



Measurement of Triple-Differential Inclusive ν_{μ} CC Cross Section at MicroBooNE

London Cooper-Troendle (University of Pittsburgh)

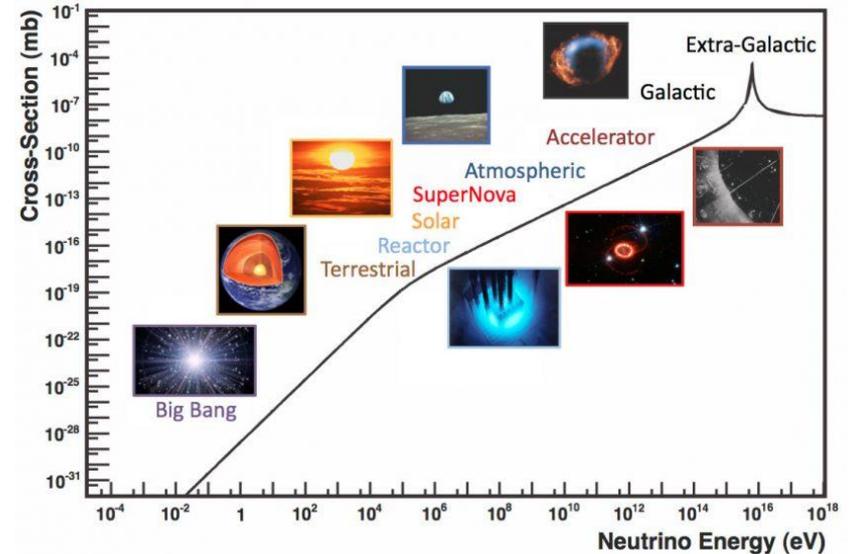
on behalf of the MicroBooNE Collaboration

Phenology Symposium, May 8, 2022

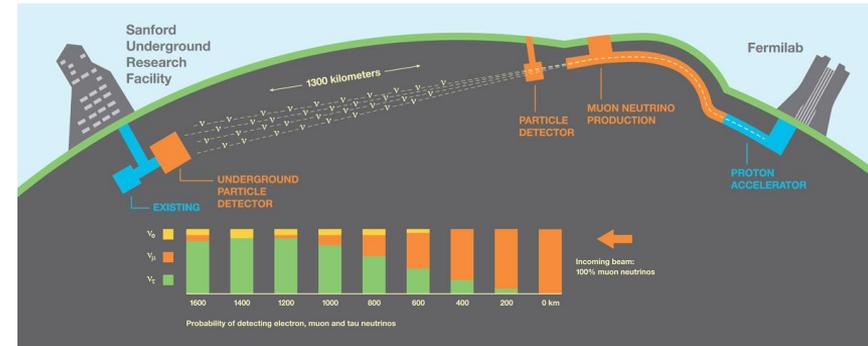
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Neutrino Physics

- Nearly massless
 - Oscillate between flavors
- Many unknown neutrino parameters
 - Potential charge-parity violation through δ_{CP}
 - Absolute neutrino masses and mass hierarchy
 - θ_{23} octant
- DUNE and Hyper-K aim to determine remaining parameters
 - Oscillations measured through use of near+far detectors
 - Oscillations driven by E_ν
 - Uncertainties limit sensitivity: flux, detector systematics, **cross section**



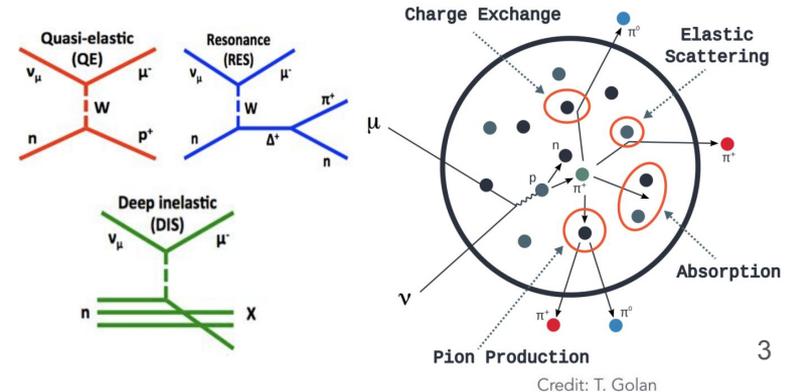
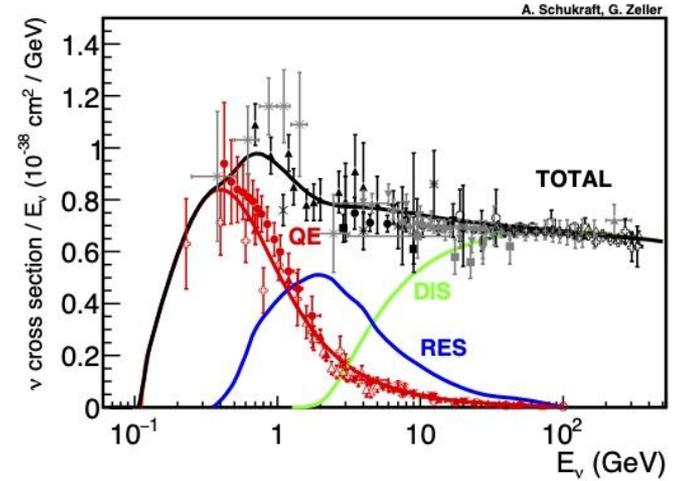
Source: [From eV to EeV: Neutrino Cross Sections Across Energy Scales. Formaggio and Zeller](#)



Deep Underground Neutrino Experiment (DUNE) conceptual design

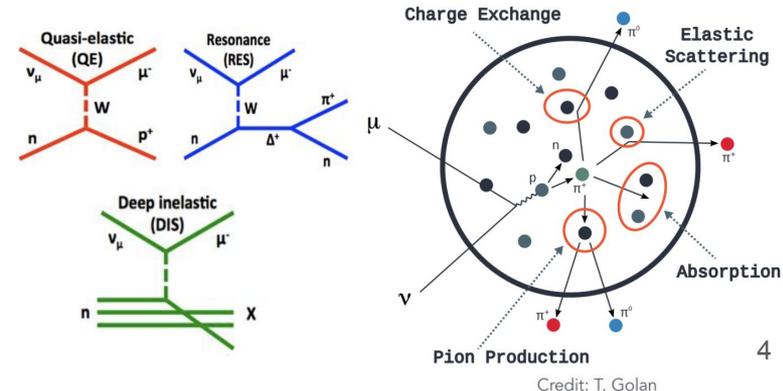
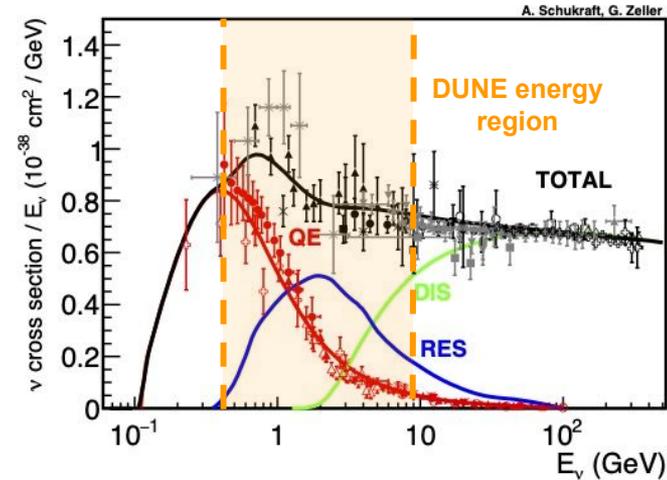
Importance of Cross Section Measurements

- Neutrino experiments rely on models to account for biases
 - Selection impurities
 - Selection efficiency loss
 - Bin migration from imperfect reconstruction
- Neutrino interaction modeling is very complicated
 - Relies on XS measurements to guide development
 - Gives predictions over kinematics of final state particles: energies & angles
 - Best when data matches intended use case - same target nucleus, energy region, interaction channel(s), ...



Why we are interested in E_ν -dependent Cross Sections

- Inclusive ν_μ CC channel, able to tag neutrino flavor, is an important channel for DUNE oscillation measurements
- Kinematics of inclusive ν_μ CC defined by 3 degrees of freedom: $\{E_\nu, P_\mu, \theta_\mu\}$
 - E_ν can be reconstructed with additional E_{had} measurement
- Inclusive ν_μ CC in the DUNE energy range consists of several major interaction modes (**QE**, **RES**, **DIS**, ...)
 - Final-state particles useful in separation up to nuclear effects (2p2h, FSI, ...)
 - E_ν -dependent cross sections give additional discrimination capabilities

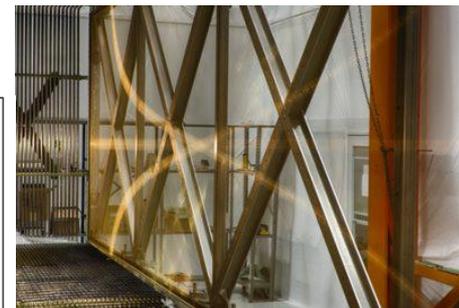


The MicroBooNE Experiment

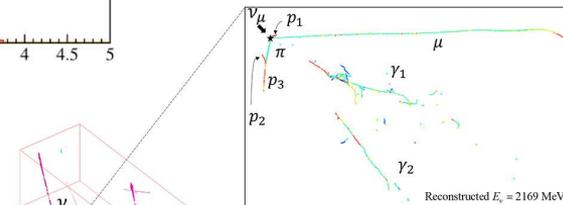
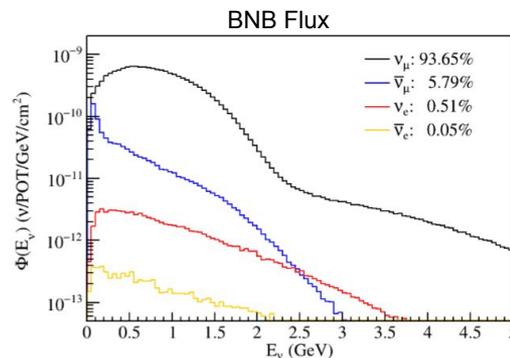
- 85-tonne Liquid Argon Time Projection Chamber (LArTPC). Primary goals of:
 - Address MiniBooNE Low-Energy Excess ([PRL 128, 241801](#))
 - R&D for future LArTPC experiments
 - Measurement of ν -Ar cross sections
- Situated on-axis on BNB neutrino beam line
 - 0.1-4 GeV, peak at 0.8 GeV
- 1.5×10^{21} POT from data taken over 2015-2021
 - 70k inclusive ν_{μ} CC events
 - This analysis uses half of the data taken



Detector



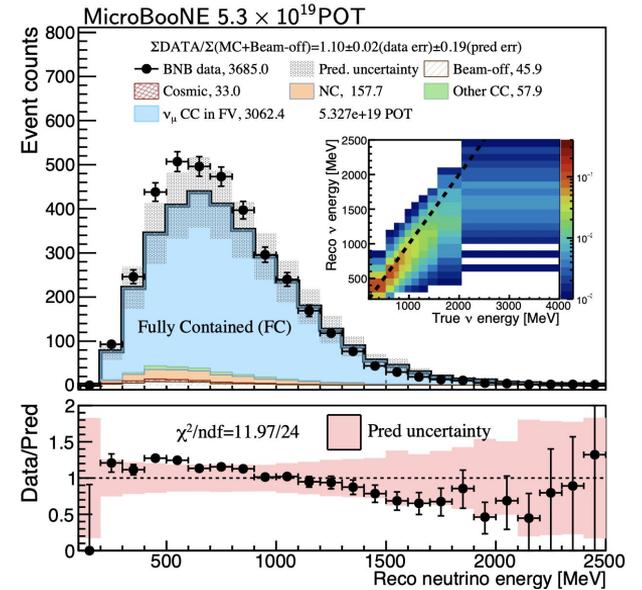
Wire Planes



Bee Event Display

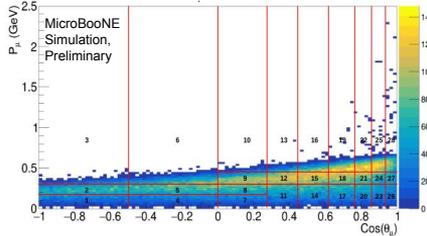
Event Selection and Binning

- Signal definition: Inclusive ν CC interactions
 - E_ν in [0.2,4] GeV, P_μ in [0,2.5] GeV/c
- Begin with 1:20,000 ν :cosmic ray
 - Rejection at $\sim 99.9997\%$ level
 - ν CC selection purity of 92%, efficiency of 68%
 - High efficiency maintained throughout 3D phase space
- 138 analysis bins in total over $\{E_\nu, P_\mu, \cos(\theta_\mu)\}$

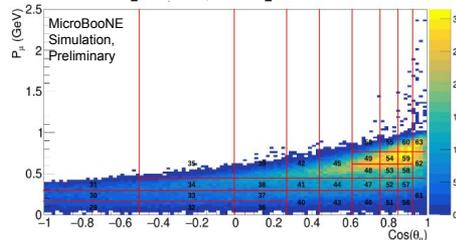


Wire-Cell reconstruction: [JINST 16 \(2021\) 06_P06043](#)
 Cosmic-ray rejection: [Phys. Rev. Applied 15.064071 \(2021\)](#)

