$$

Now solving the field equations (15) - (19) for the power-law volumetric expansion (33) by considering (29) - (31), we obtain the scale factor as follows

$$a_1 = D_2 t^n exp(D_3 t^{1-3n}) \tag{40}$$

$$a_2 = \frac{D_2}{m_2} t^n exp(D_3 t^{1-3n}) \tag{41}$$

$$a_3 = \frac{D_2}{m_1} c_3^{\frac{-l_1}{c_1}} t^n exp(-2D_3 t^{1-3n}) \tag{42}$$

where $D_3 = \left(k c_1 c_3^{\frac{l_1}{c_1}} \right)^{\frac{1}{3}}$ and $D_3 = \frac{l_1}{3c_1(1-3n)}$

For this model we obtained the energy density and pressure of the wet dark fluid with help of equations (1), (3) and (33) as follows

$$\rho_{WDF} = \frac{\omega}{1+\omega}\rho^{*} + \frac{D_1}{(c_1 t^{3n})^{1+\omega}} \quad (43)$$

$$p_{WDF} = -\frac{\omega}{1+\omega}\rho^{*} + \frac{\omega D_1}{(c_1 t^{3n})^{1+\omega}} \quad (44)$$

In this case the geometry of the universe takes the form

$$ds^2 = dt^2 - \left(D_2 t^n exp(D_3 t^{1-3n})\right)^2 dx^2 - \left(\frac{D_2}{m_2} t^n exp(D_3 t^{1-3n})\right)^2 e^{-2mx} dy^2$$

$$-\left(\frac{D_2}{m_1} t^n exp(-2D_3 t^{1-3n})\right)^2 dz^2 (45)$$

- The anisotropy paramter of the expansion $\Delta \equiv \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$ is found as

$$\Delta \equiv \frac{2l_1^2}{l^2 c_1^2} e^{-6lt} \tag{46}$$

- The shear scalar $\sigma^2 = \frac{1}{3}\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3} = \frac{1}{3}(H_1 - H_3)^2$ is found as

$$\sigma = \frac{\sqrt{3}l_1}{c_1} e^{-3lt} \tag{47}$$

- The deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$ is found as $q = -1$
- The average scale factor $a = (a_1 a_2 a_3)^{\frac{1}{3}} = c_1^{\frac{1}{3}} e^{lt}$
- The state finder paramters are found as

$$r = \frac{\dddot{a}}{aH^3} = 1, s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} = 0 \tag{48}$$

- Look back time
  The look back time t_L is defined as the elapsed time between the present age of the universe t_0 and the time t when the light from a cosmic source at a particular redshift z was emitted. In the context of our model it is given by

$$t_L = t_0 - t = \int_a^{a_0} \frac{da}{\dot{a}} \tag{49}$$

  where a_0 is the present day scale factor of the universe and $\frac{a_0}{a} = 1 + z$. For the model (39), we have
  $H_0(t_0 - t) = z - (1 + \frac{q}{2})z^2 + (1 + \frac{2q}{3})z^3 - (1 + \frac{3q}{4})z^4 + \cdots\cdots$
  where q is the deceleration parameter. This gives the look back time as a power series in the redshift.

- Proper distance

  The proper distance $d(z)$ is defined as the distance between a cosmic source emitting light at any instnat $t = t_I$ located at $r = r_1$ with redshift z an observer at $r = 0$ and $t = t_0$ receiving the light from the source emitted i. e.

$$d(z) = r_1 a_0 \tag{50}$$

where $r_1 = \int_{t_1}^{t_0} \frac{dt}{a} = c_1^{\frac{-1}{3}} H_0^{-1} a_0^{-1} z$

$$d(z) = c_1^{\frac{-1}{3}} H_0^{-1} z \tag{51}$$

The proper distance $d(z)$ is linear with redshift. From it we observe that $d(z = \infty)$ is always infinite.

Cosmological Parameters Continued

- Luminosity distance

  The apparent luminosity distance of a source at radial coordinate r_1 with redshift z of any size is defined as

$$l = \frac{L}{4\pi r_1^2 a_0^2 (1+z)^2} \tag{52}$$

where L is the absolute luminosity distance, it is convenient to introduce a luminosity distance d_L, which is defined as

$$d_L = \left(\frac{L}{4\pi L}\right)^2 = a_0 r_1 (1+z) \tag{53}$$

From equations (50) and (53), we get

$$d_L = d(z)(1+z) \tag{54}$$

which together with (51)

$$d_L = c_1^{\frac{-1}{3}} H_0^{-1} z(1+z) \tag{55}$$

- Angular diameter distance
  There is another sort of distance, which is what we measure when we compare angular sizes with physical dimensions. The angular diameter distance d_A is dfined so that θ is given by the usual relation Eulidean geometry.

$$\theta = \frac{s}{d_A} \tag{56}$$

we see that

$$d_A = a(t_1)r_1 \tag{57}$$

Comparison of this result with (53) shows that the ratio of the luminosity and angular diameter distances is simply a function of redshift:

$$d_A = d(z)(1+z)^{-1} = d_L(1+z)^{-2} \tag{58}$$

Now using (55) and (58), we get

$$d_A = c_1^{\frac{-1}{3}} H_0^{-1} z(1+z)^{-1} \tag{59}$$

- Jerk Parameter
  One of the convenient methods to describe models close to ΛCDM is based on the cosmic jerk parameter (j), a dimensionless third derivative at the scale factor with respect to the cosmic time.
  $$j = \frac{1}{H^3 a}\frac{d^3(a(t))}{dt^3} \quad (60)$$
  $$j = 1 \quad (61)$$
  This value over laps with flat ΛCDM models.
- Cosmic snap parameter
  The snap parameter in cosmology is defined as the dimensionless forth derivative of the scale factor with respect to cosmic time.
  $$s = \frac{1}{H^4 a}\frac{d^4 a(t)}{dt^4} \quad (62)$$
  $$s = 1 \quad (63)$$

Concluding Remarks

In this work, we studied a spatially homogeneous and anisotropic Bianchi type-III space time geometry filled with dark energy (DE) in the form of wet dark fluid (WDF) within the framework of newly established theory $f(R, T)$ garvity (Harko et al. 2011). The exact solutions of the field equations have been obtained by assuming two different volumetric expansion law in a way to cover all possible expansion: namely, exponential expansion and power law expansion. The directional Hubble parameters gradually decrease to constant l as $t \to \infty$ in the model with exponential expansion and in the power law expansion the directional Hubble parameters and mean Hubble parameter gradually decreases to zero as $t \to \infty$. It is observed that, the expansion anisotropic decays to zero monotonically in the models with the exponential expansion for $l \geq 0$ and in the poewr law expansion when $n \geq \frac{1}{3}$

Concluding Remarks Continued

For the model exponential expansion, the deceleration parameter $q = -1$ which is compartible with the recent supernovae Ia observations that the universe is undergoing a late time acceleration.

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THANK YOU