Nambu–Jona-Lasinio Models with Supersymmetry and Phenomenology

— *towards a viable model of completely dynamical EWSB*

— **talk at XII SILAFAE (Nov 2018)**

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BIG PICTURE :-

building models **BSM**

- **quest : architecture principles**

  Simplicity and Beauty

- **my dream scenario : gauge symmetry, dynamical SB, supersymmetry, . . .**

Standard Model :-

Beautiful Theory

Vs

phenomenological model

Standard Model as theory of EW Symmetry Breaking

- only phenomenological model (*cf.* Ginsburg-Landau Th)
- where is the BCS theory ?

$\Longrightarrow$ Nambu–Jona Lasinio Model

Experimentally Viable Option, with Supersymmetry

$\Longrightarrow$ HSNJL model

Physics is ONLY about
Effective (Field) Theories

⋆ gauge symmetry fixes spin 1 sector

⋆ **The Story of the spin $\frac{1}{2}$ fermion sector** $\cdots$

— **3 families** of **15** spin $\frac{1}{2}$ quantum fields (**Weyl 2-spinors**)
under $SU(3)_C \times SU(2)_L \times U(1)_Y$

- $(3, 2, 1)$: u u u d d d
- $(\bar{3}, 1, -4)$: $\bar{u}$ $\bar{u}$ $\bar{u}$
- $(\bar{3}, 1, 2)$: $\bar{d}$ $\bar{d}$ $\bar{d}$
- $(1, 2, -3)$: ν_L e^-_L
- $(1, 1, 6)$: e^+_R

— minimal chiral set free from all anomalies
complete nontrivial cancellation (Vs vector-like pairing)

SM fermion field spectrum for one family :-

minimal chiral set with completely nontrivial anomaly cancellation

Geng & Marshak (89)

— less than appreciated well enough

• taking $SU(3)_C \times SU(2)_L \times U(1)_Y$

• assuming a $(3, 2, 1)$ multiplet

— $SU(3)$ requires $(\bar{3}, 1, a)$ and $(\bar{3}, 1, b)$

— $SU(2)$ requires an extra $(1, 2, c)$

— $U(1)$ anomalies have no solution

$\longrightarrow$ adding a $(1, 1, k)$ give *the* unique solution

⋆ idea extended to derive the 3-family spectrum O.K. MPLA11, PRD55 (97)

Principle of Gauge-Chiral Fields

Why there is what there is — why the list ?

- gauge symmetry / canceled anomaly $\Longrightarrow$ full Lagrangian
- massless (before symmetry breaking)

— if massive, at model cut-off scale / decoupled

Georgi : survival hypothesis (79)

— no (non-chiral) scalars (SUSY $\Rightarrow$ chiral scalar)

- 'chiral matter' + gauge bosons (DICTATED)

—all fields massless by gauge symmetry

- **SM — two problems**

— needs EWSB : dynamical symmetry breaking; & SUSY (?)

— the most fundamental mystery : *Why Three Families ?*

- against vectorlike pair – Georgi's survival hypothesis
  invariant mass at cutoff scale
- SM $\rightarrow$ BSM — hierarchy/fine-tuning problem
  scalar field is somewhat sick
- scalar field content — only part arbitrary (*cf.* gauge symmetry)
- SUSY — **technically** natural hierarchy
  scalar as (part of) **chiral** superfield (constrained as fermions)
  Vs

BUT μ-problem — vectorlike pair of Higgs superfields

- **SNJL models solve our problem**

— and avoid fine-tuning of "four-quark" coupling(s)

Symmetry breaking w/o put-in hierarchy

hierarchy problem — no input mass scale

EFT has cut-off scale ; Vs conformal theories

- **dynamical symmetry breaking**

— NJL : bifermion condensate / SNJL : superfield condensate

HSNJL model— interesting viable(?) version for MSSM

O.K. *et.al.* PRD81 (10), JHEP01 (12), PRD87 (13)

- holomorphic four-superfield interaction

— simple origin : integrating out vectorlike pair

Summary of Basic NJL Models :-

- NJL (1961)

$$\mathcal{L} = i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ + i\bar{\psi}_- \sigma^\mu \partial_\mu \psi_- + g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_-$$

- SNJL (1984) — dim 6 four-superfield interaction

$$\mathcal{L} = \int d^4\theta \left(\Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_- \right) (1 - \tilde{m}^2 \theta^2 \bar{\theta}^2) + \int d^4\theta \; g^2 \Phi_+^\dagger \Phi_-^\dagger \Phi_+ \Phi_- (1 - \tilde{m}_c^2 \theta^2 \bar{\theta}^2)$$

