A non-canonical scalar field model of Dark Energy

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Plan of the talk

- Motivation
  - Non-canonical scalar field model
- Brief description of the model
- Dynamical system study
- Results

Motivation

- **The universe is accelerating at present.**
- Cause of acceleration ⇒ Repulsive anti-gravity effect
- In FRW background, the dynamics of the Universe is described by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\Pi G T_{\mu\nu} \tag{1}$$

which gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2}$$

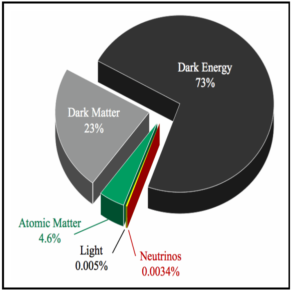
Motivation

- Logically there are two possible ways to explain the current acceleration!

  1. Modification of the geometry part of the Einstein's equations.

  2. Inclusion of Dark Energy: An unknown form of energy that provides the repulsive force.

  We shall concentrate on the second possibility.



Scalar field models :

- The most general form of Lagrangian for a scalar field model is :

$$\mathcal{L} = f(\phi)F(X) - V(\phi), \qquad X = \frac{1}{2}\dot{\phi}^2 \tag{3}$$

- $V(\phi) = 0 \Rightarrow$ k-essence.
- $f(\phi)$ = constant and F(X) = X $\Rightarrow$ quintessence.
- For a quintessence model, the field equations are:

$$3H^2 = \rho_m + \underbrace{\boxed{\frac{1}{2}\dot{\phi}^2 + V(\phi)}}_{\hookrightarrow \rho_\phi} \tag{4}$$

$$2\dot{H} + 3H^2 = \underbrace{\boxed{-\frac{1}{2}\dot{\phi}^2 + V(\phi)}}_{\hookrightarrow -p_\phi} \tag{5}$$

Noncanonical scalar field models :

- For non-canonical scalar field models :

  $f(\phi) =$ constant and F(X) $\Rightarrow$ nonlinear function of X.

- Also the conservation equation for the scalar field is :

$$\ddot{\phi}\left[\left(\frac{\partial \mathcal{L}}{\partial X}\right) + (2X)\left(\frac{\partial^2 \mathcal{L}}{\partial X^2}\right)\right] + \left[3H\left(\frac{\partial \mathcal{L}}{\partial X}\right) + \dot{\phi}\left(\frac{\partial^2 \mathcal{L}}{\partial X \partial \phi}\right)\right]\dot{\phi} - \left(\frac{\partial \mathcal{L}}{\partial \phi}\right) = 0 \tag{6}$$

A Toy model:

- For the present toy model, we choose $F(X) = X^2$.
- Einstein's field equations take the form

$$3H^2 = \rho_m + \underbrace{\boxed{\frac{3}{4}\dot{\phi}^4 + V(\phi)}}_{\hookrightarrow \rho_\phi}$$

$$2\dot{H} + 3H^2 = \underbrace{\boxed{-\frac{1}{4}\dot{\phi}^4 + V(\phi)}}_{\hookrightarrow -p_\phi}$$

$$\boxed{\dot{\rho}_\phi + 3H(1 + \omega_\phi)\rho_\phi = 0}$$

$$\boxed{\dot{\rho}_m + 3H\rho_m = 0}$$

Interacting scenario :

- For the present toy model, we choose $F(X) = X^2$.
- Einstein's field equations take the form

$$3H^2 = \rho_m + \underbrace{\boxed{\frac{3}{4}\dot{\phi}^4 + V(\phi)}}_{\hookrightarrow \rho_\phi}$$

$$2\dot{H} + 3H^2 = \underbrace{\boxed{-\frac{1}{4}\dot{\phi}^4 + V(\phi)}}_{\hookrightarrow -p_\phi}$$

$$\boxed{\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi = Q = \alpha H\dot{\phi}^4}$$

$$\boxed{\dot{\rho}_m + 3H\rho_m = -Q = -\alpha H\dot{\phi}^4}$$

Toy Model

- We made the following ansatz to close the system of equations.

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{X^2 - V}{3X^2 + V} = \omega(\text{a constant}) \tag{7}$$

- At present $\omega_\phi \simeq -1$ and $\ddot{a} > 0 \Rightarrow -1 < \omega_\phi < -\frac{1}{3}$
- This gives

$$X^2 = \frac{1+\omega}{1-3\omega} V \qquad \Rightarrow \dot{\phi}^4 = 4\left(\frac{1+\omega}{1-3\omega}\right) V(\phi) \tag{8}$$

- Finally one gets

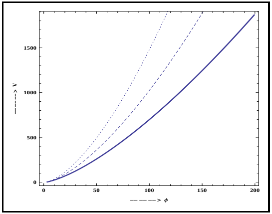
$$\frac{dV(a)}{da}\dot{a} + \epsilon\frac{\dot{a}}{a}V(a) = 0$$

