# Torsional regularization and finite bare charge

Nikodem Popławski

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# Problems of general relativity

General relativity describes gravity as curvature of spacetime.

- Singularities: points with infinite density of matter.
- Incompatible with quantum mechanics. We need quantum gravity. It may resolve the singularity problem.
- Field equations contain the conservation of orbital angular momentum, contradicting Dirac equation which gives the conservation of total angular momentum (orbital + spin) and allows spin-orbit exchange in QM.

Simplest extension of GR to include QM spin:
**Einstein-Cartan theory**. It also eliminates the singularity problem.

# Einstein-Cartan-Sciama-Kibble gravity

- Spacetime has curvature and **torsion**. $S^k{}_{ij} = \Gamma^{\ \ k}_{[i\ j]}$

- Lagrangian density is proportional to curvature scalar (as in GR).

- Cartan equations:

**Torsion** is proportional to **spin** density of fermions. ECSK differs significantly from GR at densities > $10^{45}$ kg/m$^3$; passes all tests.

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2}\kappa s_{ikj}$$

arXiv.org > gr-qc > arXiv:0911.0334

- Einstein equations:

**Curvature** is proportional to **energy and momentum** density.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2}\kappa^2\left(s^{ij}{}_j s^{kl}{}_l - s^{ij}{}_l s^{kl}{}_j - s^{ijl} s^k{}_{jl} + \frac{1}{2}s^{jli} s_{jl}{}^k + \frac{1}{4}g^{ik}(2s_j{}^l{}_m s^{jm}{}_l - 2s_j{}^l{}_l s^{jm}{}_m + s^{jlm} s_{jlm})\right)$$

# Universe with spin fluid

Dirac particles can be averaged macroscopically as a spin fluid.

Einstein-Cartan equations for a homogeneous and isotropic Universe become Friedmann equations for the scale factor *a*.

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa\Big(\epsilon - \frac{1}{4}\kappa s^2\Big)a^2$$

$$\frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + 1 = -\kappa\Big(p - \frac{1}{4}\kappa s^2\Big)a^2$$

$$s^2 = \frac{1}{8}(\hbar c n)^2$$

Spin and torsion modify the energy density and pressure with a negative term proportional to the square of the fermion number density $n$, which acts like repulsive gravity and prevents singularities. The Big Bang is replaced by a non-singular Big Bounce.

# Problems of quantum field theory

- Ultraviolet divergence: Feynman diagrams involve divergent integrals in the four-momentum space arising from high-energy contributions.
- This unphysical result requires regularization: a mathematical method of turning singular quantities into finite quantities. Most common: adding fictitious particles, changing dimensions.
- Renormalization: the original (bare) values of mass and charge absorb divergent terms, giving the measured (dressed) values.
- Dirac was critical about renormalization and expected a realistic regularization based on the principles of physics.
- Solution: torsional regularization, renormalization is finite.

# Torsion and noncommutativity of momentum

- Consider two infinitesimal four-vectors *dx* and *dx'*.
- In the presence of torsion the parallel transport of *dx* along *dx'* and the parallel transport of *dx'* along *dx* do not form a closed parallelogram:

$$\delta dx^i = -\Gamma^{\,i}_{\,jk} dx^j \, dx'^k \qquad \delta dx'^i = -\Gamma^{\,i}_{\,jk} dx^j \, dx'^k \qquad \delta dx'^i - \delta dx^i = -S^i_{\ jk} dx^j \, dx'^k$$

- Since the momentum is a generator of translation, described by the parallel transport, its operator in quantum mechanics is given by the covariant derivative:

$$p_k = i\hbar\nabla_k$$

- In the presence of torsion, translations do not commute and therefore the four-momentum components do not commute:

$$[p_i, p_j] = 2i\hbar S^k_{\ ij} p_k$$

# Integration in momentum space becomes summation over momentum eigenstates

- The classical and quantum partition functions in statistical physics: $$\int dq \int dp\, f(H(q,p)) \leftrightarrow 2\pi \sum_{\text{eigenstates}} f(E)\, |[q,p]|$$
- One can choose locally a frame of reference in which only the space momentum components do not commute:

$$[p_x, p_y] = iQp_z, \;\; [p_y, p_z] = iQp_x, \;\; [p_z, p_x] = iQp_y \qquad Q = -2\hbar A^0 \qquad A^i = \frac{1}{6}\epsilon^{ijkl} S_{jkl}$$

- Einstein-Cartan gravity gives: $Q = Up^3$ (*U* is const ~ $M^{-2}_{Pl}$)
- We obtain a relation analogous to the angular momentum:

$$[n_x, n_y] = in_z, \;\; [n_y, n_z] = in_x, \;\; [n_z, n_x] = in_y \qquad \mathbf{n} = \frac{\mathbf{p}}{Q}$$

