Newton gauge cosmological perturbations for static spherically symmetric modifications of the de-Sitter metric

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Overview

- Static vs. Comoving Coordinates
- de Sitter
- Cosmological Perturbation Theory (CPT)
- SDS Metric: Static vs. Comoving
- SSS Metric: Static vs. Comoving
- Gauge-Invariant Turn Around Radius
- SSS Metrics

Static vs. Comoving

- Static Coordinates
  - Compute observables
  - Birkhoff’s Theorem
  - Schwarzschild de Sitter Metric

- Comoving Coordinates
  - Cosmological Perturbation Theory (CPT)
  - Modified Gravity Theories
  - McVittie’s Metric

de Sitter Background

- Static Coordinates

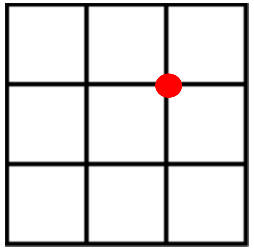
$$ds^2 = (1 - H^2 R^2) dT^2 - (1 - H^2 R^2)^{-1} dR^2 - R^2 d\Omega^2$$

- Coordinate Transformation

$$R = a(t) r \qquad T = t - \frac{1}{2H} \log(r^2 a^2(t) - H^{-2})$$

- Scale factor

$$a(t) = e^{Ht}$$



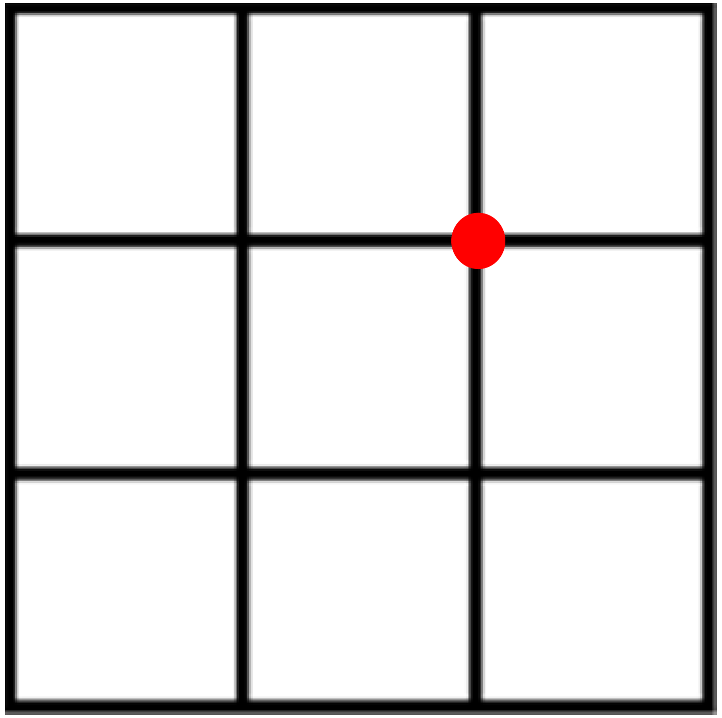
de Sitter Background

- Static Coordinates

$$ds^2 = (1 - H^2R^2)dT^2 - (1 - H^2R^2)^{-1}dR^2 - R^2d\Omega^2$$

- Coordinate Transformation

$$R = a(t)r$$



- Scale factor

$$a(t) = e^{Ht}$$

de Sitter Background

- Static Coordinates

$$ds^2 = (1 - H^2 R^2) dT^2 - (1 - H^2 R^2)^{-1} dR^2 - R^2 d\Omega^2$$

- Coordinate Transformation

$$R = a(t) r \qquad T = t - \frac{1}{2H} \log(r^2 a^2(t) - H^{-2})$$

- Isotropic

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2)$$

Cosmological Perturbations

- Spherical Symmetry: 4 DOF

$$ds^2 = a^2\left[(1+2\psi)d\tau^2 - \left(1 - 2\phi + \frac{2}{3}\mathcal{E}\right)dr^2 - 2\omega' d\tau dr - \left(1 - 2\phi - \frac{1}{3}\mathcal{E}\right)r^2 d\Omega^2\right]$$

