Traversable Wormholes in AdS

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I. Introduction

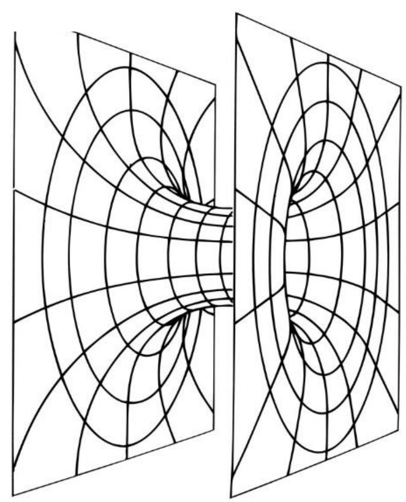
Wormhole is a vacuum solution of Einstein's equation

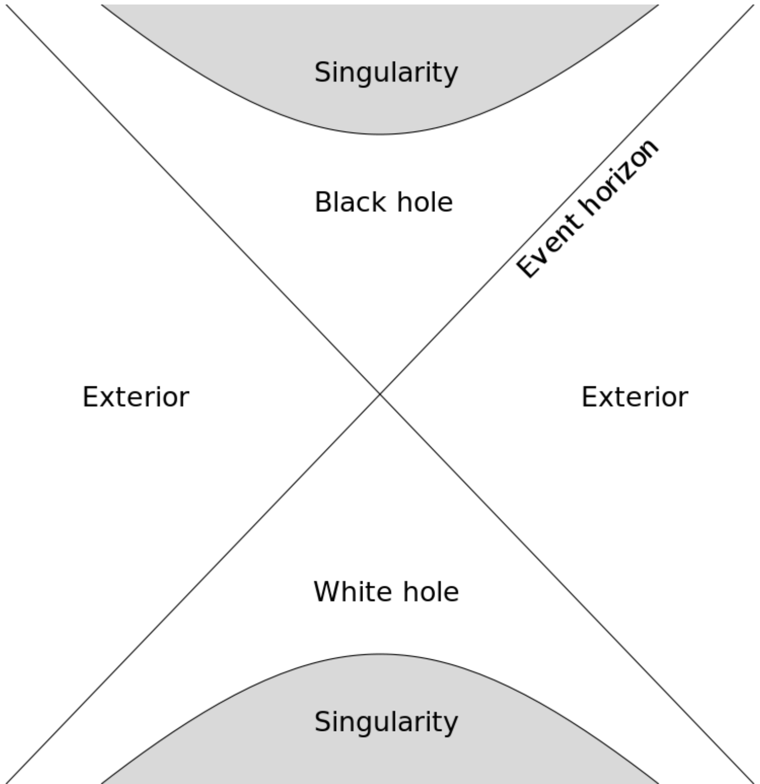
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

On a time-slice, two interiors of black holes are connected via a tube or "throat" called Einstein-Rosen bridge.

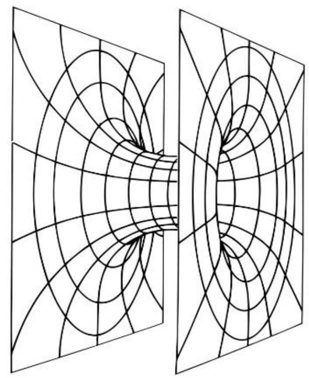
The Maximally Extended Schwarzschild Solution in Kruskal coordinates is:

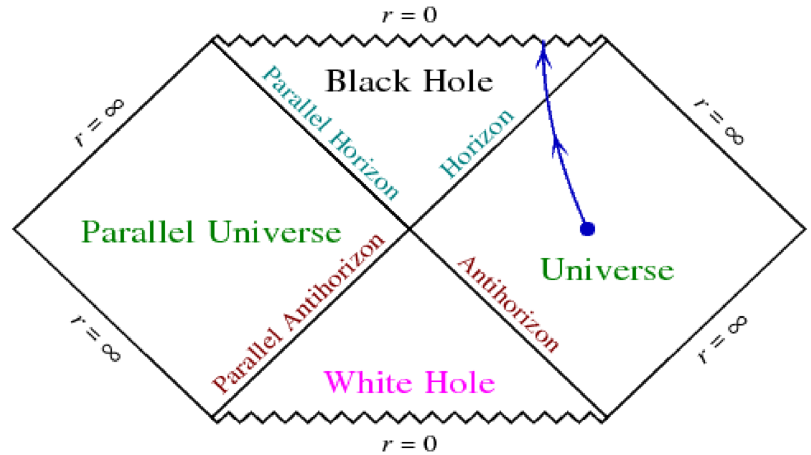
$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dU dV + r^2 d\Omega_2^2$$





