Extra dimensions' influence on the structure and stability of compact objects with a linear equation of state

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Equation of state:

$$p = (\rho - 4\mathscr{B})/3, \quad \text{with} \quad \mathscr{B}\, G_d = 60[\mathrm{MeV}/\mathrm{fm}^3], \tag{1}$$

where G_d is the universal constant.

Interior:

$$ds^2 = -e^{\nu}\, dt^2 + e^{\lambda}\, dr^2 + r^2 d\Omega^2. \tag{2}$$

Exterior: Schwarzschild-Tangherlini spacetime[1]

$$ds^2 = -\left(1 - \frac{2M\, G_d}{(d-3)r^{d-3}}\right) dt^2 + \left(1 - \frac{2M\, G_d}{(d-3)r^{d-3}}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{3}$$

with $\frac{M\, G_d}{d-3}$ being the mass of the star in d-dimensions.

[1] F. L. Tangherlini, Nuovo Cimento **27**, 636 (1963) & J. P. S. Lemos and V. T. Zanchin, Phys. Rev. D **80**, 024010 (2009).

Stellar structure equations[2]

$$\frac{dm}{dr} = S_{d-2}\rho r^{d-2}, \tag{4}$$

$$\frac{dp}{dr} = -(p+\rho)G_d \left[\frac{\frac{S_{d-2}pr}{(d-3)} + \frac{m}{r^{d-2}}}{1 - \frac{2mG_d}{(d-3)r^{d-3}}} \right], \tag{5}$$

$$\frac{d\nu}{dr} = -\frac{2}{(p+\rho)}\frac{dp}{dr}, \tag{6}$$

where

$$e^{-\lambda} = 1 - \frac{2mG_d}{(d-3)r^{d-3}}. \tag{7}$$

Boundary conditions:

- In the center $G_d m(0) = 0$ and $G_d \rho(0) = G_d \rho_c$.
- The surface of the star is at $r = R$ defined by $p(r = R) = 0$.

[2] J. Ponce de Leon and N. Cruz, Gen. Relativ. Gravit. **32**, 1207 (2000) & T. Harko and M. Mak, J. Math. Phys. **41**, 7 (2000).

Radial oscillations equations

$$\frac{d\xi}{dr} = \frac{\xi}{2}\frac{d\nu}{dr} - \frac{1}{r}\left((d-1)\xi + \frac{\Delta p}{p\Gamma}\right); \qquad \Gamma = (1+\frac{\rho}{p})\frac{dp}{d\rho}, \tag{8}$$

$$\frac{d\Delta p}{dr} = \frac{\xi r e^{\lambda}}{e^{\nu}}(p+\rho)\omega^2 + \frac{(p+\rho)r\xi}{4}\left(\frac{d\nu}{dr}\right)^2 - \left(S_{d-2}G_d\frac{re^{\lambda}(p+\rho)}{d-3} + \frac{1}{2}\frac{d\nu}{dr}\right)\Delta p$$

$$-2(d-2)\xi\frac{dp}{dr} - 2S_{d-2}G_d(p+\rho)e^{\lambda}r\xi\left(\frac{p}{d-3}\right). \tag{9}$$

Boundary conditions:

- It is required in the center $\Delta p = -(d-1)\,(\xi\Gamma p)_{\text{center}}$.
- In the center, for normalized eigenfunction, we have $\xi(r=0) = 1$.
- At the surface of the object we have $(\Delta p)_{\text{surface}} = 0$.

Solving the stellar structure equations

- The structure equations are integrated using the fourth order Runge - Kutta method for a $\rho_c\, G_d$ and d.

Solving the radial oscillation equations

- We solve the radial oscillation equation by means of the shooting method.
- The oscillation equations are integrated for a trial value of ω^2. If after the integration the condition $(\Delta p)_{\mathrm{surface}} = 0$ is not satisfy, the trial value ω^2 is corrected in order to match this condition in the next integration.
- The values of ω^2 that satisfy the boundary condition at the surface are the eigenfrequencies.

