# Diphoton production at Tevatron and the LHC in the NLO$^\star$ approximation of the Parton Reggeization Approach.

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# Introduction

This talk is based on the results, presented in [M. A. Nefedov, V. A. Saleev, Diphoton production at Tevatron and the LHC in the NLO$^\star$ approximation of the Parton Reggeization Approach; arXiv:hep-ph/1505.01718].

Diphoton production is interesting as from the point of view of the New Physics searches (e. g. irred. background to $H \to \gamma\gamma$) and also as an excellent test for the techniques in pQCD:

- Fixed-order canculations in the Collinear Parton Model (CPM)
- Soft gluon/logarithmic resummation techniques
- $k_T$-factorization, TMD factorization

# Inclusive diphotons in $pp$ or $p\bar{p}$ collisions.

Let's consider the reaction:

$$p(P_1) + p(P_2) \to \gamma(q_3) + \gamma(q_4) + X,$$

where the photons are **hard** ($q_{T3,4} \gtrsim 10$ GeV), **isolated** in the ($\Delta y$, $\Delta\phi$) plane, both from each other, and from hadronic activity, and **prompt**, i. e. have to come from the primary collision vertex.
There are many **differential** observables to meashure:

$$\frac{d\sigma}{dM}, \frac{d\sigma}{dp_T}, \frac{d\sigma}{dY}, \frac{d\sigma}{d\Delta\phi}, \frac{d\sigma}{d\cos\theta^*}, \frac{d\sigma}{dz}, \dots,$$

where $M^2 = (q_3 + q_4)^2$, $p_T^2 = (\mathbf{q}_{T3} + \mathbf{q}_{T4})^2$, $z = q_{T3}/q_{T4}$, $\theta^*$- Collins angle, and the process itself is **multiscale**:

$$p_T, \ M, \ E_{Tmin}^{(L)}, \ E_{Tmin}^{(SL)}, \ E_T^{(ISO)}$$

# Approach of the Collinear Parton Model in the fixed order.

Factorization formula of the CPM:

$$d\sigma = \sum_{p_1,p_2} \int_0^1 dx_1 \int_0^1 dx_2 f_{p_1}(x_1,\mu_F^2) f_{p_2}(x_2,\mu_F^2) d\hat{\sigma}_{p_1 p_2}(q_1,q_2,\mu_F,\mu_R),$$

where $q_1 = x_1 P_1$, $q_2 = x_2 P_2$, $P_{1,2}^2 = 0$, $f_p(x,\mu_F)$ – (integrated) PDF of the parton $p$ in proton, $d\hat{\sigma}$ – hard-scattering cross-section. For the sufficiently inclusive **single-scale** observables (e. g. $d\sigma/dydQ^2$ in Drell-Yan), it is proven (see e. g.[Collins, 2011]), that the factorization-breaking terms are power-supressed (e. g. $\sim 1/Q^2$), and large-$\alpha_s \log(Q^2)$ perturbative corrections are resummed through the $\mu_F$-dependence of the PDFs, using the **DGLAP** evolution equation.
The LO ($O(\alpha_s^0)$) subprocess is:

$$q + \bar{q} \to \gamma + \gamma,$$

The NLO ($O(\alpha_s^1)$) subprocesses:

$q + g \to \gamma + \gamma + q$; $q + \bar{q} \to \gamma + \gamma + g$; **1-loop virtual corrections**

NNLO ($O(\alpha_s^2)$):

$g + g \to \gamma + \gamma$; $2 \to 4$ **real**; $2 \to 3$ **real-virtual**; **2-loop virtual** corections.

# Direct and fragmentation contributions.

The **experimental definition** of the $\sigma$ requires no hadrons (QCD radiation) with $E_T > E_T^{(ISO)}$ in the isolation cone of the photon $\delta = \sqrt{\Delta\phi^2 + \Delta y^2} \leq R$. This definition is not **collinear-safe** due to collinear singularity of the $q \to q\gamma$ splitting. Ways out:

- Introduce the quark$\to \gamma$ FF: $f_{\gamma/q}(z, \mu^2)$, which will absorb the collinear singularity. $\Rightarrow \sigma_{obs} =$ **direct component** (with collinear singularity subtracted in the $\overline{MS}$ scheme)+ **fragmentation component**. Problems: FFs should be fitted to data and introduction of the FFs complicates the analytcs **a lot**.
- There is a way to define collinear-safe direct cros-section – **Frixione isolation condition** [Frixione, 1998].

