Variable gravity: A suitable framework for quintessential inflation

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Plan:

- Quintessential inflation
- Variable gravity frame work
- Inflation
- Late time acceleration
- Summary

What is it?

A unified description of inflation and late time cosmic acceleration:

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A unified description of inflation and late time cosmic acceleration:

- Scalar field behaves like inflaton at early epochs $\Longrightarrow$ Inflation.
- Same scalar field behaves like quintessence field at the late times $\Longrightarrow$ Late time cosmic acceleration.

Slow roll conditions:

$$\epsilon = \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}\right)^2 \ll 1, \qquad \eta = \frac{M_{\mathrm{Pl}}^2}{V}\frac{\mathrm{d}^2 V}{\mathrm{d}\phi^2} \ll 1$$

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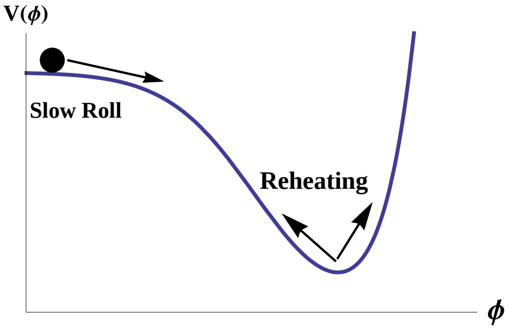

Figure : Schematic diagram of a potential needed for inflation

Scalar Field in Flat FLRW Background

A slowly rolling scalar field $\Longrightarrow$ QUINTESSENCE $\Longrightarrow$ Energy density varies with time.

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$\Longrightarrow$ $w_\phi \approx 1$ when $\dot{\phi}^2 \gg 2V$. $\Longrightarrow$ $w_\phi \approx -1$ when $\dot{\phi}^2 \ll 2V$.

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- $\rho_\phi = \rho_{\phi 0} \mathrm{e}^{-3\int (1+\mathrm{w}_\phi)\mathrm{da}/\mathrm{a}}$.

  $\Longrightarrow \rho_\phi \sim a^{-6}$ when $w_\phi = 1 \Longrightarrow$ Kinetic energy dominated regime.
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- Two kind of behavior $\Longrightarrow$ Tracker and Thawing.

Scalar field tracks the background during the radiation and matter era and take over matter at recent past $\Longrightarrow$ Late time solution is an attractor for a wide range of initial conditions

P. J. Steinhardt, L. -M. Wang and I. Zlatev, PRD **59**, 123504 (1999)

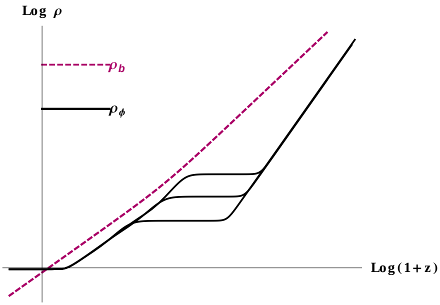


Figure : Schematic diagram of tracker behavior

- All paths are converging to a common evolutionary track.
- Not all potential can give rise to tracker behavior $\Longrightarrow$ A limitation.
- $\Gamma > 1$ where $\Gamma = \dfrac{V''V}{V'^2}$.
- Runaway potentials like $\dfrac{1}{\phi^n}$ or exponential potential $\mathrm{e}^{\mathrm{M}/\phi}$ can give rise to tracker solution.
- Field's EoS goes towards -1.

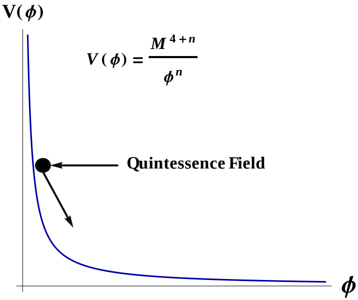


Figure : Schematic diagram of inverse power law potential, a runaway potential.

- Steep nature of the potential is needed. Hubble friction $3H\dot{\phi}$ increases since $\dot{\phi}$ increases while rolling down the steep region of the potential $\Longrightarrow$ Field's evolution freezes and energy density becomes comparable with the background energy density $\Longrightarrow$ Field starts evolving and follow the background up to recent past.

- Some potentials which reduce to inverse power law and exponential nature asymptotically can also give tracker solution $\Longrightarrow$ Example: Double exponential or cos hyperbolic potential.