- HSNJL (2010) — dim 5 four-superfield interaction

$$\mathcal{L} = \int d^4\theta \left(\Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_- \right) (1 - \tilde{m}^2 \theta^2 \bar{\theta}^2) - \int d^2\theta \; \frac{G}{2} \Phi_+ \Phi_- \Phi_+ \Phi_- (1 + B\theta^2)$$

(M)SSM from HSNJL:-

- consider $W = G\,\varepsilon_{\alpha\beta}\,\hat{Q}^{\alpha}\hat{T}^{c}\hat{Q}'^{\beta}\hat{B}^{c}\,(1 + B\theta^2)$

$$W \;\longrightarrow\; W - \mu\,(\hat{H}_d - \lambda_t\hat{Q}\hat{U}^c)(\hat{H}_u - \lambda_b\hat{Q}'\hat{D}^c)(1 + B\theta^2)$$
$$= (-\mu\hat{H}_d\hat{H}_u + y_t\hat{Q}\hat{H}_u\hat{T}^c + y_b\hat{H}_d\hat{Q}'\hat{B}^c)(1 + B\theta^2)$$

- two composites — $\hat{H}_u = \frac{y_b}{\mu}\,\hat{Q}'\hat{B}^c$ and $\hat{H}_d = \frac{y_t}{\mu}\,\hat{Q}\hat{T}^c$

- low energy effective theory looks like MSSM ($A_t = A_b = B$)
- symmetric role for $\hat{H}_u$ and $\hat{H}_d$ $\left(\text{also}:\ \mu\lambda_{\rm t}\lambda_{\rm b} = \frac{\rm y_t y_b}{\mu} = {\rm G}\right)$

— numerical lifted through non-universal soft masses

— expect $\langle h_u \rangle \gtrsim \langle h_d \rangle$ (Vs UBB in D-flat)

HOWEVER :-

- (H)SNJL needs input soft mass(es)
- (literature) models with hidden sector,
  mediating sector, . . .
- WANT completely dynamical mass generation
- WANT simple Vs contrived model

⋆ ‘NJL’ SUSY breaking → soft masses

Simple Model of DSSB Generating Soft Masses :-

- dim 6 four-superfield interaction with spin one composite

$$\mathcal{L} = \int d^4\theta \Phi^\dagger \Phi - \frac{g^2}{2} \Phi^\dagger \Phi \Phi^\dagger \Phi$$

- $\langle \Phi^\dagger \Phi|_D \rangle \neq 0$ gives soft mass, and breaks supersymmetry
- $\mathcal{L}_s = \int d^4\theta \frac{1}{2}(\mu U + g\Phi^\dagger \Phi)^2$

$$\implies \qquad \mathcal{L} + \mathcal{L}_s = \int d^4\theta \Phi^\dagger \Phi + \frac{\mu^2}{2} U^2 + \mu g U \Phi^\dagger \Phi$$

- EOM for U gives $U = -\frac{g}{\mu} \Phi^\dagger \Phi$
- works also with $\frac{m}{2}\Phi^2$ superpotential

- **U is a real superfield with tree-level mass μ**

$$U(x,\theta,\bar\theta) = \frac{C(x)}{\mu} + \sqrt{2}\theta\frac{\chi(x)}{\mu} + \sqrt{2}\bar\theta\frac{\bar\chi(x)}{\mu} + \theta\theta\frac{N(x)}{\mu} + \bar\theta\bar\theta\frac{N^*(x)}{\mu}$$
$$+\sqrt{2}\theta\sigma^\mu\bar\theta v_\mu(x) + \sqrt{2}\theta\theta\bar\theta\bar\lambda(x) + \sqrt{2}\bar\theta\bar\theta\theta\lambda(x) + \theta\theta\bar\theta\bar\theta D(x)$$