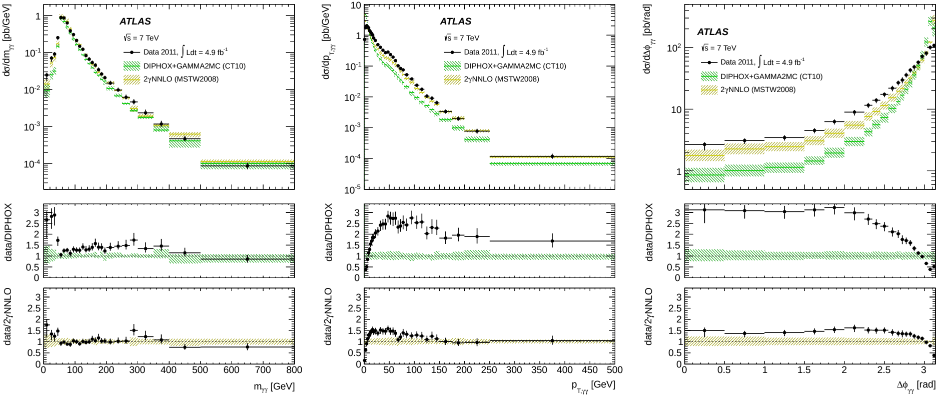
The trick is to modify the isolation condition:

$$E_T \leq E_T^{(ISO)}\chi(\delta),\ \ \chi(\delta) = \left(\frac{1-\cos\delta}{1-\cos R}\right)^n,\ n \geq 1/2.$$

Recent studies [Cieri, de Florian, 2013] show, that the direct contribution with Frixione isolation condition ($n = 1$) is a very good (within $1 - 3\%$) estimate of the full direct+fragmentation cross-section with standard isolation at NLO. Since higher-order effects are very important ($O(100\%)$ in some kinematical regions), it is reasonable to proceed with Frixione isolation.

# NLO and NNLO results, ATLAS data.

Plots are extracted from [arXiv:hep-ex/1211.1913v2]:




$LO + NLO + NNLO \simeq$ experinent $\Rightarrow N^3LO \ll NNLO$ ???

## $k_T$-factorization and PRA.

In the CPM, the transverse momentum $\mathbf{q}_T$ of initial-state parton is **neglected** in the hard part and **integrated over** in the PDFs. This approximation leads to large higher-order corrections to the $p_T$-spectra in CPM. The **$k_T$-factorization** scheme [Gribov *et. al.* 1983; Collins *et. al.* 1991; Catani *et. al.* 1991] is introduced to improve the description in the kinematical region where:

$$q^{\pm} = x\sqrt{S} \sim |\mathbf{q}_T| \ll q^{\mp}$$

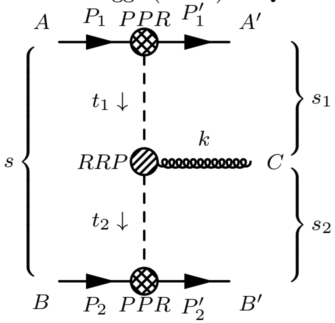
Here, most of the initial state radiation is **highly separated in rapidity** from the central region, and can be factorized, but **the transverse-momentum of the initial-state parton can not be neglected in the hard-scattering part, and should be taken ito account in the gauge-inveriant way**.
In present time, three methods proposed to solve the last problem:

- The old $k_T$-factorization prescription for gluons ($\varepsilon^\mu(q) = \frac{q_T^\mu}{|\mathbf{q}_T|}$).
- The parton Reggeization approach (PRA).
- Methods based on the extraction of the multi-Regge asymptotics of the amplitudes in the spinor-helicity representation [van Hameren *et. al.*, 2013].

In fact, the last two are closely related to each-other.

# Reggeization of amplitudes in QCD.

PRA is based on the Reggeization of amplitudes in gauge theories (QED, QCD, Gravity). The *high energy asymptotics* of the $2 \to 2 + n$ amplitude is dominated by the diagram with $t$-channel exchange of the effective (Reggeized) particle and Multi-Regge (MRK) or Quasi-Multi-Regge Kinematics (QMRK) of final state.




In the limit $s \to \infty$, $s_{1,2} \to \infty$, $-t_1 \ll s_1$, $-t_2 \ll s_2$ (Regge limit), $2 \to 3$ amplitude has the form:

$$\mathcal{A}_{AB}^{A'B'C} = 2s\gamma_{A'A}^{R_1} \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{1}{t_1} \times$$

$$\times \Gamma_{R_1 R_2}^{C}(q_1, q_2) \times \frac{1}{t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \gamma_{B'B}^{R_2}$$

$\Gamma_{R_1 R_2}^{C}(q_1, q_2)$ - $RRP$ effective production vertex,

$\gamma_{A'A}^{R}$ - $PPR$ effective scattering vertex,

$\omega(t)$ - Regge trajectory.

Two approaches to obtain this asymptotics:

- BFKL-approach (Unitarity, renormalizability and gauge invariance), see e. g. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995].

