QCD at high energy in the Parton Reggeization Approach

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Outline.

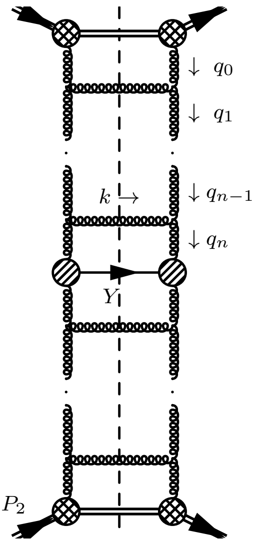
Introduction

The hard processes (i. e. the inelastic processes involving high momentum transfer $Q^2 \gg 1$ GeV^2) are the major tool to study the fundamental interactions, both QCD and EW, at hadron colliders, since the most interesting fundamental particles ($W^\pm$, Z^0, H, t, b, $\tilde{t}$, ...) are heavy.
Thanks to asympthotic freedom of QCD, and a special kinematics of the hard collision, it is possible to separate the perturbative and nonperturbative dynamics, systematically parametrize nonperturbative part, calculate hard subprocess in the perturbation theory, and therefore put the whole problem under quantitative control.
Currently, the studies of the hard processes in pQCD are developing along the lines of four *complementary* approaches:

- Fixed-order canculations in the Collinear Parton Model (CPM)
- Soft gluon/logarithmic resummation techniques
- LO, NLO, (NNLO) + Parton Shower Monte-Carlo techniques
- Soft-Collinear Effective Theory (SCET), TMD factorization
- k_T-factorization

The talk will be devoted mostly to the last class of approaches, which try to generalize the conventional Collinear Parton Model.

Collinear factorization, DGLAP evolution.



$f(x_0, Q_0^2)$

$$\sim \int_{Q_0^2}^{q_n^2} \frac{dk_T^2}{k_T^2}\alpha_s = \alpha_s ln\left(\frac{q_n^2}{Q_0^2}\right)$$

$$P_1^2 = P_2^2 = 0,\; 2P_1P_2 = S \gg \Lambda^2_{QCD}$$

$$q_0 = x_0 P_1 + q_{0T},\; q_0^2 = q_{0T}^2 = Q_0^2 \sim \Lambda^2_{QCD}$$

$$\ldots$$

$$q_n = xP_1 + q_{nT},\; q_n^2 = q_{nT}^2 = Q^2 \gg \Lambda_{QCD}.$$

Collinear factorization for the amplitude ($k_T \to 0$):

$$\overline{|\mathcal{M}_{n+1}|^2} = \frac{1}{k_T^2}\left(\frac{\alpha_s}{2\pi}P_{ij}(z)\overline{|\mathcal{M}_n|^2} + O(k_T^2)\right),$$

$\Rightarrow$ n collinear radiations will give: $\alpha_s^n \log^n(Q^2)$, which can be resummed into PDF by solving DGLAP equation:

$$\frac{\partial f_i(x,\mu^2)}{\partial \log(\mu^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} P_{ij}(z) f_j(x/z, \mu^2)$$

Approach of the Collinear Parton Model in the fixed order.

Factorization formula of the CPM:

$$d\sigma = \sum_{p_1,p_2} \int_0^1 dx_1 \int_0^1 dx_2 f_{p_1}(x_1, \mu_F^2) f_{p_2}(x_2, \mu_F^2) d\hat{\sigma}_{p_1 p_2}(q_1, q_2, \mu_F, \mu_R), +O\left(\frac{1}{(\mu_F^2)^\alpha}\right)$$

where $q_1 = x_1 P_1$, $q_2 = x_2 P_2$, $f_p(x, \mu_F)$ – (integrated) PDF of the parton p in proton, $d\hat{\sigma}$ – hard-scattering cross-section.
For the sufficiently inclusive **single-scale** observables (e. g. $d\sigma/dydQ^2$ in Drell-Yan or $F_2(x, Q^2)$ in DIS), it is proven (see e. g.[Collins, 2011]), that the factorization-breaking terms are power-supressed.
Now we can start to do perturbation theory. Possible problems:

- PT is complicated, LO – tree level, NLO – 1-loop+IR cancellations between real and virtual part, NNLO – 2-loops+ mutch more complicated IR cancellations, ...
- The PT expansion can be slow-convergent due to soft-gluon effects.
- In the case of multiscale processes, the large logarithms of the scale ratios come in – $\alpha_s \log(\mu_1/\mu_2)$.

Light-cone decomposition.

Let's introduce the Sudakov (or light-cone) notation. The protons are flying along the z-axis. For any 4-vector q:

$$q_\mu = \frac{1}{2}(q^+ n^-_\mu + q^- n^+_\mu) + q_{T\mu},$$

where $n^+ = \frac{2P_2}{\sqrt{S}}$, $n^- = \frac{2P_1}{\sqrt{S}}$, $n^+ n^- = 2$, $q^\pm = n^\pm q = q^0 \pm q^3$, $q_T n^\pm = 0$, and $\forall\ q, k$:

$$qk = \frac{1}{2}(q^+ k^- + q^- k^+) - \mathbf{q}_T \mathbf{k}_T,\ q^2 = q^+ q^- - \mathbf{q}_T^2.$$

