Glueball masses within an anomalous modified AdS/QCD model

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Summary of the talk:

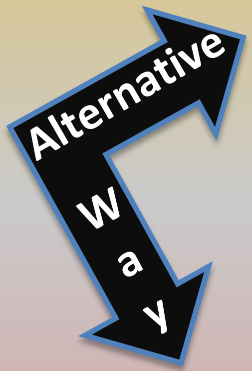
- Brief Review: Gleuballs in QCD
- Brief Review: AdS/CFT correspondence and AdS/QCD models
- The Dynamical Soft-wall model
- Modified Softwall model
- Anomalous Modified Softwall model
- Results achieved
- Last comments

Quantum Chromodynamics - QCD

✓ used as the standard theory to explain the phenomenology of strong interactions.

❑ at the low-energy limit ($g_{YM} > 1$) the QCD cannot be treated perturbatively.

❖ Regge trajectories are an example of nonpertubative behavior of strong interactions: difficult to model it using QCD.



AdS/CFT correspondence

ANTI–DE SITER/CONFORMAL FIELD THEORY

Glueballs in QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}\left(\not{D} - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \,,$$

$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_{YM} f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \,,$$

where $\mathcal{A}^a_\nu$ are the gluon fields, with $a = 1, \cdots, 8$, f^{abc} are the structure constants of $SU(3)$ group and g_{YM} is the coupling constant of Yang-Mills (strong) interactions.

- Gluons do not carry electric charges, but they have color charge;
- Due to this fact, they coupled to each other;
- The bound states of gluons predicted by QCD, but not detect so far, are called glueballs;
- Glueballs states are characterised by J^{PC}, where J is the total angular momentum,

and P and C are the P-parity (spatial inversion) and the C-parity (charge conjugation) eigenvalues, respectively.

Regge Trajectories

Strongly interacting particles (Hadrons) obey approximate relations between Angular Momentum (J) and quadratic masses (m^2)

$$J(m^2) \approx \alpha_0 + \alpha' m^2$$

Where α_0 and α' are constants

Extended for glueball: J^{PC}

Glueball Masses

Table 1: Lightest scalar glueball and its radial excitation masses expressed in GeV from lattice. The numbers in parenthesis represent the uncertainties. The last column represents mean values for each state.

	$N_c = 3$	$N_c = 3$ anisotropic lattice		$N_c = 3$	$N_c \to \infty$	Average
J^{PC}	ref. [20]	ref. [18]	ref. [21]	ref. [22]		
0^{++}	1.475(30)(65)	1.730(50)(80)	1.710(50)(80)	1.58(11)	1.48(07)	1.595
0^{++*}	2.755(70)(120)	2.670(180)(130)		2.75(35)	2.83(22)	2.751
0^{++**}	3.370(100)(150)					3.370
0^{++***}	3.990(210)(180)					3.990

Table 2: Higher even spin glueball states masses expressed in GeV from lattice and constituent model approaches. The numbers in parenthesis represent the uncertainties related to lattice results. The last column represents mean values for each state.

	$N_c = 3$	$N_c = 3$ anisotropic lattice		Constituent models		Average
J^{PC}	ref. [20]	ref. [18]	ref. [21]	ref. [28]	ref. [29]	
2^{++}	2.150(30)(100)	2.400(25)(120)	2.390(30)(120)	2.42	2.59	2.39
4^{++}	3.640(90)(160)			3.99	3.77	3.80
6^{++}	4.360(260)(200)				4.60	4.48

AdS/CFT correspondence, J. Maldacena, 1997

(simplified version of a particular useful case)

SUPERSTRING THEORY in the $AdS_5 \times S^5$ spacetime.

DUAL

YANG-MILLS THEORY

- Supersymmetric $\mathcal{N} = 4$
- Conformal
- SU(N) symmetry, with $N \to \infty$

in a 4-dimensional Minkowski spacetime ($AdS_5 \times S^5$ boundary).

At low energies string theory is represented by an effective supergravity theory → **gravity / gauge duality**

Other versions of the Correspondence: $AdS_4 \times S^7$ or $AdS_7 \times S^4$ (M-theory in 11 dimensions)

- After breaking the conformal symmetry one can build phenomenological models that describe approximately QCD. So, AdS/QCD models (hardwall, softwall, Witten BH, etc.)
- Weak coupling theory ⇔ Strong coupling theory.

