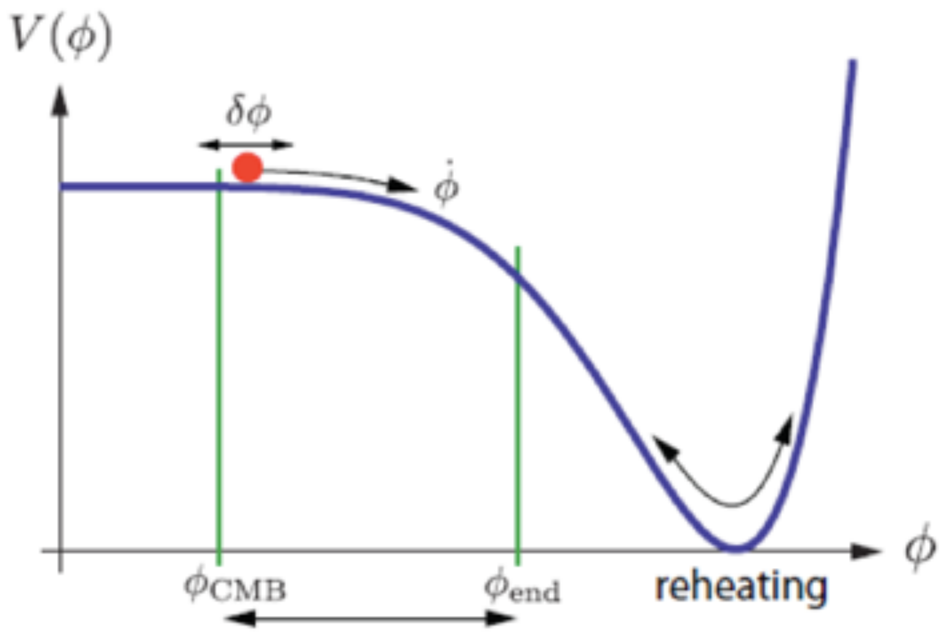
Supergravity Inflation

Subhendra Mohanty
Physical Research Laboratory, Ahmedabad

Slow roll inflation



[Source - arXiv:0907.5424]

$$\ddot\phi + 3\,\mathcal{H}\,\dot\phi + V'(\phi) = 0$$

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta = \left(\frac{V''}{V}\right)$$

$$\xi^2 = \left(\frac{V'V'''}{V^2}\right)$$

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k) \simeq \frac{1}{2\sqrt{3}\,\pi}\left(\frac{V^{3/2}}{|V'|}\right)_{k=a\mathcal{H}}$$

$$n_{\rm s} - 1 \equiv \frac{{\rm d}\ln \mathcal{P}_{\mathcal{R}}(k)}{{\rm d}\ln k} \simeq 2\,\eta - 6\,\epsilon$$

$$\frac{{\rm d}\,n_{\rm s}}{{\rm d}\ln k} \simeq 16\,\epsilon\,\eta - 24\,\epsilon^2 - 2\,\xi^2$$

$$r = \frac{\mathcal{P}_{\mathcal{T}}(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 16\,\epsilon$$

$$N_* = \int_{t_*}^{t_e} {\rm d}t\; H \approx \frac{1}{M_{\rm pl}^2}\int_{\phi_*}^{\phi_e} {\rm d}\phi\; \frac{V}{V_\phi}$$

BICEP2 messurement of $r \sim 0.16$
does not directly contradict PLANCK bound $r < 0.11$

BICEP2 is most sensitive
at $l \sim 150$ corresponding to a hub of $k \sim 0.01\, Mpc^{-1}$