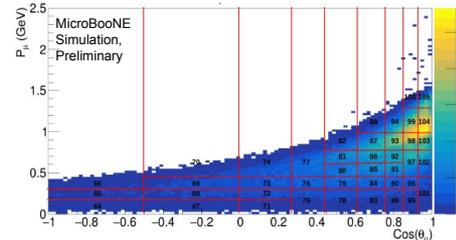
Binning for E_ν in [0.2,0.705] GeV



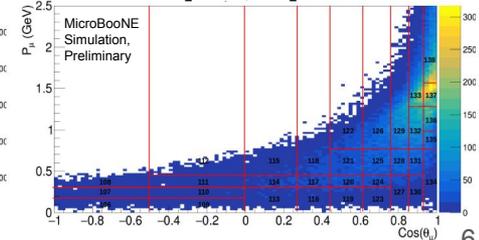
Binning for E_ν in [0.705,1.05] GeV



Binning for E_ν in [1.05,1.57] GeV



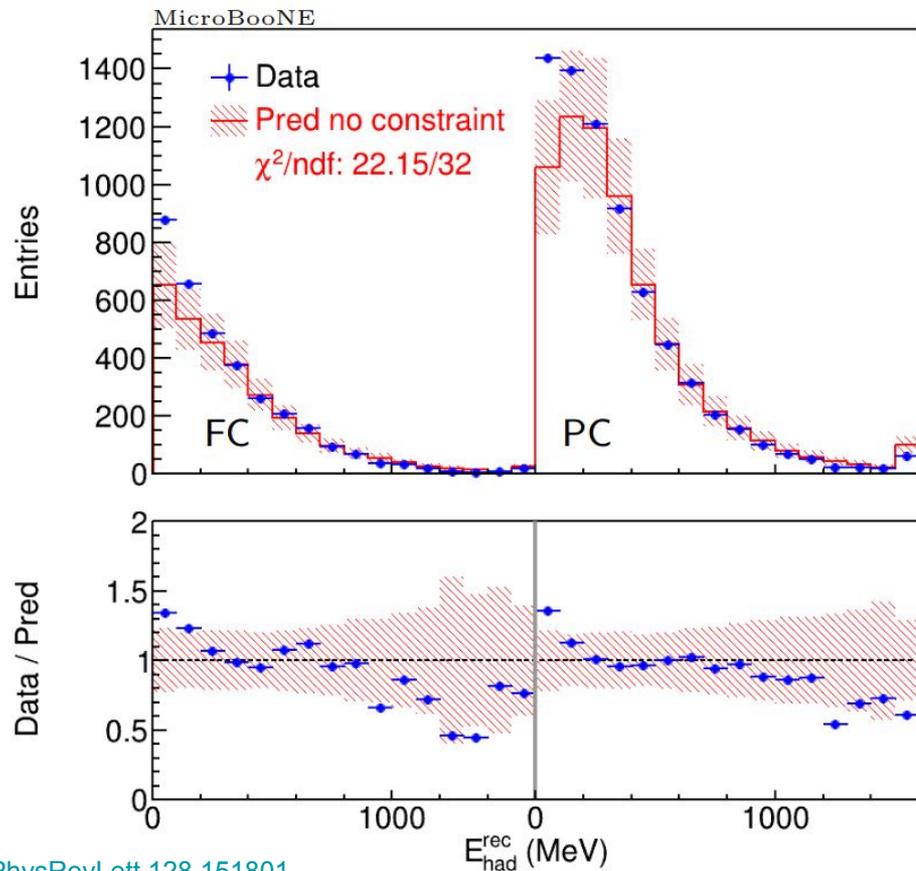
Binning for E_ν in [1.57,4.0] GeV



Key Analysis Validation Before Unfolding

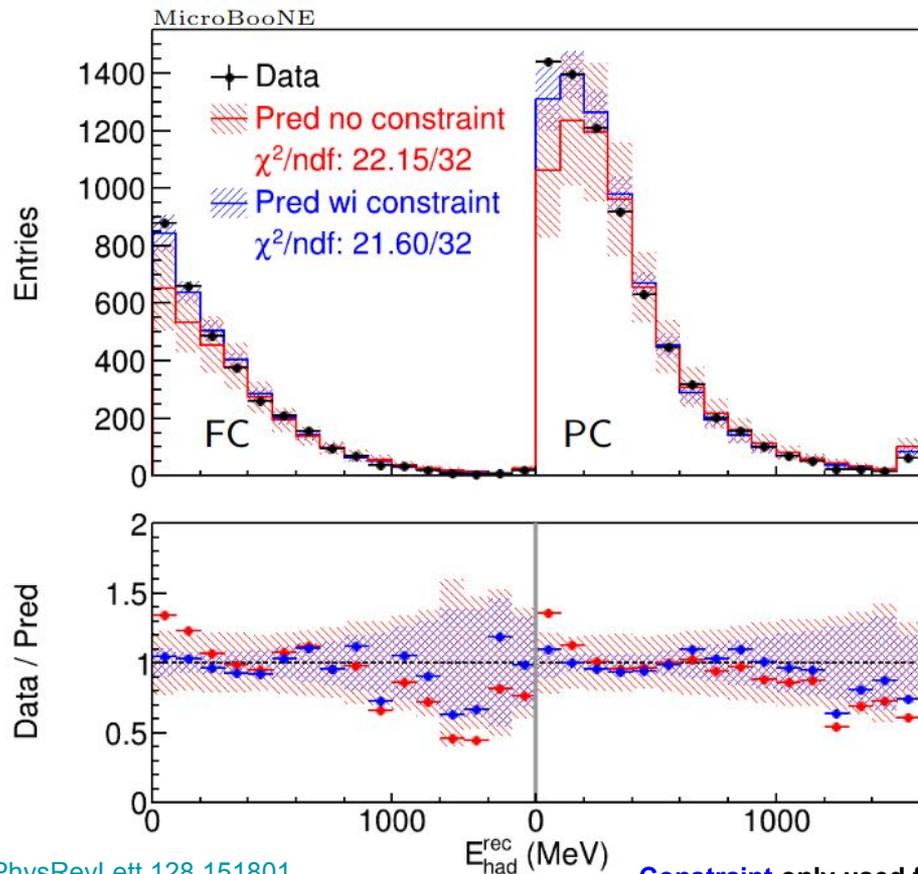
1. Validate modeling of missing hadronic energy
 - a. Novel validation test using conditional constraint
 - b. Allows confident unfolding to true E_ν
2. Unfold and present results

Model Validation: $M(E_{had}^{vis})$ vs $\mu(E_{had}^{vis} | E_\nu, E_\mu^{reco})$



- New method to validate the modeling of neutrino energy
 - Uses LArTPC measurements of lepton kinematics and hadronic energy
- Data/MC goodness of fit tested with χ^2/ndf
 - Muon kinematics used to constrain model prediction of hadronic energy under conditional constraint formalism
- E_{had} includes all visible non-muon particles: largely protons and pions
 - Proton threshold: 35 MeV

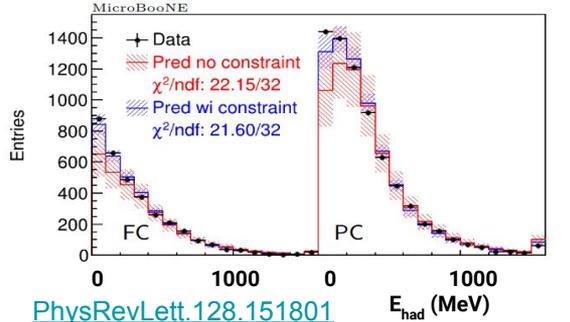
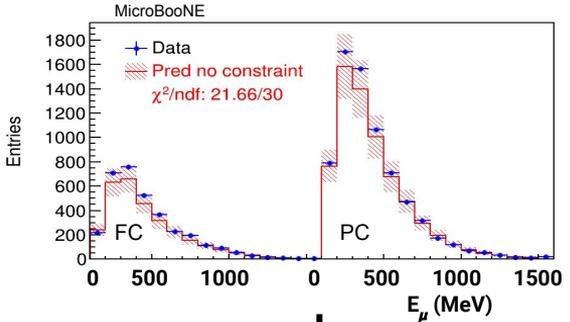
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- E_{had}^{vis} includes all visible non-muon particles: largely protons and pions
 - Proton threshold: 35 MeV
- Reduced systematic uncertainties in **constrained prediction**

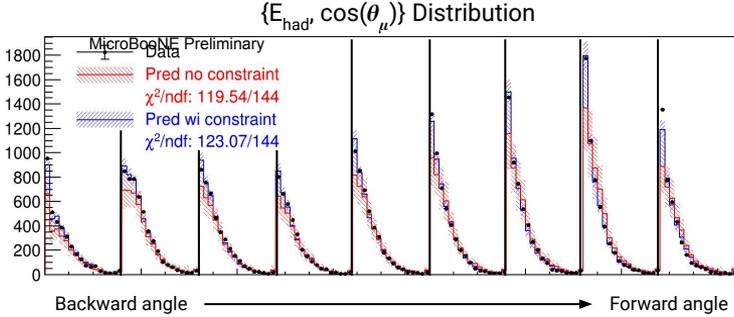
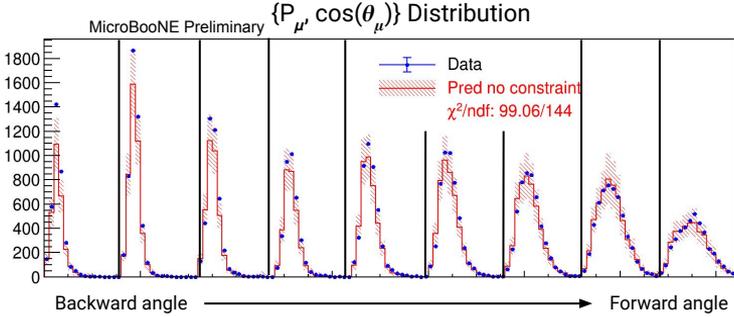
Model Validation in Single & Multiple Dimensions

Model Validation in 1D



[PhysRevLett.128.151801](https://arxiv.org/abs/1801.15181)

Model Validation in Multiple Dimensions



Constraint only used for validation, not unfolding

- 2D distribution w/ constraint covers 3D phase space
- Real data passes validation test in 1D and 2D
- Therefore model uncertainty is sufficient to cover potential bias introduced in unfolding

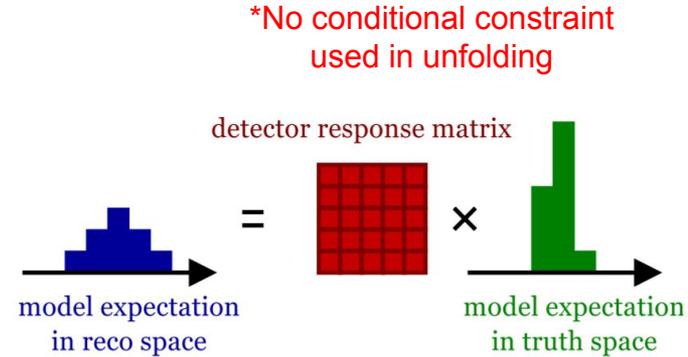
9 angle slices in $\cos(\theta_\mu)$:
 {-1, -0.5, 0, 0.27, 0.45, 0.62, 0.76, 0.86, 0.94, 1}
 16 P_μ bins within each angle slice

Wiener SVD Unfolding and Regularization

- Nominal flux-averaged XS unfolded with Wiener SVD method ([JINST 12 P10002](#))
 - Maximizes the overall signal to noise ratio through the application of the Wiener filter
- Regularized using derivatives computed along each of E_ν , P_μ , $\cos(\theta_\mu)$, combined in quadrature:

$$T_{\text{reg}}^2 = T_{\text{reg},E_\nu}^2 + T_{\text{reg},P_\mu}^2 + T_{\text{reg},\cos(\theta)}^2$$

- Bias introduced in regularization and unfolding captured in smearing matrix A_C
 - Given with unfolded measurement for bias-free model comparisons