# The field content of the effective theory.

Light-cone vectors:

$$n^+ = \frac{2P_2}{\sqrt{S}},\ n^- = \frac{2P_1}{\sqrt{S}},\ n^+n^- = 2$$

$$x^\pm = n^\pm x = x^0 \pm x^3,\ \partial_\pm = 2\frac{\partial}{\partial x^\mp}$$

Lagrangian of the effective theory $L = L_{kin} + \sum_y \left(L_{QCD} + L_{ind}\right)$, $v_\mu = v_\mu^a t^a$, $\left[t^a, t^b\right] = f^{abc}t^c$. Each subinterval in rapidity $(1 \ll \eta \ll Y)$ has it's own set of QCD fields:

$$L_{QCD} = -\frac{1}{2}tr\left[G_{\mu\nu}^2\right],\ G_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + g\left[v_\mu, v_\nu\right].$$

Different rapidity intervals communicate via Reggeized gluons $(A_\pm = A_\pm^a t^a)$ with the kinetic term:

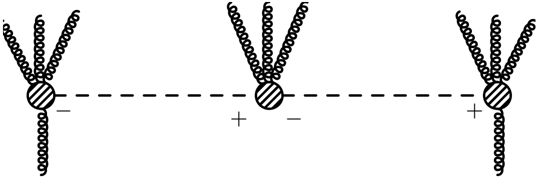
$$L_{kin} = -\partial_\mu A_+^a \partial^\mu A_-^a,$$

and the kinematical constraint:

$$\partial_- A_+ = \partial_+ A_- = 0 \Rightarrow$$

$$A_+ \text{ has } k_- = 0 \text{ and } A_- \text{ has } k_+ = 0.$$

## The effective action for high energy processes in QCD.




Particles and Reggeons interact via *induced interactions*:

$$L_{ind} = - tr\left\{ \frac{1}{g}\partial_+ \left[ P\exp\left( -\frac{g}{2}\int\limits_{-\infty}^{x^-} dx'^- v_+(x') \right)\right] \cdot \partial_\sigma\partial^\sigma A_-(x) + \right.$$

$$\left. + \frac{1}{g}\partial_- \left[ P\exp\left( -\frac{g}{2}\int\limits_{-\infty}^{x^+} dx'^+ v_-(x') \right)\right] \cdot \partial_\sigma\partial^\sigma A_+(x) \right\}$$

Wilson lines generate the infinite chain of the induced vertices:

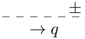
$$L_{ind} = tr\left\{\left[v_+ - gv_+\partial_+^{-1}v_+ + g^2v_+\partial_+^{-1}v_+\partial_+^{-1}v_+ - ...\right]\partial_\sigma\partial^\sigma A_- + \right.$$

$$\left. + \left[v_- - gv_-\partial_-^{-1}v_- + g^2v_-\partial_-^{-1}v_-\partial_-^{-1}v_- - ...\right]\partial_\sigma\partial^\sigma A_+\right\}$$

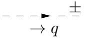
# Feynman rules. Quarks, gluons and photons.

Feynman Rules for Reggeized gluons [Antonov, Cherednikov, Kuraev, Lipatov, 2005]
Feynman Rules for Reggeized quarks [Lipatov, Vyazovsky, 2001]

Initial state factors:

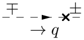


$= \frac{q^\pm}{2\sqrt{-q^2}},$

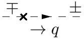


$= u(q^{\parallel}).$

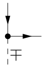
Propagators ($\hat{P}_\pm = \frac{1}{4}\hat{n}^\mp\hat{n}^\pm$):



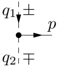
$= \hat{P}_\pm \frac{i\hat{q}}{q^2},$



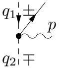
$= \frac{i\hat{q}}{q^2}\hat{P}_\pm.$



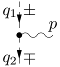
$= -ig_s T^a \hat{n}^\pm,$



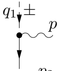
$= -ig_s T^a \left(\hat{n}^\pm + 2\frac{\hat{q}_1}{q_2^\mp}\right),$



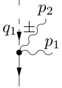
$= -2ieg_s T^a \frac{\hat{q}_1 n^\mp_\mu}{p^\mp q_2^\mp},$



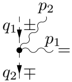
$= -ie\left(\gamma_\mu + \hat{q}_1\frac{n^\mp_\mu}{p^\mp} + \hat{q}_2\frac{n^\pm_\mu}{p^\pm}\right),$



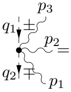
$= -ie\left(\gamma_\mu + \hat{q}_1\frac{n^\mp_\mu}{p^\mp}\right),$



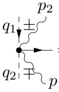
$= -ie^2\hat{q}_1\frac{n^\mp_{\mu_1}n^\mp_{\mu_2}}{p_1^\mp p_2^\mp}.$



$= ie^2\left(\hat{q}_2\frac{n^\pm_{\mu_1}n^\pm_{\mu_2}}{p_1^\pm p_2^\pm} - \hat{q}_1\frac{n^\mp_{\mu_1}n^\mp_{\mu_2}}{p_1^\mp p_2^\mp}\right),$



