CMB Power Spectrum in Delta Gravity

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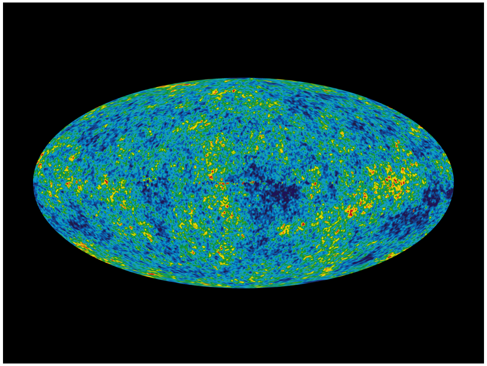
November 29, 2018

Outline

The CMB

- The Cosmic Microwave Backgroung (CMB) are photons which we can detect from all directions of space, whose distribution of temperature is practically isotropic: $T_{\mathbf{CMB}} \sim 2,725K$.

- The CMB is a “photograph” of the early universe, which corresponds with the period in which the photons decoupled from matter. This time corresponds to a Red-shift: $z \sim 1100$.

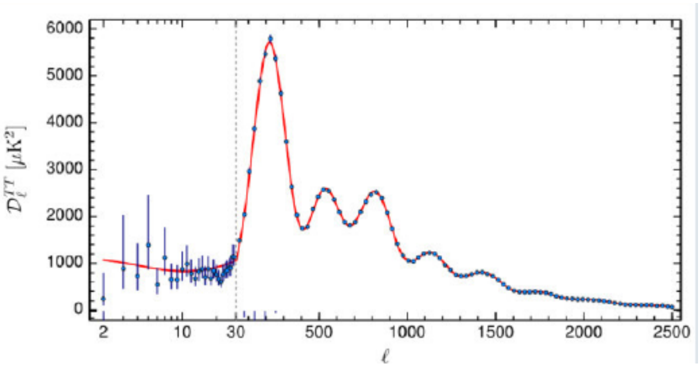


The CMB

- Its small anisotropies, $\Delta T \sim 10^{-4}$, provide valuable information about the formation and evolution of the universe.

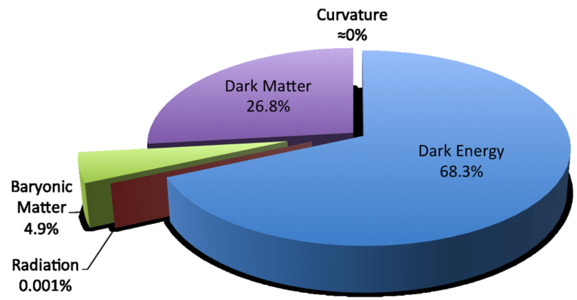
- This information is mainly extracted from the multipolar distribution of correlations in temperature anisotropies T, polarization E and B, ...

$$\langle \Delta T_l, \Delta T_l \rangle \propto D_l^{TT}$$



What do we know about the components of our Universe?

- Last discoveries in cosmology have revealed that most part of matter is in form of unknown matter, dark matter, and that the dynamics of the expansion of the Universe is governed by a mysterious component that accelerates its expansion, the so called dark energy.



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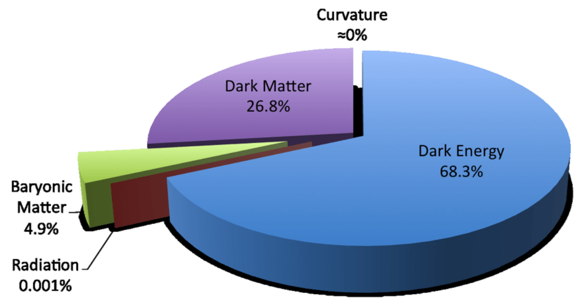

- Although General Relativity (GR) is able to accommodate both dark matter and dark energy, the interpretation of the dark sector in terms of fundamental theories of elementary particles is problematic.

What do we know about our Universe?

- There are some candidates that could play the role of dark matter, however none have been detected yet.

