Renormalization and Unitarity in Higher-Order Lorentz-Violating Models

Carlos M. Reyes

Centro de Ciencias Exactas-UBB, Chillán, Chile

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Outline

Lorentz and CPT symmetry violation

At m_P several conceptual issues arise and several theoretical limitations put in question the perturbative scheme of the standard model $\rightarrow$ need for a more fundamental theory or **quantum gravity**.

At these energies,

a) The continuum of spacetime may be lost and a sort of discreetness or spacetime foam may appear.

b) Compactified extra dimensions may become important.

c) Supersymmetry.

$\vdots$

However, all these effects are very suppressed at low energies and it is unrealistic today to reach these energies close to the Planck scale. The possibility of **Lorentz invariance violation** provides an alternative route to test quantum gravity effects with ultrahigh sensitivity experiments.

Effective framework

A **general framework** within the language of **effective field theory**, the standard model extension (SME), has been proposed to to describe Lorentz symmetry violation in all the sectors of the standard model and gravity.

Several experimental tests provide a robust framework to study Lorentz violations. The SME can be classified according to:

- Minimal sector with mass dimension $d \leq 4$ [D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998); S. R. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).]
- Non minimal sector with **higher-order operators**, $d > 4$, [R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003); V. A. Kostelecky and M. Mewes, Phys. Rev. D **80**, 015020 (2009); Phys. Rev. D **85**, 096005 (2012); M. Schreck, Phys. Rev. D **93**, no. 10, 105017 (2016).]

Alternatively, studies of CPT and Lorentz-invariance violation have been given in

- Modified dispersion relations [G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Nature **393**, 763 (1998).]
- String/M theory [V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); V. A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991).]
- Loop quantum gravity [R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999); J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000); H. Sahlmann and T. Thiemann, Class. Quant. Grav. **23**, 867 (2006).]
- Horava gravity [P. Horava, Phys. Rev. D **79**, 084008 (2009).]

Lorentz violating extensions

The Lorentz-invariance violating (LIV) terms in the Lagrangian can be classified according to its mass dimension d

$$S = \int d^4x \left(\mathcal{L}^{SM} + \underbrace{\delta\mathcal{L}_{LIV}^{(d=3,4)}}_{\text{minimal extension}} + \underbrace{\delta\mathcal{L}_{LIV}^{(d=5)} + \dots}_{\text{non-minimal}} \right) . \quad (1)$$

With experiments the LIV parameters can be bounded [V. A. Kostelecky and N. Russell, Rev. Mod. Phys. **83**, 11 (2011)],

Table D13. Photon sector, $d = 3$

Combination	Result	System
k_{AF}^{Z}	$< 10^{-28}$ GeV	PVLAS
"	$< 10^{-19}$ GeV	Hydrogen spectroscopy
$\lvert k_{(V)10}^{(3)} \rvert$	$< 16 \times 10^{-21}$ GeV	Schumann resonances
$\lvert k_{(V)11}^{(3)} \rvert$	$< 12 \times 10^{-21}$ GeV	"
$\lvert \mathbf{k}_{\mathbf{AF}}^{(3)} \rvert \equiv \left(6 \lvert k_{(V)11}^{(3)} \rvert^2 + 3 \lvert k_{(V)10}^{(3)} \rvert^2 \right)^{1/2} / \sqrt{4\pi}$	$(10^{+4}_{-8}) \times 10^{-43}$ GeV	CMB polarization
$\lvert \mathbf{k}_{\mathbf{AF}}^{(3)} \rvert$	$(15 \pm 6) \times 10^{-43}$ GeV	"
$\lvert \mathbf{k}_{\mathbf{AF}}^{(3)} \rvert$	$(0.57 \pm 0.70) H_0$	Astrophysical birefringence
$\lvert k_{(V)00}^{(3)} \rvert$	$< 14 \times 10^{-21}$ GeV	Schumann resonances
$k_{(V)00}^{(3)}$	$(1.1 \pm 1.3 \pm 1.5) \times 10^{-43}$ GeV	CMB polarization

The Chern-Simons and axial term [S. M. Carroll, G. B. Field and R. Jackiw, Phys. Rev. D **41**, 1231 (1990); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).]

$$\begin{aligned} \delta\mathcal{L}^{(3)}_{photon} &= -\frac{1}{4} k_\mu \epsilon^{\mu\nu\rho\lambda} A_\nu F_{\rho\lambda}\,, \\ \delta\mathcal{L}^{(3)}_{fermion} &= \bar{\psi} b_\mu \gamma^5 \gamma^\mu \psi\,. \end{aligned} \quad (2)$$

The Myers and Pospelov model [R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003)]

$$\begin{aligned} \delta\mathcal{L}^{(5)}_{fermion} &= \frac{1}{m_P} \bar{\psi} (\eta_1 \not{n}_1 + \eta_2 \not{n}_2 \gamma_5)(n\cdot\partial)^2 \psi\,, \\ \delta\mathcal{L}^{(5)}_{photon} &= \frac{\xi}{2m_P} n_\mu \epsilon^{\mu\nu\rho\lambda} A_\nu (n\cdot\partial)^2 F^{\rho\lambda}\,. \end{aligned} \quad (3)$$

In general the Lorentz symmetry breakdown is implemented with a preferred four vector b^μ, k^μ, n^μ, which is believed to arise from expectation values of tensor fields in an underlying theory.