Rapidity – natural parameter for boosts along the z-axis:

$$y = \frac{1}{2} \log\left(\frac{q^+}{q^-}\right),$$

is closely related to pseudorapidity $\eta = -\log\tan(\theta/2)$. For massless particle:

$$\eta = y.$$

TMD factorization

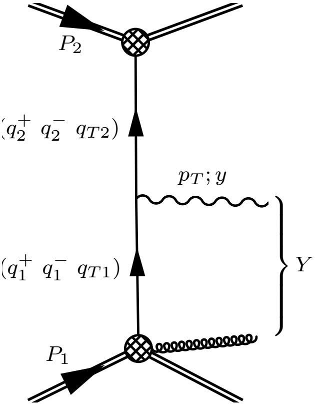
For the multiscale processes, like the Drell-Yan $d\sigma/dQ^2dp_T$, the large log corrections of the form $\alpha_s \log(p_T^2/Q^2)$ are accumulated for $\Lambda_{QCD}^2 \ll p_T^2 \ll Q^2$. For this kind of processes, the TMD-factorization theorem is proven in all orders (see e. g. [Collins, 2011]):

$$d\sigma = \int dx_1 dx_2 \int d^2\mathbf{q}_{T1} F(x_1, \mathbf{q}_{T1}^2, \mu_F^2, \mu_Y^2) F(x_2, \mathbf{q}_{T2}^2, \mu_F^2, \mu_Y^2) C_{low\ p_T}(x_1, x_2) + \\ + \int dx_1 dx_2 f(x_1, \mu_F^2) f(x_2, \mu_F^2) C_{high\ p_T}(x_1, x_2) + \ power\ corrections,$$

where $\mathbf{q}_{T2} = \mathbf{p}_T - \mathbf{q}_{T1}$, F-TMD PDF where new log corrections are absorbed to. The hard-scattering coefficients $C_{low\ p_T}$ and $C_{high\ p_T}$ are free from large logarithms, **do not depend on $\mathbf{q}_T$**, and calculable in the PT. Regions of $p_T^2 < Q^2$ and $p_T^2 > Q^2$ are treated separately. The region of high p_T can be taken into account only order by order in PT. The factorization scale μ_F and rapidity evolution scale μ_Y can be considered separately.

TMD factorization and classic k_T-factorization.

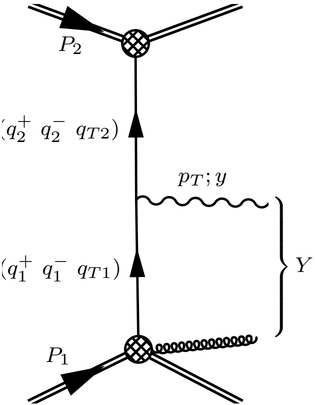
Momentum-flow diagram:



- Collinear factorization:
  - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y – arbitrary,
  - q_1^-, $\mathbf{q}_{T1}$ – integrated out $\Rightarrow f(x, \mu_F)$,
  - $\alpha_s \log(\mu_F)$ – resummed into PDF.
- TMD factorization:
  - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y – arbitrary
  - q_1^- – integrated out $\Rightarrow F(x, \mathbf{q}_T, \mu_F, \mu_Y)$,
  - $\alpha_s \log(\mu_F)$, $\alpha_s \log^2(|\mathbf{q}_T|/\mu_F)$ – resummed into TMD PDF.
- k_T-factorization [Gribov *et. al.* 1983; Collins *et. al.* 1991; Catani *et. al.* 1991] ("Semihard processes"):
  - $|\mathbf{q}_{T1}| \sim \mu_F \ll \sqrt{S}$, $Y \gg 1 \Rightarrow q_1^+ \gg q_1^-$,
  - q_1^- – integrated out $\Rightarrow \Phi(x, \mathbf{q}_T)$,
  - $\alpha_s Y \sim \alpha_s \log(1/x)$ – resummed into unPDF.

Parton Reggeization Approach.

Momentum-flow diagram:



- Collinear factorization:
  - $q_1^+ \gg q_1^-$, $|\mathbf{q}_{T1}| \ll \mu_F$, Y – arbitrary,
  - q^-, $\mathbf{q}_{T1}$ – integrated out $\Rightarrow f(x, \mu_F)$,
  - $\alpha_s \log(\mu_F)$ – resummed into PDF.
- PRA:
  - $|\mathbf{q}_{T1}|/\mu_F$ – arbitrary, Y – arbitrary, combines the TMD and k_T-factorization regions,
  - q_1^- – integrated out $\Rightarrow \Phi(x, \mathbf{q}_T, \mu_F)$,
  - $\alpha_s \log(\mu_F)$, $\alpha_s \log^2(|\mathbf{q}|/\mu_F)$ and $\alpha_s \log(1/x)$ – resummed into the unintegrated PDF.
- Next step – factorization with fully unintegrated PDFs: $\Phi(x^+, x^-, \mathbf{q}_T, \mu_F)$.

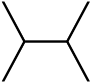
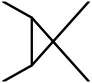
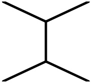
Gauge-invariant amplitudes for the k_T-factorization.

In QCD, off-shell Green functions are not gauge-invariant, in general, so the separation of the contributions between hard subprocess and unPDF seems to be ill-defined.
The Reggeization of the amplitudes in QCD solves this problem. In present time three main approaches to generate the gauge-invariant amplitudes for k_T-factorization are proposed, which are related with Reggeization in one or another way:

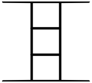
- The old k_T-factorization prescription for gluons ($\varepsilon^\mu(q) = \frac{q_T^\mu}{|\mathbf{q}_T|}$). This prescription gives the result for $g^\star g^\star \to q\bar{q}$ amplitude, coinsiding with $RR \to q\bar{q}$ amplitude in PRA, constructed with the use of Lipatov vertex.
- The parton Reggeization approach (PRA).
- Methods based on the extraction of the multi-Regge asymptotics of the amplitudes in the spinor-helicity representation [van Hameren *et. al.*, 2013]. This metod is equvivalent to the PRA at tree level.

Example: Reggeization in ϕ^3-model.

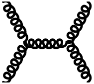
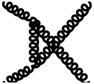
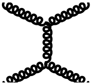
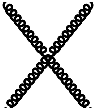
Let's consider the amplitude of the process $\phi\phi \to \phi\phi$ in the limit $s \to \infty$, t-fixed (Regge limit). The scaling of the tree-level diagrams is obvious:

 $\sim \frac{1}{s}$,  $\sim \frac{1}{4m^2 - t - s} \sim \frac{1}{s}$,  $\sim \frac{1}{t}$

Leading loop corrections:

$\sum_n$  $= \frac{g^2}{t} \sum_n \frac{1}{n!} \omega^n(t) \log^n(s) = \frac{g^2}{t} s^{\omega(t)},$

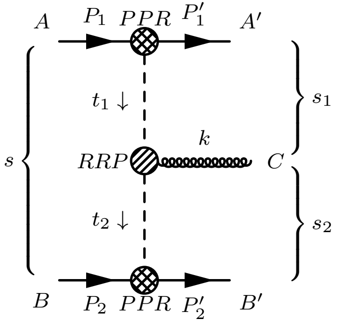
where $\omega(t)$– Regge trajectory [Landshoff, Olive, Polkinghorne, 1966]. Naively, in QCD the same power-counting is possible:

 $\sim \frac{1}{s}$,  $\sim \frac{1}{s}$,  $\sim \frac{1}{t}$, 

But the contribution of different diagrams is not gauge-invariant.