# Integration in momentum space becomes summation over momentum eigenstates

- We propose that the integration in *n*-space satisfying

$$[n_x, n_y] = i n_z, \quad [n_y, n_z] = i n_x, \quad [n_z, n_x] = i n_y$$

- Is replaced with the summation:

$$\int dn_x \int dn_y \int dn_z \, f(\mathbf{n}^2) \to 4\pi \sum_{\text{eigenstates}} f(\mathbf{n}^2)\, |n_z|$$

$$\to 4\pi \sum_{l=1}^{\infty} \sum_{m=-l}^{l} f(\mathbf{n}^2)\, |m| = 4\pi \sum_{l=1}^{\infty} f(\mathbf{n}^2)\, l(l+1)$$

- Torsional regularization – NP, arXiv:1712.09997

# Integration in momentum space becomes summation over momentum eigenstates

- Apply TR to a logarithmically divergent integral:

$$\int \frac{d^4p}{(p^2+\mu^2)^2} = \int \frac{dp_0\, d\mathbf{p}}{(p^2+\mu^2)^2} = \int \frac{dp_0\, J\, d\mathbf{n}}{(p^2+\mu^2)^2} \rightarrow 4\pi \int_{-\infty}^{\infty} dp_0 \sum_{l=1}^{\infty} \frac{J}{(p^2+\mu^2)^2}\, l(l+1)$$

$$J = \partial(p_x, p_y, p_z)/\partial(n_x, n_y, n_z)$$

$$p^2 = p_0^2 + U^2 n^2 p^6$$

$$\frac{\partial p}{\partial n_x} = \frac{U^2 p^5 n_x}{1 - 3U^2 n^2 p^4}$$

$$\frac{\partial p_x}{\partial n_x} = \frac{\partial (Q n_x)}{\partial n_x} = Q + 3U n_x p^2 \frac{\partial p}{\partial n_x}$$

$$\frac{\partial p_x}{\partial n_y} = \frac{\partial (Q n_x)}{\partial n_y} = 3U n_x p^2 \frac{\partial p}{\partial n_y}$$

$$dp_0/dp = (1 - 3U^2 n^2 p^4)/(1 - U^2 n^2 p^4)^{1/2}$$

$$n = \sqrt{l(l+1)}$$

$$J = \det \begin{pmatrix} \partial p_x/\partial n_x & \partial p_x/\partial n_y & \partial p_x/\partial n_z \\ \partial p_y/\partial n_x & \partial p_y/\partial n_y & \partial p_y/\partial n_z \\ \partial p_z/\partial n_x & \partial p_z/\partial n_y & \partial p_z/\partial n_z \end{pmatrix} = \frac{Q^3}{1 - 3U^2 n^2 p^4}$$

# Torsion eliminates ultraviolet divergence

$$4\pi\int_{-\infty}^{\infty}dp_0\sum_{l=1}^{\infty}\frac{Q^3n^2}{(1-3U^2n^2p^4)(p^2+\mu^2)^2}=4\pi\int dp\frac{dp_0}{dp}\sum_{l=1}^{\infty}\frac{Q^3n^2}{(1-3U^2n^2p^4)(p^2+\mu^2)^2}$$

$$=4\pi\int_{-1/\sqrt{Un}}^{1/\sqrt{Un}}dp\sum_{l=1}^{\infty}\frac{Q^3n^2}{(1-U^2n^2p^4)^{1/2}(p^2+\mu^2)^2}=8\pi\int_0^{1/\sqrt{Un}}dp\sum_{l=1}^{\infty}\frac{U^3p^9n^2}{(1-U^2n^2p^4)^{1/2}(p^2+\mu^2)^2}$$

$$=8\pi\int_0^1 d\xi\sum_{l=1}^{\infty}\frac{U^3\xi^9n^2(Un)^{-5}}{(1-\xi^4)^{1/2}[\xi^2/(Un)+\mu^2]^2}=8\pi\int_0^1 d\xi\sum_{l=1}^{\infty}\frac{\xi^9n^{-1}}{(1-\xi^4)^{1/2}[\xi^2+U\mu^2n]^2}$$

$$=4\pi\int_0^1 d\zeta\sum_{l=1}^{\infty}\frac{\zeta^4n^{-1}}{(1-\zeta^2)^{1/2}[\zeta+U\mu^2n]^2}=4\pi\sum_{l=1}^{\infty}\int_0^{\pi/2}d\phi\frac{\sin^4\phi\, n^{-1}}{[\sin\phi+U\mu^2n]^2}$$

$$=4\pi\sum_{l=1}^{\infty}\int_0^{\pi/2}d\phi\frac{\sin^4\phi\,[l(l+1)]^{-1/2}}{[\sin\phi+U\mu^2\sqrt{l(l+1)}]^2},\qquad Unp^2=\xi^2=\zeta=\sin\phi$$

The logarithmically divergent integral is replaced with a sum that converges as $l^{-3}$.