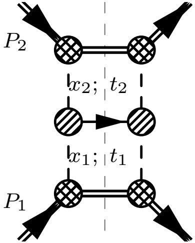
$= -ie^3\left(\hat{q}_2\frac{n^\pm_{\mu_1}n^\pm_{\mu_2}n^\pm_{\mu_3}}{p_1^\pm p_2^\pm p_3^\pm} + \hat{q}_1\frac{n^\mp_{\mu_1}n^\mp_{\mu_2}n^\mp_{\mu_3}}{p_1^\mp p_2^\mp p_3^\mp}\right),$



$= -2ie^2 g_s T^a \frac{\hat{q}_1 n^\mp_{\mu_1}n^\mp_{\mu_2}}{p_1^\mp p_2^\mp q_2^\mp}.$

# Factorization of the cross-section.

**Factorization:**




**Collinear limit** holds for the amplitude:

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2}\to 0} \overline{|\mathcal{M}|^2}_{PRA} = \overline{|\mathcal{M}|^2}_{CPM}$$

$k_T$-factorization formula:

$$d\sigma = \int \frac{d^2\mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times$$
$$\times \int \frac{d^2\mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where $\Phi$ - Unintegrated PDFs. The factorization is known to hold in the LLA ($\alpha_s \log(1/x)$) [**BFKL**, 1978], and NLLA ($\alpha_s^2 \log(1/x)$) [Fadin, Lipatov, 1998; Camici, Ciafaloni, 1998; Bartels, *et. al.*, 2006].

Normalization of the unPDF:

$$\int^{\mu^2} dt \Phi(x, t, \mu^2) = x f(x, \mu^2),$$

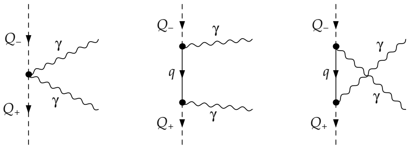
where $f(x, \mu^2)$ - collinear PDF.

# Subprocesses in the PRA.

- LO ($O(\alpha_s^0)$):
$$Q(q_1) + \bar{Q}(q_2) \to \gamma(q_3) + \gamma(q_4), \tag{1}$$
where $Q(\bar{Q})$-Reggeized quark (antiquark), $q_{1,2} = x_{1,2}P_{1,2} + q_{T1,2}$, $q_{1,2}^2 = -t_{1,2}$.
- NLO ($O(\alpha_s^1)$):
$$Q(q_1) + R(q_2) \to \gamma(q_3) + \gamma(q_4) + q(q_5), \tag{2}$$
$$Q(q_1) + \bar{Q}(q_2) \to \gamma(q_3) + \gamma(q_4) + g(q_5), \tag{3}$$
where $R$- Reggeized gluon. Also the 1-loop real-virtual corrections to 1 should be included.
- NNLO ($O(\alpha_s^2)$):
$$R(q_1) + R(q_2) \to \gamma(q_3) + \gamma(q_4), \tag{4}$$
$$\ldots$$

# $Q\bar{Q} \to \gamma\gamma$ subprocess.

$Q_- \; Q_+ \; \to \; \gamma \; \; \gamma$




The answer is known for a long time [Saleev, 2009]:

$$\overline{|M(Q\bar{Q} \to \gamma\gamma)|^2} \quad = \quad \frac{32}{3}\pi^2 e_q^4 \alpha^2 \frac{x_1 x_2}{a_3 a_4 b_3 b_4 S \hat{t}\hat{u}} \Big( w_0 + w_1 S + w_2 S^2 + w_3 S^3 \Big),$$

where $a_3 = q_3^+/\sqrt{S}$, $a_4 = q_4^+/\sqrt{S}$, $b_3 = q_3^-/\sqrt{S}$, $b_4 = q_4^-/\sqrt{S}$, $\hat{s} = (q_1+q_2)^2$, $\hat{t} = (q_1 - q_3)^2$, $\hat{u} = (q_1 - q_4)^2$, $x_1 = a_3 + a_4$, $x_2 = b_3 + b_4$,