The Penrose diagram of the Maximally Extended Schwarzschild Solution is



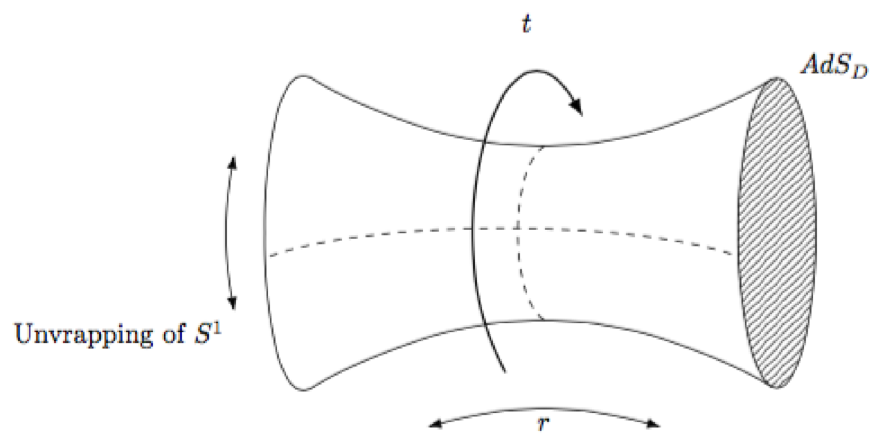


Anti-de Sitter space is a solution of Einstein's equation with negative cosmological constantan

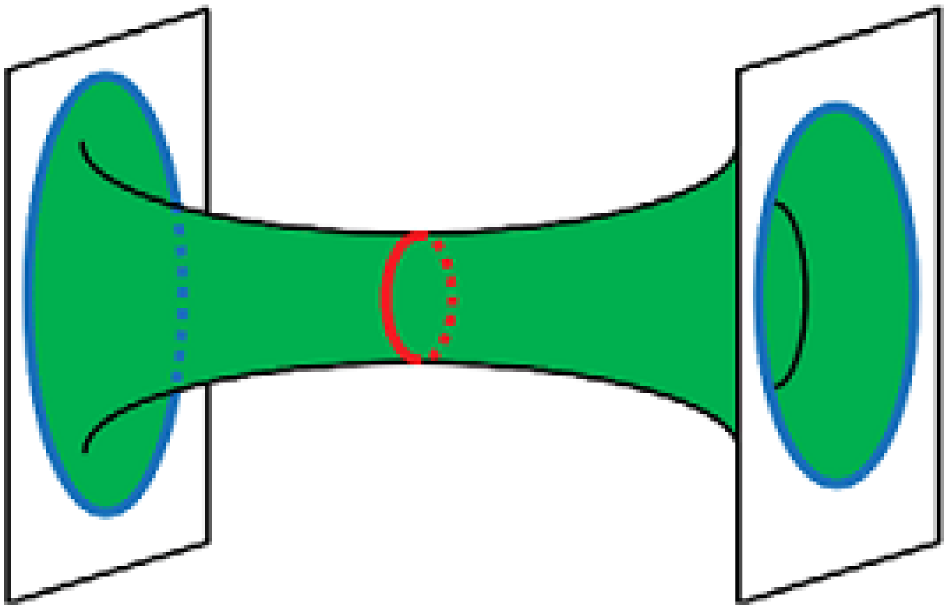
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