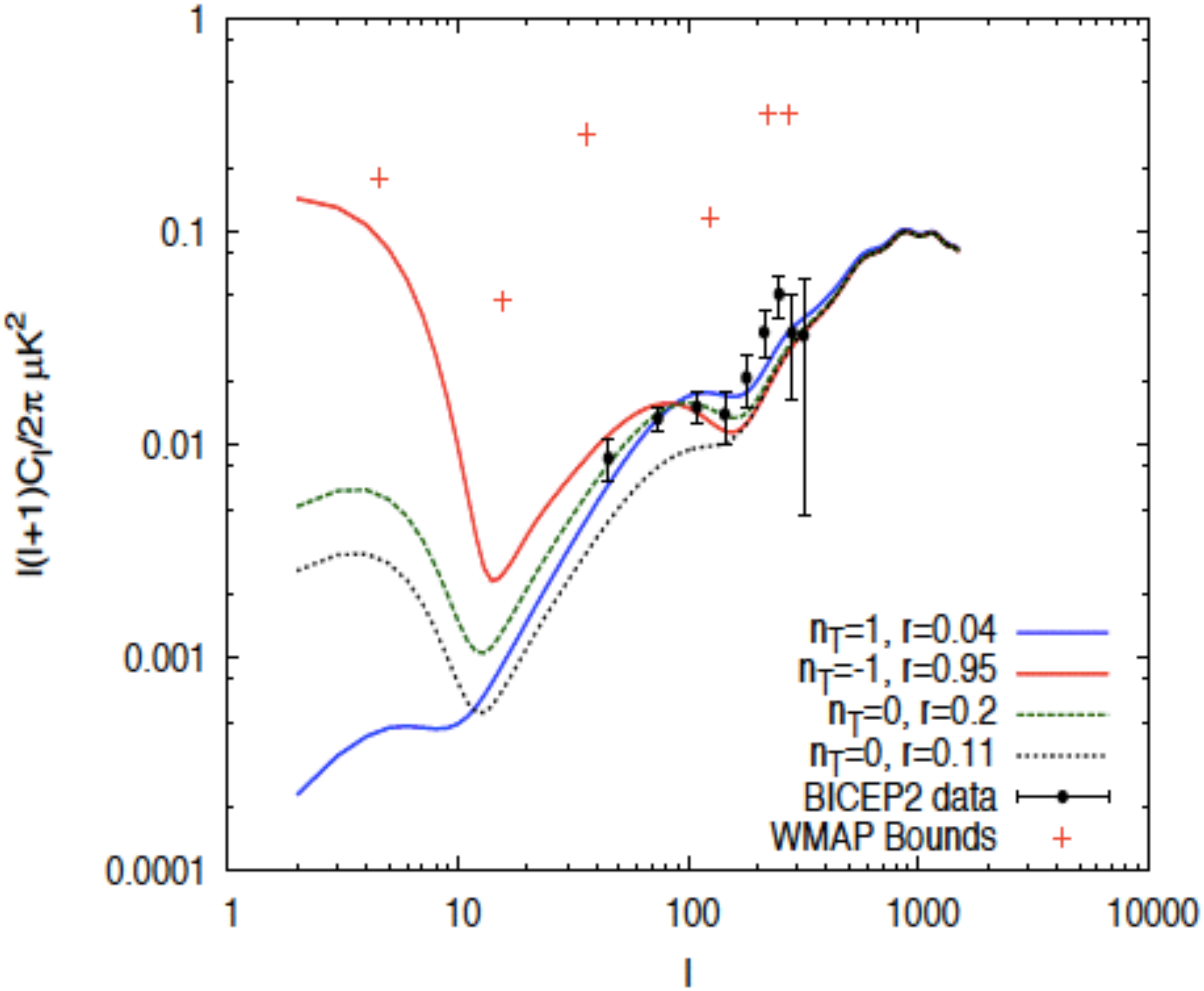
PLANCK 2013 bound on r uses the hub
$k = 0.002 Mpc^{-1}$ which corresponds to $l \sim 30$..

PLANCK bound and BICEP2 measurement can both be explained in a model if :