- In GR, dark energy can be explained if a small cosmological constant (Λ) is present. At early times, this constant is irrelevant, but at the later stages of the evolution of the Universe Λ will dominate the expansion, explaining the observed acceleration.

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- In order to understand the nature of dark energy in the context of a fundamental physical theory is that there has been various proposals to explain the observed acceleration of the Universe.

$\tilde{\delta}$ Gravity

- $\tilde{\delta}$ Gravity is a model of gravitation based on two symmetric tensors.
- In its construction, we consider an important point.
    - This theory is a type of gauge theories, $\tilde{\delta}$ gauge theories (DGT), which main properties are:
        - A new kind of field $\tilde{\phi}_I$ is introduced, different from the original set ϕ_I.
        - The action is obtained through the extension of the original gauge symmetry of the model, introducing an extra symmetry that we call $\tilde{\delta}$ symmetry, since it is formally obtained as the variation of the original symmetry.
- When we apply this prescription to GR we obtain $\tilde{\delta}$ Gravity.

Equation of motion

Let's consider the Einstein-Hilbert Action:

$$S_0 = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa} + L_M\right), \tag{1}$$

where $L_M = L_M(\phi_I, \partial_\mu)$ is the lagrangian of the matter fields ϕ_I.
Using the δ theories, this action becomes:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa} + L_M - \frac{1}{2\kappa}\left(G^{\alpha\beta} - \kappa T^{\alpha\beta}\right)\tilde{g}_{\alpha\beta} + \tilde{L}_M\right), \tag{2}$$

where $\kappa = \frac{8\pi G}{c^2}$, $\tilde{g}_{\mu\nu} = \tilde{\delta} g_{\mu\nu}$.

And:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \left[\sqrt{-g} L_M\right] \tag{3}$$

$$\tilde{L}_M = \tilde{\phi}_I \frac{\delta L_M}{\delta \phi_I} + (\partial_\mu \tilde{\phi}_I) \frac{\delta L_M}{\delta(\partial_\mu \phi_I)}, \tag{4}$$

where $\tilde{\phi}_I = \tilde{\delta}\phi_I$ are the $\tilde{\delta}$ matter fields. From this action, we can obtain the equations of motion of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$.

- The equations of motion are:

$$G^{\mu\nu} = \kappa T^{\mu\nu} \tag{5}$$

$$F^{(\mu\nu)(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} + \frac{1}{2} g^{\mu\nu} R^{\alpha\beta} \tilde{g}_{\alpha\beta} - \frac{1}{2} \tilde{g}^{\mu\nu} R = \kappa \tilde{T}^{\mu\nu}. \tag{6}$$

$\tilde{\delta}$ Gravity action and equation of motion

- On the other side, it is possible to demonstrate that:

$$\tilde{\delta}\left[G_{\mu\nu}\right] = F_{(\mu\nu)}^{(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} + \frac{1}{2} g_{\mu\nu} R^{\alpha\beta} \tilde{g}_{\alpha\beta} - \frac{1}{2} \tilde{g}_{\mu\nu} R. \quad (7)$$

This means that $(6)_{\mu\nu} = \tilde{\delta}\left[(5)_{\mu\nu}\right]$.

- Besides, we have two conservation rules:

$$D_\nu T^{\mu\nu} = 0 \quad (8)$$

$$D_\nu \tilde{T}^{\mu\nu} = \frac{1}{2} T^{\alpha\beta} D^\mu \tilde{g}_{\alpha\beta} - \frac{1}{2} T^{\mu\beta} D_\beta \tilde{g}^\alpha_\alpha + D_\beta(\tilde{g}^\beta_\alpha T^{\alpha\mu}). \quad (9)$$

It is easy to see that (9) is $\tilde{\delta}\left(D_\nu T^{\mu\nu}\right) = 0$. In conclusion, the equations of our model are (5), (6), (8) and (9).