- Gauge Transformations

$$\tilde{x}^0 = x^0 + \zeta$$
$$\tilde{x}^i = x^i + \partial^i \beta$$

$$\tilde{\phi} = \phi - \frac{1}{3}\nabla^2\beta + \frac{a_\tau}{a}\zeta,$$
$$\tilde{\omega} = \omega + \zeta + \beta_\tau,$$
$$\tilde{\psi} = \psi - \zeta_\tau - \frac{a_\tau}{a}\zeta,$$
$$\tilde{\chi} = \chi + 2\beta\,,$$

arXiv:astro-ph/0702555

Cosmological Perturbations

- Spherical Symmetry: 4 DOF

$$ds^2 = a^2\left[(1+2\psi)d\tau^2 - \left(1-2\phi+\frac{2}{3}\mathcal{E}\right)dr^2 - 2\omega' d\tau dr - \left(1-2\phi-\frac{1}{3}\mathcal{E}\right)r^2 d\Omega^2\right]$$

- Gauge Transformations

$$\tilde{x}^0 = x^0 + \zeta$$
$$\tilde{x}^i = x^i + \partial^i\beta$$

→

$$\tilde{\phi} = \phi - \frac{1}{3}\nabla^2\beta + \frac{a_\tau}{a}\zeta,$$
$$\tilde{\omega} = \omega + \zeta + \beta_\tau,$$
$$\tilde{\psi} = \psi - \zeta_\tau - \frac{a_\tau}{a}\zeta,$$
$$\tilde{\chi} = \chi + 2\beta,$$

→ $\mathcal{E} = \chi'' - \frac{\chi'}{r}$

arXiv:astro-ph/0702555

Cosmological Perturbations

- Gauge Freedom: 2 DOF
- Newton Gauge

$$\omega_N = \chi_N = 0$$

- Bardeen Potentials: Gauge-Invariant

$$\Psi_B = \psi - \frac{1}{a}\left[a\left(\frac{\chi_\tau}{2} - \omega\right)\right]_\tau$$

$$\Phi_B = \phi + \frac{1}{6}\nabla^2\chi - \frac{a_\tau}{a}\left(\omega - \frac{\chi_\tau}{2}\right)$$

J. M. Bardeen, Phys. Rev. D22, 1882 (1980).

Schwarzschild-de Sitter

- Spherically Symmetric Ansatz

$$ds^2 = F(T,R)dT^2 - H(T,R)dR^2 - R^2 d\Omega^2$$

- Birkhoff's Theorem

$$ds^2 = \left(1 - \frac{2m}{R} - H^2R^2\right)dT^2 - \left(1 - \frac{2m}{R} - H^2R^2\right)^{-1} dR^2 - R^2 d\Omega^2$$

Schwarzschild-de Sitter

- Coordinate Transformation

$$t = \tilde{t} + \gamma(R) \qquad \frac{d\gamma}{dR} = -\frac{HR^2}{\sqrt{R-m}(1 - \frac{m}{R} - H^2R^2)}$$

arXiv:gr-qc/0411033.

Schwarzschild-de Sitter (Exact)

- Coordinate Transformation

$$t = \tilde{t} + \gamma(R) \qquad R = e^{H\tilde{t}}r + m + \frac{m^2}{4e^{H\tilde{t}}r}$$

- Isotropic

$$ds^2 = \left(\frac{1 - \frac{m}{2ar}}{1 + \frac{m}{2ar}}\right)^2 d\tilde{t}^2 - a^2 \left(1 + \frac{m}{2ar}\right)^4 \left[dr^2 + r^2 d\Omega^2\right]$$

arXiv:gr-qc/0411033.