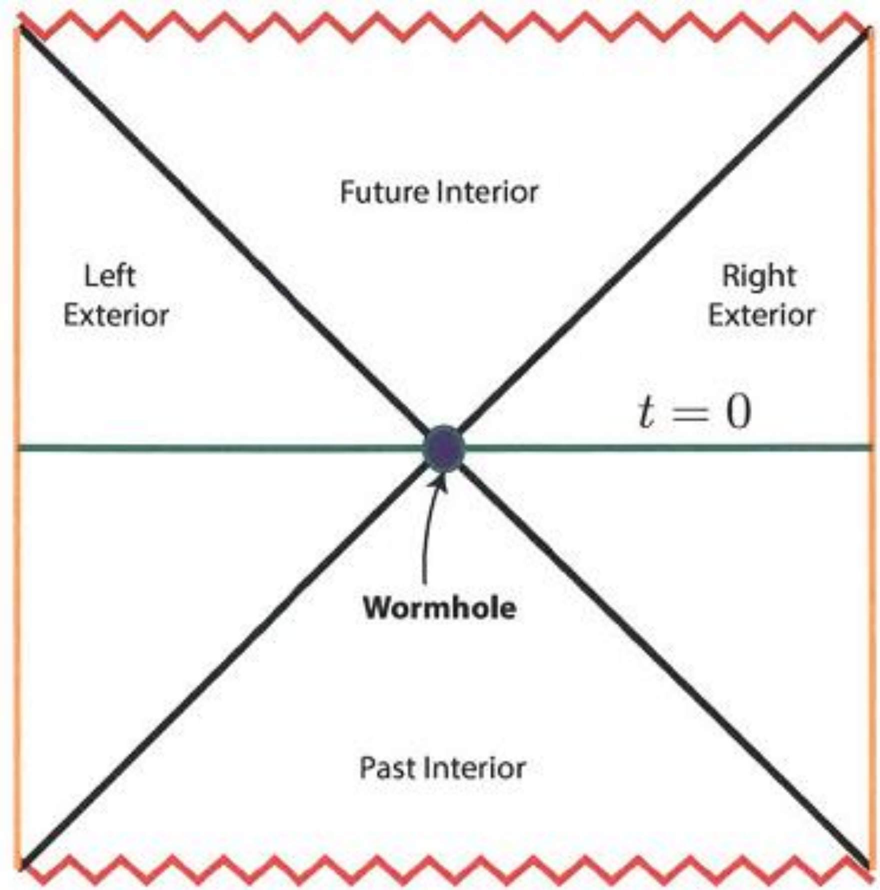
$$ds^2 = (1 + r^2/a^2)\, dT^2 - (1 + r^2/a^2)^{-1}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$3/\Lambda = -a^2$$

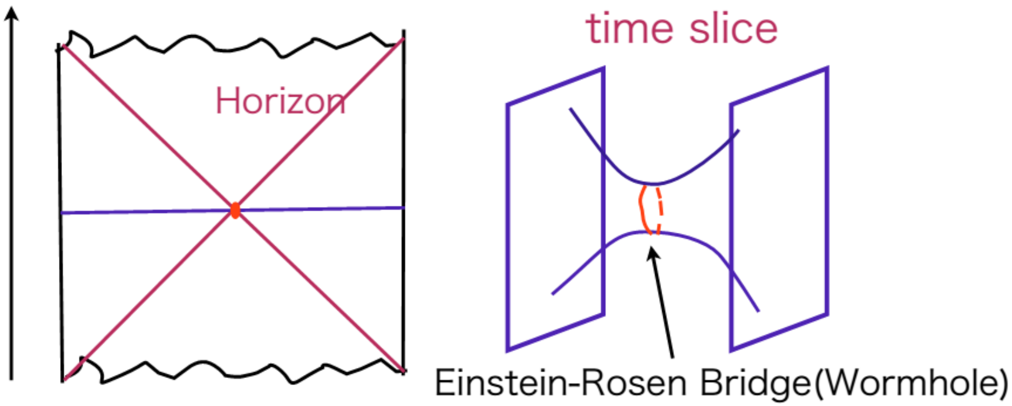


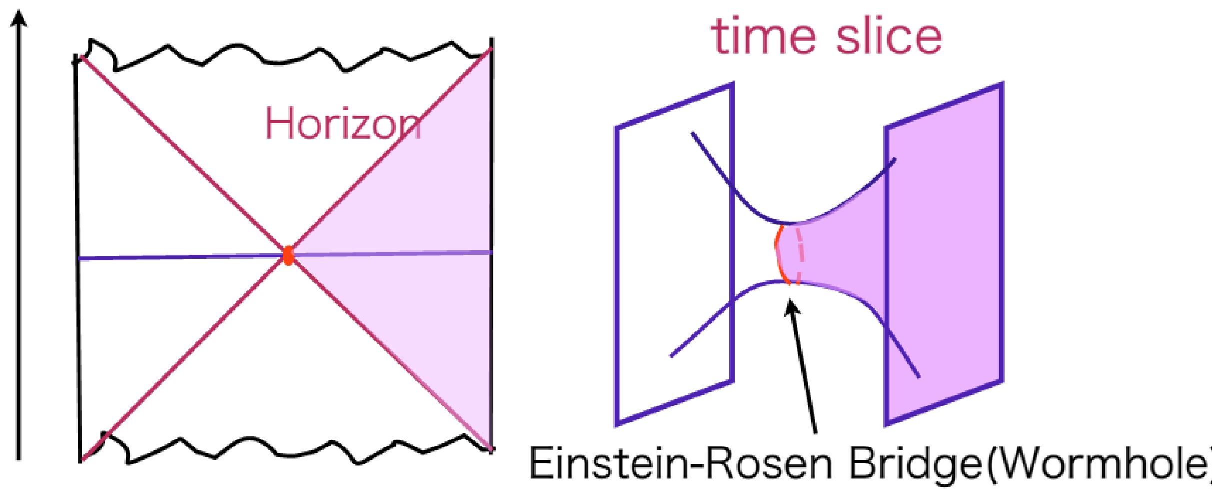
The Penrose diagram of the eternal Anti-de Sitter space in Kruskal coordinates is:

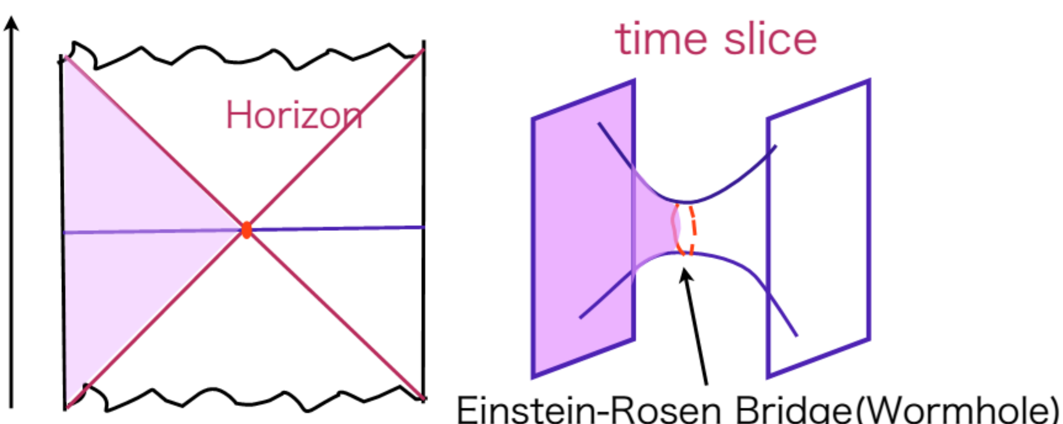




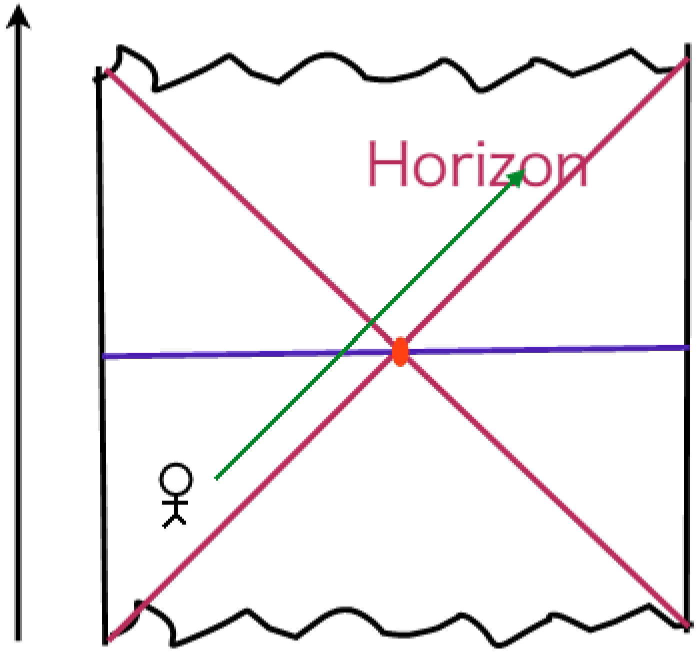
Borrowed from T. Numasawa

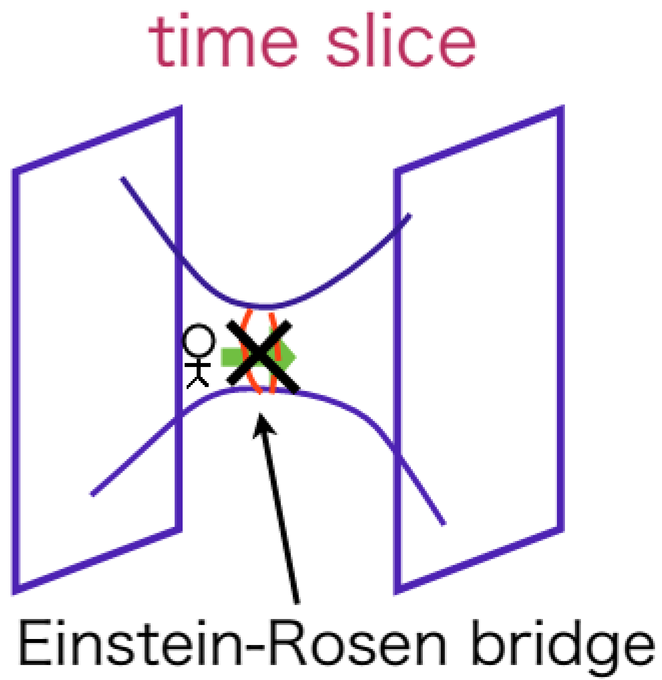






Since the Einstein-Rosen bridge is instable an we can not go to the other side.