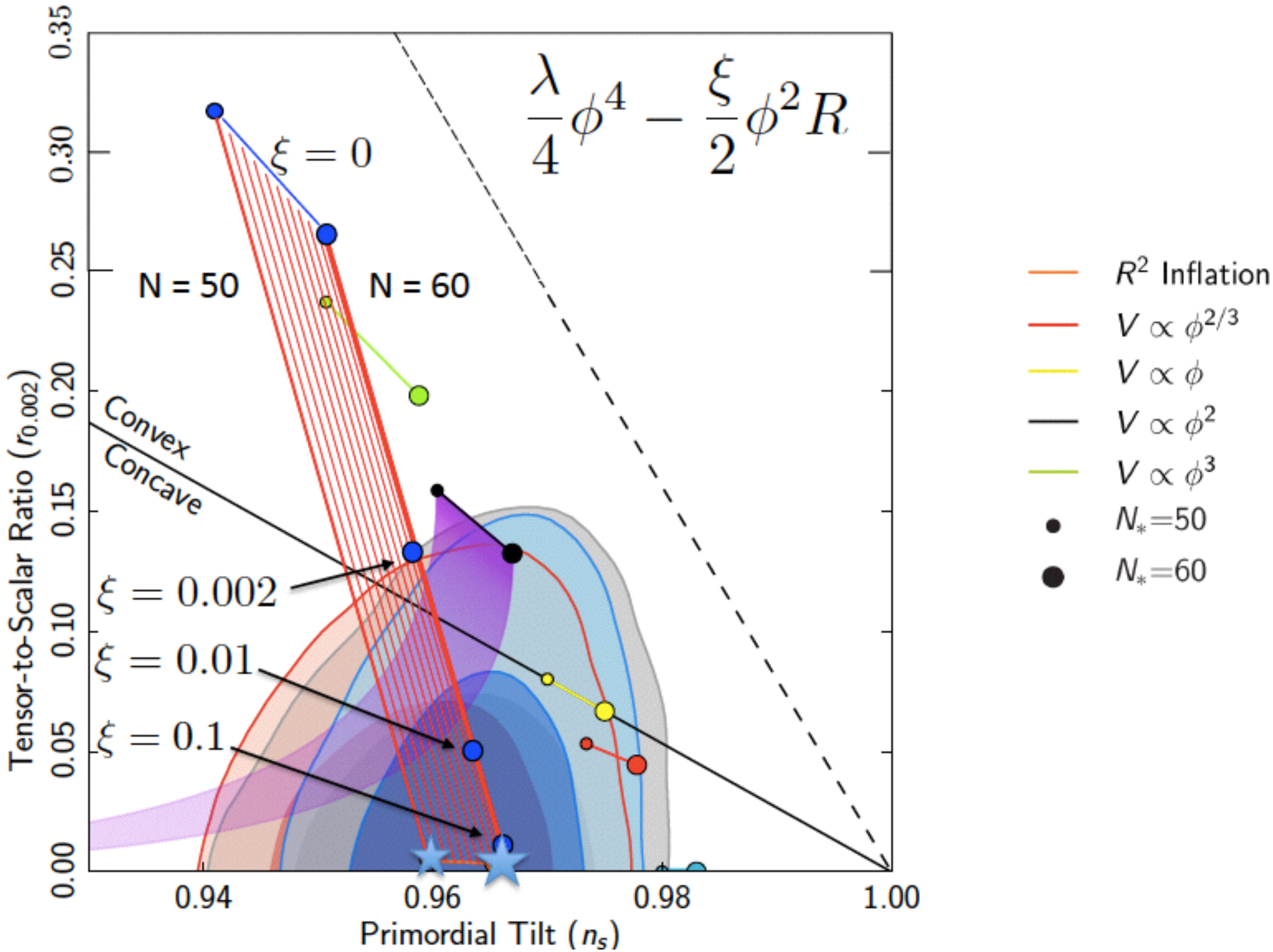
- Tensor specturm has a large blue tilt ~1

or

- There is a running of the scalar spectral index of ~ -0.002

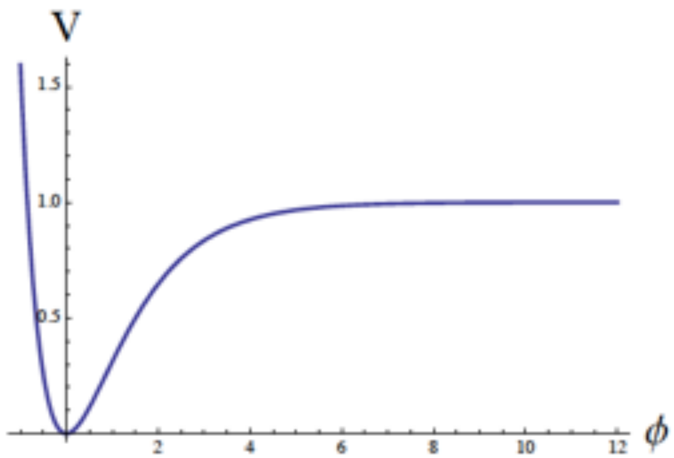


Akhilesh Nautiyal , SM 1404.2222



Kallosh, Linde 1306.3211

Starobinsky model



$$L = \sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right)$$

is equivalent to the scalar model

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\varphi}\right)^2\right]$$

$$n_s = 1 - 2/N\,, \quad r = 12/N^2$$

$$n_s \sim 0.964,\ r \sim 0.004 \text{ for } N \sim 55$$

Curvature coupling of inflation

$$\mathcal{L}_{\rm J} = \sqrt{-g}[\tfrac{1}{2}\Omega(\phi)R - \tfrac{1}{2}(\partial\phi)^2 - V_J(\phi)]\,,$$

$$\Omega(\phi) = 1 + \xi f(\phi)\,, \qquad V_J(\phi) = \lambda^2 f^2(\phi)\,.$$

$$g_{\mu\nu} \to \Omega(\phi)^{-1} g_{\mu\nu}.$$

$$\mathcal{L}_{\rm E} = \sqrt{-g}[\tfrac{1}{2}R - \tfrac{1}{2}\Big(\Omega(\phi)^{-1} + \tfrac{3}{2}(\log\Omega(\phi))'^2\Big)(\partial\phi)^2 - V_E(\phi)]$$