Schwarzschild-de Sitter (CPT)

- Static Coordinates

$$ds^2 = \left(1 - \frac{2m}{R} - H^2R^2\right)dT^2 - \left(1 - \frac{2m}{R} - H^2R^2\right)^{-1}dR^2 - R^2d\Omega^2$$

- de Sitter Transformation

$$R = a(t)r \qquad T = t - \frac{1}{2H}\log(r^2a^2(t) - H^{-2})$$

- Conformal Time $d\tau = dt/a(t)$

Schwarzschild-de Sitter (CPT)

- Static Coordinates

$$ds^2 = \left(1 - \frac{2m}{R} - H^2R^2\right)dT^2 - \left(1 - \frac{2m}{R} - H^2R^2\right)^{-1}dR^2 - R^2d\Omega^2$$

- de Sitter Transformation

$$R = a(t)r \qquad\qquad T = t - \frac{1}{2H}\log(r^2a^2(t) - H^{-2})$$

- Sub-horizon Approx. $m \ll R$

Schwarzschild-de Sitter (CPT)

- Comoving-Conformal Coordinates

$$ds^2 = a^2 \left\{ \left[1 - \frac{2m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}\right] d\tau^2 - \left[1 + \frac{2m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}\right] dr^2 - \left[\frac{8Hm}{\left(H^2r^2a^2-1\right)^2}\right] d\tau dr - r^2 d\Omega^2 \right\}$$

- Scalar Perturbations

$$\psi = -\frac{m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}$$

$$\omega' = \frac{4Hm}{\left(H^2r^2a^2-1\right)^2}$$

$$\phi = -\frac{H^2mr^2a^2+m}{3ra\left(H^2r^2a^2-1\right)^2}$$

$$\mathcal{E} = \frac{2\left(H^2mr^2a^2+m\right)}{ra\left(H^2r^2a^2-1\right)^2}$$

Schwarzschild-de Sitter (CPT)

- Comoving-Conformal Coordinates

$$ds^2 = a^2 \left\{ \left[1 - \frac{2m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}\right] d\tau^2 - \left[1 + \frac{2m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}\right] dr^2 \; - \left[\frac{8Hm}{\left(H^2r^2a^2-1\right)^2}\right] d\tau dr - r^2 d\Omega^2 \right\}$$

- Scalar Perturbations

$$\psi = -\frac{m\left(H^2r^2a^2+1\right)}{ra\left(H^2r^2a^2-1\right)^2}$$

$$\omega = \frac{2m\tanh^{-1}(Hra)}{a} - \frac{2Hmr}{H^2r^2a^2-1}$$

$$\phi = -\frac{H^2mr^2a^2+m}{3ra\left(H^2r^2a^2-1\right)^2}$$

$$\chi = \frac{2mr\left(Hra\tanh^{-1}(Hra)-1\right)}{a}$$

Schwarzschild-de Sitter (CPT)

- Bardeen Potentials

$$\Psi_B = \psi - \frac{1}{a}\left[a\left(\frac{\chi_\tau}{2} - \omega\right)\right]_\tau = -\frac{m}{ar}$$

$$\Phi_B = \phi + \frac{1}{6}\nabla^2\chi - \frac{a_\tau}{a}\left(\omega - \frac{\chi_\tau}{2}\right) = -\frac{m}{ar}$$

- Newton Gauge

$$ds^2 = a^2\left[\left(1 - \frac{2m}{ar}\right)d\tau^2 - \left(1 + \frac{2m}{ar}\right)(dr^2 + r^2 d\Omega^2)\right]$$

SSS Metrics

- Static Spherically Symmetric Metrics

$$ds^2 = \left(1 - mh_t(R) - H^2R^2\right) dT^2 - \left(1 - mh_r(R) - H^2R^2\right)^{-1} dR^2 - R^2 d\Omega^2$$