Traversable wormhole: usually require an exotic matter which violates the Average Null Energy Condition (ANEC).

The ANEC states that along a given infinite null geodesic, the integral of the null component of the stress energy is nonnegative:

$$\int_{-\infty}^{+\infty} T_{\mu\nu} k^{\mu} k^{\nu} d\lambda \geq 0$$

$T_{\mu\nu}$ -is the expectation value of the renormalized stress-energy tensor

k^{μ} - vector pointing along the direction of the null geodesic

λ -is an affine parameter along the null geodesic

- ANEC in GR ensures that gravity is an attractive force
- In quantum level ANEC can be violated.
- For all traversable wormhole it is necessary the violation of ANEC. Morris, Thorne, Yurtsever 88

II. Traversable wormhole via double trace deformation

Gao-Jefferis-Wall, 16

The Thermofield Double State (TFD)

The thermofield double state is an entangled state in the tensor product of two identical "left" and "right" CFTs:

$$|\Psi_{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta E_n}{2}} |E_n\rangle_L \otimes |E_n\rangle_R$$

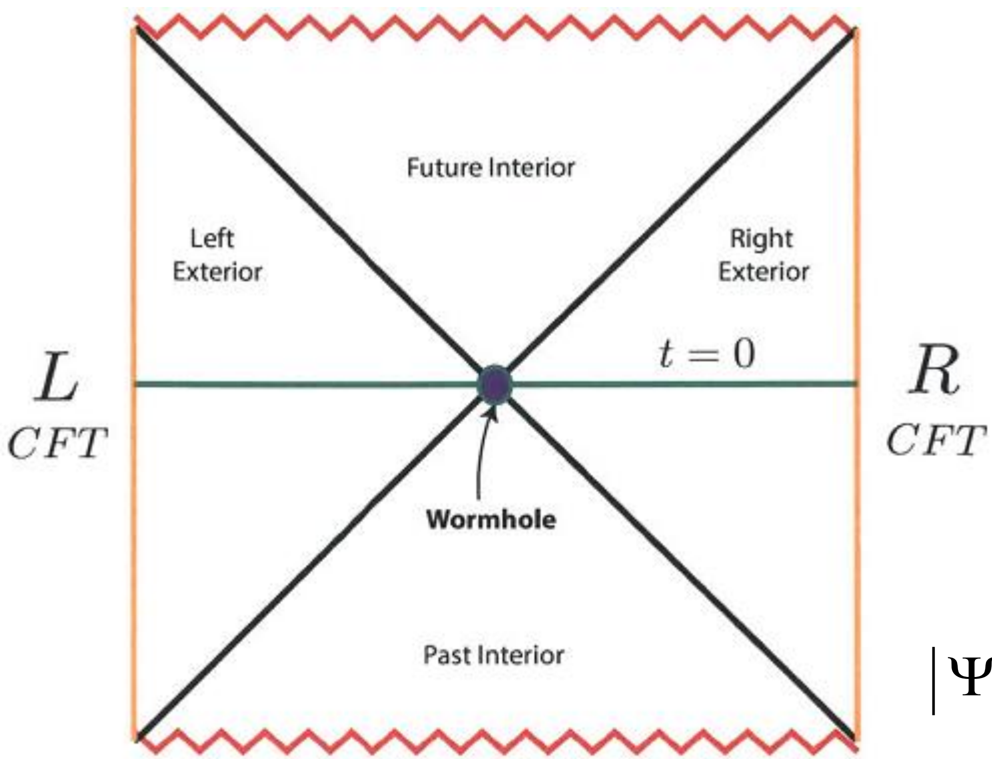
where β is the inverse temperature