$$w_0 = t_1 t_2 (t_1 + t_2) - \hat{t}\hat{u}(\hat{t} + \hat{u}),$$

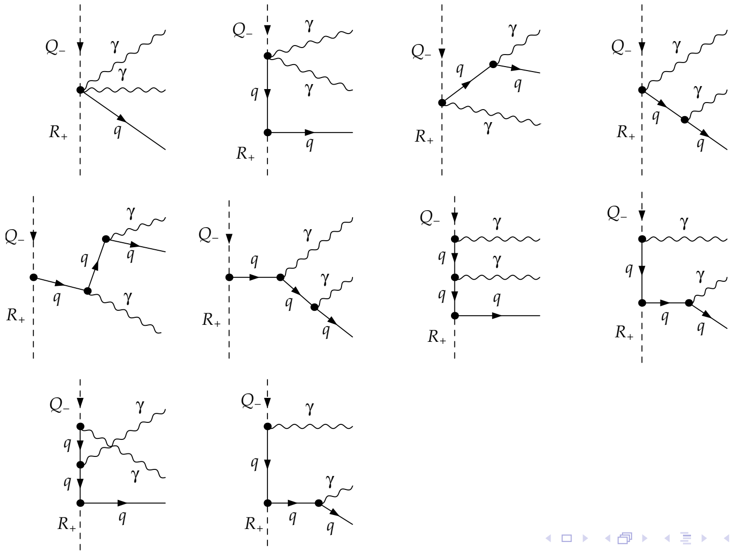
$$\begin{aligned} -w_1 \quad &= \quad t_1 t_2 (a_3 - a_4)(b_3 - b_4) + t_2 x_1 (b_4 \hat{t} + b_3 \hat{u}) + \\ &+ \quad t_1 x_2 (a_3 \hat{t} + a_4 \hat{u}) + \hat{t}\hat{u}(a_3 b_3 + 2 a_4 b_3 + 2 a_3 b_4 + a_4 b_4), \end{aligned}$$

$$-w_2 = b_3 b_4 x_1^2 t_2 + a_3 a_4 x_2^2 t_1 + a_3 b_4 \hat{t}(x_1 b_3 + a_4 b_4) + a_4 b_3 \hat{u}(a_3 b_3 + a_4 x_2),$$

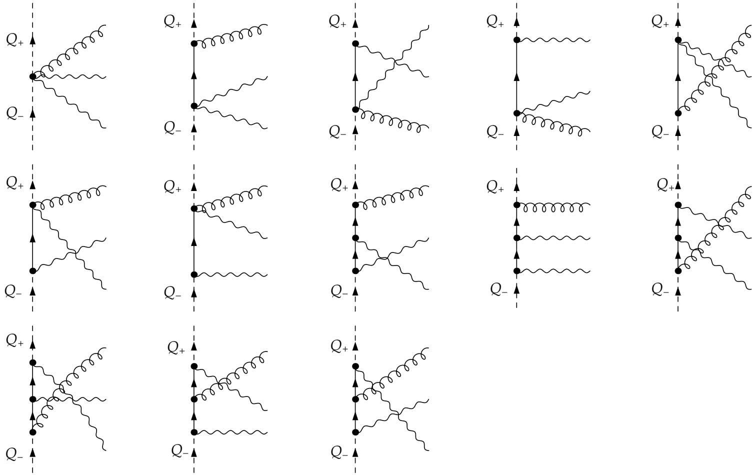
$$-w_3 = a_3 a_4 b_3 b_4 \left( a_3 b_4 \left( \frac{\hat{t}}{\hat{u}} \right) + a_4 b_3 \left( \frac{\hat{u}}{\hat{t}} \right) \right).$$