large ξ limit $\qquad \varphi = \pm\sqrt{3/2}\,\log(\Omega(\phi))\,.$

leads to the Einstein frame lagrangian

$$\mathcal{L}_{\mathrm{E}} = \sqrt{-g}\left[\tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \frac{\lambda^2}{\xi^2}(1 - e^{-\sqrt{\frac{2}{3}}\varphi})^2\right]$$

for any form of $f(\phi)$

Same prediction as the Storbinsky model for n_s *and* r

Higgs Inflation $\quad f(\phi) = \phi^2, \quad \lambda \sim 0.1, \quad \xi = 400000$

$$\frac{1}{\sqrt{-g}}\mathcal{L}^{\xi}_{\mathrm{mat}} = -\frac{1}{2}\xi\phi^2 R - \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{\lambda}{4}\phi^4$$

Bezrukov and Shaposhnikov 0710.3755

Generalisations of “Higgs Inflation” …

$$S = \int d^4x\sqrt{-g}\Big[-\frac{{M_p}^2}{2}R\left(1+\frac{\xi\Phi^a R^{b-1}}{M_p^{a+2b-2}}\right) \quad + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\lambda\Phi^4}{4}\Big]$$

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
a	3.385	3.026	2.735	2.494	2.292
b	0.277	0.439	0.571	0.679	0.770
a+2b	3.939	3.904	3.877	3.852	3.832

Parameter fits with PLANCK data assuming $\xi = 1$

model predicts $r = 0.002$

Singh, Chakravarty, SM, 1303.3870

Generalise inflaton curvature coupling models with the aim of increasing r to ~ 0.1

$$\int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2}-\frac{\xi\Phi^a R^b}{2M_p^{a+2b-4}}+\frac{\lambda\Phi^4}{4}\left(\frac{\Phi}{M_p}\right)^{4\gamma}\right)$$

The 5-parameter Higgs-Curvature model

$$\int d^4x\sqrt{-g}\left(-\frac{M_p^2 R}{2}-\frac{\xi\Phi^a R^b}{2M_p^{a+2b-4}}+\frac{\lambda\Phi^4}{4}\left(\frac{\Phi}{M_p}\right)^{4\gamma}\right)$$

is equivalent to a 2-parameter
power law Starobinsky model

$$\int d^4x\sqrt{-g}\left(\frac{-M_p^2}{2}\right)\left(R+\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right)$$

$$\beta=\frac{4b(1+\gamma)}{4(1+\gamma)-a}\quad,\quad M^2=\frac{a}{3(4(1+\gamma)-a)\lambda}\left(\frac{2\lambda(1+\gamma)}{\xi a}\right)^{\frac{4(1+\gamma)}{4(1+\gamma)-a}}$$