— **v_μ is a spin-1 vector field (not a gauge field)**

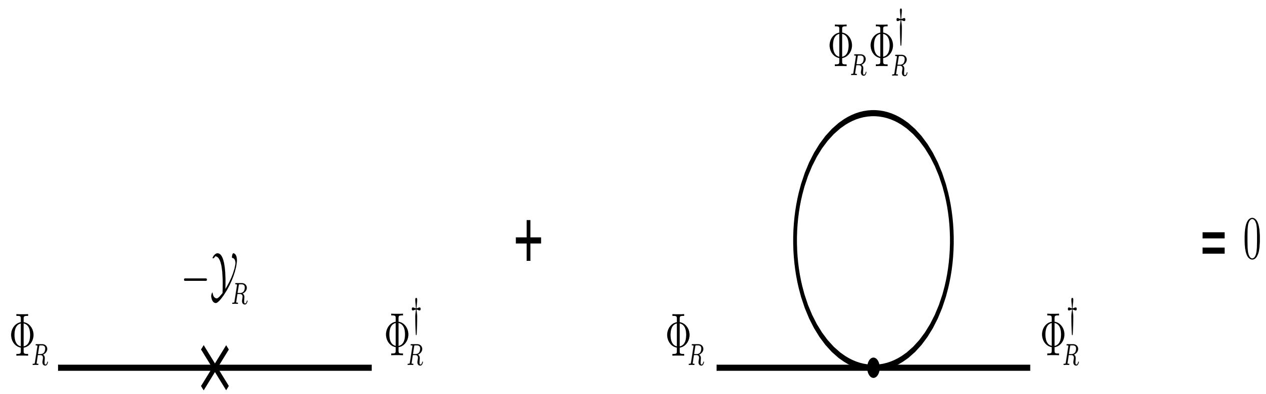
- **A-ψ-loop for $\chi\lambda$ mass cancels μ – massless Goldstino**

- **model with U like gauge multiplet with mass possible**

$$-\frac{g^2}{2}\frac{\Phi^\dagger\Phi\Phi^\dagger\Phi}{\sqrt{1+g^2\Phi^\dagger\Phi}} \quad \Rightarrow \quad U = -\frac{g}{\mu}\frac{\Phi^\dagger\Phi}{1+g^2\Phi^\dagger\Phi}$$

$$\mathcal{L}_{eff} = \int d^4\theta \;\; \tfrac{\mu^2}{2}U^2 + \Phi^\dagger\Phi\left[1 + (\mu g)U + \tfrac{(\mu g)^2}{2}U^2\right]$$

(Renormalized) Superfield Gap Equation :-



$$\mathcal{Y}_R = \frac{y}{1+y} - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2$$