# $QR \to \gamma\gamma q$

$$Q_- \quad R_+ \quad \to \quad \gamma \quad \gamma \quad q$$

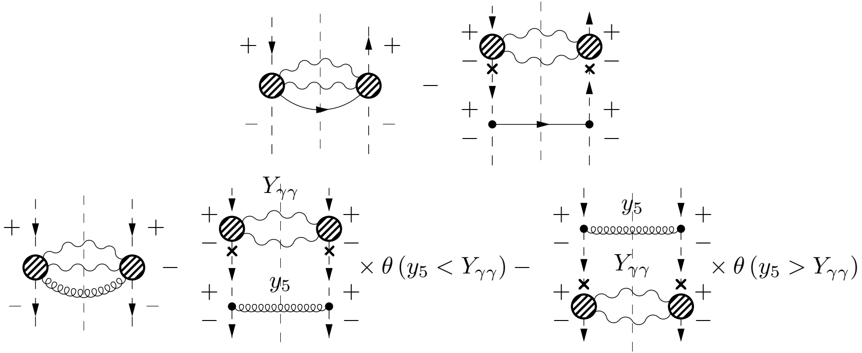


# $Q\bar{Q} \to \gamma\gamma g$

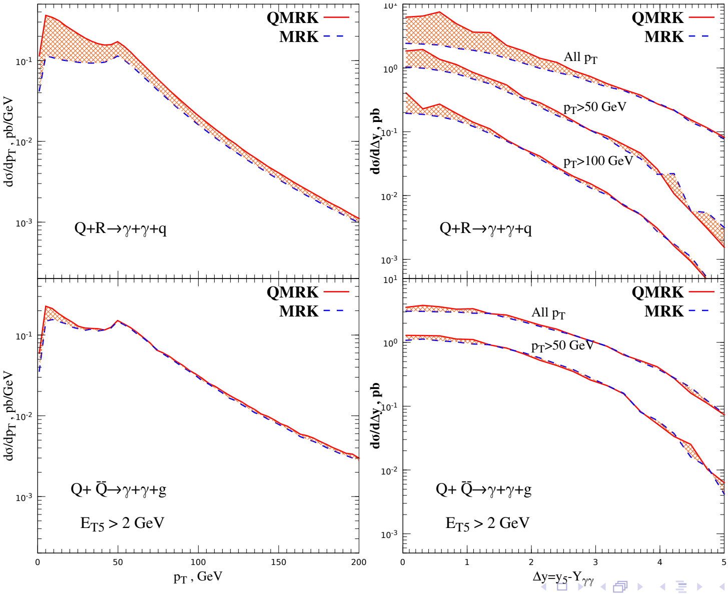


# LO/NLO double counting.

The integral over the whole phase-space of the additional parton in the $2 \to 3$ subprocesses is **finite** (unlike the CPM case), due to the Sudakov suppression of the region of small-$q_T$ in the KMR unPDF (which will be discussed below). However, there is a double-counting between LO and NLO real corections, when the additional parton goes deeply to the forward or backward rapidity regions. This double-counting should be subtracted. The appropriate subtraction technique was introduced in [Bartels, *et. al.*, 2006; Hentshinski, Vera, 2012].



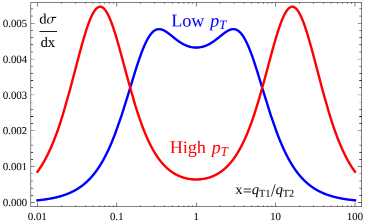
# QMRK vs. MRK. Numerical results.

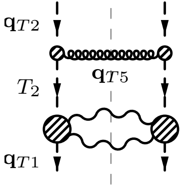


# Why region of high-$p_T$ is MRK-dominated at NLO?

Because:

- "$\hat{s}$-channel" and "$\hat{u}$-channel" contributions are power-suppressed like $\sim 1/p_T^2$.




- At high-$p_T$, the asymmetric initial states with $p_T \sim |\mathbf{q}_{T1}| \gg |\mathbf{q}_{T2}|$ dominate.
- The $T_2$ invariant:

$$T_2 = -(\mathbf{q}_{T2} - \mathbf{q}_{T5})^2 - |\mathbf{q}_{T5}|\sqrt{M^2 + p_T^2}e^{-\Delta y},$$

is minimal when $\mathbf{q}_{T2} \simeq \mathbf{q}_{T5}$ and $\Delta y \to \infty$ (Multi-Regge region).

- Therefore the MRK-configurations with:

$$p_T \sim |\mathbf{q}_{T1}| \gg |\mathbf{q}_{T2}| \simeq |\mathbf{q}_{T5}|,$$

dominate at high-$p_T$ and low-$M$, i. e. in the $M < p_T$ **region**.

# The Kimber-Martin-Ryskin unPDF.

In the present numerical computations we use the modified KMR unPDF from [Martin , Ryskin, Watt 2010].
KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton $k_T-$dependent radiation and the assumption of strong angular ordering:

$$\Phi_q(x,k_T^2,\mu^2)=\frac{1}{k_T^2}\int\limits_x^{1-\Delta} dz T_q(q^2,\mu^2)\frac{\alpha_s(q^2)}{(2\pi)}\left[P_{qg}(z)f_g\left(\frac{x}{z},q^2\right)+P_{qq}(z)f_q\left(\frac{x}{z},q^2\right)\right],$$

where $P_{qg}(z)$, $P_{qq}(z)$- LO DGLAP splitting functions, $T_q(k^2,\mu^2)$- Sudakov formfactor:

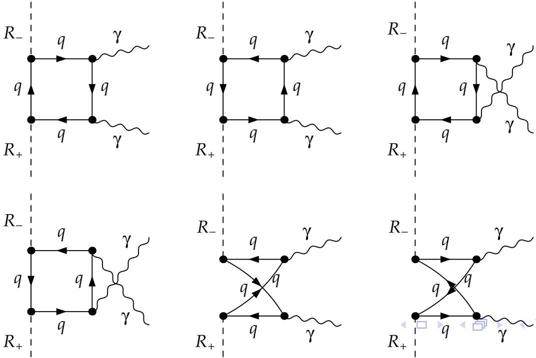
$$T_q(k^2,\mu^2)=exp\left\{-\int\limits_{k^2}^{\mu^2}\frac{dq_T^2}{q_T^2}\frac{\alpha_s(q_T^2)}{2\pi}\sum_{a'}\int\limits_0^{1-\Delta}P_{qa'}(z')dz'\right\}$$

where $\Delta=\frac{k_T}{\mu+k_T}$ ensures the **rapidity ordering of the last emission and particles produced in the hard subprocess**, and $q^2=k_T^2/(1-z)$.