Reggeization of amplitudes in QCD.

PRA is based on the Reggeization of amplitudes in gauge theories (QED, QCD, Gravity). The *high energy asymptotics* of the $2 \to 2 + n$ amplitude is dominated by the diagram with t-channel exchange of the effective (Reggeized) particle and Multi-Regge (MRK) or Quasi-Multi-Regge Kinematics (QMRK) of final state.



In the limit $s \to \infty$, $s_{1,2} \to \infty$, $-t_1 \ll s_1$, $-t_2 \ll s_2$ (MRK limit), $2 \to 3$ amplitude reads:

$$\mathcal{A}_{AB}^{A'B'C} = \gamma_{A'A}^{R_1} \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{1}{t_1} \times$$

$$\times \Gamma^{C}_{R_1 R_2}(q_1, q_2) \times \frac{1}{t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \gamma_{B'B}^{R_2}$$

$\Gamma^{C}_{R_1 R_2}(q_1, q_2)$ - RRP effective production vertex,

$\gamma_{A'A}^{R}$ - PPR effective scattering vertex,

$\omega(t)$ - Regge trajectory.

Three approaches to obtain this asymptotics:

- Direct study of the MRK limit of the amplitudes (*see examples below*).
- BFKL-approach (Unitarity, renormalizability and gauge invariance), see e. g. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995].

Example: derivation of the Reggeized gluon propagator.

The propagator of the Reggeized gluon should be universal, i. e. should not depend on the type of scattered partons. Let's consider the amplitude for the process $qq' \to qq'$:

$$\mathcal{M}(q(p_1)q'(p_2) \to q(p_1')q'(p_2')) = g_s^2 \left(\bar{u}(p_1')T^a\gamma_\mu u(p_1)\right) \frac{g_{\mu\nu}}{t} \left(\bar{u}(p_2')T^a\gamma_\nu u(p_2)\right),$$

The metric tensor can be split into the longitudinal and transversal parts:

$$g_{\mu\nu} = \frac{1}{2}(n_\mu^+ n_\nu^- + n_\mu^- n_\nu^+) + g_{\mu\nu}^\perp,$$

so that the amplitude converts into the sum of two terms $\mathcal{M} = \mathcal{M}^\parallel + \mathcal{M}^\perp$. In the Regge limit ($s \to \infty$, t-fixed), $p_1' \simeq p_1$, $p_2' \simeq p_2$ holds. Using this approximations, we obtain:

$$\mathcal{M}^\parallel = g_s^2 \left(\bar{u}(p_1')T^a \frac{\hat{n}^+}{\sqrt{2}} u(p_1)\right) \frac{1}{t} \left(\bar{u}(p_2')T^a \frac{\hat{n}^-}{\sqrt{2}} u(p_2)\right).$$

By means of standard techniques, it is easy to show, that, in the Regge limit:

$$\overline{|\mathcal{M}^\parallel|^2} \sim \frac{s^2}{t^2}, \quad \overline{|\mathcal{M}^\perp|^2} \to 0,$$

so only $\mathcal{M}^\parallel$ gives the contribution to the amplitude, which is not decreasing with energy.

Example: derivation of the Reggeized gluon propagator.

We have obtained, that:

$$\mathcal{M} = g_s^2 \left(\bar{u}(p_1') T^a \frac{\hat{n}^+}{\sqrt{2}} u(p_1) \right) \frac{1}{t} \left(\bar{u}(p_2') T^a \frac{\hat{n}^-}{\sqrt{2}} u(p_2) \right) + O\left(\frac{1}{\sqrt{s}} \right).$$

From this result it is easy to understand, that Reggeized gluon is **scalar** particle in the **8**-representation of $SU_c(3)$, which is coupled with quarks, carrying large $p^\pm$ momentum components via the effective vertices

$$\gamma_\mp^a = g_s T^a \frac{\hat{n}^\pm}{\sqrt{2}}.$$

The study of the Regge limit of the processes $qq \to qq$, $q\bar{q} \to q\bar{q}$, $qg \to qg$, $gg \to gg$ **supports** the self-consistency of the gluon Reggeization hypotesis at tree level, and allows one to derive the ggR coupling (all momenta are incoming):

$$\gamma_{\mu\nu\mp}^{abc}(k_1, k_2) = g_s f^{abc} \left(2 g_{\mu\nu} k_1^\pm + (2k_2 + k_1)_\mu n_\nu^\pm - (2k_1 + k_2)_\nu n_\mu^\pm - \frac{2k_1 k_2}{k_1^\pm} n_\mu^\pm n_\nu^\pm \right),$$

which happens to obey the Slavnov-Taylor identity:

$$k_1^\mu \varepsilon^\nu(k_2) \gamma_{\mu\nu\mp}^{abc}(k_1, k_2) = \varepsilon^\mu(k_1) k_2^\nu \gamma_{\mu\nu\mp}^{abc}(k_1, k_2) = 0.$$

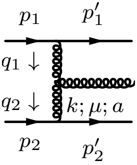
Example: derivation of the Fadin-Lipatov vertex.