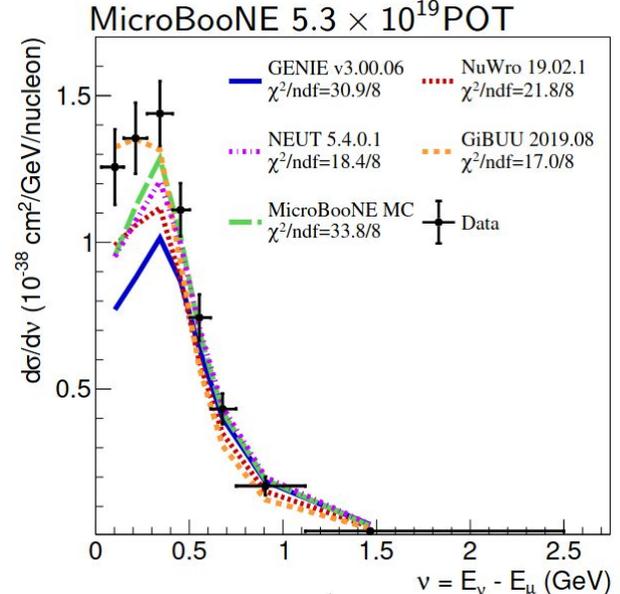
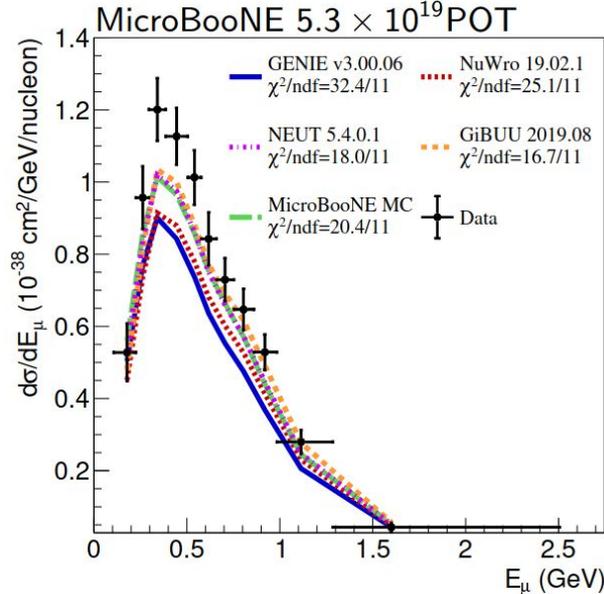
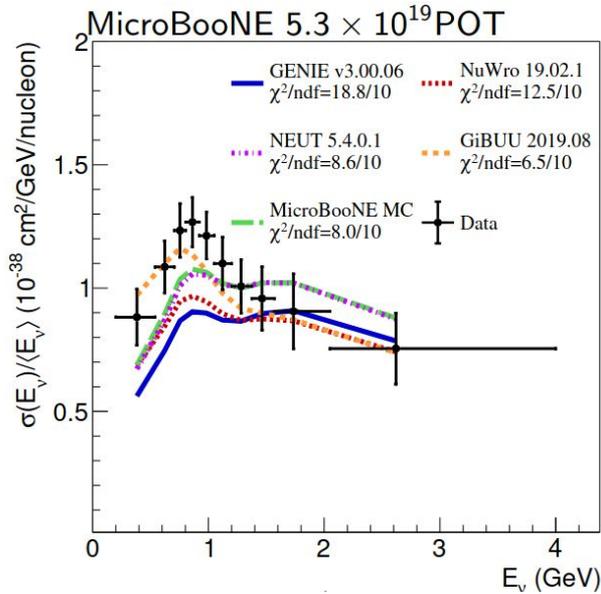


$$M_i = \sum_j R_{ij} \cdot S_j + B_i$$

Measurement M
Response matrix R
True Signal S
Background B

Reco bin index i
Truth bin index j

Previous Single-Differential Energy-Dependent XS



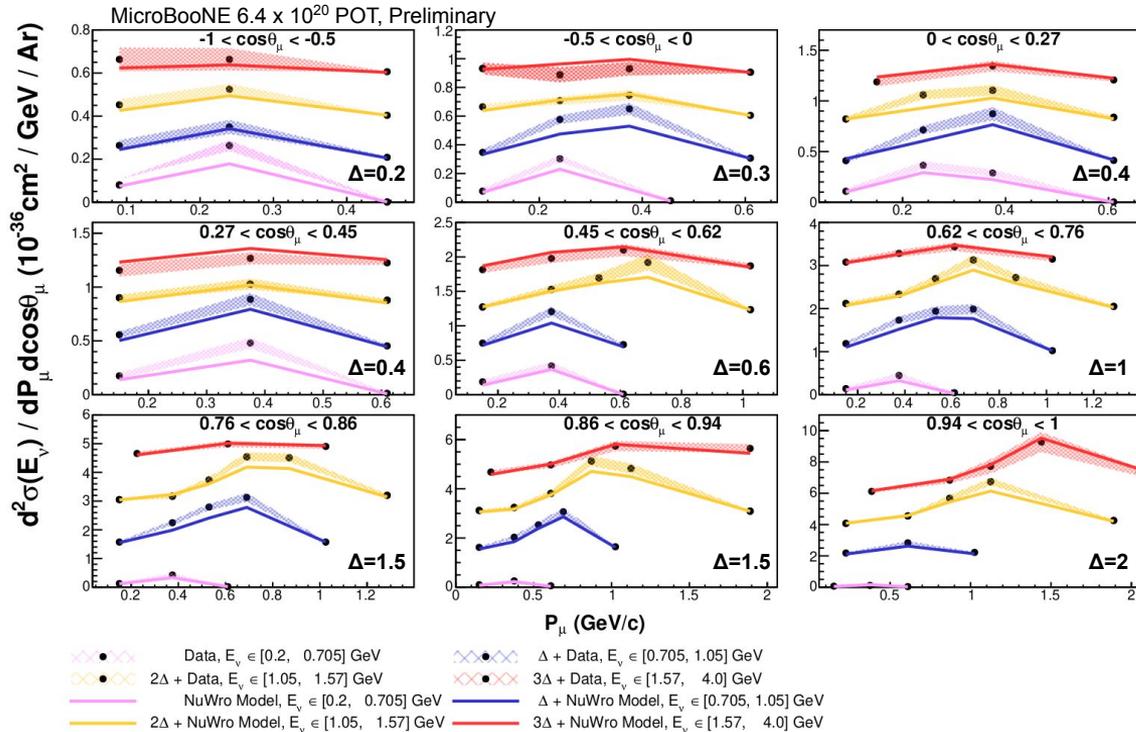
[PRL 128, 151801 \(2022\)](#)

Used 5×10^{19} POT data
($\sim 3.5\%$ of total data available)

Energy-dependent XS measurements enabled by
the new model validation procedure for

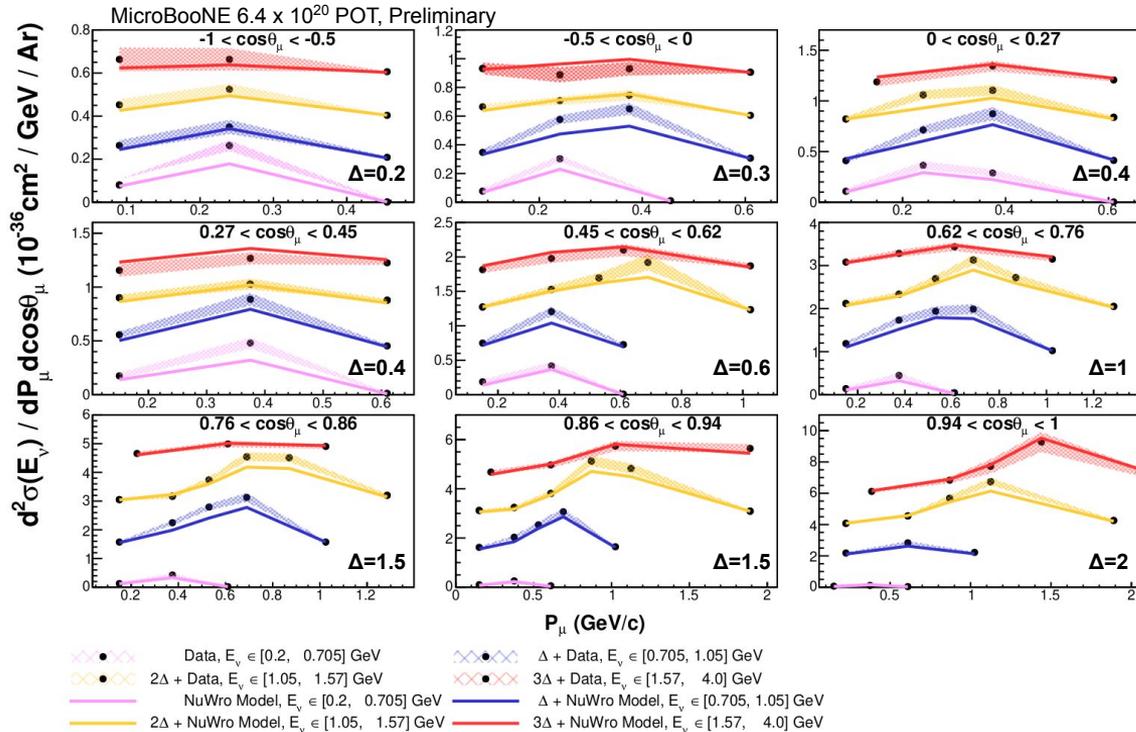
$E_{\nu}^{\text{reco}} \rightarrow E_{\nu}^{\text{true}}$ mapping

Unfolded Measurement in 3D



Data plotted against NuWro prediction
 E_ν slices overplot with offset $N\Delta$ for each angle slice
 Δ in same units of $d^2\sigma(E_\nu)/dP_\mu d\cos(\theta_\mu)(10^{-36}\text{cm}^2/\text{GeV}/\text{Ar})$

Unfolded Measurement in 3D



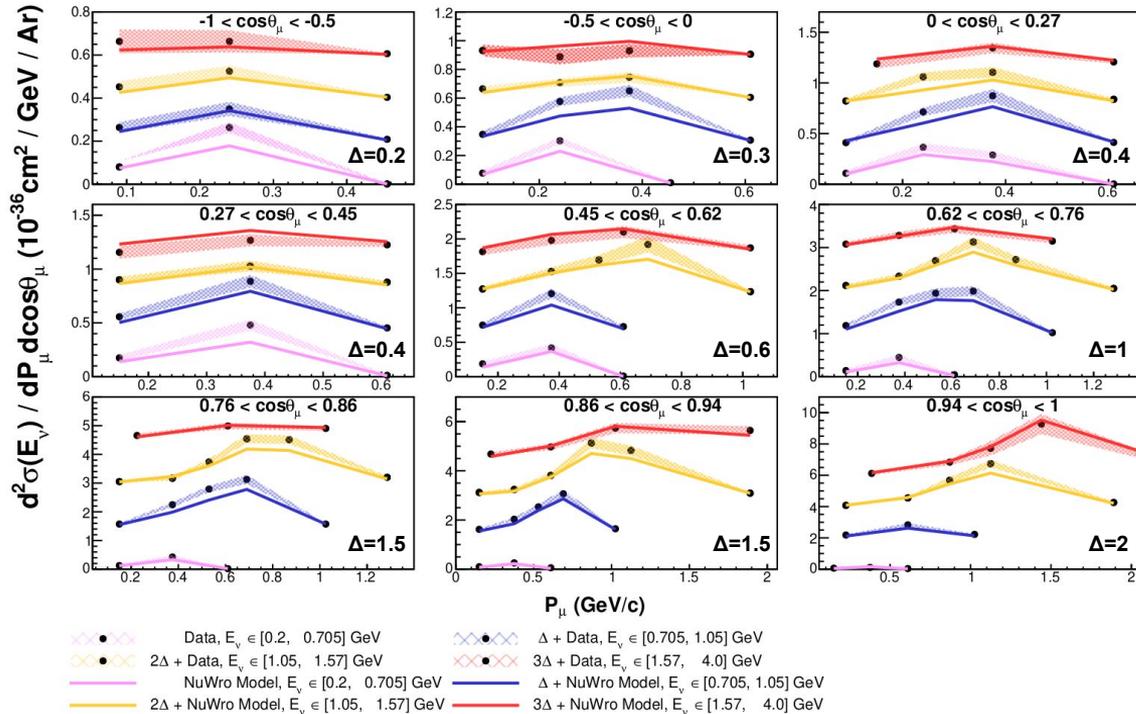
Model Generator	χ^2 (ndf=138)
Genie v2.12.10	741.1
Genie v3.0.6 (MicroBooNE Tune*)	326.1
Genie v3.0.6 (Untuned)	322.2
GIBUU 2021	269.9
NEUT v5.4.0.1	243.3
NuWro v19.02.01	212.1

Descending $\chi^2 \rightarrow$

*MicroBooNE tune for Genie v3.0.6: [PhysRevD.105.072001](https://arxiv.org/abs/1005.07200)