Perfect Fluid

- The energy-momentum tensors are

$$T_{\mu\nu} = p(\rho)g_{\mu\nu} + (\rho + p(\rho))\,U_\mu U_\nu \tag{10}$$

$$\tilde{T}_{\mu\nu} = p(\rho)\tilde{g}_{\mu\nu} + \frac{\partial p}{\partial \rho}(\rho)\tilde{\rho} g_{\mu\nu} + \left(\tilde{\rho} + \frac{\partial p}{\partial \rho}(\rho)\tilde{\rho}\right) U_\mu U_\nu + (\rho + p(\rho))\left(\frac{1}{2}\left(U_\nu U^\alpha \tilde{g}_{\mu\alpha} + U_\mu U^\alpha \tilde{g}_{\nu\alpha}\right) + U^T_\mu U_\nu + U_\mu U^T_\nu\right) \tag{11}$$

  with $U_\alpha = e^a{}_\alpha u_a$ and $U^\alpha_T = e^{\alpha a}\tilde{u}_a$, where $e^a{}_\alpha$ is the Veirbein.
- Now, we can use (10) and (11) to solve (5), (6), (8) and (9) for a perfect fluid.

Test Particles and Harmonic Gauge

- When we apply $\tilde{\delta}$ prescription to study test particles we found:
  - Free massive particles do not follow a geodesic.
  - Massless particles trajectories are null geodesics of an effective metric $\mathbf{g}_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$.
- Beside, in this prescription the harmonic gauge is extended, and for FLRW the effective metric for photons is

$$\begin{aligned} \mathbf{g}_{\mu\nu} &= g_{\mu\nu} + \tilde{g}_{\mu\nu} \\ &= -(1+3F_a(t))c^2dt^2 + R^2(t)(1+F_a(t))(dx^2+dy^2+dz^2) \quad (12) \end{aligned}$$

Photon Trajectory, Luminosity Distance and Angular Distance

When a photon emitted from a supernova travels to the Earth, the Universe is expanding. This means that the photon is affected by the cosmological Doppler effect. For this, let's use a null geodesic in a radial trajectory from r_1 to $r = 0$. So we have

$$-(1 + 3F_a(t))c^2dt^2 + R^2(t)(1 + F_a(t))dr^2 = 0.$$

In GR, we have that $cdt = -R(t)dr$. So, in the $\tilde{\delta}$ Gravity case, we can define the effective scale factor:

$$\tilde{R}(t) = R(t)\sqrt{\frac{1 + F_a(t)}{1 + 3F_a(t)}} \qquad (13)$$

such that $cdt = -\tilde{R}(t)dr$ now.

Photon Trajectory, Luminosity Distance and Angular Distance

If we integrate this expression from r_1 to 0, we obtain:

$$r_1 = c \int_{t_1}^{t_0} \frac{dt}{\tilde{R}(t)}, \tag{14}$$

where t_1 and t_0 are the emission and reception times.
So, the redshift is now:

$$1 + z(t_1) = \frac{\tilde{R}\,(t_0)}{\tilde{R}(t_1)}. \tag{15}$$

We see that $\tilde{R}\,(t)$ replaces the usual scale factor $R(t)$ to compute z.

Photon Trajectory, Luminosity Distance and Angular Distance

Furthermore, one can show that the luminosity distance is given by:

$$d_L = c\frac{\tilde{R}^2(t_0)}{\tilde{R}(t_1)}\int_{t_1}^{t_0}\frac{dt}{\tilde{R}(t)}\,. \tag{16}$$

And the angular distance is:

$$\begin{aligned} d_A &= \frac{\tilde{R}^2(t_1)}{\tilde{R}^2(t_0)}d_L \\ &= \frac{d_L}{(1+z_1)^2}\,. \end{aligned} \tag{17}$$

Therefore, the relation between d_A and d_L is the same to GR.

The CMB

In order to compute the scalar contributions we will assume both important facts:

- Hydrodynamic limit: near the time of recombination the rate of collisions of photons with free electrons was so great that photons were in local thermal equilibrium with the baryonic plasma, and so photons at these time can be treated hydro-dynamically.
- Sharp transition from thermal equilibrium to complete transparency at a moment t_L of last scattering.