Higher-order time derivative theories

Theoretically, what to expect in the presence of higher-order Lorentz violation?

1) More degrees of freedom.

2) Ghost solutions with nonstandard momentum dependence and nontrivial behavior in the complex energy plane.

3) Improved UV divergencies in the QFT.

$\Delta = \frac{1}{p^2 - m^2 - p^4/M^2} = \frac{1}{p^2 - m_1^2} - \frac{1}{p^2 - m_2^2}$, We have two poles, one at $p^2 = m_1^2(m, M)$ and the other at $p^2 = m_2^2(m, M)$.

4) An indefinite metric in Hilbert space η, which may lead to a pseudo-unitary condition for the S matrix, i.e., $S^\dagger \eta S = \eta$. [T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); T. D. Lee, G. C. Wick, Phys. Rev. D2, 1033 (1970).].

5) Non renormalizability, Large Lorentz violations [J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, Phys. Rev. Lett. **93** (2004) 191301.], modified asymptotic Hilbert space [R. Potting, Phys. Rev. D **85**, 045033 (2012); M. Cambiaso, R. Lehnert and R. Potting, Phys. Rev. D **90**, no. 6, 065003 (2014)].

In this talk we will show some approaches to prove unitarity and studies on renormalization in effective field theories with higher-order Lorentz invariance violation.

The optical theorem

The main tool to prove perturbative unitarity in our models is the unitarity constraint given by the optical theorem. Expand the S-matrix, $SS^\dagger$ as $S = 1 + iT$

$$-i(T - T^\dagger) = T^\dagger T. \tag{4}$$

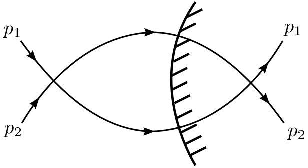
Take the expectation value between initial states $|i\rangle$ and final states $\langle f|$ and inserting a complete set of intermediate states $\langle m|$ rewrite the above equation as

$$\langle f| T |i\rangle - \langle f| T^\dagger |i\rangle = i \sum_m \int d\Pi_m \langle f| T^\dagger |m\rangle \langle m| T |i\rangle \, . \tag{5}$$

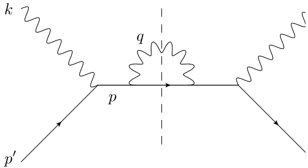
We write $\mathcal{M}_{fi} - \mathcal{M}^*_{if} = i \sum_m \int d\Pi_m \mathcal{M}_{fm} \mathcal{M}^*_{im}$, and in the special case of forward scattering $f = i$ one has

$$2\,\mathrm{Im}(\mathcal{M}_{ii}) = \sum_m \int d\Pi_m \, |\mathcal{M}_{im}|^2 \, . \tag{6}$$

Graphically, the cut equations are given by the diagrams



(a) $\frac{\lambda\phi^4}{4!}$ interaction term.



(b) QED electron self energy.

Note that the loops contain the negative-metric states, however, they should not appear in the physical (intermediate) sum of states.

The model [CMR and L. F. Urrutia, Phys. Rev. D **95**, no. 1, 015024 (2017) ;J. R. Nascimento, A. Y. Petrov and CMR, Eur. Phys. J. C **78**, no. 7, 541 (2018)]

Let us focus on the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\,\psi + g\bar{\psi}\not{n}(n \cdot \partial)^2\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{7}$$

where n is a privileged four-vector and g is a small parameter. We fix $n = (1, 0, 0, 0)$ which yields the equation of motion

$$\left(i\gamma^\mu\partial_\mu - m + g\gamma^0\partial_0^2\right)\psi(x) = 0.$$

The dispersion relations is

$$(p_0 - gp_0^2)^2 - \vec{p}^2 - m^2 = 0. \tag{8}$$

and solving we find the four solutions

$$\omega_1 = \tfrac{1-\sqrt{1-4gE}}{2g}, \qquad \omega_2 = \tfrac{1-\sqrt{1+4gE}}{2g},$$
$$W_1 = \tfrac{1+\sqrt{1-4gE}}{2g}, \qquad W_2 = \tfrac{1+\sqrt{1+4gE}}{2g},$$

where $E = \sqrt{\vec{p}^2 + m^2}$.