- Comoving-Conformal

$$\begin{aligned} ds^2 = a^2 \Bigg\{ & \frac{H^6r^6a^6 - H^4r^4a^4[mh_r(ra) + 3] - 1 + mh_t(ra)}{\left(H^2r^2a^2 - 1\right)^3} d\tau^2 \\ & + \frac{H^2r^2a^2\left[m^2h_r(ra)^2 + mh_r(ra) - mh_t(ra)\right] + 3H^2r^2a^2}{\left(H^2r^2a^2 - 1\right)^3} d\tau^2 \\ & - \left[1 + \frac{mh_r(ra)\left(H^2r^2a^2 - mh_r(ra) - 1\right) + H^2mr^2a^2\left(H^2r^2a^2 - 1\right)h_t(ra)}{\left(H^2r^2a^2 - 1\right)^3}\right] dr^2 \\ & - \frac{2Hmra\left[h_r(ra)\left(H^2r^2a^2 - mh_r(ra) - 1\right) + \left(H^2r^2a^2 - 1\right)h_t(ra)\right]}{\left(H^2r^2a^2 - 1\right)^3} d\tau dr - r^2 d\Omega^2 \Bigg\} \end{aligned}$$

SSS Metrics

- Static Spherically Symmetric Metrics

$$ds^2 = \left(1 - mh_t(R) - H^2R^2\right) dT^2 - \left(1 - mh_r(R) - H^2R^2\right)^{-1} dR^2 - R^2 d\Omega^2$$

- Subhorizon Scalar Perturbations

$$\psi = \frac{1}{2} m h_t(ra) \qquad \omega = Hma \int r[h_r(ra) + h_t(ra)]\, dr$$

$$\phi = -\frac{1}{6} m h_r(ra)[mh_r(ra) + 1] \qquad \chi = m \int k_1 \int \frac{h_r(k_2 a)}{k_2}\, dk_2\, dk_1$$

SSS Metrics

- Static Spherically Symmetric Metrics

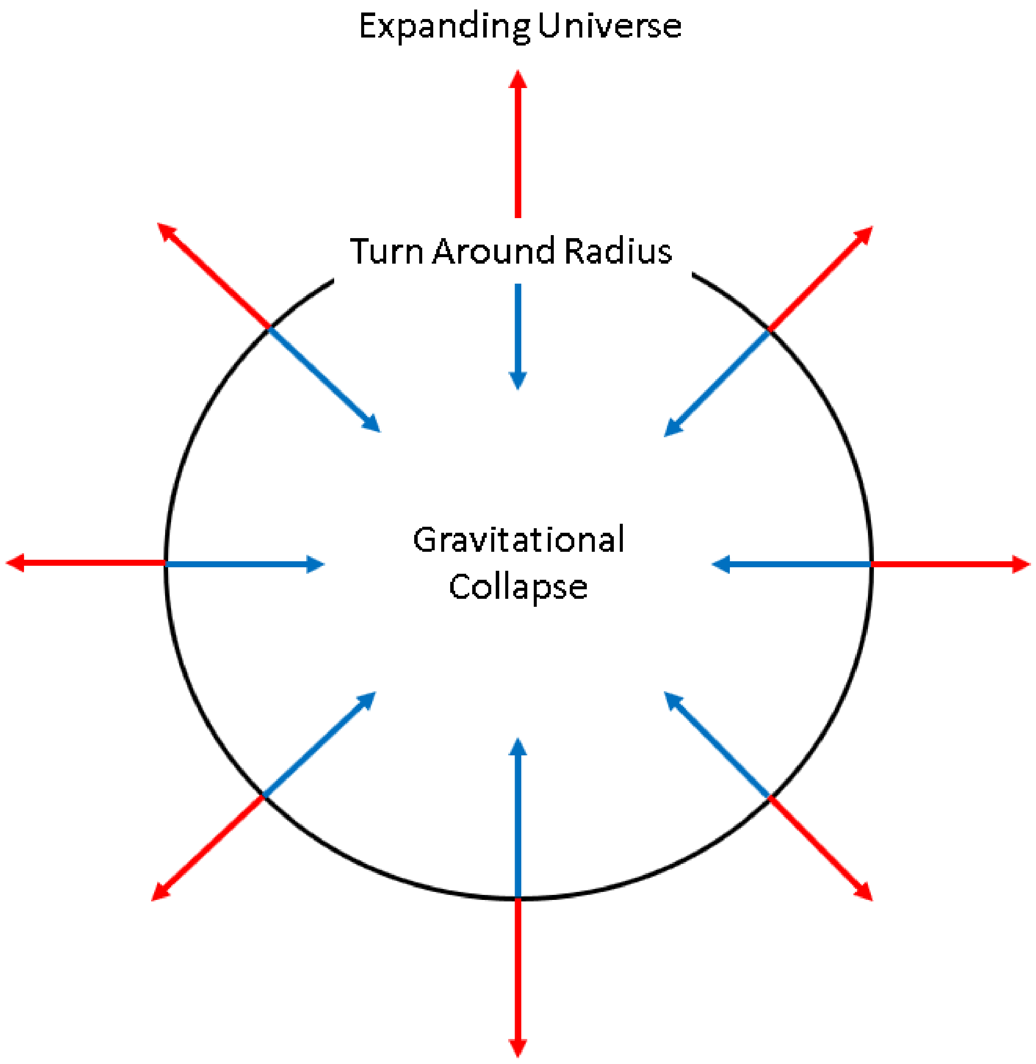
$$ds^2 = \left(1 - mh_t(R) - H^2R^2\right)dT^2 - \left(1 - mh_r(R) - H^2R^2\right)^{-1}dR^2 - R^2d\Omega^2$$