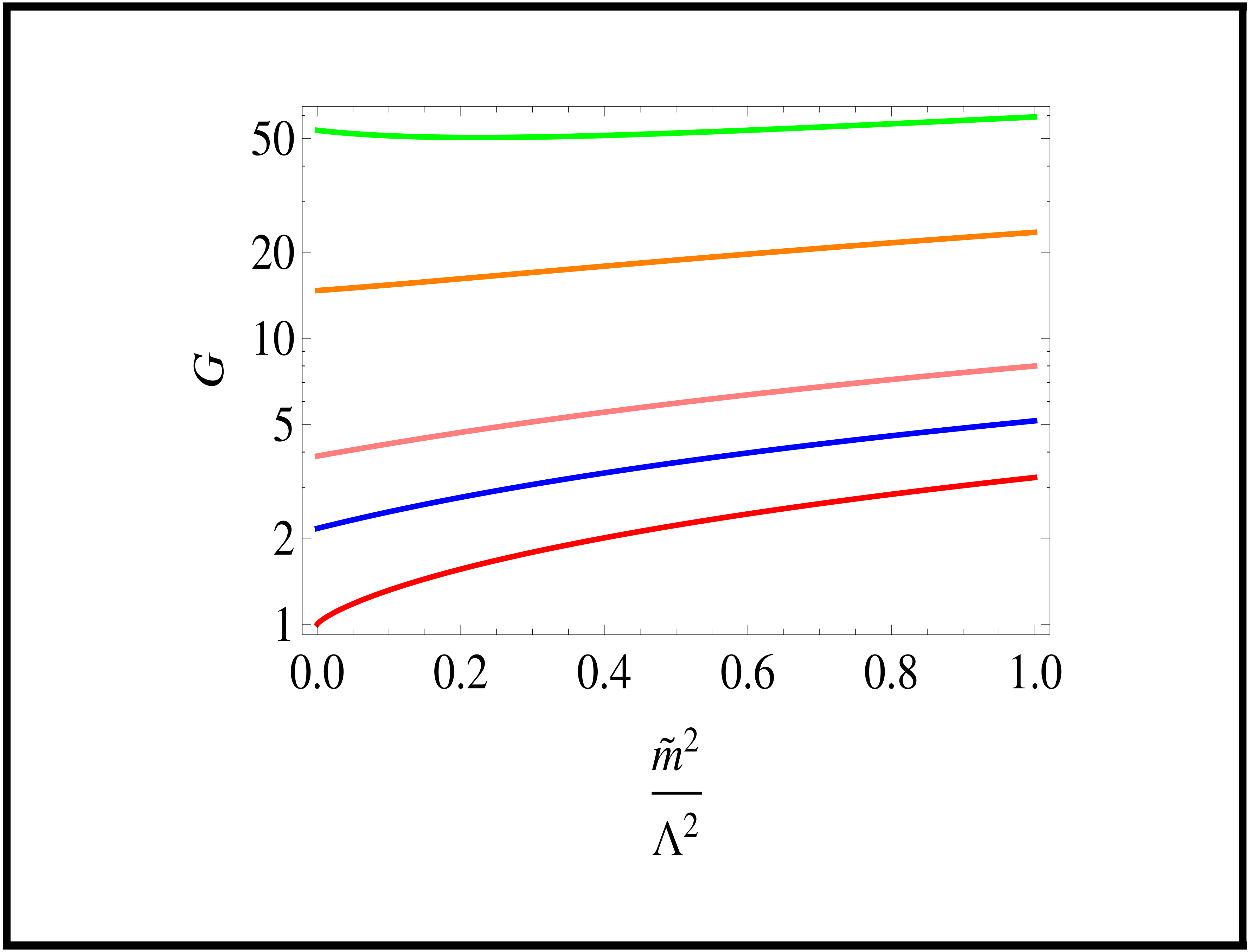
Analytical Gap Equations :-

$$\frac{y}{1+y} = -g^2 \int^E \frac{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2}$$

$$\tilde{\eta} = g^2 \tilde{\eta} \int^E \frac{(k^2 - |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2}$$

$$\tilde{m}^2 = g^2 \int^E \frac{1}{(k^2 + |m|^2)} \frac{1}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2} \cdot \left\{ \left[\tilde{m}^2 (k^2 - |m|^2) + 2k^2 |\tilde{\eta}|^2 \right] (k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) - 8k^2 |m|^2 |\tilde{\eta}|^2 \right\}$$

- need simultaneous solution for $\tilde{\eta}$ and $\tilde{m}^2$
- y is wavefunction renormalization function — no physical


50
20
10
5
2
1
G
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{\tilde{m}^2}{\Lambda^2}$