Girish Chakravarti, SM 1405.1301

Jordan frame

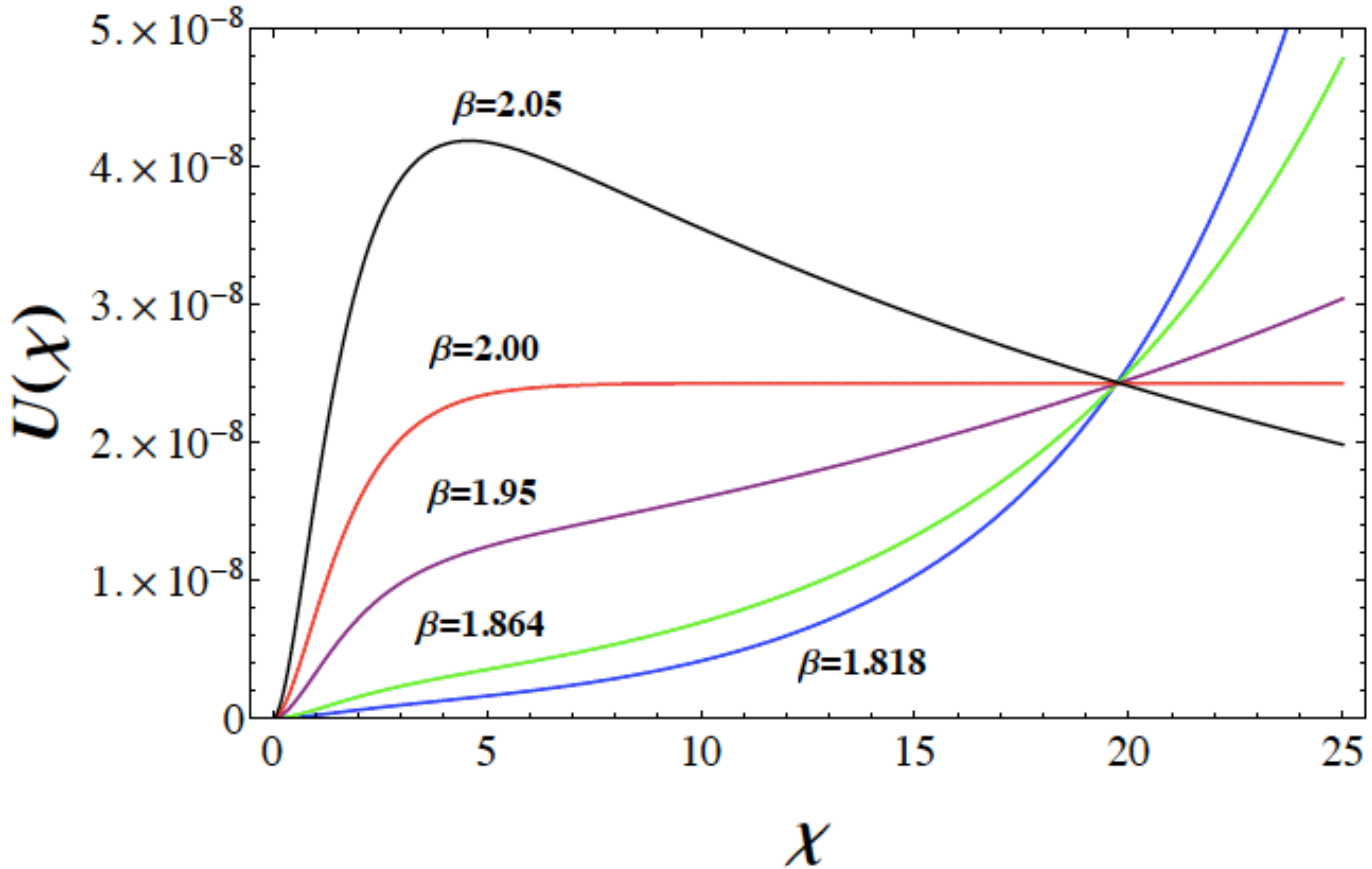
$$S = \frac{-M_p^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6M^2} \frac{R^\beta}{M_p^{2\beta-2}} \right)$$

Einstein frame action

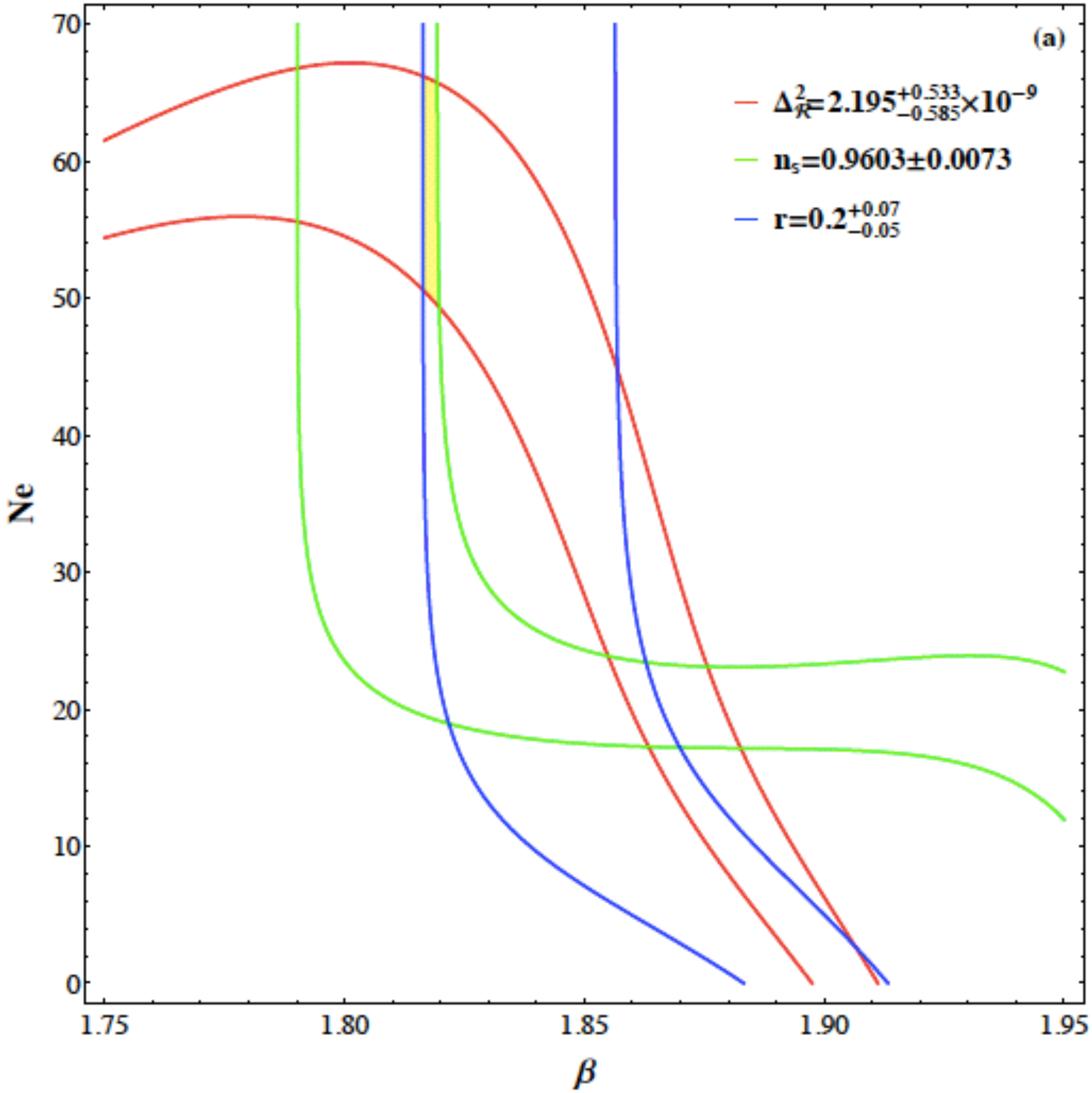
$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{-M_p^2}{2} \tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + U(\chi) \right]$$