- Bardeen Potentials

$$\Psi_B = -\frac{m}{2}h_t(ar) \qquad \Phi_B = \frac{m}{2}\int \frac{h_r(ar)dr}{r}$$

Turn Around Radius

- Maximum Structure Size



Turn Around Radius

- Spherically Symmetric Ansatz

$$ds^2 = F(T,R)dT^2 - H(T,R)dR^2 - R^2 d\Omega^2$$

- Radial Geodesics

$$\frac{d^2R}{ds^2} = \frac{1}{2}H(t,R)\dot{t}^2\frac{\partial F(t,R)}{\partial R} + \frac{\dot{R}^2}{2H(t,R)}\frac{\partial H(t,R)}{\partial R} - \frac{\dot{R}\dot{t}}{H(t,R)}\frac{\partial H(t,R)}{\partial t}$$

- Static observer $\dot{R} = 0$

Turn Around Radius

- Spherically Symmetric Ansatz

$$ds^2 = F(T,R)dT^2 - H(T,R)dR^2 - R^2 d\Omega^2$$

- Radial Geodesics

$$\frac{d^2R}{ds^2} = \frac{1}{2}H(t,R)\dot{t}^2\frac{\partial F(t,R)}{\partial R} + \frac{\dot{R}^2}{2H(t,R)}\frac{\partial H(t,R)}{\partial R} - \frac{\dot{R}\dot{t}}{H(t,R)}\frac{\partial H(t,R)}{\partial t}$$

- Static observer

$$\dot{R} = 0$$

Turn Around Radius

- Spherically Symmetric Ansatz

$$ds^2 = F(T,R)dT^2 - H(T,R)dR^2 - R^2 d\Omega^2$$

- Radial Geodesics

$$\frac{d^2R}{ds^2} = \frac{1}{2}H(t,R)\dot{t}^2\frac{\partial F(t,R)}{\partial R}$$