Holomorphic Vs Old Model (for MSSM) :-

• bottom together with (vs only) top mass at quasi-fixed point

⋆ both (vs one) Higgs superfields as composites

• large (vs small) $\tan\beta$

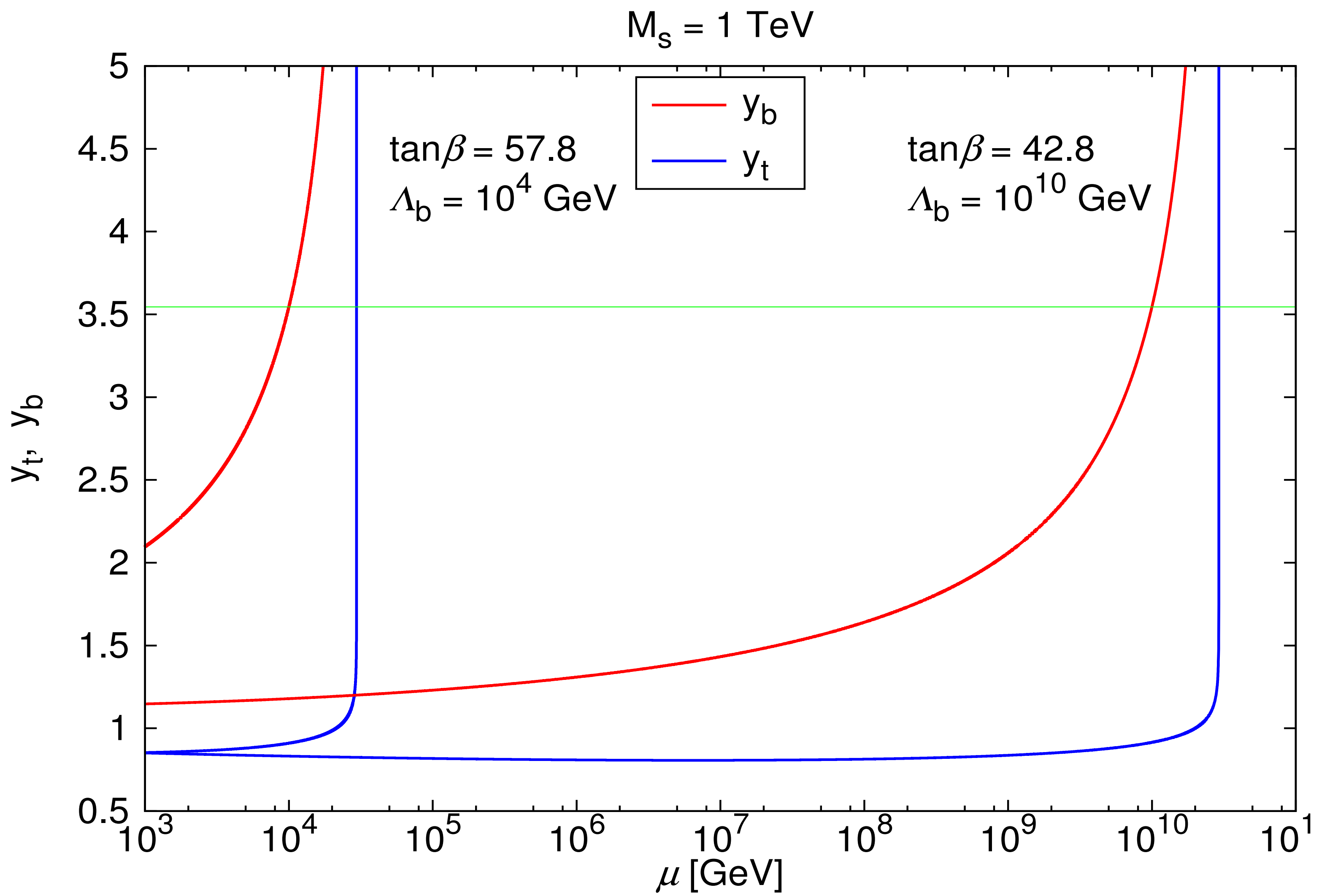
• $A_t \simeq A_b \simeq B \quad$ (vs $A_t \simeq 0$)

• $m^2_{H_d} \simeq -(m^2_Q + m^2_b + |A_b|^2)$
plus (vs only) $m^2_{H_u} \simeq -(m^2_Q + m^2_t + |A_t|^2)$