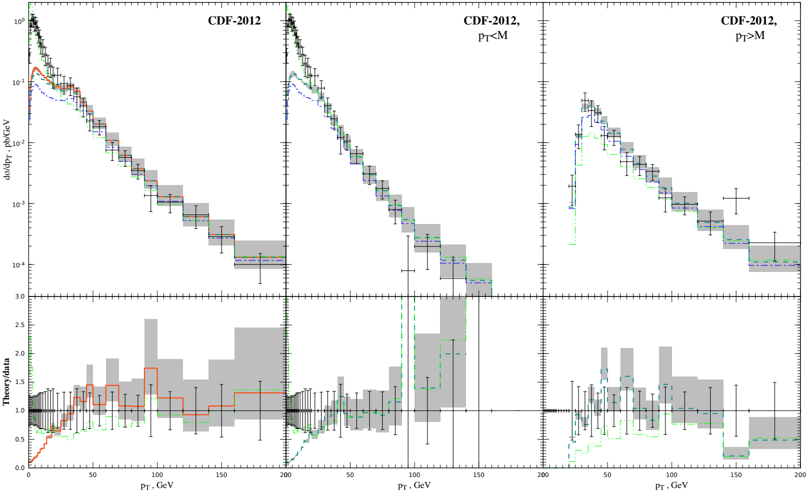
# The $RR \to \gamma\gamma$ contribution.

The $RR \to \gamma\gamma$ contribution is formally NNLO, but it is sizable, due to the enchancement of the gluon distribution at small-$x$. We have computed this contribution, keepinging the **exact** dependence on the virtuality of the initial-state partons, which was done for a first time. The main conclusion is the same as in the case of $\gamma R \to \gamma g$ subprocess in $\gamma p \to \gamma X$ [Kniehl, Nefedov, Saleev, 2014], the transverse momentum of the initial-state partons leads to the $30 - 40\%$ **suppression** of the loop-induced contributions w. r. t. corresponding CPM results.

$$R_- \; R_+ \to \gamma \; \gamma$$

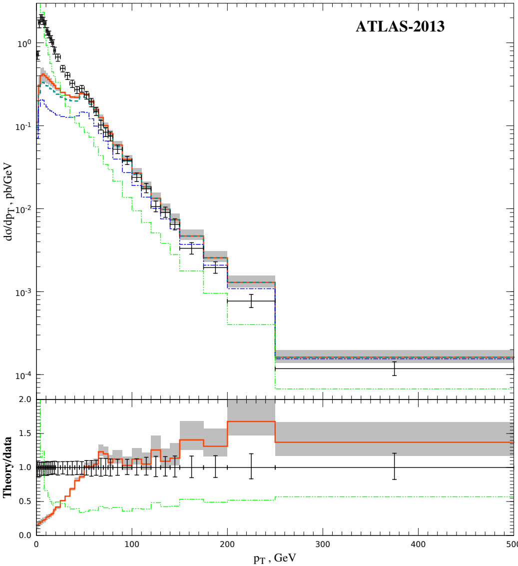


# Tevatron ($p\bar{p}$, $\sqrt{S} = 1960$ GeV), $p_T$-spectra.

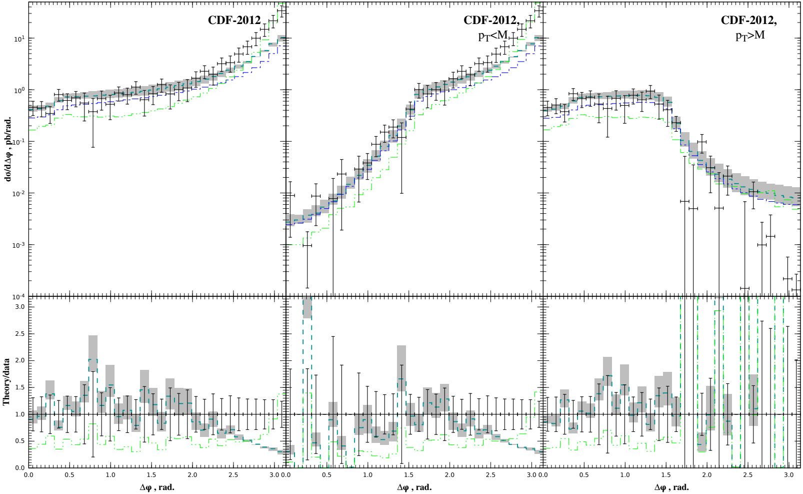



Dash-dotted curve – NLO CPM (DIPHOX). NLO$^\star$ PRA curves from bottom to top: $Q\bar{Q} \to \gamma\gamma$ + $QR \to (\gamma\gamma)q$, QMRK-MRK + $RR \to \gamma\gamma$. Gray band – scale uncertainty: $\mu_R = \mu_F = \xi M$, where $\xi = 1,\ 2^{\pm 1}$.