Now we will verify the gluon Reggeization in the Multi-Regge Kinematics, by considering the MRK limit of the process

$$q(p_1)q'(p_2) \to q(p_1')g(k)q'(p_2')$$

First, let's consider the 3g-vertex contribution. Straightforwardly applying the Feynman rules, momentum conservation and Dirac equation we get:

$\mathcal{M}_{3g} =$



$$= \frac{g_s^3 f^{cab}}{q_1^2 q_2^2} \left(\bar{u}(p_1')T^c\gamma^\nu u(p_1)\right) \left(\bar{u}(p_2')T^b\gamma^\rho u(p_2)\right) \times$$

$$\times \quad \left[-g_{\nu\rho}(q_1+q_2)_\mu + 2q_{1\rho}g_{\nu\mu} + 2q_{2\nu}g_{\rho\mu}\right] \varepsilon_a^{*\mu}(k)$$

Example: derivation of the Fadin-Lipatov vertex.

In the MRK limit, we can make the substitutions:

$$g_{\mu\nu} \to \frac{1}{2}(n_\mu^+ n_\nu^- + n_\mu^- n_\nu^+),\ q_{1\rho} \to \frac{q_1^+}{2} n_\rho^-,\ q_{2\nu} \to \frac{q_2^-}{2} n_\nu^+,$$

after which we get:

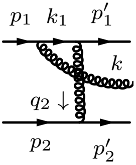
$$\begin{aligned} \mathcal{M}_{3g} &= \frac{g_s^3 f^{cab}}{q_1^2 q_2^2} \left(\bar{u}(p_1') T^c \frac{\hat{n}^+}{\sqrt{2}} u(p_1) \right) \left(\bar{u}(p_2') T^b \frac{\hat{n}^-}{\sqrt{2}} u(p_2) \right) \times \\ &\times \left[-(q_1 + q_2)_\mu + q_1^+ n_\mu^- + q_2^- n_\mu^+ \right] \varepsilon_a^{*\mu}(k) \end{aligned}$$

The obtained result has the correct t-channel factorized form, but the “effective vertex” in the square brackets is not gauge-invariant. Only the sum of t and s-channel diagrams is gauge-invariant, so let’s consider the s-channel diagrams in the MRK limit.

Example: derivation of the Fadin-Lipatov vertex.

The contribution of the $s_1 = (p_1' + k)^2$ channel has the form:

$\mathcal{M}_{s_1} =$



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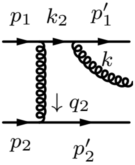


$$= \frac{i g_s^3 q_1^2}{q_1^2 q_2^2} \left(\bar{u}(p_2') T^b \gamma^\rho u(p_2) \right) \times$$

$$\times \quad g_{\rho\nu} \left(\bar{u}(p_1') \left[\frac{1}{k_1^2} (\gamma^\nu \hat{k}_1 \gamma^\sigma) T^b T^a + \frac{1}{k_2^2} (\gamma^\sigma \hat{k}_2 \gamma^\nu) T^a T^b \right] u(p_1) \right) g_{\sigma\mu} \varepsilon_a^{*\mu}(k),$$

where $k_1 = p_1 - k$, $k_2 = p_1' + k$. In the MRK limit one can make the substitutions:

$$\gamma^\nu \hat{k}_1 \gamma^\sigma \to -\hat{p}_1' \gamma^\nu \gamma^\sigma + 2(p_1')^\nu \gamma^\sigma, \;\; \gamma^\sigma \hat{k}_2 \gamma^\nu \to -\gamma^\sigma \gamma^\nu \hat{p}_1 + 2(p_1)^\nu \gamma^\sigma$$

$$k_1^2 \to 2 p_1' q_2, \;\; k_2^2 \to -2 p_1 q_2, \;\; p_{1,2}' \simeq p_{1,2}, g_{\mu\nu} \to \frac{1}{2}(n_\mu^+ n_\nu^- + n_\mu^- n_\nu^+),$$

which after the application of the Dirac equation and the Lie-algebra identity $[T^a, T^b] = -i f^{abc} T^c$ lead us to

$$\mathcal{M}_{s_1} = \frac{g_s^3 f^{cab}}{q_1^2 q_2^2} \left(\bar{u}(p_1') T^c \frac{\hat{n}^+}{\sqrt{2}} u(p_1) \right) \left(\bar{u}(p_2') T^b \frac{\hat{n}^-}{\sqrt{2}} u(p_2) \right) \left[-\frac{q_1^2}{q_2^-} n_\mu^- \right] \varepsilon_a^{*\mu}(k)$$

Example: derivation of the Fadin-Lipatov vertex.

Analogously, the MRK-limit for the s_2-channel is:

$$\mathcal{M}_{s_2} = \frac{g_s^3 f^{cab}}{q_1^2 q_2^2} \left(\bar{u}(p_1') T^c \frac{\hat{n}^+}{\sqrt{2}} u(p_1) \right) \left(\bar{u}(p_2') T^b \frac{\hat{n}^-}{\sqrt{2}} u(p_2) \right) \left[-\frac{q_2^2}{q_1^+} n_\mu^+ \right] \varepsilon_a^{*\mu}(k).$$

Collecting all results together we obtain the amplitude in the expected t-channel factorized form:

$$\mathcal{M}_{3g} + \mathcal{M}_{s_1} + \mathcal{M}_{s_2} = \left(\bar{u}(p_1') \gamma_-^c u(p_1) \right) \frac{1}{q_1^2} \left(\Gamma^{cab}_{+\mu-}(q_1, q_2) \varepsilon_a^{*\mu}(k) \right) \frac{1}{q_2^2} \left(\bar{u}(p_2') \gamma_+^b u(p_2) \right),$$

where the Fadin-Lipatov vertex has the form (q_1-incoming, q_2-outgoing):

$$\Gamma^{cab}_{+\mu-}(q_1, q_2) = g_s f^{cab} \left[-(q_1 + q_2)_\mu + n_\mu^- \left(q_1^+ - \frac{q_1^2}{q_2^-} \right) + n_\mu^+ \left(q_2^- - \frac{q_2^2}{q_1^+} \right) \right].$$

It is easy to see, that the Fadin-Lipatov vertex obeys the Slavnov-Taylor identity:

$$\Gamma^{cab}_{+\mu-}(q_1, q_2)(q_1 - q_2)^\mu = 0.$$

So the gluon Reggeization hypothesis is **non-trivially checked in the MRK**.

The field content of the effective theory.