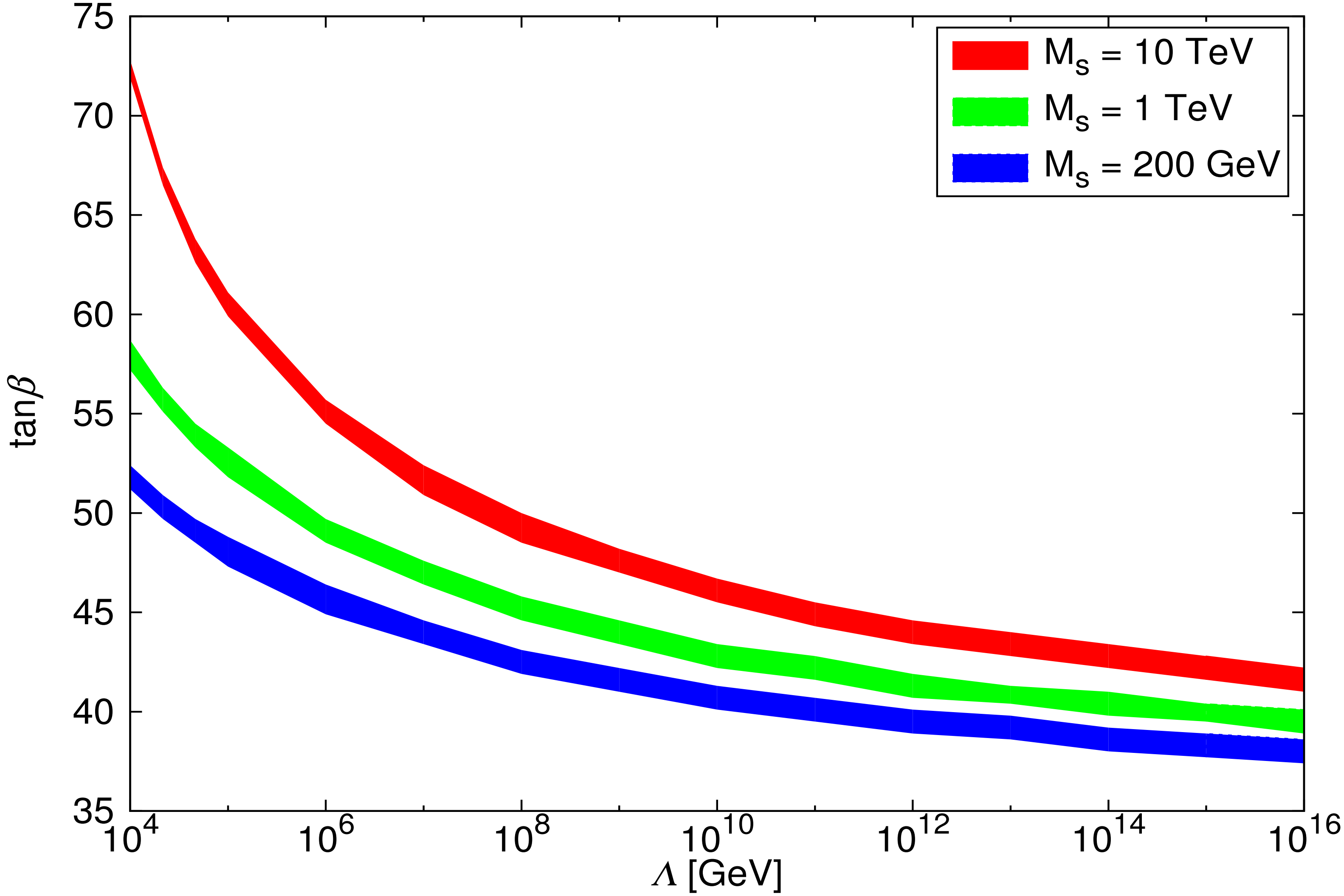
⋆ full W $[= G_{ijkh}\, Q_i U^c_j Q_k D^c_h (1 + A\theta^2) + G^e_{ij}\, Q_3 U^c_3 L_i E^c_j (1 + A\theta^2)]$
— non-holomorphic case needs similar holomorphic terms
for Yukawa couplings of down-type quarks and charged leptons

• sbottom and stop condensates for u_i and $d_i + \ell_i$ masses
(vs top condensate and stop condensates for u_i and $d_i + \ell_i$ masses)

Illustrative y_t and y_b :-



Our Solution :-



Concluding Remarks :-

- **looks like we can have SSM with *supersymmetry* and then EW symmetry broken dynamically**

- **(supersymmetric) chiral 3-family models like $N321$ may have extra symmetries dynamically broken**

- **mass pattern from operator suppression ?**

⋆ **please join the architectural firm**

3-family Models (with gauge-chiral fields ?) :-

• construction of minimal (?) chiral (fermion) spectrum
with extended (gauge) symmetry

• require consistent SM embedding
—— 1 fully chiral spectrum —— + **SSB**
$\Longrightarrow$ 3 SM families + vectorlike SM fermions

• note: extending embedding to kill all anomaly always possible
— spectrum may be huge (*esthetic !*)
— yielding new chiral SM fermion is phenomenologically fatal

• beyond $3N1$ stories, $N321$ models

O.K. MPLA430, PRD55 (97)

Back to Horizontal Symmetry

— $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$

	Scheme I $U(1)_Y$-states	*Scheme II* $U(1)_Y$-states
$(\mathbf{3},\mathbf{3},\mathbf{2})$	3 $\mathbf{1}(Q)$	3 $\mathbf{1}(Q)$
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{1})$	3 $\mathbf{2}(\bar{d})$	3 $\mathbf{-4}(\bar{u})$
$(\bar{\mathbf{3}},\mathbf{1},\mathbf{2},)$	3 $\mathbf{-3}(L)$	3 $\mathbf{-3}(L)$
$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	3 $\mathbf{-6}(\bar{E})$	3 $\mathbf{-12}(\bar{S}^{''})$
3 $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$	3 $\mathbf{-4}(\bar{u})$	3 $\mathbf{2}(\bar{d})$
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	3 $\mathbf{6}(E)$	3 $\mathbf{6}(E)$
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	3 $\mathbf{6}(E)$	3 $\mathbf{12}(S^{''})$

- simple gauge version of horizontal(/family) symmetry
- 3 SM families in one minimal chiral fermion spectrum

THANK YOU !