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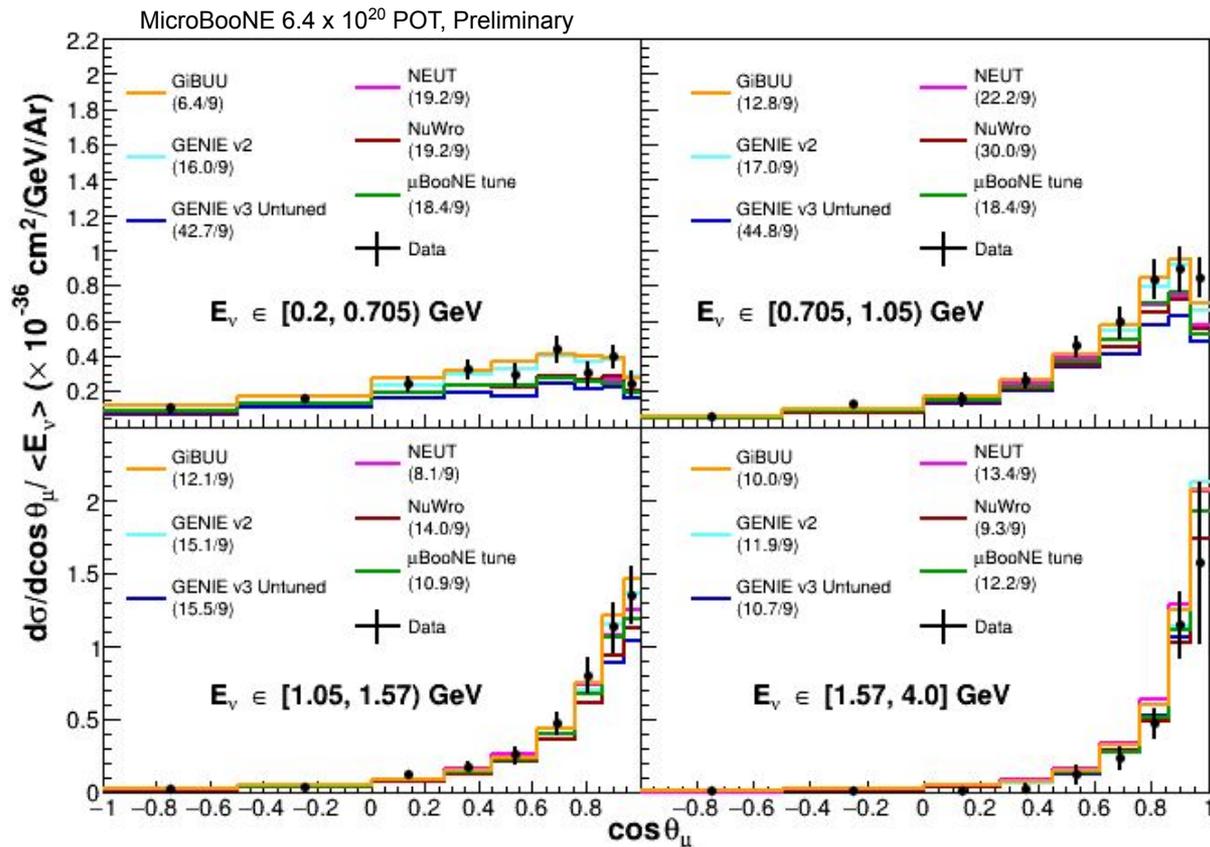
Descending χ^2 →

*MicroBooNE tune for Genie v3.0.6: [PhysRevD.105.072001](https://arxiv.org/abs/1005.07200)

3D measurement contains wealth of information → all model central value predictions are now in tension with data

More powerful than 1D measurement, which was consistent with some models

Unfolded Measurement in 2D: Integrate over P_μ



ν -interaction channels vary over energy range

- QE fraction 75% → 55% from lowest to highest E_ν bin

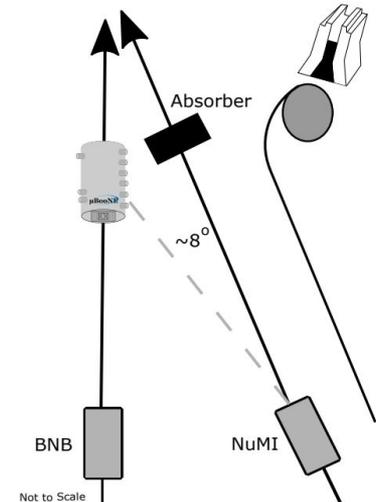
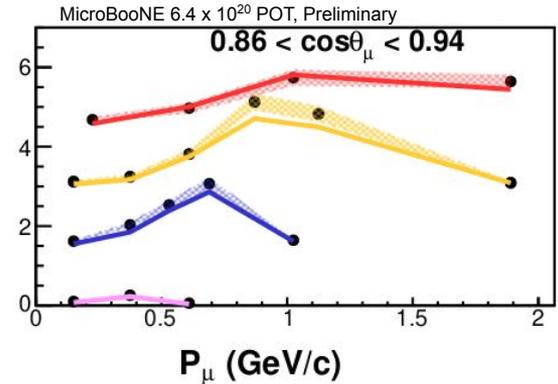
Model performances vary over E_ν

- **GiBUU** performs best at low energy
- **NuWro**, **Genie v3** give best prediction at high E_ν , forward angle, where RES fraction is higher

3D XS provides new insights for future model improvement

Summary and Outlook

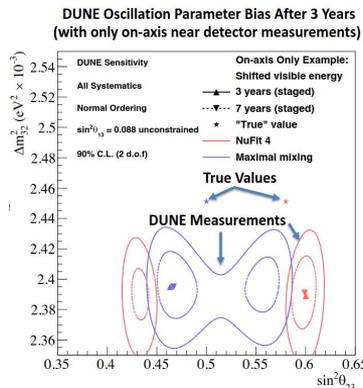
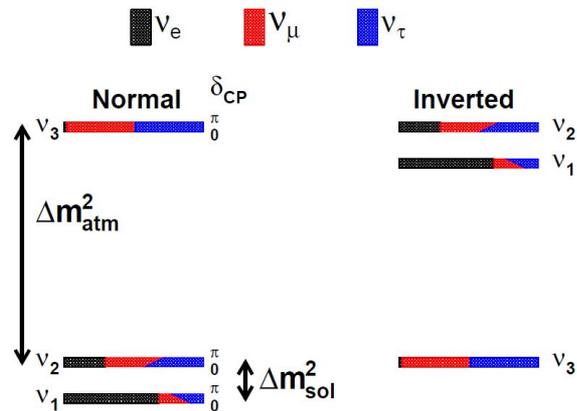
- Triple differential cross sections for inclusive ν_{μ} CC are measured with high precision in MicroBooNE with LArTPC technology
 - New model validation procedure with conditional covariance allows for a validation of model of missing energy
 - Allows for better model development for DUNE and SBN program
- More results in the future:
 - Twice as much MicroBooNE data available
 - NuMI+BNB combined measurement for improved flux uncertainty
 - Numerous valuable contributions from MicroBooNE: [40+ publications](#), tons more in progress on electron neutrinos, proton multiplicity, pion production, NuMI beam measurements, rare searches, methodology, ...



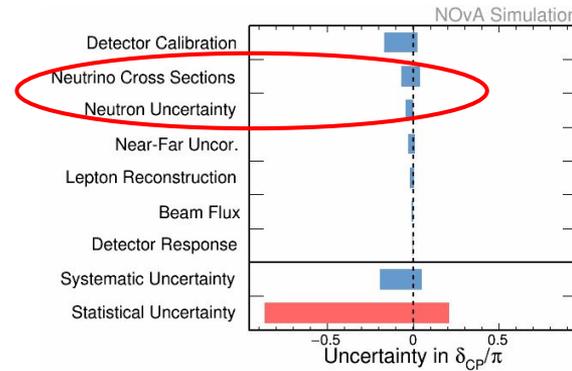
Backup

Understanding ν -nucleus Interactions for ν Oscillations

- Accelerator oscillation experiments aim to definitively answer δ_{CP} , mass hierarchy, etc.
 - DUNE with LArTPC
 - Hyper-K with Water Cherenkov
- Cross section uncertainties or mismodeling may limit the physics reach of these measurements
- Accurate knowledge of the mapping between reconstructed and true E_ν is very important



Simulation with 20% of proton energy moved to neutrons

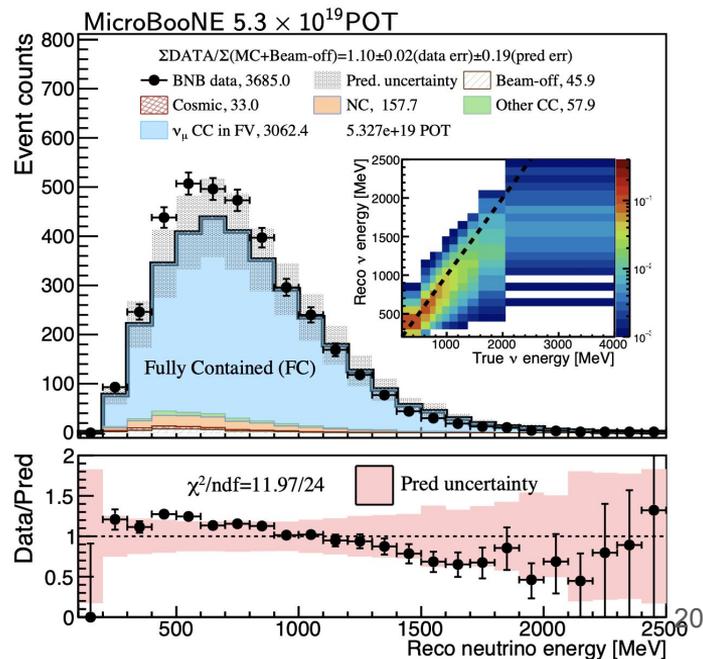
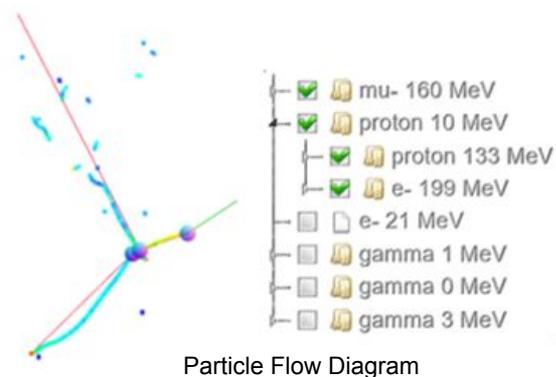


ν_μ Event Selection

- Begin with 1:20,000 ν :cosmic ray
 - Rejection at $\sim 99.9997\%$ level
- ν_μ CC selection purity of 92%, efficiency of 68%
- Reconstructed with 3D tomographic imaging, many-to-many flash-charge matching, particle flow hierarchy
 - Select both fully contained (FC) and partially contained (PC) events. FC means that all the deposited charge with a ν interaction is inside the fiducial volume

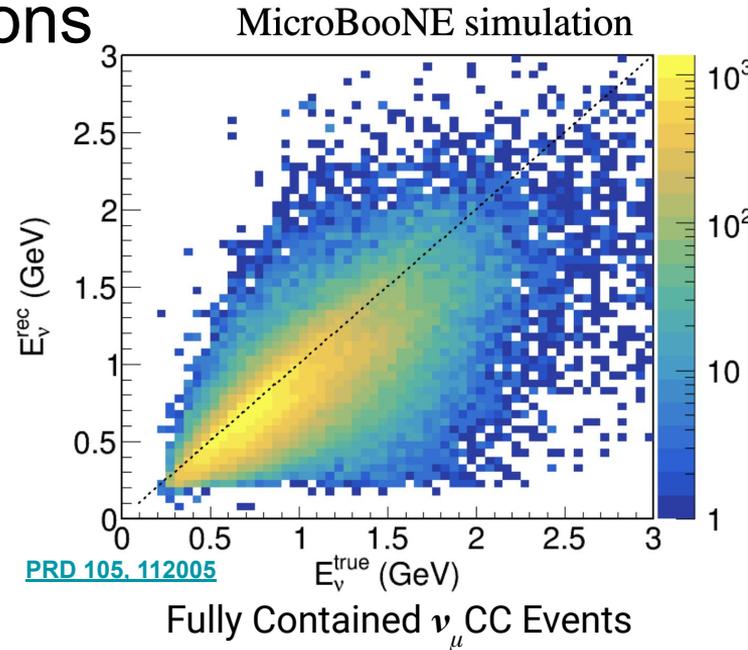
Wire-Cell reconstruction: [JINST 16 \(2021\) 06, P06043](#)

Cosmic-ray rejection: [Phys. Rev. Applied 15, 064071 \(2021\)](#)



Energy Reconstruction and Resolutions

- Calorimetry-based energy reconstruction, particle mass and binding energy included
 - **Tracks:** use range, $dQ/dx \rightarrow dE/dx$.
Calibrated and verified by stopped muons & protons
 - **Showers:** sum charge and scale.
Calibrated by π^0 invariant mass reconstruction
- Resolutions for fully contained events:
 - E_ν : 20%; P_μ : 10 %; θ_μ : $\sim 5^\circ$ at forward angles

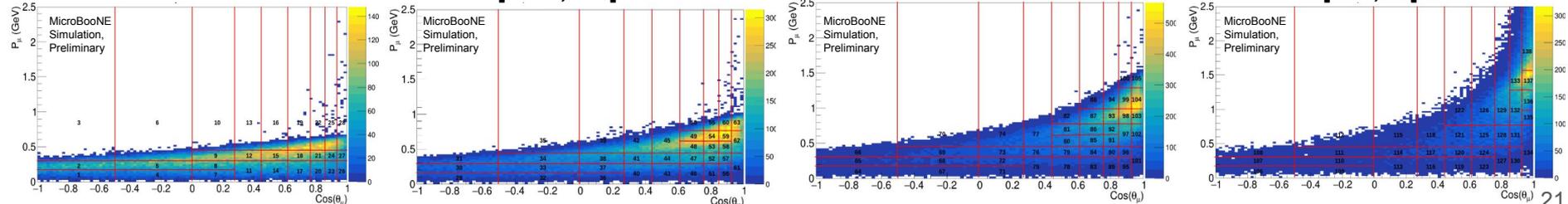


Binning for E_ν in [0.2,0.705] GeV

Binning for E_ν in [0.705,1.05] GeV

Binning for E_ν in [1.05,1.57] GeV

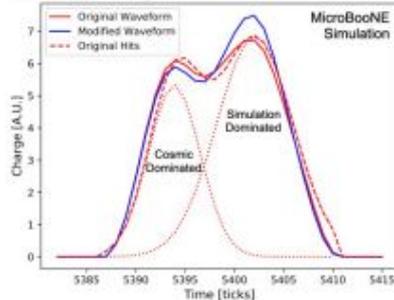
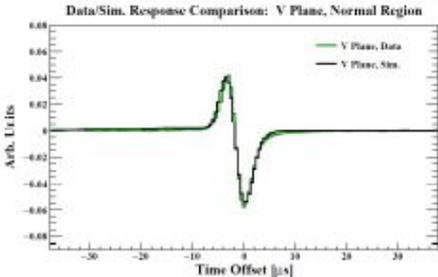
Binning for E_ν in [1.57,4.0] GeV