potential in Einstein frame :

$$U(\chi) = \frac{(\beta - 1)}{2} \left(\frac{6M^2}{\beta^\beta} \right)^{\frac{1}{\beta-1}} \exp\left[\frac{2\chi}{\sqrt{6}} \left(\frac{2-\beta}{\beta-1} \right) \right] \left[1 - \exp\left(\frac{-2\chi}{\sqrt{6}} \right) \right]^{\frac{\beta}{\beta-1}}$$



Scalar potential of the power law model



β	M
1.818	1.8×10^{-4}

$$\int d^4x\sqrt{-g}\left(\frac{-M_p^2}{2}\right)\left(R+\frac{1}{6M^2}\frac{R^\beta}{M_p^{2\beta-2}}\right)$$

Scalar potential in SUSY

superpotential $$W = W_0 + L^i \Phi_i + \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k \,.$$

Scalar potential in SUSY

$$V(\phi_i, \phi_i^*) = V_F + V_D = W_i^* W^i + \frac{1}{2} \sum_a g_a^2 \left(\phi^* \mathcal{T}^a \phi\right)^2$$

Scalar potential in SUGRA

Kahler function $K(\phi^i, \phi^{*\bar{i}})$

Kahler metric $K_{i\bar{j}} \equiv \dfrac{\partial^2 K}{\partial\phi^i \, \partial\phi^{*\bar{j}}}$

F-term scalar potential

$$V_F = \mathrm{e}^K \left[K^{i\bar{j}} \, \mathcal{D}_i W \, \mathcal{D}_{\bar{j}} W^* - 3 \, |W|^2 \right]$$

$$\mathcal{D}_i W \equiv W_i + W K_i$$

D-term scalar potential $V_D = \frac{1}{2}\left[\mathrm{Re} f_{ab}^{-1}\right] D^a D^b$

$$D_a = \Phi_i (T_a)^i_j \frac{\partial K}{\partial \Phi_j} + \xi_a$$

kinetic terms of the scalar fields :

$$\frac{1}{\sqrt{-g}} \mathcal{L}_{\mathrm{kin}} = -K_{ij^*} D_\mu \Phi_i D_\nu \Phi_j^* g^{\mu\nu}$$