Quintessential Inflation

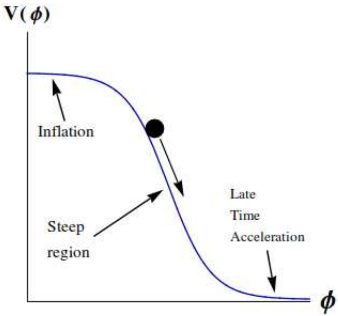
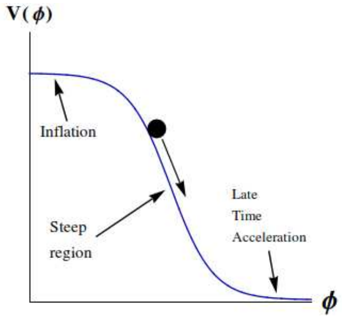


Figure : Schematic diagram of an effective potential which can give quintessential inflation.



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Problems

1. Find out a suitable potential.
2. Scalar field survives until late times $\Longrightarrow$ potential is typically of a run-away type $\Longrightarrow$ One requires an alternative mechanism of reheating *e.g.*, instant preheating.
3. Long kinetic regime enhances the amplitude of relic gravitational waves $\Longrightarrow$ violates nucleosynthesis constraints at the commencement of radiative regime.

CASE I: Brane Inflation

- Invoke Randall-Sundrum (RS) braneworld corrections to facilitate inflation with steep potential at early epochs.

- As the field rolls down to low energy regime, the braneworld corrections disappear, giving rise to a graceful exit from inflation and thereafter the scalar field has the required behavior.

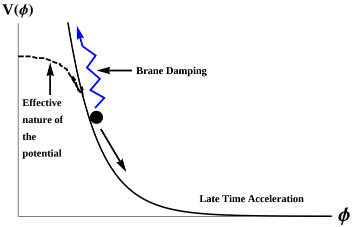

Figure : Schematic diagram of an effective potential of quintessential inflation with brane damping term.

Variable Gravity Framework (CASE II)

C. Wetterich, PRD **89**, 024005 (2014)

In Einstien frame:

- There is a coupling between massive neutrinos and non-canonical scalar field.
- This coupling is necessary to get late time acceleration.
- Dark energy scale is related with neutrino mass scale.

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Let us consider the following action,

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$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \boxed{\frac{k^2(\phi)}{2}\partial^\mu\phi\partial_\mu\phi} - V(\phi)\right] + \mathcal{S}_m + \mathcal{S}_r$$
$$\boxed{+\mathcal{S}_\nu(\mathcal{C}^2 g_{\alpha\beta};\Psi_\nu)},$$

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$$k^2(\phi) = \left(\frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2}\right)\frac{1}{1+\beta^2 \mathrm{e}^{\alpha\phi/M_{\rm Pl}}} + 1\,,$$
$$\mathcal{C}^2(\phi) = Ae^{2\tilde{\gamma}\alpha\phi/M_{\rm Pl}}, \qquad V(\phi) = M_{\rm Pl}^4 \mathrm{e}^{-\alpha\phi/M_{\rm Pl}},$$

where $\beta \Longrightarrow$ can be fixed from inflation.

Let us consider the transformation,

$$\sigma = \mathbb{k}(\phi)\,,$$
$$k^2(\phi) = \left(\frac{\partial \mathbb{k}}{\partial \phi}\right)^2$$

The action becomes,

$$\mathcal{S}_E = \int d^4x\sqrt{g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - V(\mathbb{k}^{-1}(\sigma))\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu(\mathcal{C}^2 g_{\alpha\beta};\Psi_\nu)\,.$$

$\Longrightarrow$ σ is the canonical scalar field.

Asymptotic behavior of the canonical field

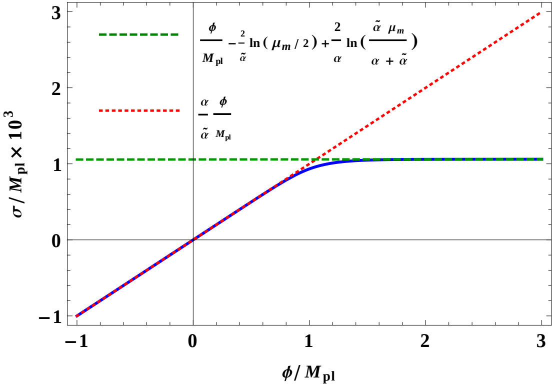

Figure : Blue (solid) line represents the behavior of σ field. Parameter values are $\alpha = 10$, $\tilde{\alpha} = 0.01$ and $\beta = 0.01$.