# LHC, ($pp$, $\sqrt{S} = 7000$ GeV), $p_T$-spectra.

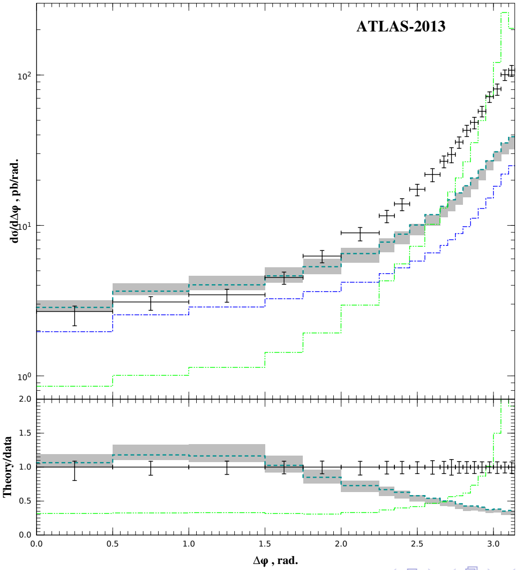


# Tevatron ($p\bar{p}$, $\sqrt{S} = 1960$ GeV), $\Delta\phi$-spectra.

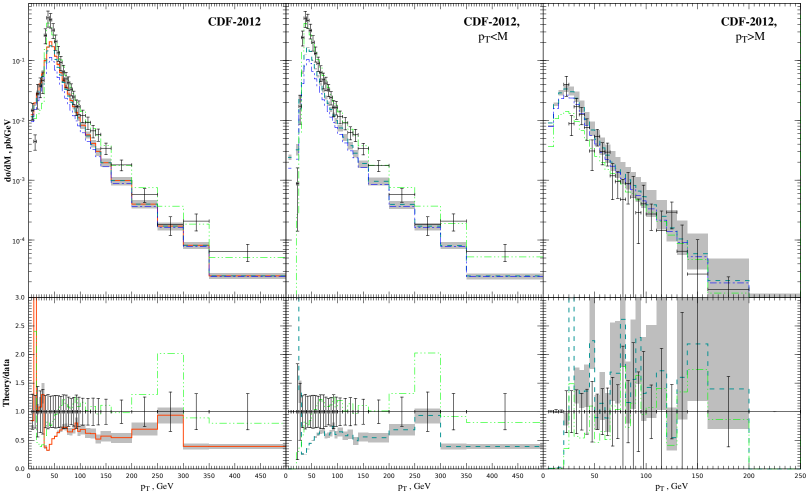



The deficit of the theoretical cross-section is observed in the region of low $p_T$ and $\Delta\phi \simeq \pi$, where the additional radiation is kinematically constrained to be very **soft**, this region is more suitable for the fixed-order calculations or SCET, than PRA.

# LHC, ($pp$, $\sqrt{S} = 7000$ GeV), $\Delta\phi$-spectra.

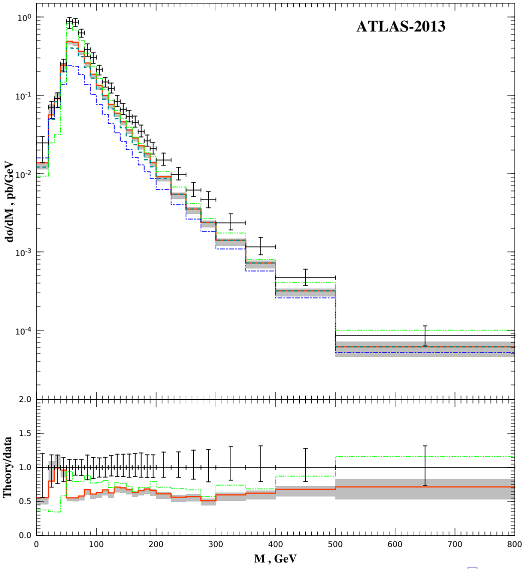


# Tevatron ($p\bar{p}$, $\sqrt{S} = 1960$ GeV), $M$-spectra.




The deficit of the inclusive cross-section is a consequence of the deficit of the cross-section in the CPM region. The data in $p_T > M$ region are well-described again.

# LHC, ($pp$, $\sqrt{S} = 7000$ GeV), $M$-spectra.



## Conclusions.

- The good description of experimental data is achieved in the $p_T > M$-region, already in the LO of PRA.
- In this kinematical region, NLO QMRK corrections are shown to be subleading, demonstrationg the self-consistency of the approach.
- Our results demonstrate, why in many other processes, like single photon, jet or heavy quarkonia production, the LO PRA calculations describe data well.
- The $RR \to \gamma\gamma$ contribution is shown to be $30 - 40\%$ smaller than in the CPM case, constituting $O(10\%)$ of the cross-section.

**Thank you for your attention!**