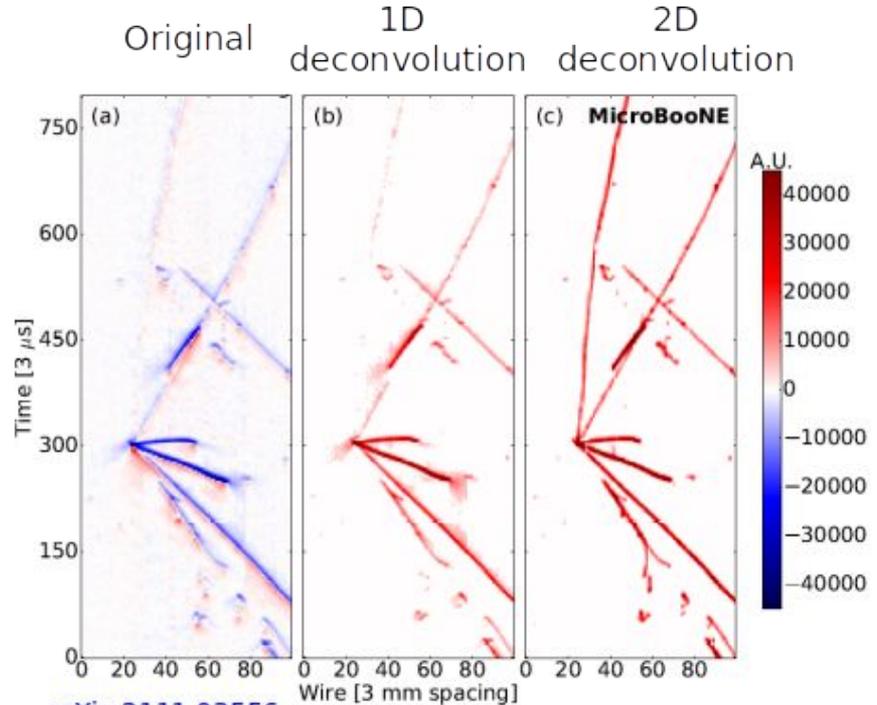
138 analysis bins in total

Improved TPC Signal Processing, Detector Simulation, and Improved Evaluation of Detector Systematics

- 2D deconvolution algorithm allows to accurately recover the ionization electrons from recorded original signals
- Improved 2D detector simulation, modeling both the long-range induction and the position-dependent effects lead to much better data/MC consistency



- Improved evaluation of detector systematic uncertainties with changes to detector modeling



[arXiv:2111.03556](https://arxiv.org/abs/2111.03556)

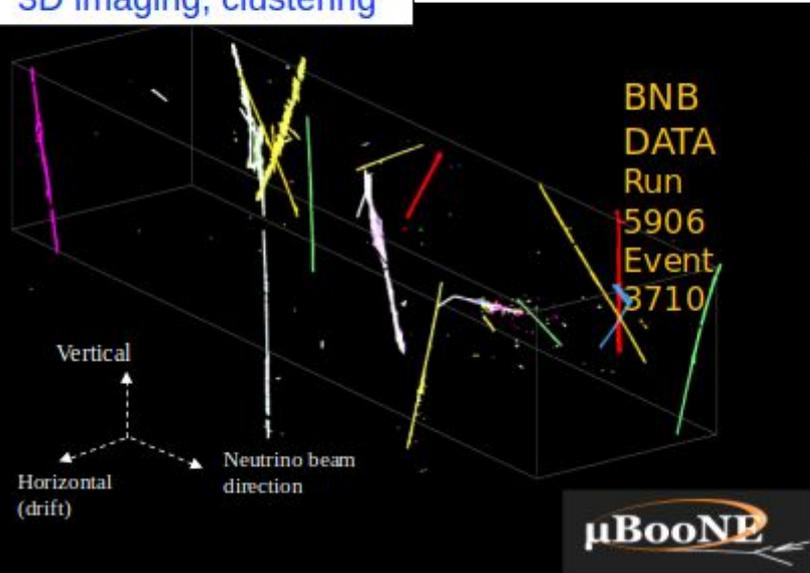
[JINST 13 P07006/7](https://inspirehep.net/literature/2111035)



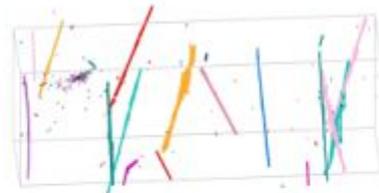
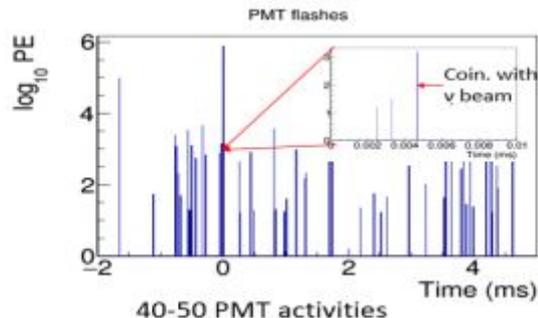
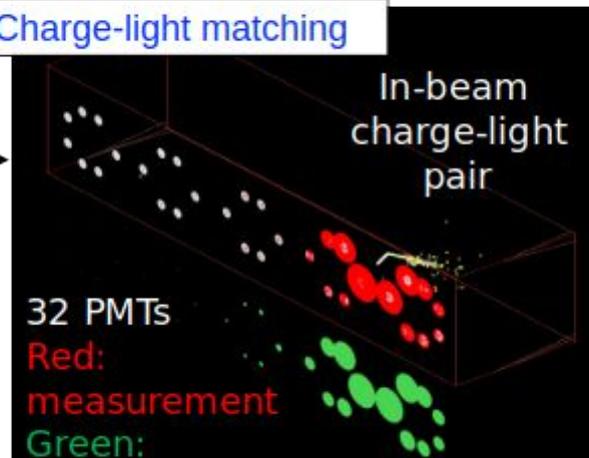
The First **Many-to-Many** Charge-Light Matching

//INST 16 P06043

3D imaging, clustering

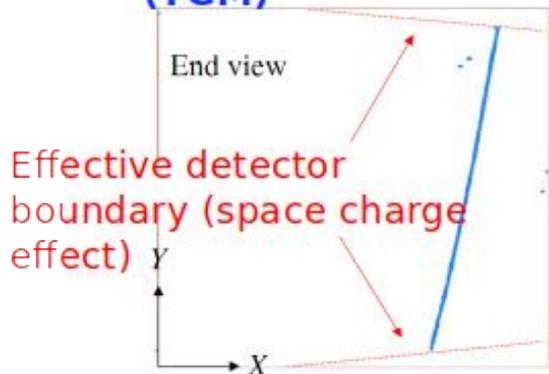


Charge-light matching

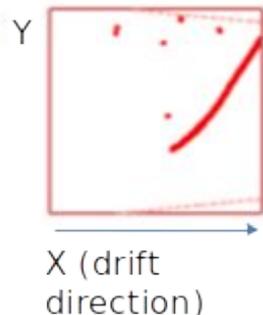
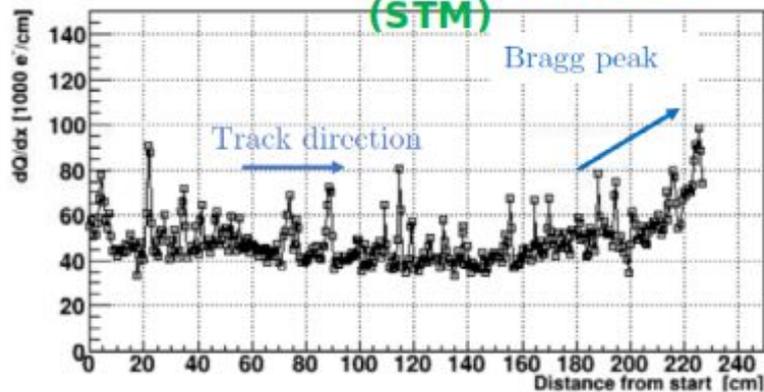


Rejecting Random Coincident Cosmic-Ray Muons

Through-going muon (TGM)



Stopping muon (STM)



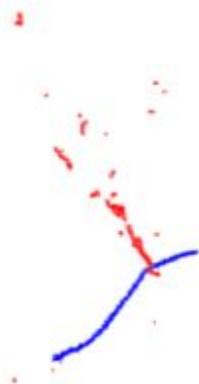
	Neutrino:Cosmic-ray	
Charge-light matching	1 : 6.4	Improved by factor of
TGM rejection	1 : 0.91	>6
STM rejection	1 : 0.36	Improved by factor of ~3
Additional Cuts	1 : 0.20	

Wire-Cell 3D Pattern Recognition

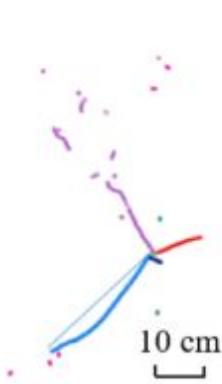
(a) Selected neutrino activity



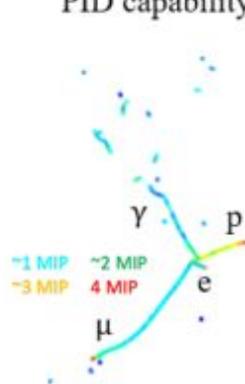
(b) Track/Shower separation



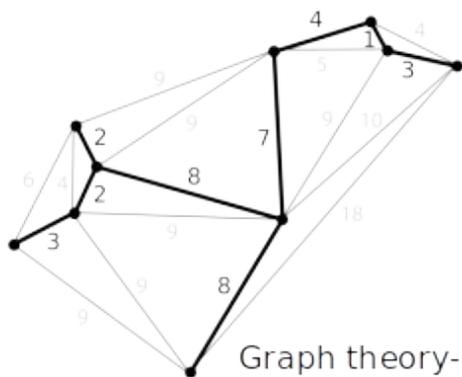
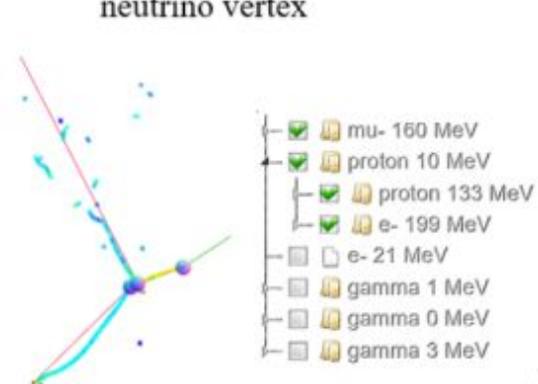
(c) Particle-level sub-clustering



(d) 3D dQ/dx displayed with PID capability

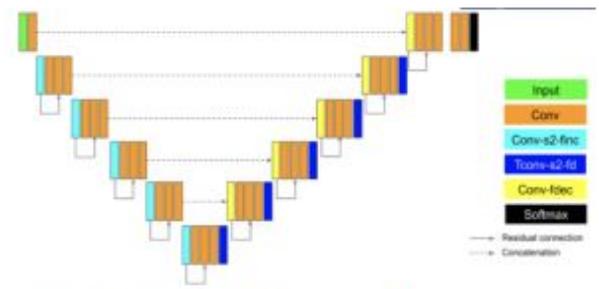


(e) Particle flow starting from neutrino vertex



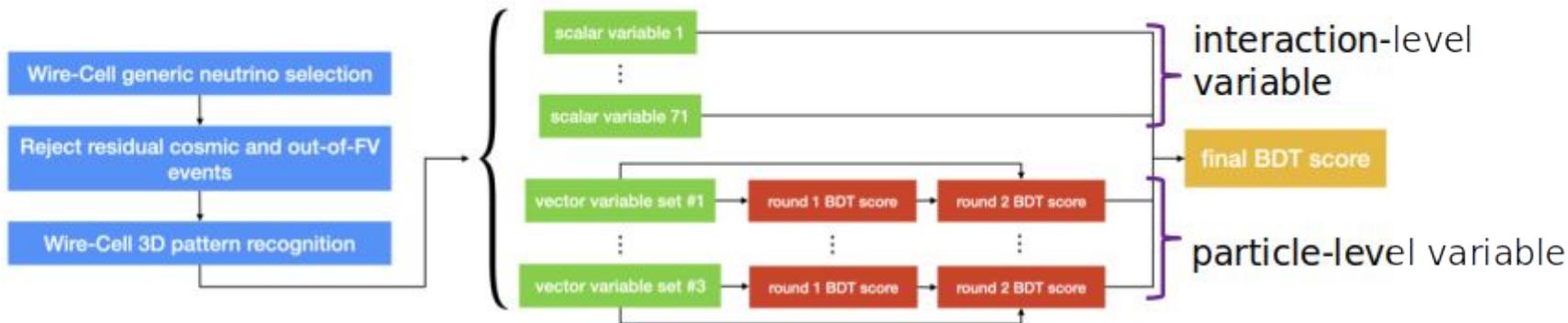
Graph theory-based multi-track fitting (e.g., Steiner tree)