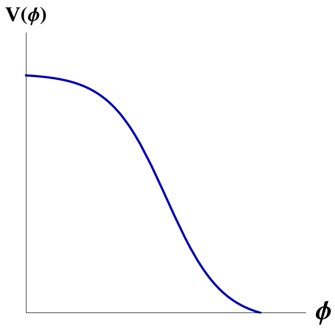

Figure : Schematic diagram of the potential for variable gravity

Effect of Neutrinos Coupled with Scalar Field

- Non-minimal coupling modifies the EoM of scalar field $\Longrightarrow \tilde{\gamma}\alpha\left(\rho_\nu - 3p_\nu\right)$.
- $p_\nu = \frac{1}{3}\rho_\nu \Longrightarrow$ Neutrinos behave like radiation $\Longrightarrow$ No modification.
- Modification comes into play only when neutrinos become nonrelativistic $\Longrightarrow p_\nu = 0$ $\Longrightarrow$ Effective potential forms $\Longrightarrow V_{\text{eff}} = V(\sigma) + f(\sigma)$ where $f(\sigma)$ is a growing function.

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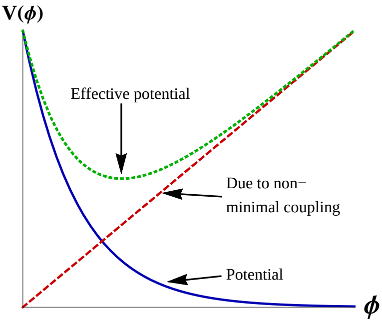

Figure : Schematic diagram of an effective potential.

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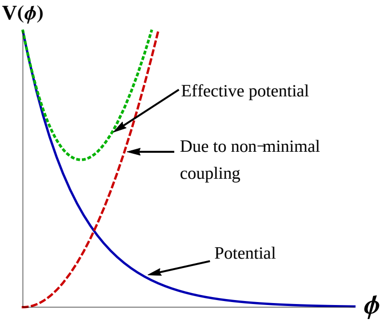

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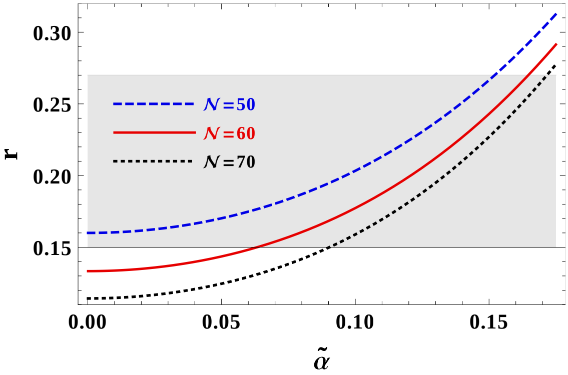

Figure : The tensor-to-scalar ratio (r) versus the model parameter $\tilde{\alpha}$, for different e-foldings $\mathcal{N}$. Blue (dashed), red (solid) and black (dotted) lines correspond to $\mathcal{N} = 50$, 60 and 70 respectively. The shaded region marks the BICEP2 constraint on r at 1σ confidence level, that is $r = 0.2^{+0.07}_{-0.05}$.

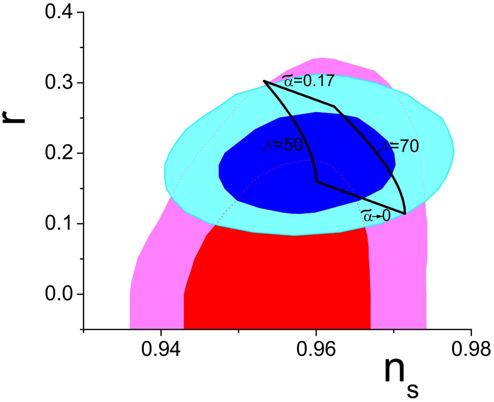

Figure : 1σ (blue) and 2σ (cyan) contours for *Planck* + *WP* + *highL* + *BICEP*2 data, and 1σ (red) and 2σ (pink) contours for *Planck* + *WP* + *highL* data, on the $n_s - r$ plane. The black solid curves bound the region predicted in our model for efoldings between $\mathcal{N} = 50$ and $\mathcal{N} = 70$ and for the parameter $\tilde{\alpha}$ between 0^+ and 0.175. The lower line ($\tilde{\alpha} \to 0$) is for $\mathcal{N}$ from 50 to 70, the left curve ($\mathcal{N} = 50$) is for $\tilde{\alpha}$ from 0^+ to 0.17, the right curve ($\mathcal{N} = 70$) is for $\tilde{\alpha}$ from 0^+ to 0.17, and the upper line ($\tilde{\alpha} = 0.17$) is for $\mathcal{N}$ from 50 to 70.