- Turn Around Radius

$$\partial_R F(R_{TA}) = 0$$

Turn Around Radius

- Newton Gauge

$$ds^2 = a^2\left[(1+2\Psi)dt^2 - (1-2\Phi)(dr^2 + r^2 d\Omega^2)\right]$$

- Turn Around Radius

$$\ddot{a}r - \frac{\Psi'_N}{a} = 0$$

arXiv:1508.00475

Turn Around Radius

- SSS Metrics

$$ds^2 = \left(1 - mh_t(R) - H^2R^2\right) dT^2 - \left(1 - mh_r(R) - H^2R^2\right)^{-1} dR^2 - R^2 d\Omega^2$$

- Turn Around Radius

$$F(R) = 1 - mh_t(R) - H^2R^2 \qquad \partial_R F(R_{TA}) = 0$$

$$2H^2R + mh_t'(R) = 0$$

SSS Metrics

SDS

- SSS functions

$$h_t = h_r = \frac{2}{R}$$

- Bardeen Potentials

$$\Psi = \Phi = -\frac{m}{ar}$$

SDS

- SSS functions

$$h_t = h_r = \frac{2}{R}$$

- Turn Around Radius

$$2H^2R - 2\frac{m}{R^2} = 0 \Longrightarrow R_{TA} = \sqrt[3]{\frac{m}{H^2}}$$

Brans-Dicke

- Static Metric

$$ds^2 = \left[1 - (1+\epsilon)\frac{2m}{R} - (1-2\epsilon)H^2R^2\right]dt^2 - \left[1 - (1-\epsilon)\frac{2m}{R} - (1-4\epsilon)H^2R^2\right]dR^2 - R^2 d\Omega^2$$

- Bardeen Potentials

$$\Psi_B = H^2R^2\epsilon - \frac{m(1+\epsilon)}{R} \qquad \Phi_B = -H^2R^2\epsilon - \frac{m(1-\epsilon)}{R}$$

- Newton Gauge

$$ds^2 = a^2\left[\left(1 - \frac{2m(1+\epsilon)}{ar} + 2\epsilon H^2r^2a^2\right)\tau^2 - \left(1 - \frac{2m(1+\epsilon)}{ar} - 4\epsilon H^2r^2a^2\right)dr^2 \; - 8Hm\,d\tau dr - r^2 d\Omega^2\right]$$

arXiv:1611.05055

Brans-Dicke

- Static Metric

$$ds^2 = \left[1 - (1+\epsilon)\frac{2m}{R} - (1-2\epsilon)H^2R^2\right]dt^2 - \left[1 - (1-\epsilon)\frac{2m}{R} - (1-4\epsilon)H^2R^2\right]dR^2 - R^2 d\Omega^2$$

- Turn Around Radius

$$R_{TA} = a r_{TA} = \sqrt[3]{\frac{m}{H^2}}(1+\epsilon)$$

arXiv:1611.05055

Power Law

- SSS Functions

$$h_t(R) = \lambda_1 R^{n_1}$$

$$h_r(R) = \lambda_2 R^{n_2}$$

- Bardeen Potentials

$$\Psi_B = -\frac{m\lambda_1 (ra)^{n_1}}{2}$$

$$\Phi_B = \frac{m\lambda_2 (ra)^{n_2}}{2n_2}$$

Power Law

- SSS Functions

$$h_t(R) = \lambda_1 R^{n_1} \qquad h_r(R) = \lambda_2 R^{n_2}$$

- Turn Around Radius

$$R_{TA} = \left(-\frac{m\lambda_1 n_1}{2H^2}\right)^{\frac{1}{2-n_1}}$$

Exponential

- SSS Functions

$$h_t(R) = \lambda_1 e^{b_1 R}$$

$$h_r(R) = \lambda_2 e^{b_2 R}$$

- Bardeen Potentials

$$\Psi_B = -\frac{1}{2}\lambda_1 m e^{b_1 r a}$$

$$\Phi_B = \frac{1}{2}\lambda_2 m \mathrm{Ei}(b_2 r a)$$

Exponential

- SSS Functions

$$h_t(R) = \lambda_1 e^{b_1 R} \qquad h_r(R) = \lambda_2 e^{b_2 R}$$

- Bardeen Potentials

$$Ei(z) = -\int_{-z}^{\infty} \frac{e^{-t}}{t} dt \qquad \Phi_B = \frac{1}{2}\lambda_2 m \mathrm{Ei}(b_2 r a)$$

