Inflation after Planck

Supratik Pal

Indian Statistical Institute Kolkata

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Outline

- The ABC of Inflation
- CMB à la WMAP9 and Planck 2013
- Test of inflationary predictions
- Status of inflationary models

The ABC of Inflation

Puzzles of standard Big Bang Cosmology

- Horizon
- Flatness
- Monopole
- Structure formation...

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Wayout

Super-fast accelerated expansion at the beginning $\Longrightarrow$ Inflation

Dynamics? $\longrightarrow$ Scalar field

EM tensor components $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$; $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

Governing Equations

$$H^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

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Employ slow roll condition

$$\dot{\phi}^2 << V(\phi) \quad ; \quad |\ddot{\phi}| << 3H|\dot{\phi}|, V'(\phi)$$

Slow roll parameters $\epsilon_V = \frac{M_P^2}{2}\left[\frac{V'}{V}\right]^2 \ll 1$

$$\eta_V = M_P^2\left[\frac{V''}{V}\right] \ll 1$$

For sufficient inflation $N = \ln\frac{a_f}{a_i} \approx -\frac{1}{M_P^2}\int_{\phi_i}^{\phi_f}\frac{d\phi}{\sqrt{2\epsilon_V}} \approx 56 - 70$

Solves first 3 puzzles at a single go.

Structure formation? $\longrightarrow$ Perturbations

Quantum fluctuations of inflaton are transformed to classical perturbations

Perturbations in the metric

$\Downarrow$

$$\boxed{\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}}$$

$\swarrow$ $\downarrow$ $\searrow$

Scalar perturbation | Vector perturbation | Tensor perturbation

Scalar perturbation $\Downarrow$ Matter

Tensor perturbation $\Downarrow$ Gravitational waves

Solves Puzzle No.4

Structure formation? ⟶ Perturbations

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Scalar perturbation	Vector perturbation	Tensor perturbation
$\Downarrow$		$\Downarrow$
Matter		Gravitational waves

Solves Puzzle No.4

First impression: Too good to be true!!

Inflationary predictions

Observable parameters	Scalar modes	Tensor modes
Power spectrum	$P_R(k) = \frac{k^3}{2\pi^2} \frac{\lvert v\rvert^2}{z^2}\rvert_*$	$P_T(k) = 2 \times \frac{k^3}{2\pi^2} \frac{2}{M_P^2} \frac{\lvert u\rvert^2}{a^2}\rvert_*$
Tensor to scalar ratio		$r = \frac{P_T\rvert_*}{P_R\rvert_*}$
Spectral index	$n_S = 1 + \frac{d \ln P_R(k)}{d \ln k}\rvert_*$	$n_T = \frac{d \ln P_R(k)}{d \ln k}\rvert_*$
Running of S.I.	$\alpha_S = \frac{dn_s}{d \ln k}\rvert_*$	$\alpha_T = \frac{dn_s}{d \ln k}\rvert_*$

$* \Rightarrow k = aH$

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+ Consistency relation $r = 16\epsilon = -8n_T$

more or less generic for

- slow roll
- single scalar, canonical, minimally coupled
- $c_s \approx 1$

Inflationary predictions

- Perturbations generate specific peaks in CMB $\Rightarrow$ Give $\Omega^{\prime s}$
- Spatially flat universe $\Rightarrow \Omega_{\rm tot} \approx 1 \pm 10^{-4}$
- Adiabatic perturbations $\Rightarrow$ all species share a common perturbation
- Gaussian perturbations $\Rightarrow$ stochastic properties completely determined by spectrum

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- $(n_s - 1) =$ small $\Rightarrow$ nearly scale invariant power spectrum
  $(n_s - 1) \neq 0 \Rightarrow$ perturbations originated from dynamics of scalar field
- Generic spectrum $P_R(k) \simeq P_R(k_*)(\frac{k}{k_*})^{n_s-1+n_s' \ln(k/k_s)}$
  $n_s' \neq 0 \Rightarrow$ deviation from power law

Inflationary predictions

- Tensor modes would be small but bear strong physical significance.
  A small fraction of CMB photons get polarized due to quadrupole anisotropies. $\Rightarrow$ 2 polarization modes (E & B)

  $\boxed{\text{B modes} \rightarrow \text{Gravitational waves}}$ + NG + Lensing...