The CMB

It is possible to derive formulas for the scalar temperature fluctuations by following photon trajectories, without needing to use the Boltzmann equation formalism.
We write the perturbed metric in the form

$$\begin{aligned} \bar{\mathbf{g}}_{00} &= -((1+3F(t))c^2 + E(\mathbf{x},t) + \tilde{E}(\mathbf{x},t)) \\ \bar{\mathbf{g}}_{i0} &= 0 \\ \bar{\mathbf{g}}_{ij} &= R^2(t)(1+F(t))\delta_{ij} + h_{ij}(\mathbf{x},t) + \tilde{h}_{ij}(\mathbf{x},t) \ , \end{aligned} \tag{18}$$

where

$$\begin{aligned} h_{ij} &= (1+F)R^2\left[A\delta_{ij} + \frac{\partial^2 B}{\partial x^i \partial x^j}\right] \\ \tilde{h}_{ij} &= (1+F)R^2\left[\tilde{A}\delta_{ij} + \frac{\partial^2 \tilde{B}}{\partial x^i \partial x^j}\right] \end{aligned} \tag{19}$$

The CMB

The redshift is now

$$\frac{\nu_0}{\nu_L} = \frac{\delta\tau_L}{\delta\tau_0} = \frac{\tilde{R}(t_L)}{\tilde{R}(t_0)} \left[1 + \frac{1}{2} \left(E(r_L\hat{n}, t) - E(0, t_0) \right) \right.$$
$$\left. - \int_{t_L}^{t_0} \left(\frac{\partial}{\partial t} N(r\hat{n}, t) \right)_{r=s(t)} dt - \tilde{R}(t)(\delta u^r_\gamma(r_L\hat{n}, t) + \delta\tilde{u}^r_\gamma(r_L\hat{n}, t)) \right] \quad (20)$$

Where

$$N = \frac{1}{2} \left[A + \frac{\partial^2 B}{\partial r^2} + \left(\tilde{A} + \frac{\partial^2 \tilde{B}}{\partial r^2} \right) - \frac{E}{1+3F} - \frac{\tilde{E}}{1+3F} \right] \quad (21)$$

The CMB

The temperature observed at the present time t_0 coming from direction $\hat{n}$ is

$$T(\hat{n}) = \left(\frac{\nu_0}{\nu_L}\right)\left(\bar{T}(t_L) + \delta T(r_L\hat{n}, t_l)\right) . \tag{22}$$

Likewise, in absence of perturbations the temperature observed in all directions would be

$$T_0 = \left(\frac{\tilde{R}(t_L)}{\tilde{R}(t_0)}\right)\bar{T}(t_L) , \tag{23}$$

so the fractional shift in the radiation temperature observed is

$$\begin{aligned} \frac{\Delta T(\hat{n})}{T_0} &\equiv \frac{T(\hat{n}) - T_0}{T_0} = \frac{\nu_0 \tilde{R}(t_0)}{\nu_L \tilde{R}(t_L)} - 1 + \frac{\delta T(r_L\hat{n}, t_L)}{\bar{T}(t_L)} \\ &= \frac{1}{2}\left(E(r_L\hat{n}, t) - E(0, t_0)\right) - \int_{t_L}^{t_0} dt \left(\frac{\partial}{\partial t} N(r\hat{n}, t)\right)_{r=s(t)} \\ &- \tilde{R}(t)(\delta u^r_\gamma(r_L\hat{n}, t) + \delta \tilde{u}^r_\gamma(r_L\hat{n}, t)) + \frac{\delta T(r_L\hat{n}, t_L)}{\bar{T}(t_L)} . \end{aligned} \tag{24}$$

The CMB

After a few steps more we get

$$\left(\frac{\Delta T(\hat{n})}{T_0}\right)^S = \left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{early}} + \left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{late}} + \left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{ISW}} \quad (25)$$

where the three terms are Gauge Invariant!