AdS/CFT Dictionary

Isometries in the bulk ↔ Simmetries in the boundary field theory

Field $(\phi,\ g_{\mu\nu}\ldots.)$	$\leftrightarrow$	Operator $(TrF^2,\ T_{\mu\nu}\ldots.)$
Radial distance, u	$\leftrightarrow$	Energy
Minimal area	$\leftrightarrow$	Wilson loop
$\vdots$		$\vdots$
Minimal volume	$\leftrightarrow$	Entanglement entropy

Bulk field mass ↔ boundary operator scaling dimension

$$\phi: \quad \Delta(\Delta - d) = m^2$$

$$\psi: \quad |m| = \Delta - \frac{d}{2}$$

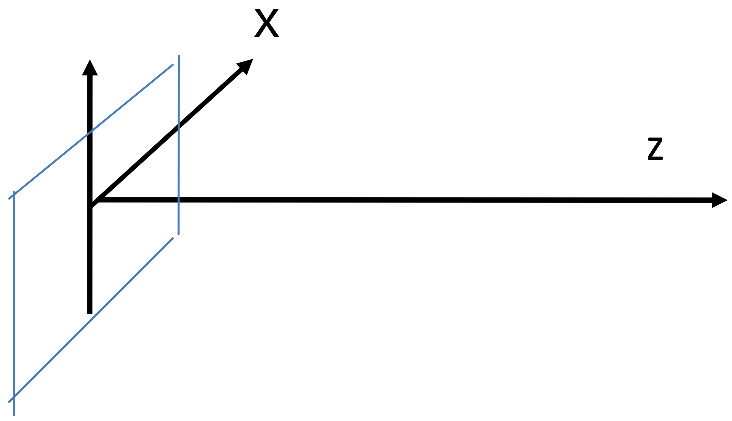
$$A_\mu: \quad m^2 = (\Delta - 1)(\Delta + 1 - d)$$

The AdS_5 Spacetime

Disregarding the S^5 space, the AdS_5 Space in Poincaré coordinates is given by:

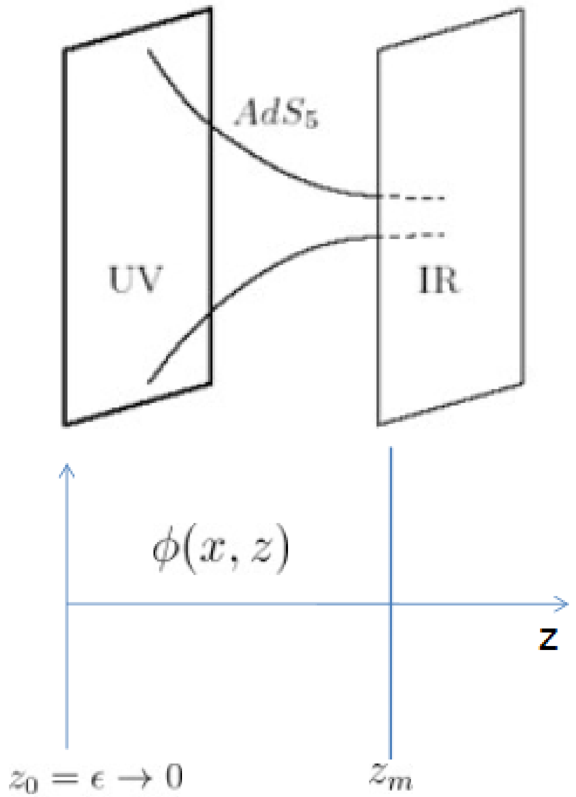
$$ds^2 = \frac{R^2}{(z)^2}(dz^2 + (d\vec{x})^2 - dt^2)$$

The 4-dim boundary is at z = 0



Fifth dimension z ~ 1 / E where E = Energy in 4-dim boundary

AdS/QCD models: Hardwall and Softwall Models



UV IR

Finite region in AdS space

$0 \leq z \leq z_{max}$ with $z_{max} = 1/\Lambda_{QCD}$

$$S = \int d^{10}x\sqrt{-g}e^{-\phi(z)}\mathcal{L}, \quad \phi(z) = kz^2.$$

With k ~$\Lambda^2{}_{QCD}$

Hard-wall Model

Polchinski & Strassler 2001/2002

Scattering of Glueballs using the AdS/CFT correspondence

Finite region in AdS space $0 < z < z_{max}$

$z_{max} \sim 1/E$ where E is the Energy scale in boundary theory

HBF & Braga JHEP 2003, EPJC 2004

Masses of Glueball states 0++ and its radial excited states 0++*, 0++**, 0++***, …

Brodsky, Teramond PRL 2005, 2006; Erlich, Katz, Son & Stephanov PRL 2005.