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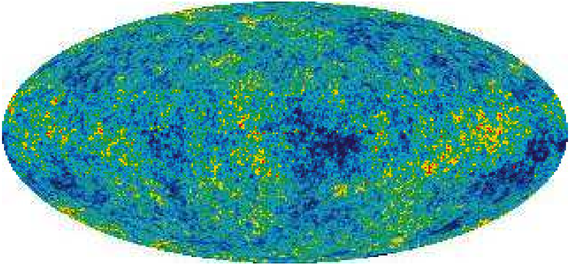
Most of these predictions can be violated with more complicated models

The CMB sky



Background temperature $T_0 = 2.725K$ at all directions
$\Rightarrow$ The Universe is homogeneous and isotropic at largest scale
How many parameters to describe the Universe? $\longrightarrow$ 6 (or 7?)

All about CMB temperature

Background : $T_0 = 2.725K \longrightarrow$ Blackbody spectrum

Fluctuations : $-200\mu K < \Delta T < 200\mu K$

$$\Delta T_{rms} \sim 70\mu K$$
$$\Delta T_{pE} \sim 5\mu K$$
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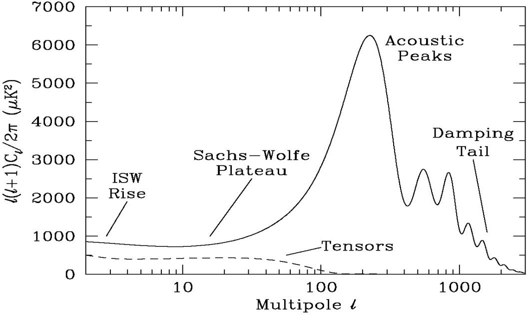
How to decode information?

- Temperature anisotropy T + two polarization modes E & B
  $\Rightarrow$ Four CMB spectra: $C_l^{TT}, C_l^{EE}, C_l^{BB}, C_l^{TE}$
- Parity violation/systematics $\Rightarrow$ Two more spectra: C_l^{TB}, C_l^{EB}

$\Delta T(n) = \sum a_{lm} Y_{lm}(n) \Rightarrow$ 2-point correlation fn.
$C_l = \frac{1}{2l+1} \sum |a_{lm}|^2$

$$C_l = \int \frac{dk}{k} P_R(k) T_l^2(k)$$



Peak positions, heights and ratios give cosmological parameters $\Rightarrow$ imprints of both early universe and late universe

Cosmological parameters from C_l

Fundamental/ fit parameters

$\Omega_b h^2$ = baryonic matter density
$\Omega_c h^2$ = dark matter density
Ω_X = dark energy density
P_R = primordial scalar power spectrum
n_s = scalar spectral index
τ = optical depth
r = tensor-to-scalar ratio

Altogether 6 (or 7 if $r \neq 0$)

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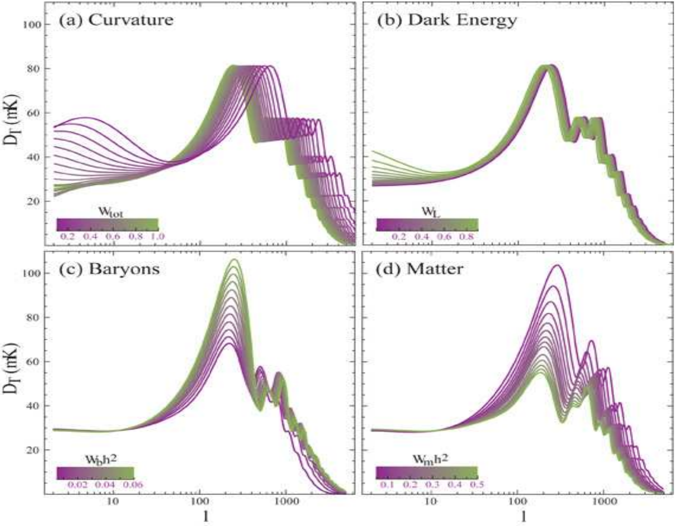
Derived parameters