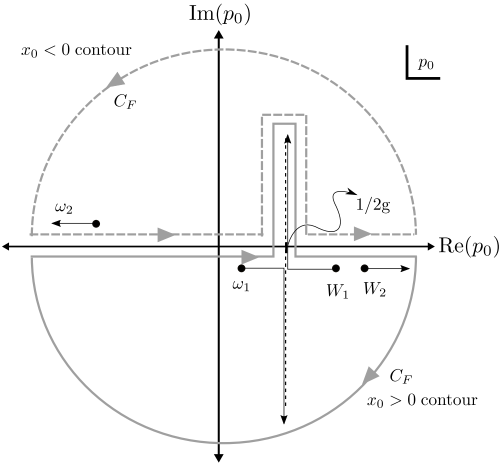


Figure 1: The poles in the complex plane

Implementation: Lee-Wick prescription

Some central points to satisfy the perturbative constraint are

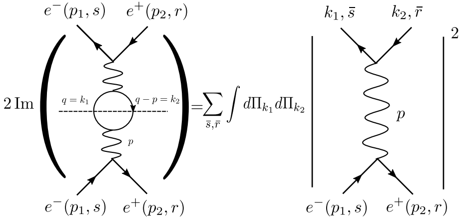


Figure 2: The scattering diagram

1) The residues in the **loop** diagram are computed with an Euclidean Wick rotated contour path.

2) The sum over physical states in the **cut** diagram are carried out only over positive metric states. This is the Lee-Wick prescription

We should compare the imaginary part of the integral

$$M(p) = \frac{-e^4 J_1^\mu(p) J_2^\nu(p)}{p^4} \int \frac{d^3q}{(2\pi)^4}\ Tr\left(\gamma_\mu(\not{R}+m)\gamma_\nu(\not{Q}+m)\right) I(q_0,p_0)\,,$$

where $R^\mu = ((p_0+q_0)(1-g(p_0+q_0)), \vec{q})$, $J_1^\mu(p) = \bar{v}^r(p_2)\gamma^\mu u^s(p_1)$
$J_2^\nu(p) = \bar{u}^s(p_1)\gamma^\nu v^r(p_2)$ and

$$\begin{aligned} I(q_0,p_0) &= -i\int_C \frac{dq_0}{g^4(q_0-\omega_1)(q_0-\omega_2)(q_0-W_1)(q_0-W_2)} \\ &\times \frac{1}{(q_0-p_0-\omega_1)(q_0-p_0-\omega_2)(q_0-p_0-W_1)(q_0-p_0-W_2)}. \end{aligned}$$

with the amplitude

$$\mathcal{A} = \sum_{\text{phys}\to\omega_1,\omega_2}\ \sum_{r,r'} \int \frac{d^3k_2}{(2\pi)^3}\frac{1}{E_2}\frac{d^3k_1}{(2\pi)^3}\frac{1}{E_1}(2\pi)^4\delta^4(k_2+k_1-p)|M|^2\,,$$

where according to the Lee-Wick prescription the sum is only over positive metric states.

Renormalization and asymptotic states

It has been shown that in the presence of Lorentz invariance violation new operators induced via radiative corrections in the effective Lagrangians may modify the pole masses of the two-point functions. [R. Potting, Phys. Rev. D **85**, 045033 (2012); M. Cambiaso, R. Lehnert and R. Potting, Phys. Rev. D **90**, no. 6, 065003 (2014)].

To explain the idea consider the standard Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + \lambda\bar{\psi}\phi\psi + \psi(i\partial\!\!\!/ - M)\psi\,, \tag{9}$$

and let us compute the one-loop radiative correction to the scalar self energy Σ_2.

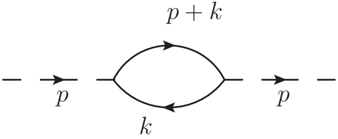


Figure 3: Scalar self energy

The standard computation gives divergencies proportional to p^2 and M^2 which can be cancelled by counterterms.
One can write the two-point function (or Green function) as

$$G(p^2) = \frac{1}{p^2 - m_R^2 + \Sigma(p^2)}\,, \tag{10}$$

with

$$\Sigma(p^2) = \Sigma_2(p^2) + p^2\delta_\phi - (\delta_\phi + \delta_m)m_R^2\,, \tag{11}$$

The on-shell conditions are

$$\Sigma(m_{Pole}^2) = m_R^2 - m_{Pole}^2\,, \qquad \left.\frac{d\Sigma(p^2)}{dp^2}\right|_{p^2 = m_{Pole}^2} = 0 \tag{12}$$

1) The pole mass m_{Pole} defines the mass of asymptotic states.
2) In the presence of Lorentz violations one has a different structure that may lead to modifications in the renormalization conditions and pole mass.