Extension to Mesons and Baryons

HBF, Braga & Carrion PRD 2006 –higher even spin glueball

EFC & HBF PRD 2013 –higher odd spin glueball

Diego, EFC & HBF PRD 2017 – Twist two - higher even spin glueball

Original Softwall model

Soft cutoff (Karch, Katz, Son, Stephanov PRD 2006)

$$\int d^5x \sqrt{-g}\,\mathcal{L} \quad \Longrightarrow \quad \int d^5x \sqrt{-g}e^{-\Phi}\mathcal{L}\,. \quad ; \quad \Phi(z) = cz^2$$

spectrum of vector mesons $\qquad m^2_{V_n} = 4c(n+1)\,,$

Glueballs in the soft-wall (T=0)
[Colangelo, De Fazio, Jugeau, Nicotri PLB(2007)]

The corresponding glueball spectrum is

$$m^2_{G_n} = 4c(n+2)\,.$$

EFC, Henrique Boschi-Filho, PLB (2016).

↳ **Original SW model seems not work well for glueballs.**

Dynamical Softwall model

D. Li, M. Huang, JHEP (2013)

Dilatonic field became dynamical satisfying the Einstein equations in five dimensions.

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_s}\, e^{-2\Phi(z)} (R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi))$$

String frame

$$ds^2 = g^s_{mn} dx^m dx^n = b_s^2(z)(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu); \quad b_s(z) \equiv e^{A_s(z)}$$

$$S = \frac{1}{16\pi G_5} \int \mathrm{d}^5 x \sqrt{-g_E}\, \left(R_E - \frac{4}{3}\partial_M \Phi \partial^M \Phi - V^E(\Phi)\right)$$

Einstein frame

EOM

$$-A_E'' + A_E'^2 - \frac{4}{9}\Phi'^2 = 0,$$

$$\Phi'' + 3A_E'\Phi' - \frac{3}{8} e^{2A_E} \partial_\Phi V^E(\Phi) = 0,$$

$$A_s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) - \log\left({}_0F_1\left(\frac{5}{4}, \frac{\Phi^2}{9}\right)\right)$$

Modified Softwall model

EFC, Henrique Boschi-Filho, PLB (2016).

seeking for analytical solutions we propose a modified softwall model.

$$A_s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) - \log\left({}_0F_1\left(\frac{5}{4}, \frac{\Phi^2}{9}\right)\right)$$

$$\Rightarrow \quad A_M^s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z) \quad \Rightarrow \quad m_n^2 = [4n+4]k\,.$$

Masses expressed in GeV for the glueball states J^{PC} of the lightest scalar glueball and its radial excitations from the modified softwall model using Eq. (32) for $k =$ 0.2, 0.85 and 1 GeV^2.

	Glueball states J^{PC}				k
	0^{++}	0^{++*}	0^{++**}	0^{++***}	
n	0	1	2	3	
m_n	0.89	1.26	1.55	1.79	0.20
m_n	1.84	2.61	3.19	3.69	0.85
m_n	2.00	2.83	3.46	4.00	1.00

Masses expressed in GeV for the glueball states J^{PC} with even J from the original SW using Eq. (14) with $k = 1$ and 2 GeV^2 and from the modified SW using Eq. (33) with $k = 0.2$ GeV^2.

	Glueball states J^{PC}						k
	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}	
Masses	2.83	3.46	4.00	4.47	4.90	5.29	1.00
Masses	4.00	4.90	5.67	6.32	6.93	7.48	2.00
Masses	0.89	2.19	3.30	4.38	5.44	6.49	0.20

Masses expressed in GeV for the glueball states J^{PC} with odd J from SW using eq. (18) and $k = 1$ and 2 GeV^2 and from the modified SW using eq. (34) and $k =$ 0.2 GeV^2.

	Glueball states J^{PC}						k
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}	
Masses	3.74	4.24	4.69	5.10	5.48	5.83	1.00
Masses	5.29	6.00	6.63	7.21	7.75	8.24	2.00
Masses	2.82	3.94	5.03	6.11	7.19	8.26	0.20

We could not from a single mass equation and one value for the dilaton constant fit the scalar and even higher spin glueball at same time.

Anomalous Modified Softwall model (1)

Motivation: Can we unify the spectra of the scalar and higher even spin glueball states with just one dilaton constant value (k = 0.85 GeV^2)?