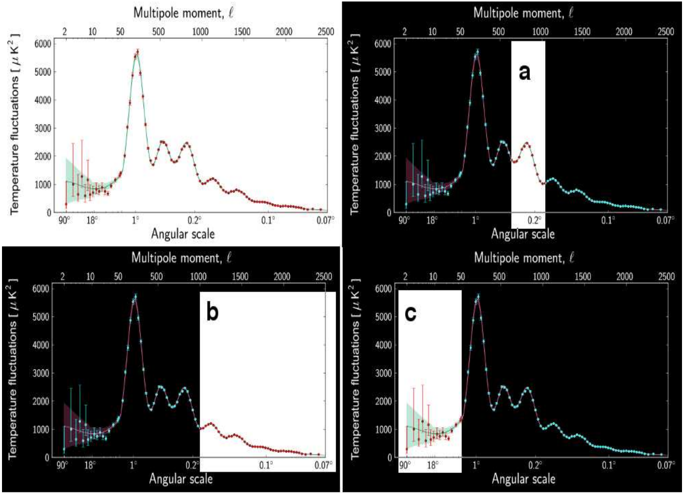
$t_0,\ H_0,\ \Omega_b,\ \Omega_c,\ \Omega_m,\ \Omega_k,\ \Omega_{\rm tot},\ \sigma_8,\ z_{\rm eq},\ z_{\rm reion}\ldots$

Parameters	WMAP 9	Planck 2013
P_R	$(2.464 \pm 0.072) \times 10^{-9}$	$(2.196^{+0.051}_{-0.060}) \times 10^{-9}$
n_s	0.9606 ± 0.008	0.9603 ± 0.0073
n'_s	-0.023 ± 0.001	-0.013 ± 0.009
r	< 0.13	< 0.11
Ω_b	0.04628 ± 0.00093	
Ω_c	$0.2402^{+0.0088}_{-0.0087}$	$\Omega_b + \Omega_c = 0.315 \pm 0.017$
Ω_X	$0.7135^{+0.0095}_{-0.0096}$	$0.685^{+0.018}_{-0.016}$
τ	0.088 ± 0.015	$0.089^{+0.012}_{-0.014}$
H_0	69.32 ± 0.80 km/s/Mpc	67.3 ± 1.2 km/s/Mpc
t_0	13.772 ± 0.059 Gyr	13.817 ± 0.048 Gyr

How sensitive to parameters the CMB TT plot is?



Planck 2013 highlights



Inflationary parameters	Planck 2013 results
P_R	$(2.196^{+0.051}_{-0.060}) \times 10^{-9}$
n_s	0.9603 ± 0.0073
n_s'	-0.013 ± 0.009
r	< 0.11
n_T	> -0.048 at $95\% CL$
$100\Omega_k$	$-0.05^{+0.65}_{-0.66}$
$f_{\rm NL}^{\rm loc}$	2.7 ± 5.8
$f_{\rm NL}^{\rm eq}$	-42 ± 75
$f_{\rm NL}^{\rm ortho}$	-25 ± 39

Test of predictions post-Planck 2013

Boring universe, 6 parameters suffice

More matter, less energy (slightly altered in Planck 2014?)

Little bit older universe (13.771 Gyr $\longrightarrow$ 13.817 Gyr)

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Outliners are still there $\Rightarrow$ physical origin?
Large scale anomalies : hemispherical asymmetry?
Big cold spot $\Rightarrow$ superstructure?

Have we "seen" inflation in the sky?

No!!

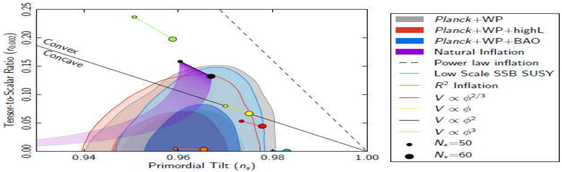
Can be claimed only when we detect

- r conclusively (BICEP2: $r \approx 0.2$ or dust??)
- n_T independently and verify consistency relation $r = -8n_T$ for
  - * slow roll
  - * single field, canonical, minimally coupled
  - * $c_s \approx 1$
- α_T (or confirm it is zero)
- $f_{\rm NL}$ for single field vs multi field debate

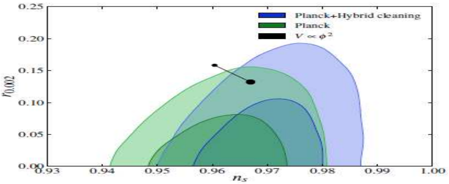
... but of course we are zeroing in!

What can we say about the inflationary models?

Chaotic + minimal copuling   P.A.R.Ade et.al., 1303.5082

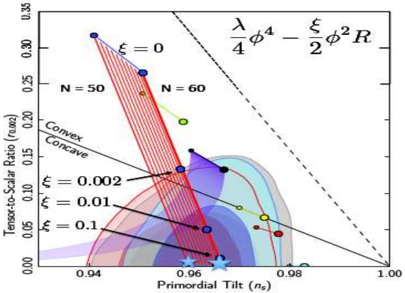


Tightly constrained. Different cleaning: Spergel et.al., PRD:2015



ϕ^2 marginally consistent.