To produce the amplitudes for the arbitrary QMRK processes, the effective-action approach is very useful [Lipatov, 1995]. Light-cone coordinates and derivatives:

$$x^{\pm} = n^{\pm}x = x^0 \pm x^3, \; \partial_{\pm} = 2\frac{\partial}{\partial x^{\mp}}$$

Lagrangian of the effective theory $L = L_{kin} + \sum_y \left(L_{QCD} + L_{ind}\right)$, $v_\mu = v_\mu^a t^a$, $\left[t^a, t^b\right] = f^{abc}t^c$. The rapidity space is sliced into the subintervals, corresponding to the groups of final-state particles, close in rapidity. Each subinterval in rapidity $(1 \ll \eta \ll Y)$ has it's own set of QCD fields:

$$L_{QCD} = -\frac{1}{2}tr\left[G_{\mu\nu}^2\right], \; G_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + g\left[v_\mu, v_\nu\right].$$

Different rapidity intervals communicate via the **gauge invariant fields** of Reggeized gluons $(A_\pm = A_\pm^a t^a)$ with the kinetic term:

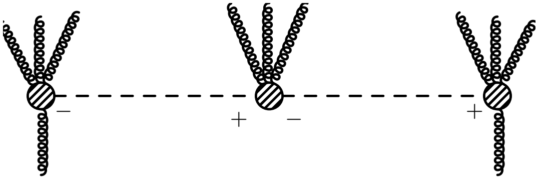
$$L_{kin} = -\partial_\mu A_+^a \partial^\mu A_-^a,$$

and the kinematical constraint:

$$\partial_- A_+ = \partial_+ A_- = 0 \Rightarrow$$

$$A_+ \text{ has } k_- = 0 \text{ and } A_- \text{ has } k_+ = 0.$$

The effective action for high energy processes in QCD.



Particles and Reggeons interact via *induced interactions*:

$$L_{ind} = - \quad tr\left\{ \frac{1}{g}\partial_+ \left[P\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} dx'^- v_+(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_-(x) + \right.$$
$$\left. + \quad \frac{1}{g}\partial_- \left[P\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} dx'^+ v_-(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_+(x) \right\}$$

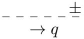
Wilson lines generate the infinite chain of the induced vertices:

$$L_{ind} = tr\left\{ \left[v_+ - g v_+ \partial_+^{-1} v_+ + g^2 v_+ \partial_+^{-1} v_+ \partial_+^{-1} v_+ - \ldots \right] \partial_\sigma \partial^\sigma A_- + \right.$$
$$\left. + \quad \left[v_- - g v_- \partial_-^{-1} v_- + g^2 v_- \partial_-^{-1} v_- \partial_-^{-1} v_- - \ldots \right] \partial_\sigma \partial^\sigma A_+ \right\}$$

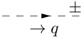
Feynman rules. Quarks, gluons and photons.

Feynman Rules for Reggeized gluons [Antonov, Cherednikov, Kuraev, Lipatov, 2005]
Feynman Rules for Reggeized quarks [Lipatov, Vyazovsky, 2001]

Initial state factors:

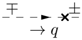


$$= \frac{q^{\pm}}{2\sqrt{-q^2}},$$

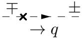


$$= u(q^{\parallel}).$$

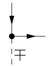
Propagators ($\hat{P}_{\pm} = \frac{1}{4}\hat{n}^{\mp}\hat{n}^{\pm}$):



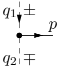
$$= \hat{P}_{\pm}\frac{i\hat{q}}{q^2},$$



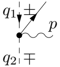
$$= \frac{i\hat{q}}{q^2}\hat{P}_{\pm}.$$



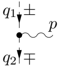
$$= -ig_s T^a \hat{n}^{\pm},$$



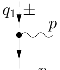
$$= -ig_s T^a \left(\hat{n}^{\pm} + 2\frac{\hat{q}_1}{q_2^{\mp}}\right),$$



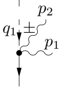
$$= -2ieg_s T^a \frac{\hat{q}_1 n_\mu^{\mp}}{p^{\mp} q_2^{\mp}},$$



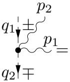
$$= -ie\left(\gamma_\mu + \hat{q}_1\frac{n_\mu^{\mp}}{p^{\mp}} + \hat{q}_2\frac{n_\mu^{\pm}}{p^{\pm}}\right),$$



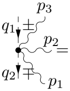
$$= -ie\left(\gamma_\mu + \hat{q}_1\frac{n_\mu^{\mp}}{p^{\mp}}\right),$$



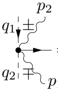
$$= -ie^2\hat{q}_1\frac{n_{\mu_1}^{\mp} n_{\mu_2}^{\mp}}{p_1^{\mp} p_2^{\mp}}.$$



$$= ie^2\left(\hat{q}_2\frac{n_{\mu_1}^{\pm} n_{\mu_2}^{\pm}}{p_1^{\pm} p_2^{\pm}} - \hat{q}_1\frac{n_{\mu_1}^{\mp} n_{\mu_2}^{\mp}}{p_1^{\mp} p_2^{\mp}}\right),$$



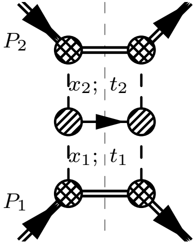
$$= -ie^3\left(\hat{q}_2\frac{n_{\mu_1}^{\pm} n_{\mu_2}^{\pm} n_{\mu_3}^{\pm}}{p_1^{\pm} p_2^{\pm} p_3^{\pm}} + \hat{q}_1\frac{n_{\mu_1}^{\mp} n_{\mu_2}^{\mp} n_{\mu_3}^{\mp}}{p_1^{\mp} p_2^{\mp} p_3^{\mp}}\right),$$



$$= -2ie^2 g_s T^a \frac{\hat{q}_1 n_{\mu_1}^{\mp} n_{\mu_2}^{\mp}}{p_1^{\mp} p_2^{\mp} q_2^{\mp}}.$$

Factorization of the cross-section.