Scale of Inflation

Energy scale of inflation $\Longrightarrow V_{\rm in}^{1/4}$ where,

$$V_{\rm in} = \frac{2.5 \times 10^{-7} \tilde{\alpha}^2 M_{\rm Pl}^4}{\left(1 - e^{-\tilde{\alpha}^2 \mathcal{N}}\right)} \,.$$

For $\mathcal{N} = 60$ we can have $r = 0.2$ for $\tilde{\alpha} = 0.12$

$\Longrightarrow V_{\rm in}^{1/4} = 2.46 \times 10^{16}\ {\rm GeV}$.

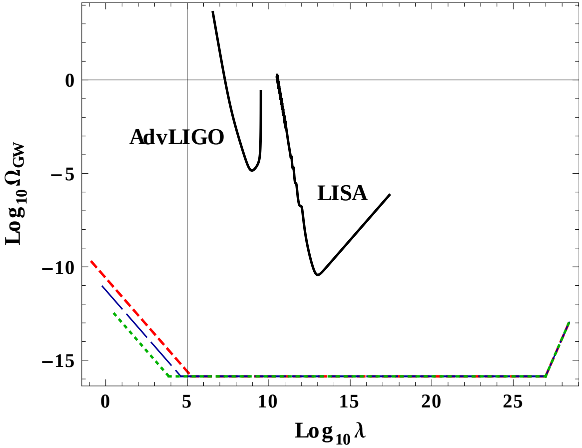

Figure : Spectral energy density of relic gravity wave background for different reheating temperatures. Red (dashed), Blue (long dashed) and Green (dotted) lines for $g = 5 \times 10^{-4}$, 0.01 and 0.3 respectively. α is taken to be 10. Also we have considered $\mathcal{N} = 70$ for this plot but it is checked that the behavior does not change significantly for the variation of $\mathcal{N}$ from 50 to 70. Black solid curves represent the expected sensitivity curves of Advanced LIGO and LISA.

Evading Lyth Bound

MWH, R. Myrzakulov, M. Sami and E. N. Saridakis, arXiv:1405.7491.

For variable gravity scenario,

$$\Longrightarrow \delta\phi \gtrsim \left(\mathcal{N} M_{\rm Pl}\sqrt{\frac{r_\star}{8}}\right)\boxed{\frac{\tilde{\alpha}}{\alpha}}.$$

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For canonical scalar field:

For $r_\star \gtrsim 0.1$ and $\mathcal{N} = 50$

$\Longrightarrow$ $\boxed{\delta\phi \gtrsim 5 M_{\rm Pl}}$ $\Longrightarrow$ Super-Planckian field.

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For variable gravity scenario:

From early dark energy constraint $\Longrightarrow \alpha \gtrsim 20$.

For $\mathcal{N} = 60$, $r_\star = 0.15$, $\alpha = 20$ and $\tilde{\alpha} = 0.06 \Longrightarrow \delta\phi \geq 0.0246 M_{\rm Pl}$

$\Longrightarrow$ Sub Planckian

From the ratio $V_{\rm end}/V_{\rm in}$ it can be shown that $\delta\phi \approx 5/\alpha \Longrightarrow$ For $\alpha = 20$

$\Longrightarrow \delta\phi = 0.25 M_{\rm Pl} \Longrightarrow$ respects the Lyth bound.

Late Time Acceleration

Effective potential

$$V_{\rm eff}(\phi) = V(\phi) + \hat{\rho}_\nu e^{(\tilde{\gamma}\alpha\phi/M_{\rm Pl})}\,.$$

where $\hat{\rho}_\nu = \rho_\nu e^{-(\tilde{\gamma}\alpha\phi/M_{\rm Pl})}$ is independent of ϕ.

Effective potential at the minimum,

$$V_{\rm eff,min} = (1+\tilde{\gamma})\rho_\nu(\phi_{\rm min})\,.$$

$\Longrightarrow \rho_{\rm DE} \approx V_{\rm eff,min} \sim \rho_\nu \Longrightarrow$ Sets dark energy scale through neutrino mass scale.