Chaotic + non-minimal coupling   Bezrukov et.al., JHEP:2013



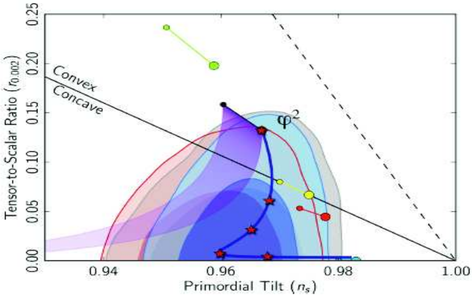
Allowed, even ϕ^4 for $\xi/2 > 10^{-3}$

but issues, e.g. candidate inflaton? Higgs?

Polynomial (SUGRA?)

Kallos, Linde, JCAP: 2010

$$V(\phi) = \frac{m^2\phi^2}{2}(1 - a\phi + a^2 b\phi^2)^2$$

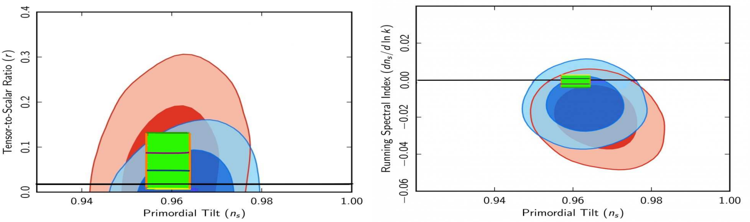


Allowed, with $b = 0.34, a = 0, 0.03, 0.05, 0.1, 0.13$

but issues, e.g. SUGRA origin is debatable

MSSM(inflection point)   Choudhury, Majumdar, SP, JCAP:2013

$$V(\phi) = \alpha + \beta(\phi - \phi_0) + \gamma(\phi - \phi_0)^3 + \kappa(\phi - \phi_0)^4 + \cdots$$



Planck+WP9+BAO: Blue: ΛCDM+$r(\alpha_S)$, Red: ΛCDM+$r + \alpha_S$

Allowed, better fit for low l

Starobinsky model   Starobinsky, Sov. Astron. Lett: 1983

$$L = \sqrt{-g}\left(\frac{R}{2} + \frac{R^2}{12M^2}\right), \quad M \ll M_p$$

can be reduced to canonical gravity + scalar field by field redefinition and metric transformation

$$N \sim 60 \Rightarrow n_S \sim 0.967, r \sim 0.003$$
$$N \sim 60 \Rightarrow n_S \sim 0.964, r \sim 0.004.$$

Allowed

Many models, except a few with very high r, are still allowed.

Two open questions

- All models lead to same predictions matching with Planck.
  Can they be incorporated into a common platform?
  Superconformal theory??
  Universal attractor?? Linde, 1402.0526

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- Among all allowed models, which ones are more probable?

Most probable models

Model selection algorithm Liddle et.al., astro-ph/0608184

Consider 2 models

- $\mathcal{M}_1$ with one model parameter θ
- $\mathcal{M}_2$ with two model parameters α and β

How do they fair against some data D? $\Rightarrow$ maximum likelihood

$\mathcal{L}_1 = \mathcal{L}_{1,\max} \exp^{-\chi^2(\theta)/2}$; $\mathcal{L}_2 = \mathcal{L}_{2,\max} \exp^{-\chi^2(\alpha,\beta)/2}$

But this does not distinguish between "complexity" of the models.

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Occam's razor : penalize complex models.

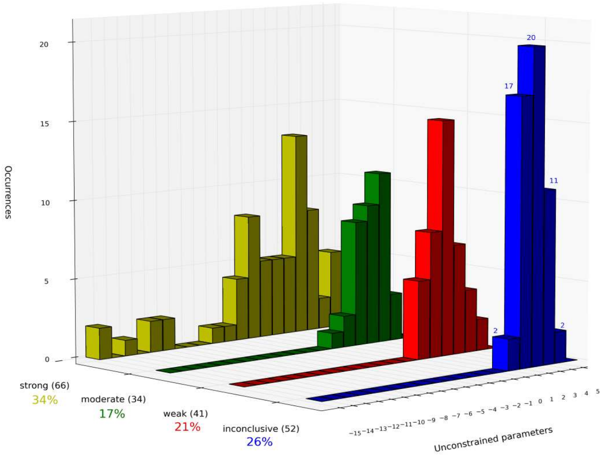
Best models are those who can make best compromise between simplicity and quality of fits