Factorization:



Collinear limit holds for the amplitude:

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2}\to 0} \overline{|\mathcal{M}|^2}_{PRA} = \overline{|\mathcal{M}|^2}_{CPM}$$

k_T-factorization formula:

$$d\sigma = \int \frac{d^2\mathbf{q}_{T1}}{\pi} \int \frac{dx_1}{x_1} \Phi(x_1, t_1, \mu_F) \times$$
$$\times \int \frac{d^2\mathbf{q}_{T2}}{\pi} \int \frac{dx_2}{x_2} \Phi(x_2, t_2, \mu_F) d\hat{\sigma}_{PRA}$$

Where Φ - Unintegrated PDFs. The factorization is known to hold in the LLA ($\alpha_s \log(1/x)$) [**BFKL**, 1978], and NLLA ($\alpha_s^2 \log(1/x)$) [Fadin, Lipatov, 1998; Camici, Ciafaloni, 1998; Bartels, *et. al.*, 2006].

Normalization of the unPDF:

$$\int^{\mu^2} dt \Phi(x, t, \mu^2) = x f(x, \mu^2),$$

where $f(x, \mu^2)$ - collinear PDF.

The Kimber-Martin-Ryskin unPDF.

In the present numerical computations we use the modified KMR unPDF from [Martin , Ryskin, Watt 2010].
KMR prescription to obtain unintegrated PDF from collinear one is based on the mechanism of last step parton k_T−dependent radiation and the assumption of strong angular ordering:

$$\Phi_q(x,k_T^2,\mu^2)=\frac{1}{k_T^2}\int\limits_x^{1-\Delta} dz T_q(q^2,\mu^2)\frac{\alpha_s(q^2)}{(2\pi)}\left[P_{qg}(z)f_g\left(\frac{x}{z},q^2\right)+P_{qq}(z)f_q\left(\frac{x}{z},q^2\right)\right],$$

where $P_{qg}(z)$, $P_{qq}(z)$- LO DGLAP splitting functions, $T_q(k^2,\mu^2)$- Sudakov formfactor:

$$T_q(k^2,\mu^2)=exp\left\{-\int\limits_{k^2}^{\mu^2}\frac{dq_T^2}{q_T^2}\frac{\alpha_s(q_T^2)}{2\pi}\sum_{a'}\int\limits_0^{1-\Delta}P_{qa'}(z')dz'\right\}$$

where $\Delta=\frac{k_T}{\mu+k_T}$ ensures the **rapidity ordering of the last emission and particles produced in the hard subprocess**, and $q^2=k_T^2/(1-z)$.

Selected results in LO PRA.

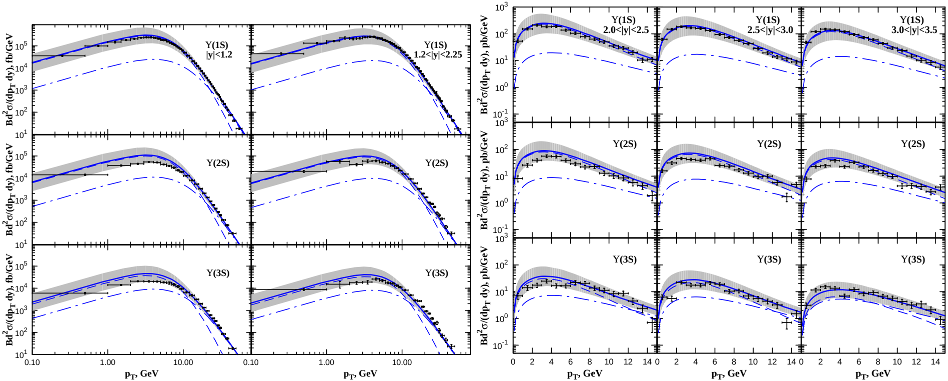
1. $pp \to J/\Psi(\Upsilon)X$ at Tevatron and the LHC
2. DY pair production
3. Single jet and prompt photon production
4. $D(B)$-meson production
5. $b-$jet production
6. Pair correlations in PRA,
7. Diphoton production in NLO* PRA.

$pp \to J/\Psi(\Upsilon)X$ at Tevatron and the LHC

- B. A. Kniehl, V. A. Saleev and D. V. Vasin, "Bottomonium production in the Regge limit of QCD," Phys. Rev. D **74**, 014024 (2006) [hep-ph/0607254].
- B. A. Kniehl, D. V. Vasin and V. A. Saleev, "Charmonium production at high energy in the k_T -factorization approach," Phys. Rev. D **73**, 074022 (2006) [hep-ph/0602179].
- V. A. Saleev, M. A. Nefedov and A. V. Shipilova, "Prompt J/psi production in the Regge limit of QCD: From Tevatron to LHC," Phys. Rev. D **85**, 074013 (2012) [arXiv:1201.3464 [hep-ph]].
- M. Nefedov, V. Saleev and A. Shipilova, "Prompt $\Upsilon(nS)$ production at the LHC in the Regge limit of QCD," Phys. Rev. D **88**, no. 1, 014003 (2013) [arXiv:1305.7310 [hep-ph]].

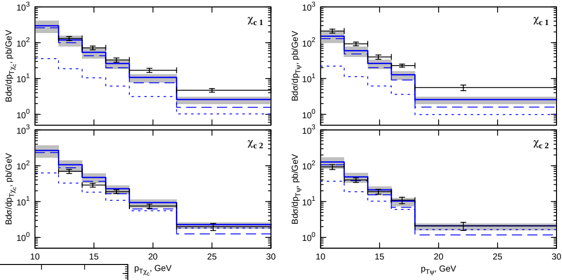
It was shown that using LO PRA and NRQCD we can describe the p_T-spectra both for S and P-wave states. The situation with polarization is discussed in [hep-ph/1410.6421]. Both Color-Singlet and Color-Octet contributions are required.

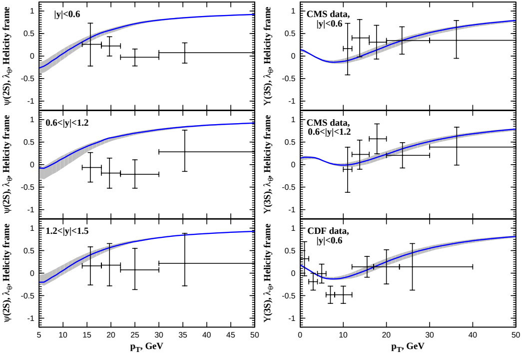
$\Upsilon(nS)$ production at the LHC ($\sqrt{S} = 7$ TeV).