Basically: Modified Softwall model + anomalous dimension

$$S = \int d^5x \sqrt{-g} e^{-\Phi(z)} \left[g^{MN} \partial_M \mathcal{G} \partial_N \mathcal{G} + M_5^2 \mathcal{G}^2 \right]$$

$$A_M^s(z) = \log\left(\frac{R}{z}\right) + \frac{2}{3}\Phi(z)$$

$$-\psi''(z) + \left[k^2 z^2 + \frac{15}{4z^2} - 2k + M_5^2 \left(\frac{R}{z}\right)^2 + \frac{4kz^2}{3} M_5^2 \left(\frac{R}{z}\right)^2 \right] \psi(z) = (-q^2)\psi(z)$$

$$m_n^2 = \left[4n + 2\sqrt{4 + M_5^2 R^2} + \frac{4}{3} M_5^2 R^2 \right] k \quad (n = 0, 1, 2, \cdots)$$

Anomalous Modified Softwall model (2)

AdS/CFT dictionary

$$M_5^2 R^2 = \Delta(\Delta - 4) - J.$$

$$\mathcal{O}_4 = Tr(F^2) = Tr(F^{\mu\nu} F_{\mu\nu}) \Longrightarrow \mathcal{O}_{4+J} = F D_{\{\mu 1 \ldots} D_{\mu J\}} F, \Longrightarrow \Delta = 4 + J$$

Taking into account the idea of an anomalous dimension (γ)

$$\Delta \rightarrow \Delta' = \Delta + \gamma(J),$$

for a massive tensor field with spin J ⟹ $M_5^2 R^2 = [4 + J + \gamma(J)][J + \gamma(J)] - J.$

For simplicity we are going to take a linear approximation for the anomalous dimension ⟹ $\gamma(J) = \gamma_0 J$

$$m_n^2 = \left[4n + 2\sqrt{4 + (4 + J + \gamma_0 J)(J + \gamma_0 J) - J} + \frac{4}{3}(4 + J + \gamma_0 J)(J + \gamma_0 J) - J\right] k \quad (n = 0, 1, 2, \cdots).$$

Results achieved

	Glueball States J^{PC}								
	0^{++}	0^{++*}	0^{++**}	0^{++***}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}
Masses (GeV)	1.84	2.60	3.18	3.67	2.64	3.52	4.42	5.31	6.21
Deviations (%)	15	5.4	5.6	7.7	10.4	7.3	1.4		

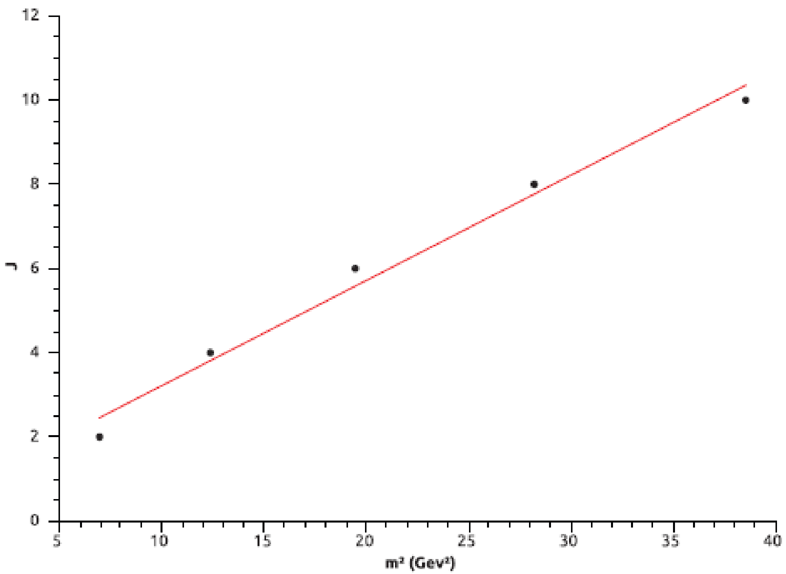


Fig. 1: (Colour online) Approximate Regge trajectory for the pomeron using data from table 3, for the states 2^{++}, 4^{++}, 6^{++}, 8^{++} and 10^{++}, from the anomalous modified softwall model using eq. (25) with $k = 0.845\,\text{GeV}^2$ and $\gamma_0 = -0.585$.

$$J(m^2) = (0.25 \pm 0.02)m^2 + (0.73 \pm 0.42),$$

$$\Downarrow$$

$$J(m^2) \approx 0.25m^2 + 1.08,$$

(Donnachie and Landshoff, 1984 and 1986)

Last comments

- In this work, we have proposed an anomalous modified softwall model (which is analytically solvable) in order to unify the spectra of the scalar and higher even spin glueball states with just one dilaton constant value ($k = 0.845 GeV^2$).
- Note that in our previous work a single mass equation for the scalar and higher even spin glueball states was obtained but to fit lattice data two different values of the dilaton constant k were needed. Here, with the introduction of an anomalous dimension in the conformal dimension of the glueball operators, this problem is overcome.
- the anomalous dimension for high spin fields is a logarithm function of spin J. Here, we just used a linear function of the spin J as an approximation for lower spins, presenting good results for our model in comparison with lattice data and other models.
- our model provided a Regge trajectory for the pomeron in agreement with the literature.

Muchas Gracias por la atención!