Hybrid of Traditional and Deep-Learning based approaches



Deep-learning neural network for neutrino vertex identification

CC Selection through XGBoost BDT

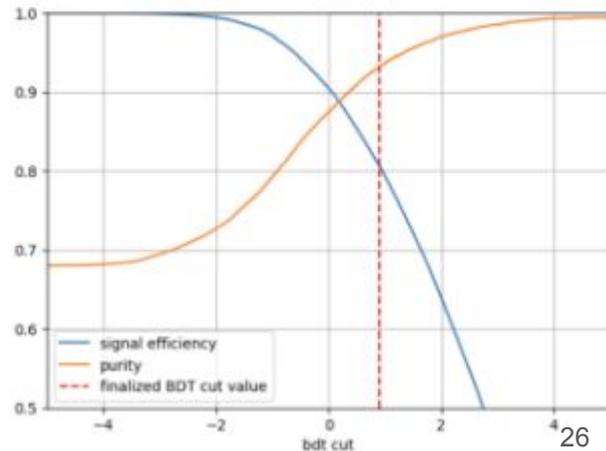
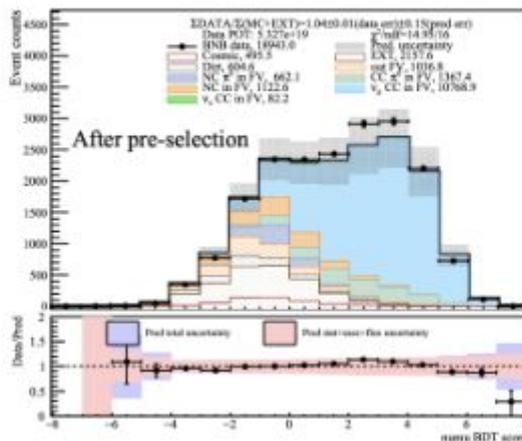


Human feature engineering

+

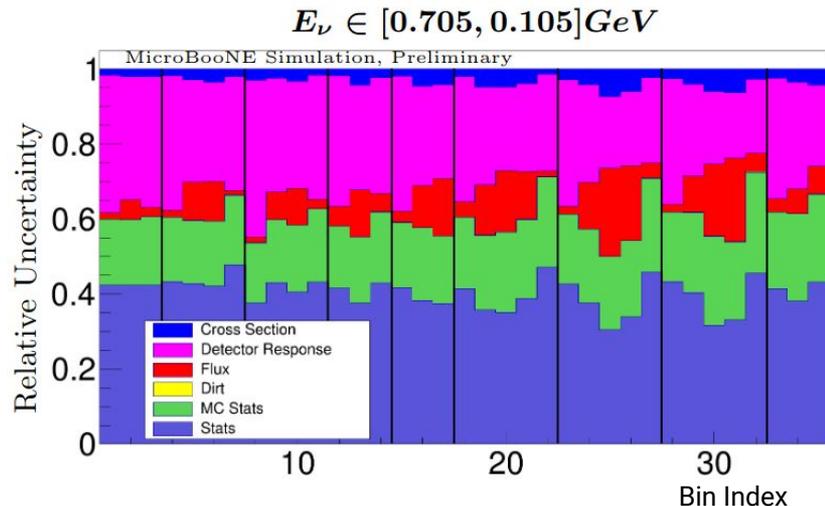
Machine learning algorithm:

XGBOOST: extreme Gradient Boosting



Systematic Uncertainties

- **MC statistical uncertainty**: estimated with Poisson likelihood with a Bayesian approach
- **Flux prediction**: MiniBooNE prediction updated to MicroBooNE baseline
 - [PRD 79, 072002](#)
- **Cross Section (Xs)**: Modeled using Genie v3.0.6_g18_10a_02_11a tuned to T2K CC0 π data
 - [PRD 105, 072001](#)
- **Detector Systematics**: TPC waveform, light yield, space charge effect, recombination
 - Estimated using bootstrapping (event resampling)
 - Many bins with limited MC events \rightarrow overestimate uncertainty
Smoothing used to address this



Breakdown of uncertainties fraction within the 2D binning of $\{P_\mu, \cos(\theta_\mu)\}$. Vertical black bars separate each angle slice, going from backward to forward scattering based on the edges $\cos(\theta_\mu)$ in $\{-1, -0.5, 0, 0.27, 0.45, 0.62, 0.76, 0.86, 0.94, 1\}$

Additional (smaller) uncertainties:

- ν interaction outside cryostat
- GEANT4 model reweighting
- POT from originating proton flux
- Number of target nuclei

How to estimate systematic uncertainties?

- Full systematic covariance

$$\Sigma^{syst} = \Sigma^{flux} + \Sigma^{xsec} + \Sigma^{det} + \dots$$

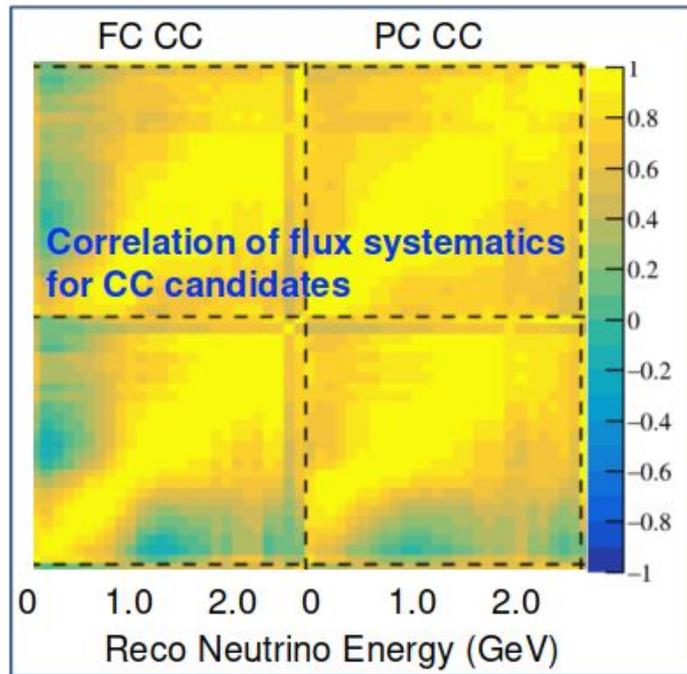


	Multisim	Unisim
# of parameter variation at a time	Many	One
Parameter(s) variation	Random	Exactly 1
# of MC run	One	Many (one per parameter)
Technical treatment	Event reweighting	Bootstrapping

Flux and cross section systematics: multisim

- Standard reweighting approach, each event has different weights from the randomization of the underlying model parameters.

Tuning parameter name	Parameter type	
π^+ hadron production	FLUX	Neutrino flux
π^- hadron production	FLUX	
K^+ hadron production	FLUX	
K^- hadron production	FLUX	
K_S^0 hadron production	FLUX	
horn current distribution	FLUX	
horn current calibration	FLUX	
nucleon total scattering X_s	FLUX	
nucleon inelastic scattering X_s	FLUX	
nucleon quasi-elastic scattering X_s	FLUX	
pion total scattering X_s	FLUX	
pion inelastic scattering X_s	FLUX	
pion quasi-elastic scattering X_s	FLUX	
MicroBooNE GENIE All	GENIE X_s (μ B tune)	
Strength of the CCQE RPA correction	GENIE X_s (μ B tune)	
Parameterization of the CCQE nucleon axial form factor	GENIE X_s	
Parameterization of the CCQE nucleon vector form factors	GENIE X_s	
Changes angular distribution of nucleon cluster in MEC	GENIE X_s (μ B tune)	
CCMEC Cross-section Shape	GENIE X_s (μ B tune)	
Angular distribution for RES $\Delta \rightarrow N + \pi$	GENIE X_s	
Angular distribution for RES $\Delta \rightarrow N + \gamma$	GENIE X_s (μ B tune)	
Scaling factor for CC coherent π production	GENIE X_s (μ B tune)	
Scaling factor for NC coherent π production	GENIE X_s (μ B tune)	
Second-class vector current	X_s	Final-state hadron-argon interaction
Second-class axial current	X_s	
π^- interactions	GEANT4	
π^+ interactions	GEANT4	
proton interactions	GEANT4	



Geant4Reweight: JINST 16 (2021) 08, P08042

Detector systematics: unisim

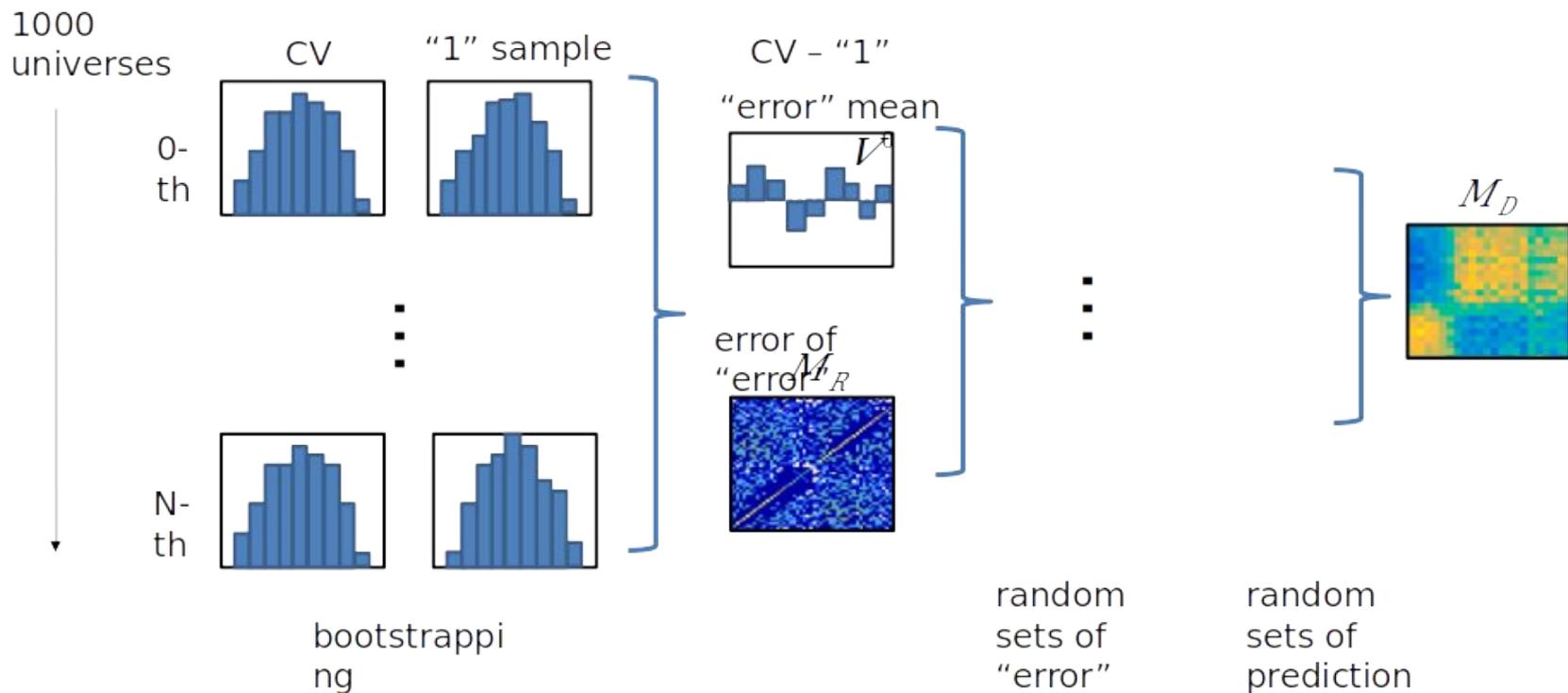
- Four major categories
 - 1) Light yield and propagation
 - 2) Charge readout detector response
 - 3) Recombination model (to conversion)
 - 4) Space charge effect (impacts on E-field)

- For each source of the systematic uncertainty, the same set of MC simulation events are re-simulated with a change to the detector modeling parameter of interest. In total, we have two samples
 - 1) One sample with nominal value of all parameters:
CV sample
 - 2) One sample with changed value of interested par: 1 σ sample

Can not calculate the covariance matrix by the two samples in traditional way, which needs many samples with different pars values:

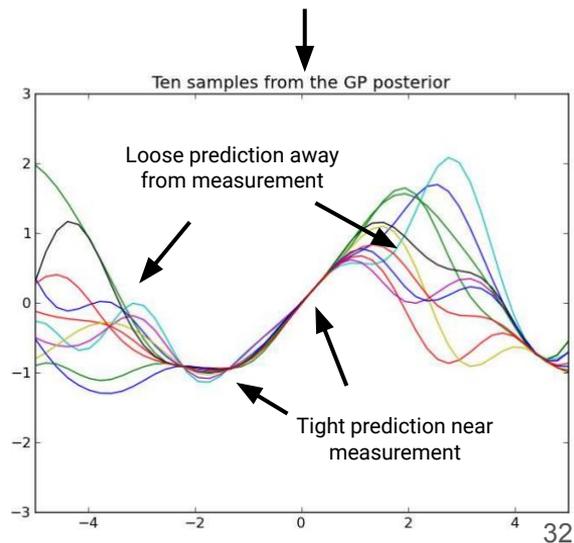
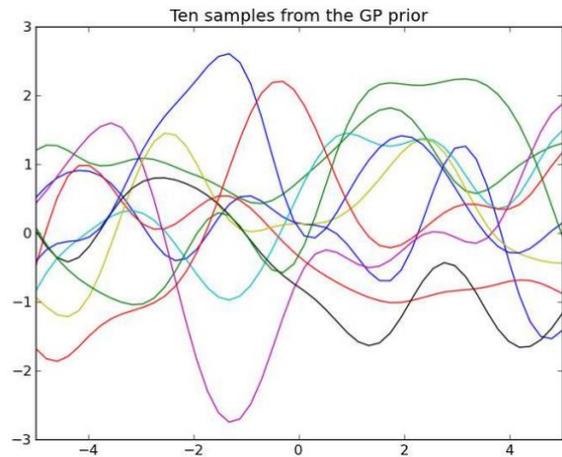
$$COV_{ij} = EXP((X_i - \bar{X})(X_j - \bar{X}))$$