# Torsion eliminates ultraviolet divergence

This procedure can be generalized to tensor integrals:

$$\int \frac{d^4p}{(p^2+\mu^2)^s} \to 8\pi \int_0^{1/\sqrt{Un}} dp \sum_{l=1}^{\infty} \frac{U^3 p^9 n^2}{(1-U^2n^2p^4)^{1/2}(p^2+\mu^2)^s}$$
$$= 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{U^3 \xi^9 n^2 (Un)^{-5}}{(1-\xi^4)^{1/2}[\xi^2/(Un)+\mu^2]^s} = 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{U^{s-2}\xi^9 n^{s-3}}{(1-\xi^4)^{1/2}[\xi^2+U\mu^2 n]^s}$$
$$= 4\pi \int_0^1 d\zeta \sum_{l=1}^{\infty} \frac{U^{s-2}\zeta^4 n^{s-3}}{(1-\zeta^2)^{1/2}[\zeta+U\mu^2 n]^s} = 4\pi U^{s-2} \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4\phi\, n^{s-3}}{[\sin\phi + U\mu^2 n]^s}$$
$$= 4\pi U^{s-2} \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4\phi\, [l(l+1)]^{(s-3)/2}}{[\sin\phi + U\mu^2\sqrt{l(l+1)}]^s}.$$

$$\int d^4p \frac{\partial}{\partial p_\nu}\left(\frac{p^\mu}{(p^2+\Delta)^s}\right) = \int d^4p \frac{\delta^{\mu\nu}}{(p^2+\Delta)^s} - 2s \int d^4p \frac{p^\mu p^\nu}{(p^2+\Delta)^{s+1}}$$

$$\int d^4p \frac{p^\mu p^\nu}{(p^2+\Delta)^s} = \frac{\delta^{\mu\nu}}{2(s-1)} \int d^4p \frac{1}{(p^2+\Delta)^{s-1}}$$

# Vacuum polarization

The vacuum polarization tensor is gauge invariant:

$$\Pi^{\mu\nu}_{\text{bubble}}(q) = -\frac{\alpha_0}{\pi^3}\int d^4p_{\rm E}\int_0^1 dx \frac{-2p^\mu_{\rm E}p^\nu_{\rm E} + p^2_{\rm E}\delta^{\mu\nu} + \Delta\delta^{\mu\nu} + 2(q^2g^{\mu\nu} - q^\mu q^\nu)x(1-x)}{(p^2_{\rm E}+\Delta)^2}$$

$$= -\frac{2\alpha_0}{\pi^3}\int d^4p_{\rm E}\int_0^1 dx \frac{x(1-x)}{(p^2_{\rm E}+\Delta)^2}(q^2g^{\mu\nu} - q^\mu q^\nu) = \Pi(q^2)q^2\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right),$$

$$\Pi(q^2) = -\frac{2\alpha_0}{\pi^3}\int d^4p_{\rm E}\int_0^1 dx \frac{x(1-x)}{(p^2_{\rm E}+\Delta)^2}$$

$$\Delta = m^2 - q^2x(1-x)$$

$$\Pi(q^2) \to -\frac{8\alpha_0}{\pi^2}\sum_{l=1}^{\infty}\int_0^1 dx\int_0^{\pi/2} d\phi \frac{\sin^4\phi\, n^{-1}x(1-x)}{[\sin\phi + U\Delta n]^2}$$

$$= -\frac{8\alpha_0}{\pi^2}\sum_{l=1}^{\infty}\int_0^1 dx\int_0^{\pi/2} d\phi \frac{\sin^4\phi\,[l(l+1)]^{-1/2}x(1-x)}{[\sin\phi + U\Delta\sqrt{l(l+1)}]^2}$$

The sum-integral in Π is finite.

# Torsion makes bare charge finite

Renormalization of the electric charge:

$$\alpha = \frac{\alpha_0}{1 - \Pi(0)} \qquad \alpha_{\rm run} = \frac{\alpha_0}{1 - \Pi(q^2)}$$

Gives the bare electric charge of an electron:

$$e_0 = \frac{e}{(1 + \Pi_{\rm R}(0))^{1/2}} = e\Big[1 - \frac{8\alpha}{\pi^2}\sum_{l=1}^{\infty}\int_0^1 dx \int_0^{\pi/2} d\phi \frac{\sin^4\phi\, n^{-1} x(1-x)}{[\sin\phi + U m^2 n]^2}\Big]^{-1/2}$$

Including all charged fermions in *Π* gives the bare charge -1.22 e.
The running coupling constant is finite.

Accordingly, the bare fine structure constant is about 1/92.1.
NP, arXiv:1712.09997

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# Summary

- The conservation law for total angular momentum (orbital + spin) in curved spacetime, consistent with Dirac equation, requires torsion.
- In the presence of torsion, the four-momentum operator components do not commute. The integration in the momentum space must be replaced with the summation over the momentum eigenvalues.
- The separation between the momentum eigenvalues increases with the magnitude of the momentum as a result of the Einstein-Cartan gravity. Consequently, ultraviolet divergent integrals turn into convergent sums.
- Torsion naturally regularizes ultraviolet divergence in QED. Renormalization in QED is finite, leading to a finite bare charge of an electron: -1.22 e.
- Future work: research how torsion affects the electroweak and strong interactions.