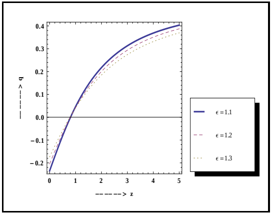
$$\Rightarrow \boxed{V(a) = V_0 a^{-\epsilon}}$$

and

$$\boxed{H^2 = \gamma a^{-\epsilon} + B a^{-3}}$$

- $\epsilon = (3 - \alpha)(1 + \omega)$
- $\gamma = \frac{4\omega V_0}{(3-\epsilon)(3\omega-1)}$





Dynamical system study

- We rewrite the equations as plane autonomous system
- We define three new variables :

$$x = \frac{\dot{\phi}^2}{2H},\ y = \frac{\sqrt{V}}{\sqrt{3}H} \text{ and } \lambda = -\frac{1}{\dot{\phi}V}\frac{dV}{d\phi}.$$

- The evolution equations for the scalar field are :

$$x' = -Wx + \frac{3}{2}x(1 + \frac{x^2}{3} - y^2) + \lambda y^2$$

$$y' = \frac{3}{2}y(1 + \frac{x^2}{3} - y^2) - \lambda xy$$

- with

$$\Omega_\phi = \frac{\rho_\phi}{3H^2} = x^2 + y^2$$

and

$$\omega_{tot} = \frac{p_\phi}{\rho_m + \rho_\phi} = \frac{x^2}{3} - y^2$$

	x^*	y^*	Nature of eigenvalues	Stability?	ω^*_{tot}	Acceleration?
i	0	0	real, unequal and opposite signs	Saddle point	0	No
ii	$\sqrt{2W-3}$	0	real, unequal and positive	Unstable node	3.88	No
iii	$-\sqrt{2W-3}$	0	real, unequal and positive	Unstable node	3.88	No
iv	b_+	$\frac{p}{2\sqrt{2\lambda}}$	real, unequal and opposite signs	Saddle point	2.87	No
v	b_+	$-\frac{p}{2\sqrt{2\lambda}}$	real, unequal and opposite signs	Saddle point	2.87	No
vi	b_-	$\frac{q_1}{\sqrt{2\lambda}}$	real, unequal and negative	Stable node	-0.898	Yes
vii	b_-	$-\frac{q_1}{\sqrt{2\lambda}}$	real, unequal and negative	Stable node	-0.898	Yes

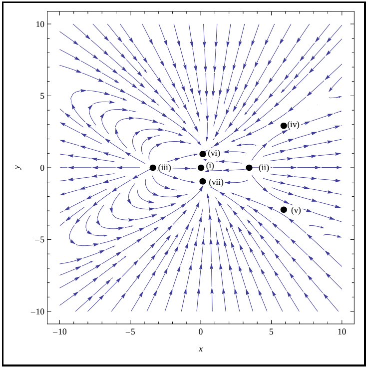
Table: The properties of the critical points. This is for $\lambda = 1$, $\epsilon = 1.1$ and $\omega_\phi = -0.9$. Here, $\alpha = 3 - \frac{\epsilon}{1+\omega_\phi} = -8.0$ and $W = \frac{2}{3}(3-\alpha) = 7.33$.

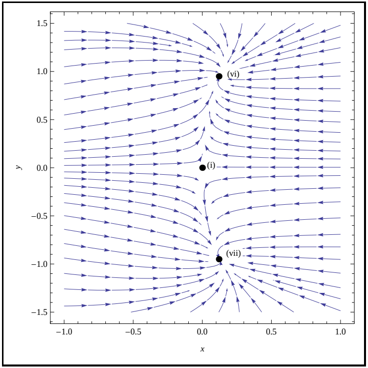
- $b_\pm = \frac{(3W+2\lambda^2)}{8\lambda} \pm \frac{\sqrt{-48\lambda^2+(3W+2\lambda^2)^2}}{8\lambda}$
- For (vi) and (vii),

$$\Omega_\phi \sim 0.9169 \text{ and}$$
$$\Omega_m \sim 0.0831, q = -1 + \frac{3}{2}\left[1 + \frac{x^2}{3} - y^2\right] = -0.84.$$

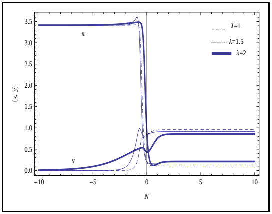
Results

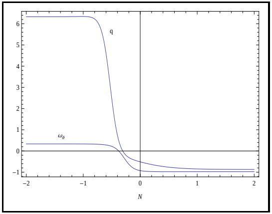
- Phase portrait for the system





- Evolution of (i) x and y and (ii) q and ω_ϕ against N for $\lambda = 1.0, 1.5, 2.0$, $\epsilon = 1.1$ and $\omega_\phi = -0.9$
- The universe enters into an accelerated phase in the recent past
- ω_ϕ was positive initially, close to -0.9 now and settles to a value -1 in future
- The present model will behave like a ΛCDM model in future





Drawbacks / Future scope

- $\omega_\phi =$ constant !!
- $Q = \alpha H \dot{\phi}^4$??
- $\lambda = -\frac{1}{\dot{\phi} V}\frac{dV}{d\phi}$

Thank You