Detector systematics: bootstrapping method



Gaussian Processes Smoothing

- Bayesian approach: uninformed gaussian prior (μ, Σ_T) updated with input from bootstrapping and kernel function K:
 - Asserts smoothness intuition: nearby bins are correlated
 - Smoothed uncertainties consistent with increased statistics in 1D test
 - Similar formalism as the model validation with conditional covariance
- Factor of 2 reduction in estimated detector systematic uncertainties \rightarrow improved model prediction for later dedicated validation tests



Gaussian Processes Regression

$$\hat{\mu}_{a|b} = \mu_a + \Sigma_{K,ab} \Sigma_{T,bb}^{-1} (x_b - \mu_b)$$

Input bins b

$$\hat{\Sigma}_{T,a|b} = \Sigma_{K,aa} - \Sigma_{K,ab} \Sigma_{T,bb}^{-1} \Sigma_{K,ba}$$

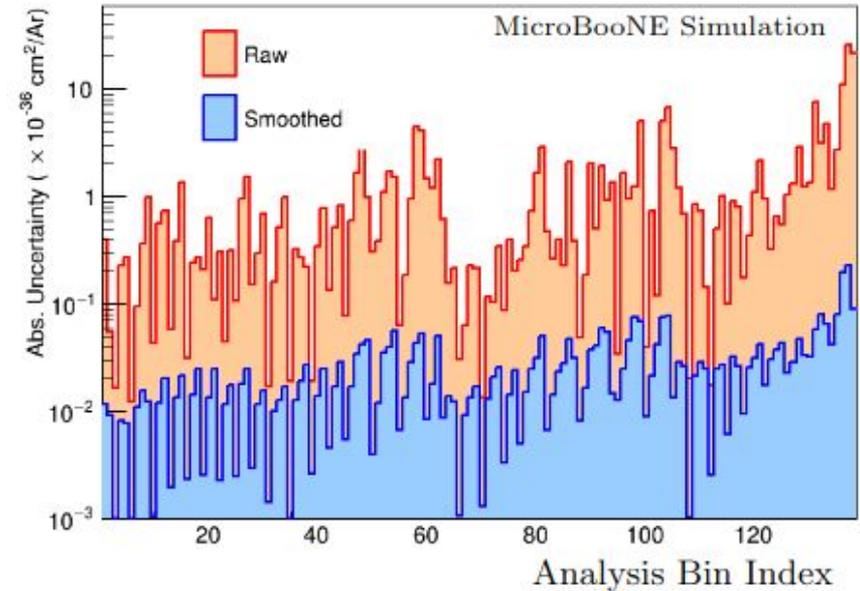
Posterior bins a

$$\Sigma_K(x_1, x_2) = e^{-|(\vec{x}_1 - \vec{x}_2) \cdot \vec{s}|^2 / 2}$$

Inverse length scales s

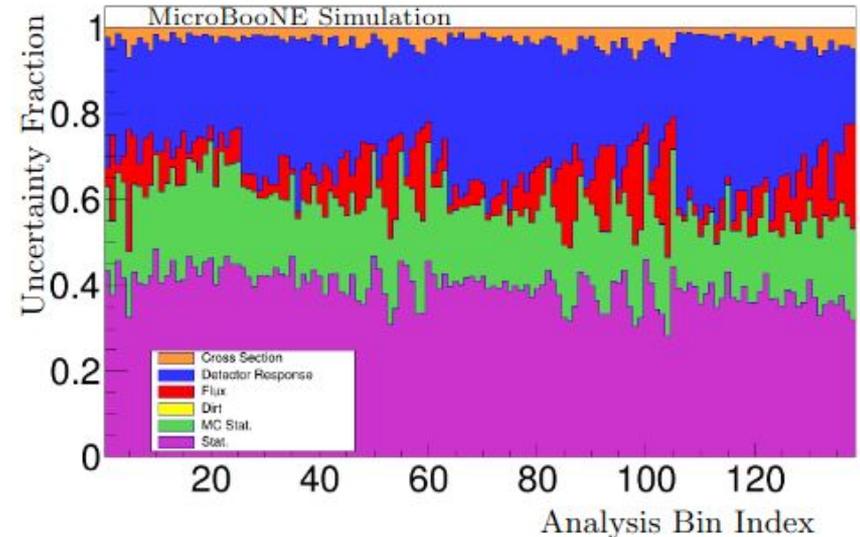
Gaussian Processes Regression (GPR) Smoothing

- Detector Response uncertainties estimated with unisim w/ bootstrapping
 - Limited MC stats + 3D binning -> statistical fluctuations
- GPR Treats nearby bins (at positions x_1, x_2) as correlated following a kernel function:
$$K(x_1, x_2) \sim e^{-|x_1 - x_2|^2}$$
- Computed correlations & ‘measurement’ (here MC) used to update an uninformed prior using Bayes’s Rule
- Result: less driven by statistical fluctuations

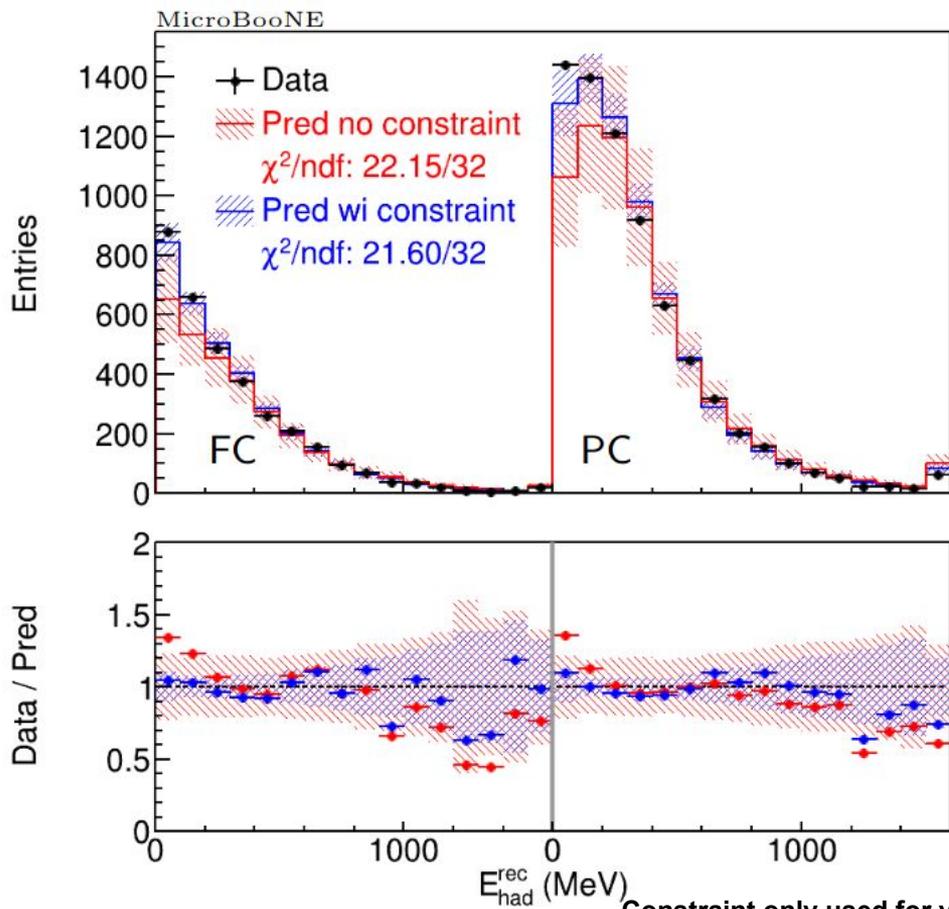


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Model Validation: $M(E_{had}^{vis})$ vs $\mu(E_{had}^{vis} | E_{\nu}, E_{\mu}^{reco})$



Given by neutrino
flux modeling

Muon kinematics
measurement

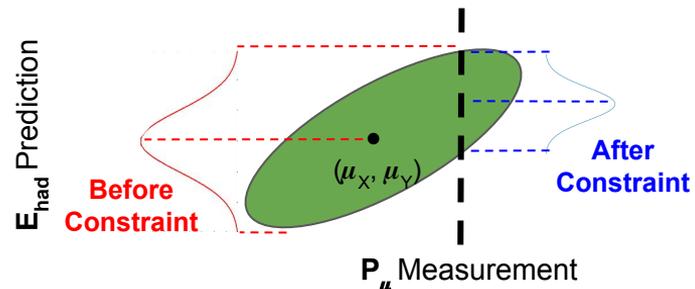
Sensitive to modeling of missing
hadronic energy through
conservation of energy:

- $E_{\nu} = E_{\mu} + E_{had}^{vis} + E_{had}^{missing}$
- E_{μ} and E_{had}^{vis} measured directly
- Constrained flux modeling \rightarrow constrained E_{ν} prediction

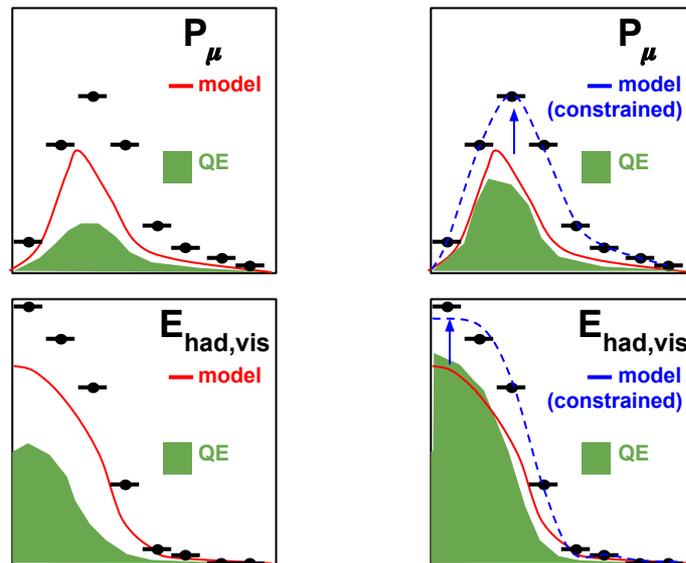
Constraint only used for validation, not unfolding

Model Validation of Missing Hadronic Energy

- Conditional constraint procedure akin to reweighting based on P_μ measurement
- QE, RES, DIS predict different P_μ , $E_{\text{had}}^{\text{missing}}$, and $E_{\text{had}}^{\text{vis}}$ distributions
 - The constrained prediction of $E_{\text{had}}^{\text{vis}}$ is sensitive to the modeling of $E_{\text{had}}^{\text{missing}}$ in each process
- Measurement of constrained $E_{\text{had}}^{\text{vis}}$ is thus sensitive to the model processes used in $E_{\text{had}}^{\text{missing}} \rightarrow$ validation of **the mapping between true and reconstructed E_ν**

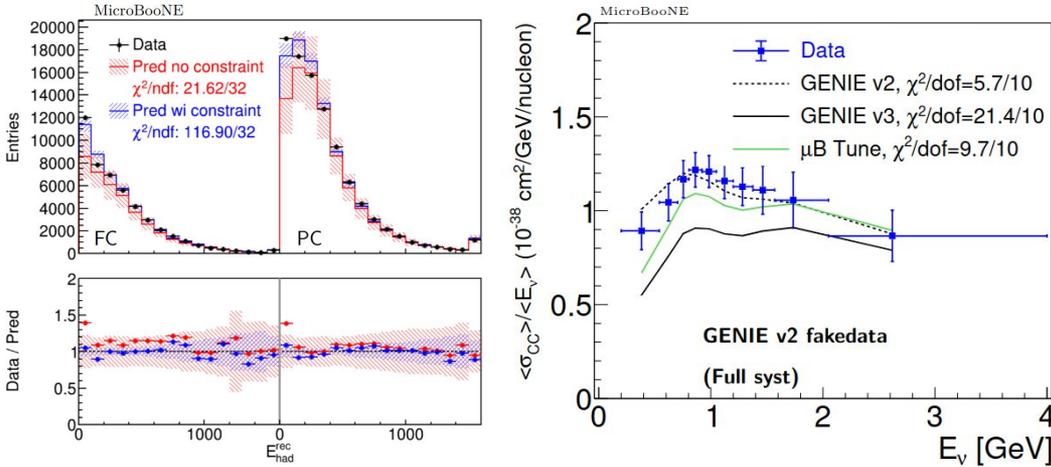


For Illustrative Purposes Only:



Constraint only used for validation, not unfolding

Testing Model Validation Procedure with Fake Data

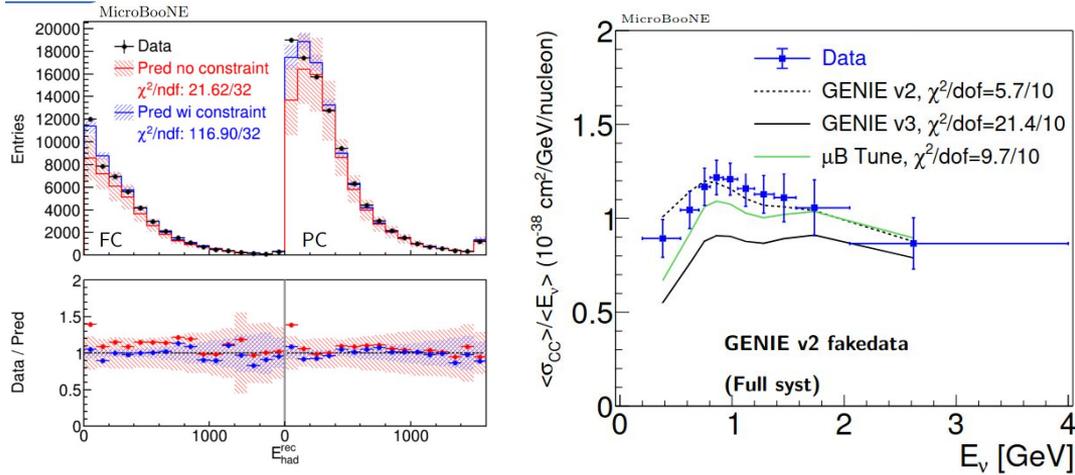


Fake Data	Model Validation GoF χ^2/ndf (p-value)	Unfolded XS w.r.t truth χ^2/ndf (p-value)
Genie v2	116.9/32 (1e-10)	5.7/10 (.84)
-30% E_p	47.1/16 (6.6e-5)	5.2/10 (.88)

- Fake data generated from scratch with Genie v2 prediction
 - 7.2×10^{20} POT exposure used
 - Generated with Poisson distribution, statistically independent
- **Constrained model prediction fails validation test** $\rightarrow E_{\text{had}}^{\text{missing}}$ modeling disagreement
- **Unfolded XS consistent with truth**
 - Xs extraction is less sensitive to data/model discrepancy than the model validation
 - Consistent with expectation
 - Similar observation in scaled proton energy fake data study, which is non-statistically independent so no bias $\rightarrow \chi^2/\text{ndf} = 0$.

Constraint only used for validation, not unfolding

Testing Model Validation Procedure with Fake Data



- Fake data generated from scratch with Genie v2 prediction
 - 7.2×10^{20} POT exposure used
- Constrained model prediction fails validation test ($\chi^2/\text{ndf} = 116.9/32$, $\text{p-value} = 1.3 \times 10^{-11}$) $\rightarrow E_{\text{had}}^{\text{missing}}$ modeling disagreement
- Unfolded XS consistent with truth ($\chi^2/\text{ndf} = 5.7/10$, $\text{p-value} = 0.84$) \rightarrow Xs extraction is less sensitive to data/model discrepancy than the model validation)
 - Consistent with expectation
 - Similar observation in other fake data sets

Fake Data	GoF χ^2/ndf	Unfolded XS w.r.t truth χ^2/ndf	Type of Uncertainties Stat. + Syst.
Genie v2	116.9/32	5.7/10	Fluctuations + Full
-15% E_p	39.5/16	4.1/10	Asimov + Xs only
-30% E_p	47.1/16	5.2/10	Asimov + Full

Equation For Unfolding

$$M_i - B_i = \sum_j R_{ij} \cdot S_j$$



$$\chi^2 = (M - B - R \cdot S)^T \cdot V^{-1} \cdot (M - B - R \cdot S)$$

$$R_{ij} = \tilde{\Delta}_{ij} \cdot \tilde{F}_j$$

$$\tilde{\Delta}_{ij} = \frac{POT \cdot T \cdot \int_j F(E_{vj}) \cdot \sigma(E_{vj}) \cdot D(E_{vj}, E_{wct}) \cdot \epsilon(E_{vj}, E_{wct}) \cdot dE_{vj}}{POT \cdot T \cdot \int_j F(E_{vj}) \cdot \sigma(E_{vj}) \cdot dE_{vj}}$$

→ a MC ratio, less sensitive to Xs uncertainty

$$\tilde{F}_j = POT \cdot T \cdot \int_j \bar{F}(E_{vj}) \cdot dE_{vj}$$

$$S_j = \frac{\int_j \bar{F}(E_{vj}) \cdot \sigma(E_{vj}) \cdot dE_{vj}}{\int_j \bar{F}(E_{vj}) \cdot dE_{vj}}$$

Not subject to prior knowledge of the Xs uncertainty

- **V** is the covariance matrix encoding:
 - Data statistical uncertainty: **M**
 - Flux uncertainty: **B, R (F)**
 - Cross-section (Xs) uncertainty: **B, R (σ)**
 - GEANT4 hadron interaction uncertainty: **B, R (D, ε)**
 - Detector-model uncertainty: **B, R (D, ε)**
 - “Dirt” uncertainty: **B**
 - POT uncertainty (2%): **M**
 - MC statistical uncertainty: **M**
- The unfolded cross section is defined based on the nominal flux
 - Easy for model comparisons
 - Simple for uncertainty calculation

Equation For Unfolding

Measurements
↓

Flux
↘

Cross section
↓

Detector response
↙

Selection efficiency
↘

Background
↓

$$M(E_{rec}) = POT \cdot T \cdot \int F(E_\nu) \cdot \sigma(E_\nu) \cdot D(E_\nu \rightarrow E_{rec}) \cdot \varepsilon(E_\nu, E_{rec}) \cdot dE_\nu + B(E_{rec})$$

$$M_i = \sum_j R_{ij} \cdot S_j + B_i$$

$$R_{ij} = \tilde{\Delta}_{ij} \cdot \tilde{F}_j$$

$$\tilde{\Delta}_{ij} = \frac{POT \cdot T \cdot \int F(E_{\nu j}) \cdot \sigma(E_{\nu j}) \cdot D(E_{\nu j}, E_{rec i}) \cdot \varepsilon(E_{\nu j}, E_{rec i}) \cdot dE_{\nu j}}{POT \cdot T \cdot \int_j F(E_{\nu j}) \cdot \sigma(E_{\nu j}) \cdot dE_{\nu j}}$$

→ a MC ratio, less sensitive to Xs uncertainty

$$\tilde{F}_j = POT \cdot T \cdot \int \bar{F}(E_{\nu j}) \cdot dE_{\nu j}$$

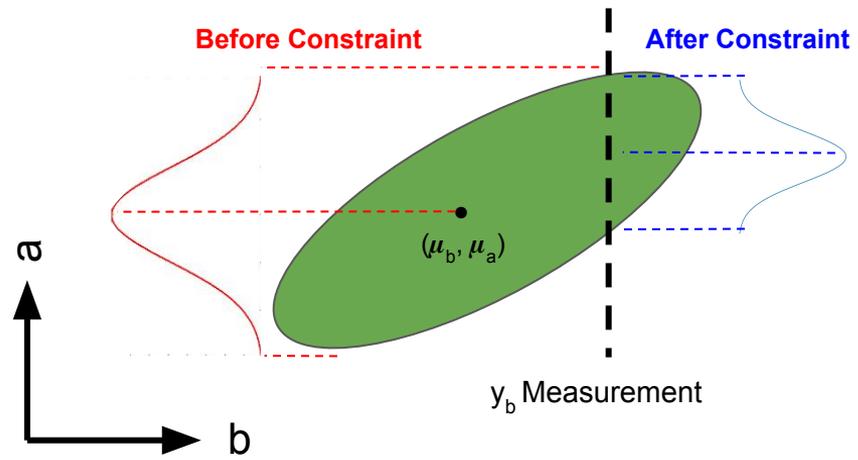
$$S_j = \frac{\int \bar{F}(E_{\nu j}) \cdot \sigma(E_{\nu j}) \cdot dE_{\nu j}}{\int_j \bar{F}(E_{\nu j}) \cdot dE_{\nu j}}$$

Not subject to prior knowledge of the Xs uncertainty

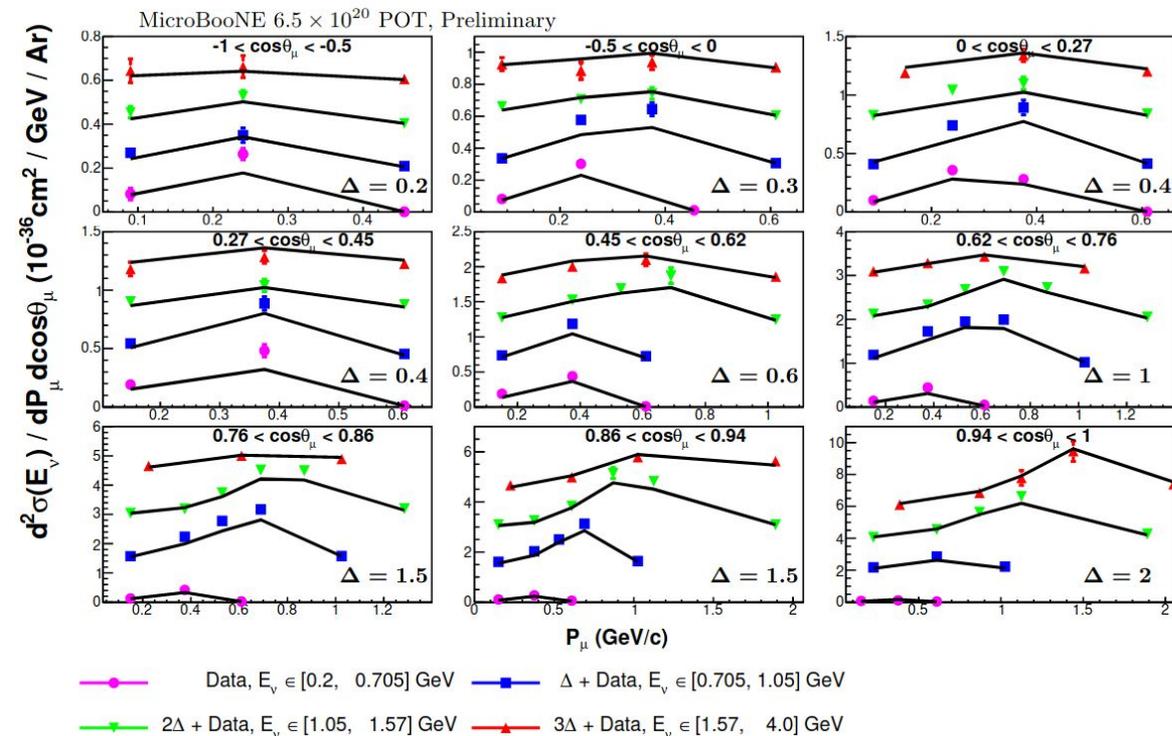
	GENIE 3.0.6	NEUT 5.4.0.1	NuWro 19.2.1	GiBUU 2019.08
Nuclear Model	LFG	LFG	LFG	LFG
QE	Valencia	Nieves	Lwlyn-Smith	standard
MEC	Valencia	Nieves	Nieves	empirical
Resonant	KLN-BS	Berger-Sehgal	Adler-Rarita-Schwinger	MAID (Spin-dependent)
Coherent	Berger-Sehgal	Rein-Sehgal	Berger-Sehgal	
FSI	hA2018 cascade	cascade	cascade	BUU transport model

Inclusive CC measurements

Experiment	Target	References	Efficiency (%)	Purity (%)
ArgoNeUT	Ar	Phys. Rev. Lett. 108 161802 Phys. Rev. D 89 112003	49.5 42.0 (59.0)	95 95.2 (91.2)
MicroBooNE	Ar	Phys. Rev. Lett. 123 131801 Phys. Rev. Lett. 128 , 151801	57.2 68	50.4 92
MINERvA	CH, C/CH, Fe/CH, Pb/CH	Phys. Rev. Lett. 112, 231801 Phys. Rev. D94, 112007 Phys. Rev. Lett. 116	24 ~ 50	60 ~ 80
MINOS	Fe	Phys. Rev. D81, 072002		
NOMAD	C	Phys. Lett. B660, 19	40.9 ~ 73.3	99.3
SciBooNE	CH	Phys. Rev. D83, 12005	34.5	~90
T2K	CH, H ₂ O, Fe	Phys. Rev. D87, 092003 Phys. Rev. D90, 052010 Phys. Rev. D93, 072002	~50 41.2 ~50 @1GeV	~86 89.4 ~97



Unfolded Measurement in 3D



Data plotted against NuWro prediction
 E_ν slices overplot with offset $N\Delta$ for each angle slice
 Δ in same units of $d^2\sigma(E_\nu)/dP_\mu d\cos(\theta_\mu)(10^{-36}\text{cm}^2/\text{GeV}/\text{Ar})$

Model Generator	χ^2/ndf
Genie v2.12.10	740.8/138
Genie v3.0.6 (MicroBooNE Tune)	313.9/138
Genie v3.0.6 (Untuned)	309.7/138
GIBUU 2021	265.6/138
NEUT v5.4.0.1	233.1/138
NuWro v19.02.01	200.9/138

Descending $\chi^2/\text{ndf} \rightarrow$

3D measurement contains wealth of information \rightarrow all model central value predictions are now in tension with data

More powerful than 1D measurement, which was consistent with some models