Starobinsky potential from SUGRA

$$K = -3\ln(T + T^* - |\phi|^2/3)$$

$$W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

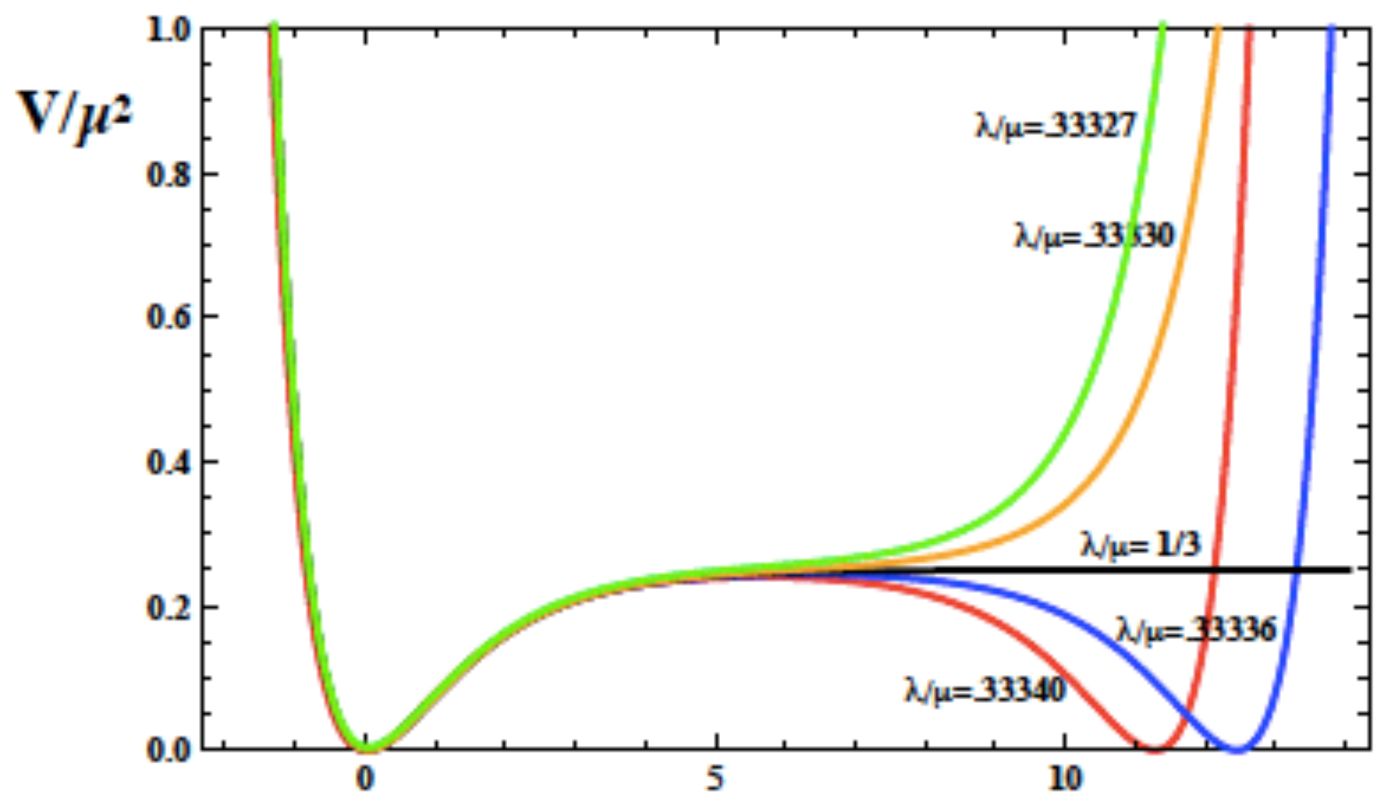
$$\mathcal{L}_{eff} = \frac{c}{(c - |\phi|^2/3)^2}|\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}$$