Finnally, the scalar coefficients of temperature fluctuations are given by

$$C_{TT,l}^{\text{obs}} = \frac{1}{4\pi}\int d^2\hat{n}\int d^2\hat{n}' P_l(\hat{n}\cdot\hat{n}')\Delta T(\hat{n})\Delta T(\hat{n'}), \tag{26}$$

with a little of algebra, we get

$$\begin{aligned} l(l+1)C_{TT,l}^{S} &= \frac{8\pi^2 T_0^2 l^3}{r_L^3}\int_1^\infty \frac{\beta d\beta}{\sqrt{\beta^2-1}} \\ &\times\left[\left(F\left(\frac{l\beta}{r_L}\right)+\tilde{F}\left(\frac{l\beta}{r_L}\right)\right)^2\right. \\ &+ \left.\frac{\beta^2-1}{\beta^2}\left(G\left(\frac{l\beta}{r_L}\right)+\tilde{G}\left(\frac{l\beta}{r_L}\right)\right)^2\right]. \end{aligned} \tag{27}$$

The CMB

Where

$$F(q) = -\frac{1}{2}\tilde{R}^2(t)\ddot{B}_q(t_L) - \frac{1}{2}\tilde{R}(t)\dot{\tilde{R}}(t_L)\dot{B}_q(t_L) + \frac{1}{2}E_q(t_L) + \frac{\delta T_q(t_L)}{\bar{T}(t_L)} \quad (28)$$

$$\tilde{F}(q) = -\frac{1}{2}\tilde{R}^2(t)\ddot{\tilde{B}}_q(t_L) - \frac{1}{2}\tilde{R}(t_L)\dot{\tilde{R}}(t_L)\dot{\tilde{B}}_q(t_L) \quad (29)$$

$$G(q) = -q\left(\frac{1}{2}\tilde{R}(t_L)\dot{B}_q(t_L) + \frac{1}{(1+3F(t_L))\tilde{R}(t_L)}\delta u_\gamma(t_L)\right) \quad (30)$$

$$\tilde{G}(q) = -q\left(\frac{1}{2}\tilde{R}(t_L)\dot{\tilde{B}}_q(t_L) + \frac{1}{(1+3F(t_L))\tilde{R}(t_L)}\delta\tilde{u}_\gamma(t_L)\right), \quad (31)$$

are the so called form factors.

Where to go next...

- We are computing the equations of motion for all the perturbations.
- For this task we need to choose a gauge, we are using the Synchronous gauge.
- As an approximation, we are interested at the the time of last scattering t_L, when the Universe was dominated by matter, so we can solve the equations for perturbations at that regime.

Summary

- Due the problems in GR with the interpretation of the dark sector in terms of fundamental theories of elementary particles, is suggestive try new theories.

[1] Riess, A. *et al.* Astrophys. J. 826 (2016) no. 1, 56 axXiv:1604.01424

Summary

- Due the problems in GR with the interpretation of the dark sector in terms of fundamental theories of elementary particles, is suggestive try new theories.
- We present a new theory, called Delta Gravity (DG), based in two symmetric tensors.

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Summary

- Due the problems in GR with the interpretation of the dark sector in terms of fundamental theories of elementary particles, is suggestive try new theories.
- We present a new theory, called Delta Gravity (DG), based in two symmetric tensors.
- DG does not need dark energy in order to explain the observed acceleration of the Universe. The age of the Universe is in accord with the accepted actual age of the Universe predicted by ΛCDM model. Also the Hubble constant is in accord with all the observed values until now, even with the last Hubble Space Telescope result $H_0 = 73.02 \pm 1.79$ km/(s Mpc)[1], where ΛCDM is out of error range.

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- Because we add a new kind of matter, delta matter, it is important to test the theory with different measurements such as supernovae data and the CMB, this will help us to test the theory and establish how important is this new matter.
- We are working on it!

Thank you!