Results

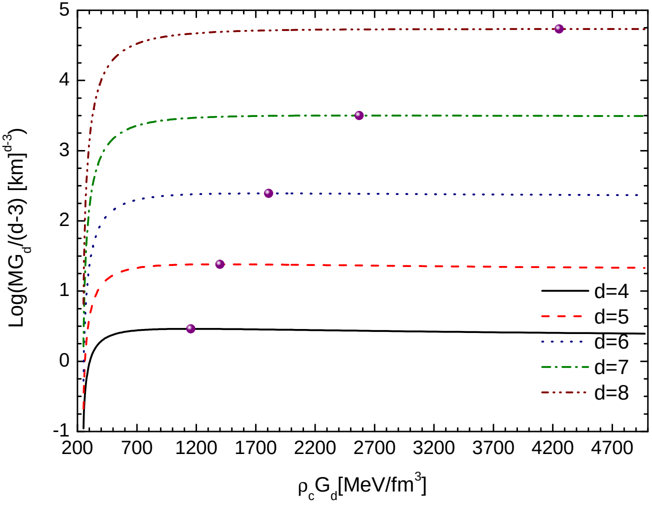

Figure: Mass of the object against the central energy density, for different spacetime dimensions. The full circles indicate the places where maximum masses are found.

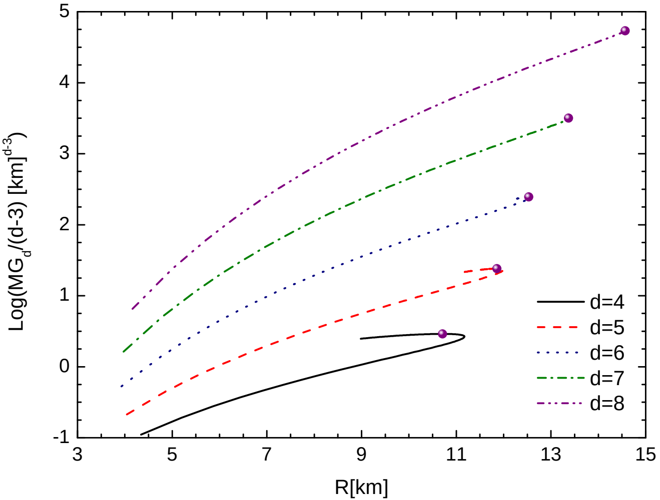

Figure: Total mass of the object against the total radius, for different spacetime dimensions. The complete circles mark the points where the maximum masses are found.

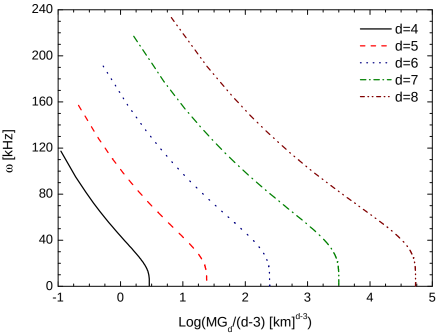

Figure: Fundamental mode eigenfrequency versus the mass of the object, for different dimensions.

- In all cases, the maximum mass point is found in $\omega = 0$.
- For $MG_d/(d-3) \times \rho_c G_d$, the points where $dM/d\rho_c = 0$ separates stable stars from the unstable ones.