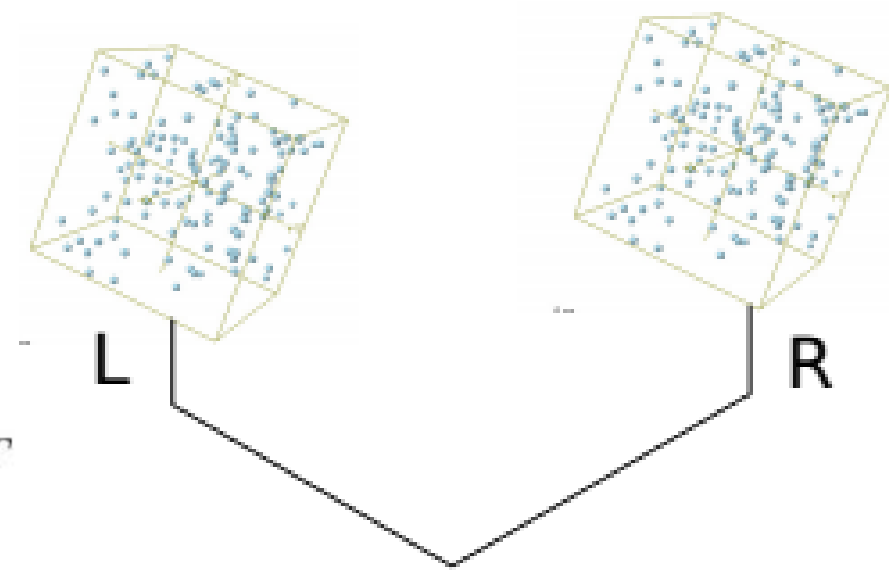
sum is over energy eigenstates.

where Z is the partition function

Assuming the AdS/CFT correspondence TFD is holographically dual to an eternal two-sided AdS black hole.

Start with the eternal AdS black hole, dual to the thermofield double state in the decoupled product of two





$$|\Psi_{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta E_n}{2}} |E_n\rangle_L \otimes |E_n\rangle_R$$

Israel, Maldacena

Gao, Jafferis and Wall showed that a wormhole in the eternal AdS black hole scenario can be made traversable by turning on a coupling between the left and right boundaries of form

$$\delta H(t_1) = -\int d^{d-1}x_1\, h(t_1, x_1)\mathcal{O}_R(t_1, x_1)\mathcal{O}_L(-t_1, x_1)$$

$$h(t_1, x_1) = \begin{cases} h\kappa^{2-2\Delta}, & t_0 \leqslant t_1 \leqslant t_f \\ 0 \quad , & \text{otherwise.} \end{cases}$$ is a small deformation.

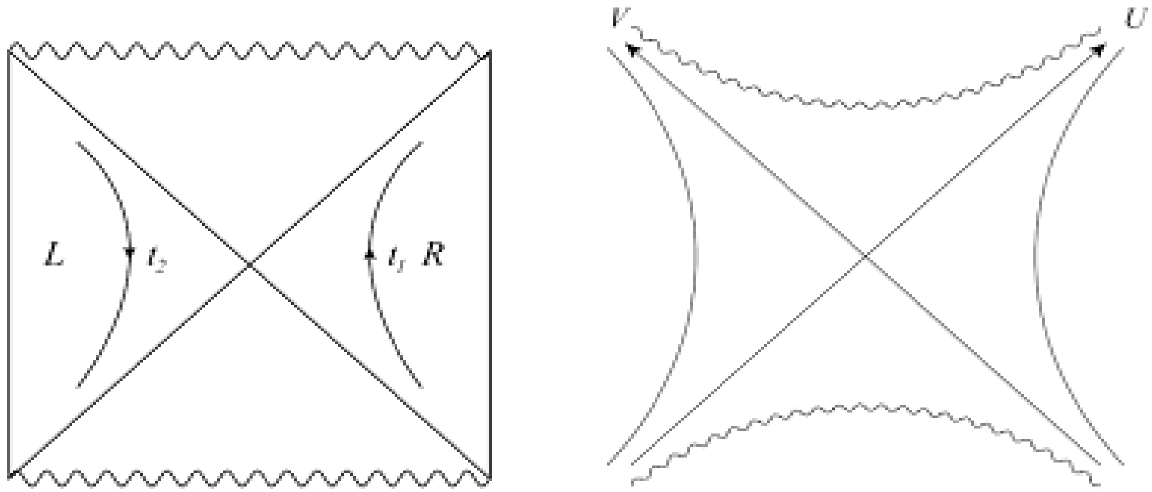
$\mathcal{O}_{L/R}$ is a scalar operator of dimension $\Delta = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$ living in the Left/Right CFT, dual to a bulk scalar field with mass m.