$$\hat{V} \equiv \left|\frac{\partial W}{\partial \phi}\right|^2$$

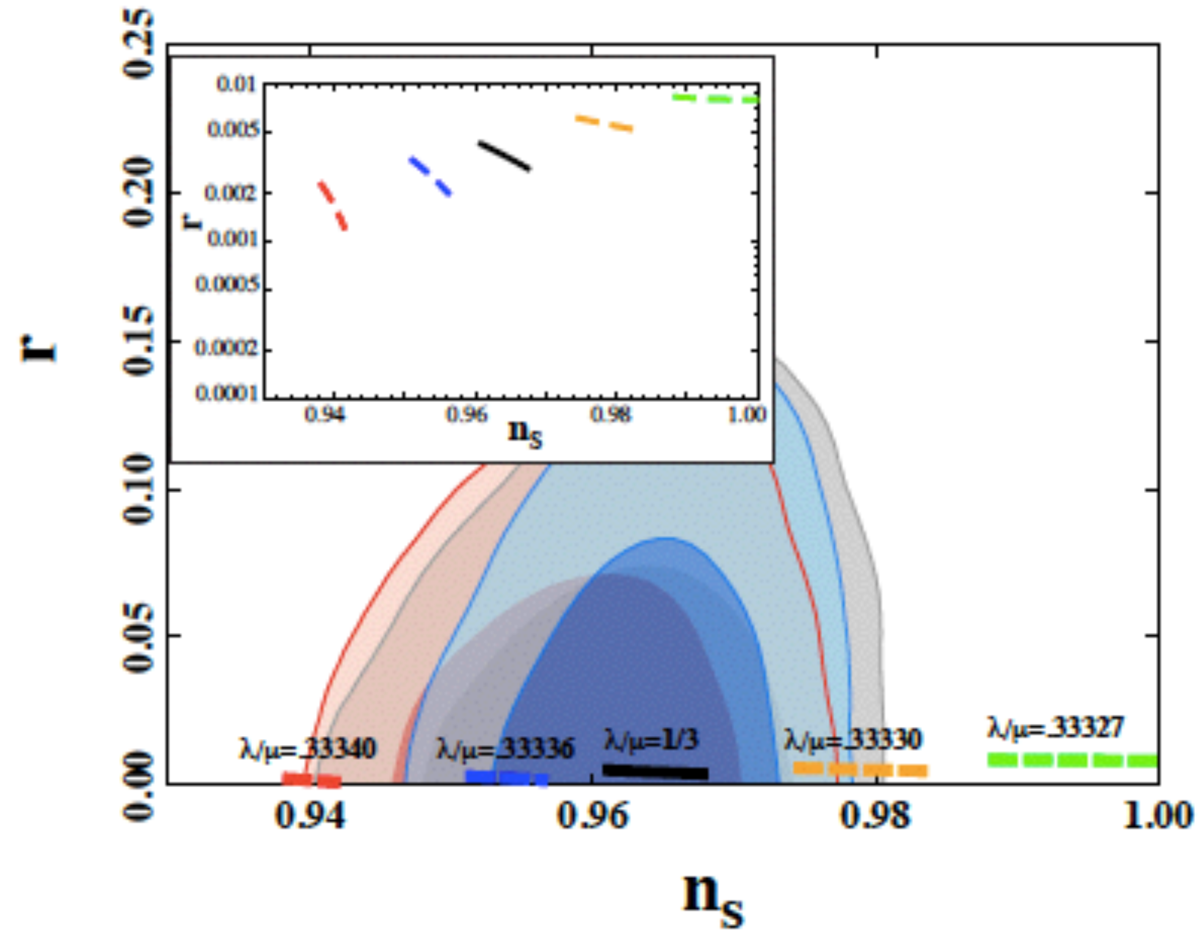
$$\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

Ellis, Nanoupolos, Olive 1305.1247

$$V = \frac{\mu^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2$$



Ellis, Nanoupolos, Olive 1305.1247



Ellis, Nanoupolos, Olive 1305.1247

SUGRA model for power law Starobinsky potential

$$K = -3\ln\left[T + T^* - \frac{(\phi + \phi^*)^n}{12}\right]$$

$$W(\Phi) = \frac{\mu}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

$$V = \frac{144\mu^2}{n(n-1)}\exp\left[\frac{2\chi}{\sqrt{6}}\left(\frac{3\sqrt{2}(2-n)}{\sqrt{n}}\right)\right] \times\left[1 - \exp\left(\frac{-2\chi}{\sqrt{3n}}\right) - \frac{9c(n^2-n-2)}{n}\exp\left(\frac{-2n\chi}{\sqrt{3n}}\right)\right]^2$$

Girish Chakravarti, SM 1405.1301

Starobinsky model and SUGRA parameters

$$\beta = \frac{2\sqrt{n} + 3\sqrt{2}(2-n)}{\sqrt{n} + 3\sqrt{2}(2-n)}$$

$$M^2 = \frac{\beta^\beta}{6}\left[\frac{96\mu^2}{n(n-1)(\beta-1)}\right]^{\beta-1}$$

β	M	n	$\mu = \frac{\lambda}{2}$	$\alpha_s = \frac{dn_s}{d\ln k}$
1.818	1.8×10^{-4}	1.927	$\pm 2.30 \times 10^{-6}$	-5.30×10^{-6}
1.864	1.8×10^{-4}	1.948	$\pm 4.97 \times 10^{-6}$	-2.76×10^{-3}
2	1.1×10^{-5}	2	$\pm 1.16 \times 10^{-6}$	-5.23×10^{-4}

Embedding Higgs Inflation in a SUGRA model

- Not possible is MSSM (Einhorn and Jones ,0912.2718)
- Possible in NMSSM with power law additions in the Kahler potential (Lee 1005.2735)

$$n_s \simeq 0.968, \quad r \simeq 3.0 \times 10^{-3}$$

Affleck Dine leptogensis with $L.H_u$ inflation.

Next time in Kolkota

Thank You