Left panel – ATLAS data (fit), right panel – LHCb data (prediction). Dashed line – CS contribution, dash-dotted line – CO contribution, unPDF – LO KMR.

χ_{cJ}-production and polarization observables for $\psi(2S)$ and $\Upsilon(3S)$.





Single jet and prompt-photon production at HERA, Tevatron and the LHC

- V. A. Saleev, "Prompt photon photoproduction at HERA within the framework of the quark Reggeization hypothesis," Phys. Rev. D **78**, 114031 (2008) [arXiv:0812.0946 [hep-ph]].
- V. A. Saleev, "Deep inelastic scattering and prompt photon production within the framework of quark Reggeization hypothesis," Phys. Rev. D **78**, 034033 (2008) [arXiv:0807.1587 [hep-ph]].
- B. A. Kniehl, V. A. Saleev, A. V. Shipilova and E. V. Yatsenko, "Single jet and prompt-photon inclusive production with multi-Regge kinematics: From Tevatron to LHC," Phys. Rev. D **84**, 074017 (2011) [arXiv:1107.1462 [hep-ph]].

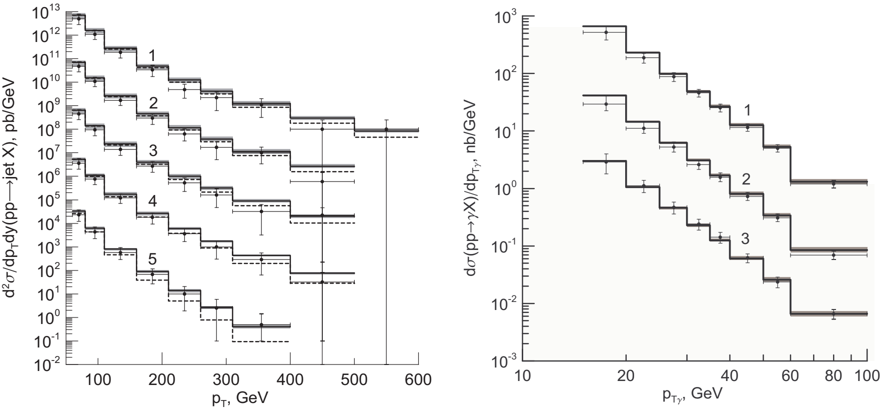
We have studied single jet and prompt-photon inclusive production, at LO PRA, in which they are dominated by $2 \to 1$ partonic subprocesses initiated by Reggeized gluons and quarks, respectively. Despite the great simplicity of our analytic expressions, we found excellent agreement with single jet [CDF,ATLAS] and prompt-photon [ZEUS,CDF,ATLAS].

$$C^{\gamma/g,\mu}_{\mathcal{Q}\overline{\mathcal{Q}}}(q_1, q_2) = C_1^{\gamma/g}\left[\gamma^\mu - \hat{q}_1\frac{(n^-)^\mu}{q_2^-} - \hat{q}_2\frac{(n^+)^\mu}{q_1^+}\right],$$

$$\overline{\left|\mathcal{M}\left(\mathcal{Q}+\overline{\mathcal{Q}}\to\gamma^\star/g^\star\right)\right|^2} = C_2^{\gamma/g}(Q^2+t_1+t_2),$$

$$\overline{|\mathcal{M}(\mathcal{R}+\mathcal{R}\to g)|^2} = \frac{3}{2}\pi\alpha_s(t_1+t_2+2\sqrt{t_1t_2}\cos\phi_{12}).$$

Single jet and prompt photon at the LHC.



Drell-Yan pair production at Tevatron and the LHC

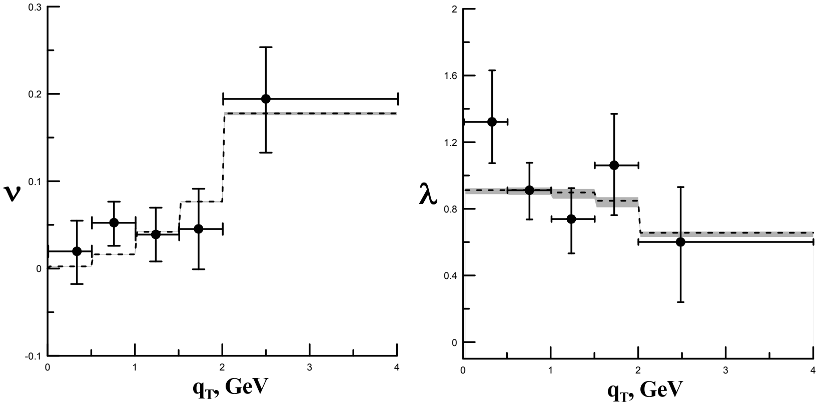
- M. A. Nefedov, N. N. Nikolaev and V. A. Saleev, "Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach," Phys. Rev. D **87**, no. 1, 014022 (2013) [arXiv:1211.5539 [hep-ph]].

$$
\begin{aligned}
w_{\mu\nu}^{PRA} &= x_1 x_2 \Big[-S g^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu) \frac{(2x_1 x_2 S - Q^2 - t_1 - t_2)}{x_1 x_2 S} + \\
&+ \frac{2}{x_2}(q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1}(q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \\
&+ \frac{4(t_1 - x_1 x_2 S)}{S x_2^2} P_1^\mu P_1^\nu + \frac{4(t_2 - x_1 x_2 S)}{S x_1^2} P_2^\mu P_2^\nu \Big].
\end{aligned}
$$

The LO PRA predictions provide an adequate numerical description of lepton pair distributions on the invariant mass (Q), lepton pair transverse momentum (q_T) and longitudinal scaling variable (x_F) as well as lepton pair angular distributions at the SPS, Tevatron and LHC Colliders.