Renormalization

We focus on the Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + \bar{\psi}\left(i\not\partial - m\right)\psi + g_2\bar{\psi}\not n(n\cdot\partial)^2\psi + g\bar{\psi}\phi\psi\,. \quad (13)$$

We impose the simplification of considering $m = M$ and choose the preferred four-vector to be purely timelike $n = (1, 0, 0, 0)$.

We have the usual dispersion relation for the scalar with solutions $p_0 = \pm\sqrt{\vec{p}^2 + m^2}$, and the modified ones for the fermion.

We will compute the diagram

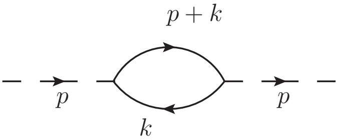


Figure 4: Scalar self energy

Radiative corrections

In this way we have

$$\Pi(p) = -\tfrac{g^2}{2}\phi(-p)\phi(p)\int \tfrac{d^4k}{(2\pi)^4}\tfrac{\mathrm{Tr}((Q_\mu\gamma^\mu+m)(R_\nu\gamma^\nu+m))}{(Q^2-m^2)(R^2-m^2)}\,, \tag{14}$$

where we define

$$\begin{aligned} Q_\mu &= k_\mu - g_2 n_\mu (n\cdot k)^2\,, \\ R_\mu &= k_\mu + p_\mu - g_2 n_\mu (n\cdot(k+p))^2\,. \end{aligned} \tag{15}$$

$$\Pi(p) = \Pi(0) + p_\mu\left(\tfrac{\partial\Pi}{\partial p_\mu}\right)_{p=0}\tfrac{1}{2}p_\mu p_\nu\left(\tfrac{\partial^2\Pi}{\partial p_\mu\partial p_\nu}\right)_{p=0} + \ldots\,. \tag{16}$$

The scalar self energy

Finally and after a lengthy calculation, one arrives at

$$\Pi(p) \;=\; -2g^2m^2q_0 - 2g^2p^2q_1 - 2g^2(n\cdot p)^2q_n\,, \tag{17}$$

with

$$\begin{aligned} q_0 &= -\tfrac{i}{48\pi^2 g_2^2 m^2} + \tfrac{i}{48\pi^2}\left(6\gamma_E - 0,46 + 12i\pi - 18\ln\left(\tfrac{g_2 m}{2}\right)\right)\,, \\ q_1 &= -\tfrac{i}{2\pi^2}\left(i\pi - \ln\left(\tfrac{g_2 m}{2}\right) - \tfrac{1}{3}\right)\,, \\ q_n &= \tfrac{i}{\pi^2}\,. \end{aligned} \tag{18}$$

The contribution for the fermion self energy follows in the along similar lines.

Renormalization conditions

We start with

$$\mathcal{L} \;=\; \tfrac{1}{2}\partial_\mu \phi_R \partial^\mu \phi_R - \tfrac{1}{2} m_R^2 \phi_R^2 \,, \tag{19}$$

Consider the renormalized two-point function

$$(\Gamma_R^{(2)})^{-1} = p^2 - m_R^2 + \Pi_R(p) \,, \tag{20}$$

where

$$\Pi_R(p) = p^2 A_\phi + m_R^2 B_\phi + (n \cdot p)^2 C_\phi \,, \tag{21}$$

and

$$\begin{aligned} A_\phi &= -2g^2 q_1 \,, \\ B_\phi &= -2g^2 q_0 \,, \\ C_\phi &= -2g^2 q_n \,. \end{aligned} \tag{22}$$

The scalar pole mass

We demand the renormalized two-point function to satisfy the condition at $\bar{P}_\phi^2 = 0$

$$(\Gamma_R^{(2)})^{-1}(\bar{P}_\phi^2 = 0) = 0\,, \tag{23}$$

and consider the ansatz

$$\bar{P}_\phi^2 = p^2 - M_{\rm ph}^2 + \bar{y}(n\cdot p)^2\,, \tag{24}$$

where $M_{\rm ph}$ and $\bar{y}$ are the unknown constants we want to find.
From (20) replacing the value of p^2 given in (24) and using the condition (23), we arrive at the equation

$$\begin{aligned} 0 &= M_{\rm ph}^2 - \bar{y}(n\cdot p)^2 - m_R^2 + A_\phi\left(M_{\rm ph}^2 - \bar{y}(n\cdot p)^2\right) + B_\phi m_R^2 \\ &\quad + C_\phi (n\cdot p)^2\,. \end{aligned} \tag{25}$$

Conclusions

- Consistency of the Lorentz violating effective field models are important to be tested.
- One can preserve unitarity at one loop order in higher-order Lorentz violating theories using the Lee-Wick prescription.
- In some LIV models the UV divergences are improved and they can be renormalized at finite number of loops, and they lead to changes in the free external states.
- Several works are planned for the near future: Kallen-Lehman representation for higher-order theories, study of more models, induced finite corrections (vertex).