Calculate Bayesian evidence

$\mathcal{E}_1 = \int \mathcal{L}_1(D/\theta)\pi(\theta)d\theta$; $\mathcal{E}_2 = \int \mathcal{L}_2(D/\alpha,\beta)\pi(\alpha,\beta)d\alpha d\beta$

Prior distributions satisfy $\int \pi(\theta)d\theta = 1$; $\int \pi(\alpha,\beta)d\alpha d\beta = 1$

Lower evidence $\Rightarrow$ More probable : Jeffrey's scale



$\sim$ 26% models are most probable. J.Martin et.al., JCAP:2014
+ Bayesian complexity $\Rightarrow \sim$ 9%. Preferred potential: pleatue type.
But it all depends on how reliable Bayesian evidence calculation is!

Take home message

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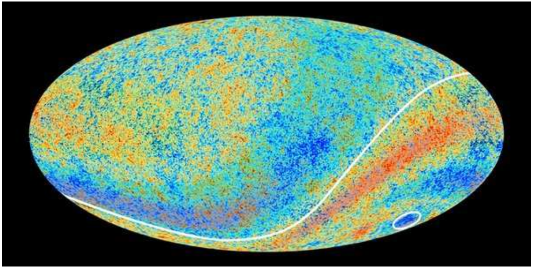
The point is not to pocket the truth but to chase it – Elio Vittorini

Some more questions

- Large scale anomalies
- Lensing
- Non-Gaussianity

Large scale anomalies



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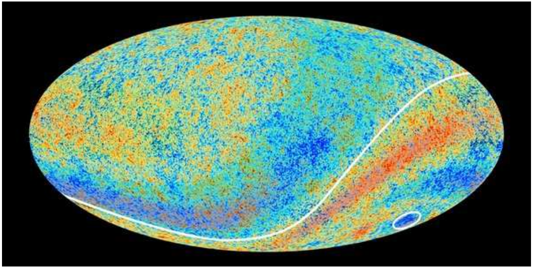

- Modifications to inflation? (Carroll, PRD:2008)
- Earlier universe preceding Big Bang? (Efstathiou,)
- Undiscovered source in solar system? (Yoho, PRD:2011)

A nice review by Huterer, 1004.5602

Lensing

Effects of lensing

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- Non-Gaussianity

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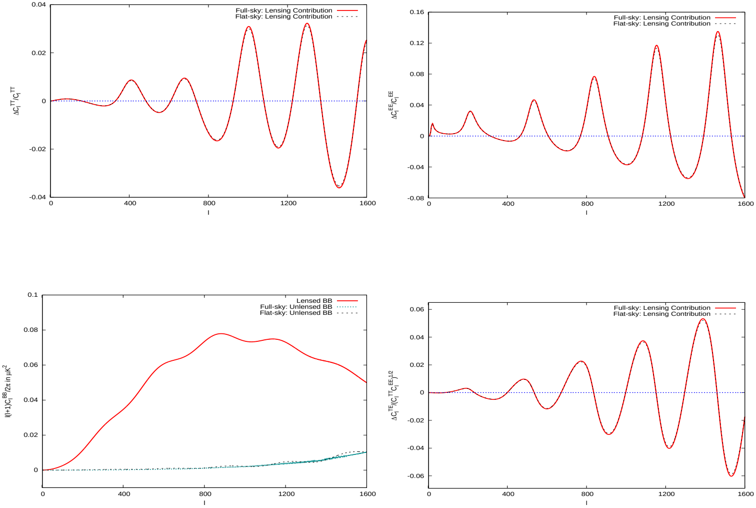
To do

- Propose delensing techniques
- Wait for Planck polorization & CMBPol data

Delensing using matrix inversion technique

Pal, Padmanabhan, SP, MNRAS:2014



Fractional difference between lensed and unlensed power spectra

Non-Gaussianity

Perturbations mostly Gaussian, described by 2-point correlation fn.
If (small) non-Gaussianities are present $\longrightarrow$ reflected via B modes
3- and 4-point correlation fn. $\Rightarrow$ bispectrum f_{NL} & trispectrum g_{NL}, τ_{NL}

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Why important?

- Maldacena limit $\Rightarrow$ single field ($|f_{NL}| < 1$) vs multifield ($|f_{NL}| > 5$)
- B modes = GW + NG + lensing $\Rightarrow$ Need to separate out NG for correct estimate of GW
- Suyama-Yamaguchi consistency relation between f_{NL} & τ_{NL}