Logarithmic

- SSS Functions

$$h_t(R) = \lambda_1 \log b_1 R \qquad h_r(R) = \lambda_2 \log b_2 R$$

- Bardeen Potentials

$$\Psi_B = -\frac{1}{2}\lambda_1 m \log\left(b_1 r a\right) \qquad \Phi_B = \frac{1}{4}\lambda_2 m \left[\log\left(b_2 r a\right)\right]^2$$

Logarithmic

- SSS Functions

$$h_t(R) = \lambda_1 \log b_1 R \qquad h_r(R) = \lambda_2 \log b_2 R$$

- Turn Around Radius

$$R_{TA} = \frac{1}{H}\sqrt{\frac{m\lambda_1}{2}}$$

Flat Rotation Curves

- Metric

$$ds^2 = \left(\frac{R}{R_c}\right)^{2v^2} dt^2 - \left[1 - v^2 f(R) - H^2 R^2\right] dR^2 - R^2 d\Omega^2.$$

- Bardeen Potential

$$\Psi_B = v^2 \log\left(\frac{R}{R_c}\right) - \frac{1}{2} H^2 R^2$$

arXiv:0709.0046

Flat Rotation Curves

- Metric

$$ds^2 = \left(\frac{R}{R_c}\right)^{2v^2} dt^2 - \left[1 - v^2 f(R) - H^2 R^2\right] dR^2 - R^2 d\Omega^2.$$

- Density Profile

$$8\pi\rho = \frac{2v^2}{a^2 r^2} - 6H^2$$

Flat Rotation Curves

- Metric

$$ds^2 = \left(\frac{R}{R_c}\right)^{2v^2} dt^2 - \left[1 - v^2 f(R) - H^2 R^2\right] dR^2 - R^2 d\Omega^2.$$

- Linear Mass Profile

$$M(R) = \int_0^R 4\pi R'^2 \rho(R') dR' = \int_0^R \left(v^2 - 3H^2 R'^2\right) dR' = v^2 R - H^2 R^3$$

Conclusions

- Developed a method to obtain the Newton Gauge of Static Spherically Symmetric metrics.
- Obtained the Bardeen Potentials of SSS Metrics
- Applied this method to several examples
  - Schwarzschild-de Sitter
  - Brans-Dicke
  - Power Law, Exponential, Logarithmic Modifications
  - Flat Rotational Curves
- Useful when observations are convenient in the framework of CPT

Thanks!

References

- S. Bhattacharya, K. F. Dialektopoulos, A. E. Romano, C. Skordis, and T. N. Tomaras, (2016), arXiv:1611.05055.
- V. Faraoni, Phys. Rev. D81, 044002 (2010), arXiv:1001.2287.
- T. Biswas and A. Notari, JCAP 0806, 021 (2008), arXiv:astro-ph/0702555.
- J. M. Bardeen, Phys. Rev. D22, 1882 (1980).
- S. Bhattacharya, K. F. Dialektopoulos, A. E. Romano, and T. N. Tomaras, Phys. Rev. Lett. 115, 181104 (2015), arXiv:1505.02375.
- S. Bhattacharya, K. F. Dialektopoulos, and T. N. Tomaras, JCAP 1605, 036 (2016),arXiv:1512.08856.
- V. Pavlidou, N. Tetradis, and T. N. Tomaras, JCAP 1405, 017 (2014), arXiv:1401.3742.
- V. Faraoni, Phys. Dark Univ. 11, 11 (2016), arXiv:1508.00475.