BTZ black hole

Bañados, Teitelboim Zanalli 92

$$ds^2 = -\frac{r^2 - r_h^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2 - r_h^2}dr^2 + r^2 d\phi^2, \quad \phi \sim \phi + 2\pi$$

Kruskal coordinates: $e^{2r_h t} = -\frac{U}{V}, \quad \frac{r}{r_h} = \frac{1-UV}{1+UV}$



The bulk stress tensor associated to the scalar field is

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \partial_\rho \Phi \partial_\sigma \Phi - \frac{1}{2} g_{\mu\nu} m^2 \Phi^2.$$

They considered correlation function $\langle \varphi_R^H(t, r, \phi) \varphi_R^H(t', r', \phi') \rangle$

First order in contribution in h

$$G_h = i\int_{t_0}^{t} dt_1 h(t_1) K_\Delta(t'+t_1-i\beta/2)[K_\Delta(t-t_1-i\epsilon)-K_\Delta(t-t_1+i\epsilon)]+(t\leftrightarrow t')$$

Bulk to boundary propagator

$$K_\Delta(r,t,\phi;0,0) = \frac{r_h^\Delta}{2^{\Delta+1}\pi}\left(-\frac{(r^2-r_h^2)^{1/2}}{r_h}\cosh r_h t + \frac{r}{r_h}\cosh r_h\phi\right)^{-\Delta}$$

The stress tensor

$$T_{UU} = \lim_{U'\to U}\partial_U\partial_{U'}G_h(U,U').$$

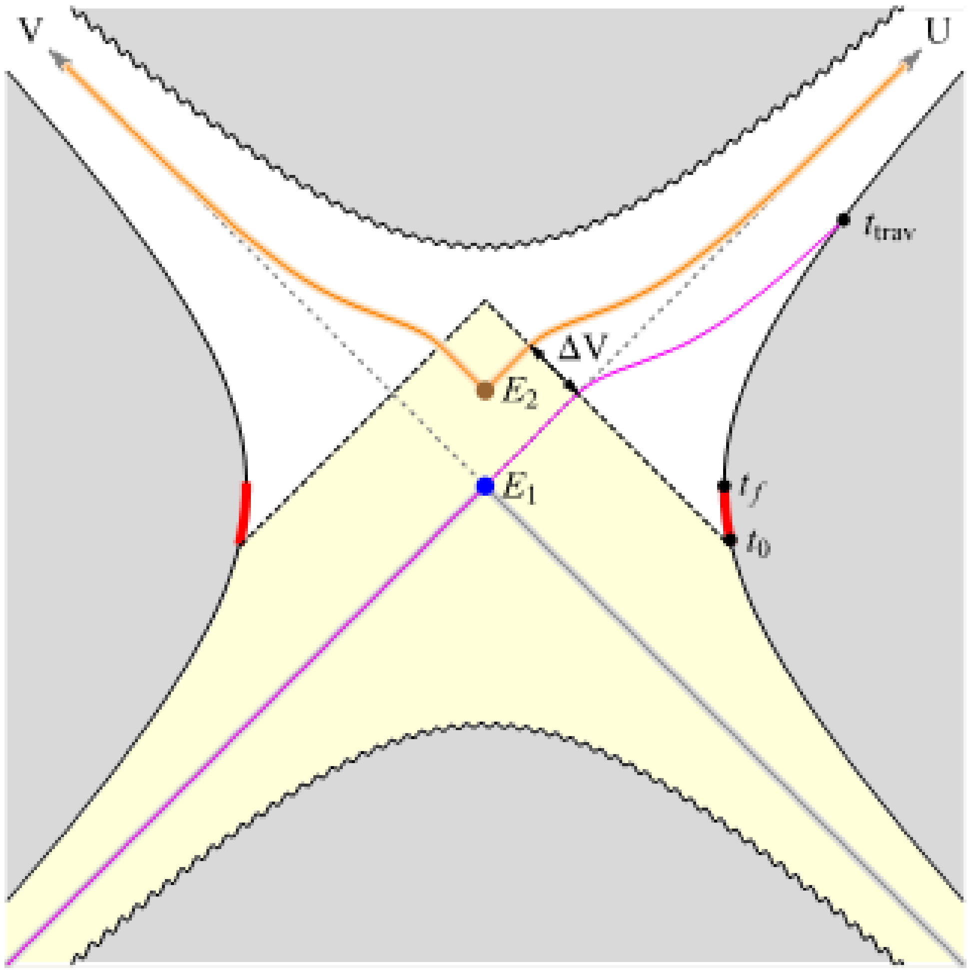
The linearized Einstein equation for the UU component for the fluctuations evaluated at V=0 gives