Polarization observables in Drell-Yan ($\sqrt{S} = 39 GeV$).

The data are from NuSea Collaboration (Fermilab).

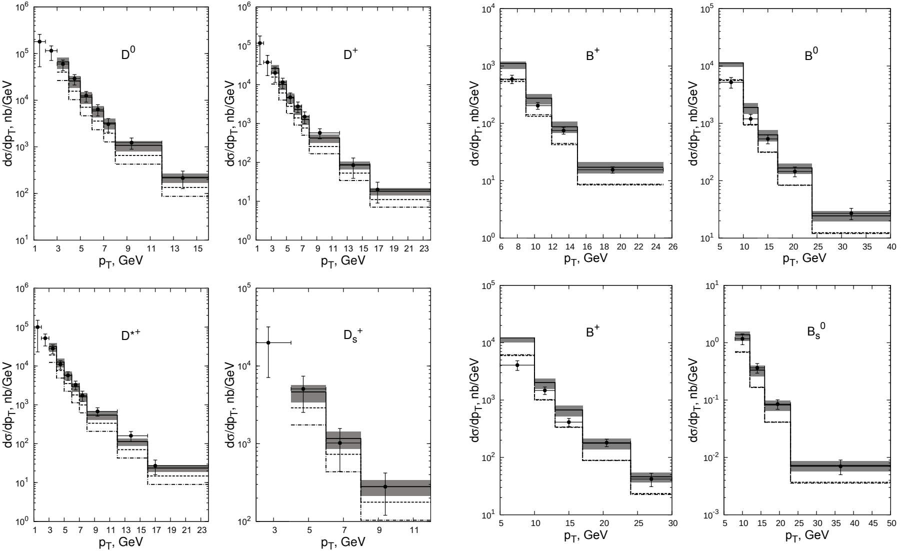


$D(B)$-meson production at Tevatron and the LHC

- A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, "B-meson production in the Parton Reggeization Approach at Tevatron and the LHC," Int. J. Mod. Phys. A **30**, no. 04n05, 1550023 (2015) [arXiv:1411.7672 [hep-ph]].
- A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, "Open charm production in the parton Reggeization approach: Tevatron and the LHC," Phys. Rev. D **91**, no. 5, 054009 (2015)
- B. A. Kniehl, A. V. Shipilova and V. A. Saleev, "Open charm production at high energies and the quark Reggeization hypothesis," Phys. Rev. D **79**, 034007 (2009) [arXiv:0812.3376 [hep-ph]].

It was shown that at high p_T region the gluon into the final heavy meson fragmentation in $R + R \to g$ with $g \to D(B)$ is dominating production mechanism instead of heavy quark fragmentation in $R + R \to c(b) + \bar{c}(\bar{b})$ **with** $c(b) \to D(B)$

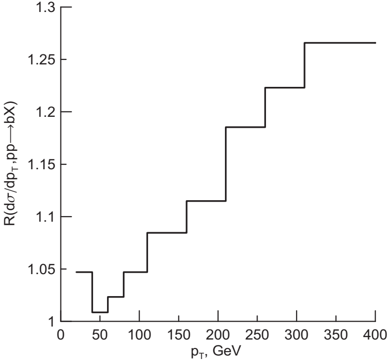
D and B mesons at the LHC. ALICE data.



b-jet production at Tevatron and the LHC

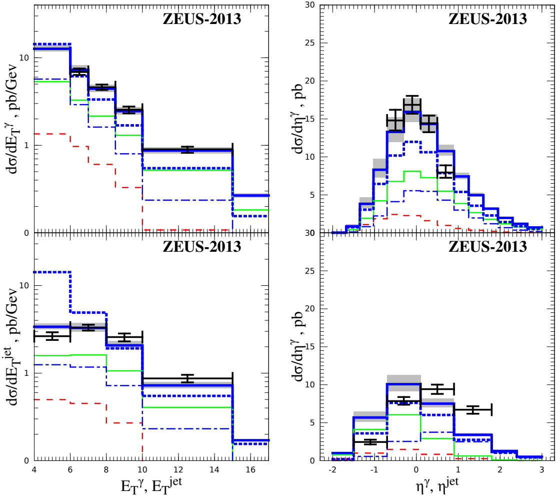
- V. A. Saleev and A. V. Shipilova, "Inclusive b-jet and $b\bar{b}$-dijet production at the LHC via Reggeized gluons," Phys. Rev. D **86**, 034032 (2012) [arXiv:1201.4640 [hep-ph]].
- B. A. Kniehl, V. A. Saleev and A. V. Shipilova, "Inclusive b and b anti-b production with quasi-multi-Regge kinematics at the Tevatron," Phys. Rev. D **81**, 094010 (2010) [arXiv:1003.0346 [hep-ph]].

It was shown that at high p_T region the gluon into the final heavy meson fragmentation in $R + R \to g$ with $g \to b$ is dominating production mechanism instead of direct $b-$quark production in $R + R \to b + \bar{b}$.



Selected results in PRA. Prompt photon + jet photoproduction at HERA ($\sqrt{S_{ep}} = 318.7$ GeV).

B. A. Kniehl, M. A. Nefedov and V. A. Saleev, “Prompt-photon plus jet associated photoproduction at HERA in the parton Reggeization approach,” Phys. Rev. D **89**, no. 11, 114016 (2014) [arXiv:1404.3513 [hep-ph]].



Jet and prompt-photon pair production.

See the talks by:

- A. Shipilova, "Pair correlations in particle and jet production at the LHC in the parton Reggeization approach"
- M. Nefedov, "Prompt photon pair production at the Tevatron and LHC in the Parton Reggeization Approach"

Conclusions.

- MRK and QMRK dominate in high energy particle production, DGLAP+BFKL
- k_T-factorization is proven in Leading and Next-to-Leadind-log$(1/x)$ approximation $\Rightarrow$ NLO calcualtions are possible.
- Gluons and quarks in $t-$channel are Reggeized at high energy
- k_T-factorization formalism + Reggeized amplitudes = Parton Reggeization Approach
- PRA is only one way to correct inclusion of NLO corrections in k_T-factorization

Thank you for your attention!