Late Time Cosmology

$$x = \frac{\dot{\sigma}}{\sqrt{6} H M_{\mathrm{Pl}}}$$

$$y = \frac{\sqrt{V}}{\sqrt{3} H M_{\mathrm{Pl}}}$$

$$\Omega_m = \frac{\rho_m}{3 H^2 M_{\mathrm{Pl}}^2}$$

$$\Omega_r = \frac{\rho_r}{3 H^2 M_{\mathrm{Pl}}^2}$$

$$\Omega_\nu = \frac{\rho_\nu}{3 H^2 M_{\mathrm{Pl}}^2}$$

$$\Omega_\sigma = \frac{\rho_\sigma}{3 H^2 M_{\mathrm{Pl}}^2}$$

$$x=\frac{\dot{\sigma}}{\sqrt{6}HM_{\rm Pl}}=\frac{\sqrt{3}}{\sqrt{2}\alpha(1+\tilde{\gamma})},$$

$$y=\frac{\sqrt{V}}{\sqrt{3}HM_{\rm Pl}}=\pm\frac{\sqrt{3+2\alpha^2\tilde{\gamma}(1+\tilde{\gamma})}}{\sqrt{2}\alpha(1+\tilde{\gamma})},$$

$$\Omega_m=\frac{\rho_m}{3H^2M_{\rm Pl}^2}=0,$$

$$\Omega_r=\frac{\rho_r}{3H^2M_{\rm Pl}^2}=0,$$

$$\Omega_\nu=\frac{\rho_\nu}{3H^2M_{\rm Pl}^2}=\frac{-3+\alpha^2(1+\tilde{\gamma})}{\alpha^2(1+\tilde{\gamma})^2},$$

$$\Omega_\sigma=\frac{\rho_\sigma}{3H^2M_{\rm Pl}^2}=\frac{\tilde{\gamma}}{1+\tilde{\gamma}}+\frac{3}{\alpha^2(1+\tilde{\gamma})^2}.$$

$$x = \frac{\dot{\sigma}}{\sqrt{6} H M_{\rm Pl}} = \frac{\sqrt{3}}{\sqrt{2}\alpha(1+\tilde{\gamma})},$$

$$y = \frac{\sqrt{V}}{\sqrt{3} H M_{\rm Pl}} = \pm \frac{\sqrt{3 + 2\alpha^2\tilde{\gamma}(1+\tilde{\gamma})}}{\sqrt{2}\alpha(1+\tilde{\gamma})},$$

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$$\Omega_\sigma = \frac{\rho_\sigma}{3H^2 M_{\rm Pl}^2} = \frac{\tilde{\gamma}}{1+\tilde{\gamma}} + \frac{3}{\alpha^2(1+\tilde{\gamma})^2}.$$

$$w_{\rm eff} = -\frac{\tilde{\gamma}}{1+\tilde{\gamma}}$$

$$w_\sigma = -\frac{\alpha^2\tilde{\gamma}(1+\tilde{\gamma})}{3 + \alpha^2\tilde{\gamma}(1+\tilde{\gamma})}$$

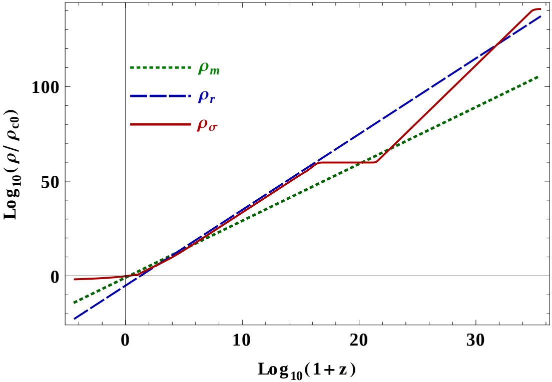

Figure : Evolutions of different energy densities (ρ). ρ_r (Blue dashed), ρ_m (Green dot-dashed), ρ_σ (Red solid (upper panel)), represent the densities of radiation, matter, scalar field and σ . ρ_{c0} is the critical energy density of universe at present. To plot this figure we have considered $\alpha = 10$, $\tilde{\gamma} = 30$ and $z_{\mathrm{dur}} = 10$.

Summary

- A unified model of inflation and dark energy is investigated in variable gravity framework.
- Model gives the tensor-to-scalar ratio and thereby the inflation scale consistent with the BICEP2.
- Blue spectrum in relic gravity wave spectrum is present due to the presence of kinetic regime.
- Lyth bound can be evaded due to the constraint on post-inflationary dynamics.
- Model has a stable late time attractor which can cause late time acceleration for large $\tilde{\gamma}$.

THANK YOU