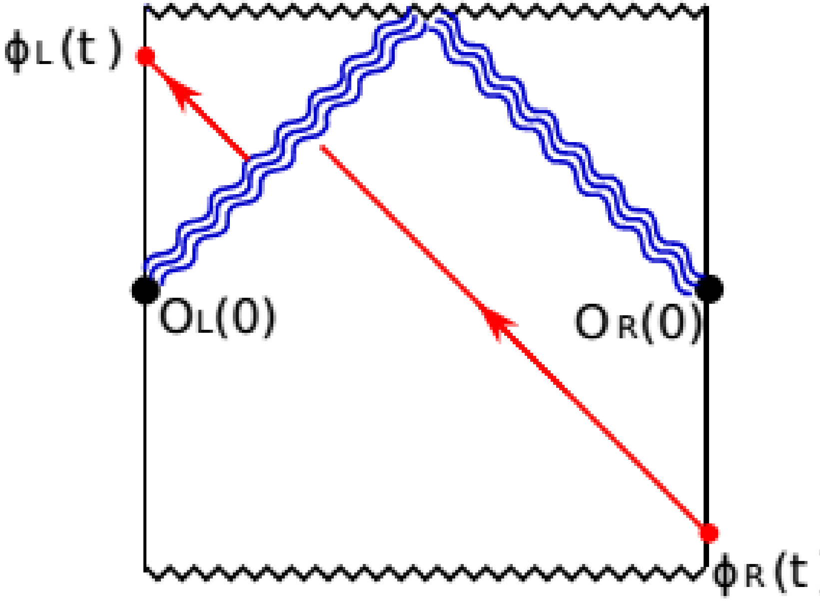
$$\int dU\left(\frac{\kappa}{2r_+}h_{UU} - \frac{r_-\partial_x h_{UU}}{r_+^2} - \frac{\partial_x^2 h_{UU}}{2r_+^2}\right) = 8\pi G_N\int dU T_{UU}$$

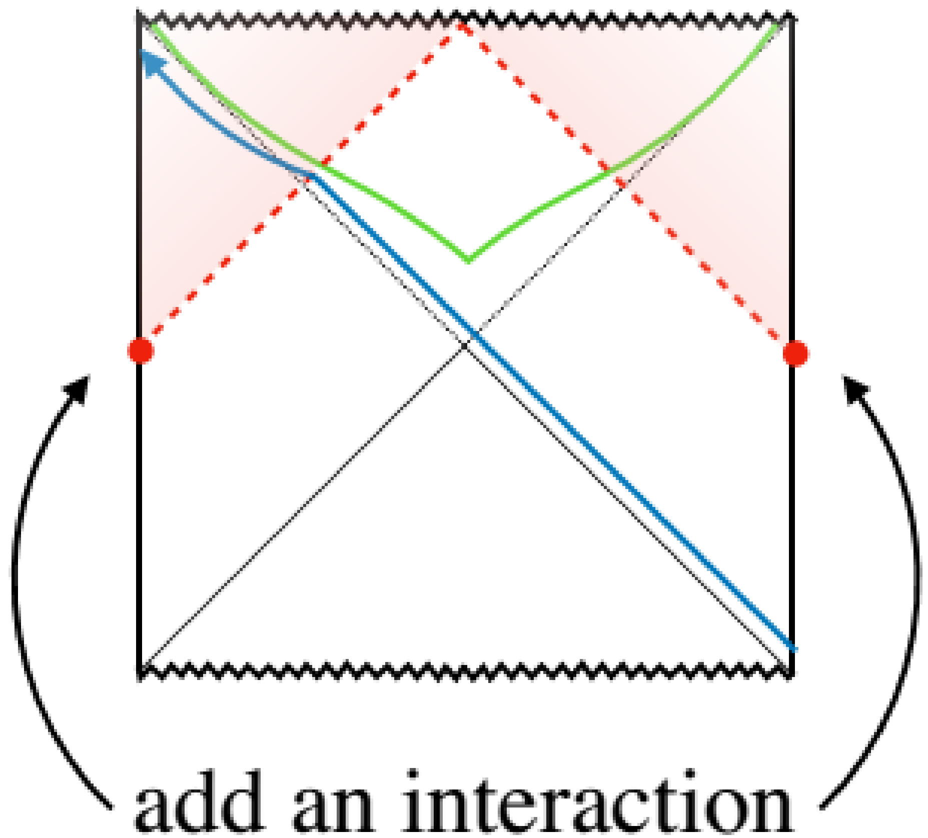
The null geodesics at the horizon caused by the interaction is

$$V(U) \sim \int dU T_{UU}$$

$$\Delta V \sim \frac{h G_N}{R^{D-2}}$$




ϕL(t)
OL(0)
OR(0)
ϕR(t


add an interaction

III. Final Comments

Rotaing E. Caceres, A. Misobuchi

Charged

In collaboration with: E. Caceres, C. Rivera and A. Misobuchi

$$\left|\Psi_{TFD}\right\rangle = \frac{1}{\sqrt{Z(\beta,\mu)}} \sum_n e^{-\frac{\beta(E_n+\mu Q_n)}{2}} \left|E_n, Q_n\right\rangle_L \otimes \left|E_n, -Q_n\right\rangle_R$$


Muchas Gracias !