Scalar contributions to the CMB

$$\begin{aligned}
\left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{early}} &= -\frac{1}{2}\tilde{R}(t_L)\dot{\tilde{R}}(t_L)\dot{B}(r_L\hat{n},t_L) - \frac{1}{2}\tilde{R}^2(t_L)\ddot{B}(r_L\hat{n},t_L) \\
&+ \frac{1}{2}E(r_L\hat{n},t_L) + \frac{\delta T(r_L\hat{n})}{\bar{T}(t_L)} \\
&- \tilde{R}(t_L)\left[\frac{\partial}{\partial r}\left(\frac{1}{2}\dot{B}(r\hat{n},t_L) + \frac{1}{(1+3F)\tilde{R}^2(t_L)}\delta u_\gamma(r\hat{n},t_L)\right)_{r=r_L}\right] \\
&- \left\{\left(\frac{1}{2}\tilde{R}(t_L)\dot{\tilde{R}}(t_L)\dot{\tilde{B}}(r_L\hat{n},t_L) + \frac{1}{2}\tilde{R}^2(t_L)\ddot{\tilde{B}}(r_L\hat{n},t_L)\right)\right. \\
&+ \left.\tilde{R}(t_L)\left[\frac{\partial}{\partial r}\left(\frac{1}{2}\dot{\tilde{B}}(r\hat{n},t_L) + \frac{1}{(1+3F)\tilde{R}^2(t_L)}\delta\tilde{u}_\gamma(r\hat{n},t_L)\right)_{r=r_L}\right]\right\} \qquad (32)
\end{aligned}$$

Scalar contributions to the CMB

$$
\begin{aligned}
\left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{late}} &= \frac{1}{2}\tilde{R}(t_0)\dot{\tilde{R}}(t_0)\dot{B}(0,t_0) + \frac{1}{2}\tilde{R}^2(t_0)\ddot{B}(0,t_0) - \frac{1}{2}E(0,t_0) \\
+ & \quad \tilde{R}(t_0)\left[\frac{\partial}{\partial r}\left(\frac{1}{2}\dot{B}(r\hat{n},t_0) + \frac{1}{(1+3F)\tilde{R}^2(t_0)}\delta u_\gamma(r\hat{n},t_0)\right)_{r=0}\right] \\
+ & \quad \left\{\left(\frac{1}{2}\tilde{R}(t_0)\dot{\tilde{R}}(t_0)\dot{\tilde{B}}(0,t_0) + \frac{1}{2}\tilde{R}^2(t_0)\ddot{\tilde{B}}(0,t_0)\right)\right. \\
+ & \quad \left.\tilde{R}(t_0)\left[\frac{\partial}{\partial r}\left(\frac{1}{2}\dot{\tilde{B}}(r\hat{n},t_0) + \frac{1}{(1+3F)\tilde{R}^2(t_0)}\delta\tilde{u}_\gamma(r\hat{n},t_0)\right)_{r=r_L}\right]\right\} \qquad (33)
\end{aligned}
$$

Scalar contributions to the CMB

$$
\begin{aligned}
\left(\frac{\Delta T(\hat{n})}{T_0}\right)^S_{\text{ISW}} &= -\frac{1}{2}\int_{t_l}^{t_0} dt \left\{ \frac{\partial}{\partial t}\left[\tilde{R}^2(t)\ddot{B}(r\hat{n},t) + \tilde{R}(t)\dot{\tilde{R}}(t)\dot{B}(r\hat{n},t)\right.\right. \\
&+ A(r\hat{n},t) - \frac{E(r\hat{n},t)}{1+3F(t)} \\
&+ \tilde{R}^2(t)\ddot{\tilde{B}}(r\hat{n},t) + \tilde{R}(t)\dot{\tilde{R}}(t)\dot{\tilde{B}}(r\hat{n},t) + \tilde{A}(r\hat{n},t) \\
&- \left.\left.\frac{\tilde{E}(r\hat{n},t)}{1+3F}\right]\right\} \qquad (34)
\end{aligned}
$$