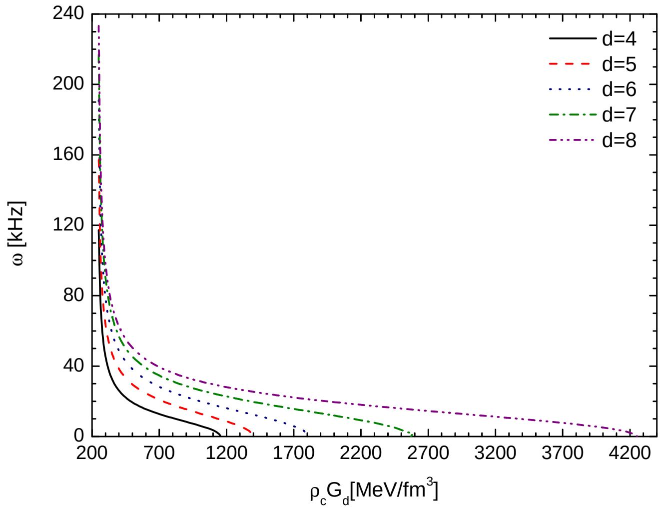

Figure: Fundamental mode eigenfrequency against the central energy density, for different dimensions.

Conclusions

- We found that the dimension affects the physical properties of the stars, such as: the total radius, the total mass, and the eigenfrequencies of oscillation of the fundamental mode.
- We found that for some values of central energy densities and total mass range, the increment of the spacetime dimension helps to grow the stability of compact stars.
- In a graphic $M\,G_d/(d-3) \times \rho_c\,G_d$ the regions constituted by stable and unstable stars can be recognized by the conditions $\frac{dM}{d\rho_c} > 0$ and $\frac{dM}{d\rho_c} < 0$, respectively.

Thanks

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Thanks

The model

$$d\Omega^2 = \sum_{i=1}^{d-2} \left(\prod_{j=1}^{i-1} \sin^2 \theta_j \right) d\theta_i^2$$

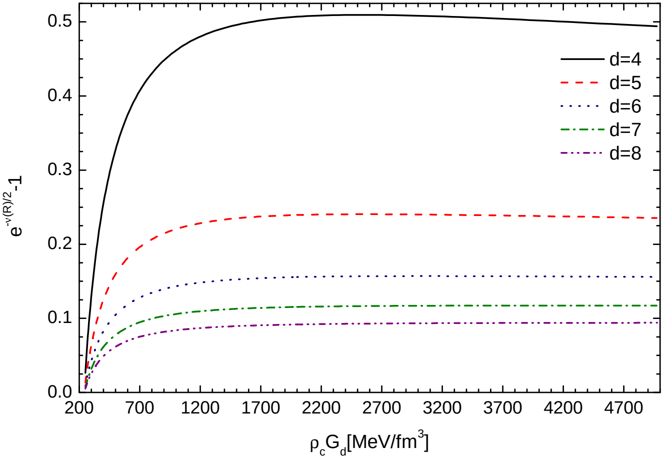

Figure: The redshift function at the object's surface, $e^{-\nu(R)/2} - 1$, as a function of the central energy density, for different spacetime dimensions.

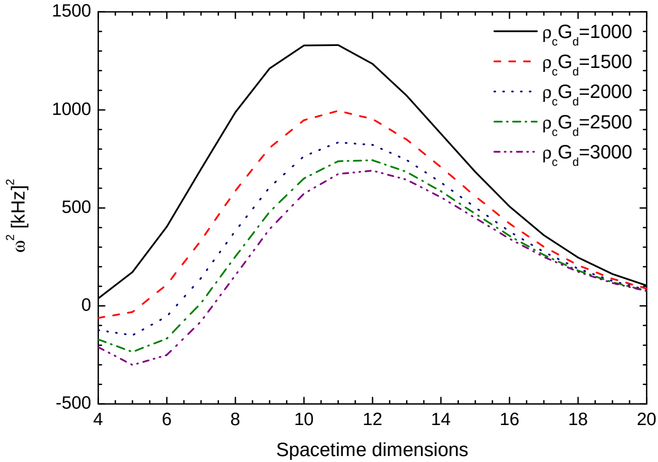

Figure: Change of the fundamental mode eigenfrequency squared with the spacetime dimensions, for five different central energy densities. The central energy density units are $[\mathrm{MeV}/\mathrm{fm}^3]$.