# Beyond $N^2$LL' $q_T$ resummation with CuTe

Thomas Lübbert
University of Hamburg

work in progress with Thomas Becher, Matthias Neubert, Daniel Wilhelm

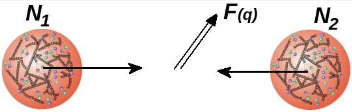
Dubna CALC 2015
July 2015

# Outline

# Observable

Consider:

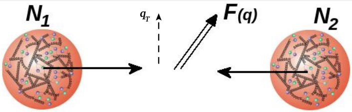
$N_1 + N_2 \to F(q) + X$




- $F = l^+ l^-, Z, W, H, Z', \ldots$
- Test SM to high precision.

# Observable

Consider:
$N_1 + N_2 \to F(q) + X$

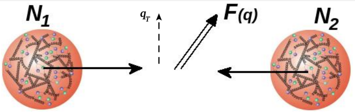



- $F = l^+ l^-, Z, W, H, Z', \ldots$
- Test SM to high precision.
- $d\sigma/dq_T$ in region $q_T^2 \ll M^2$.

# Observable

Consider:

$N_1 + N_2 \to F(q) + X$

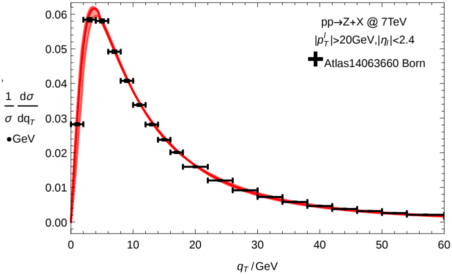



- $F = l^+l^-, Z, W, H, Z', \ldots$
- Test SM to high precision.
- $d\sigma/dq_T$ in region $q_T^2 \ll M^2$

$\Rightarrow$ Need to resum large logarithms.

$\Rightarrow$ Transverse PDFs (TPDFs) (Beam functions).

$\Rightarrow$ $F$ recoils against initial state radiation

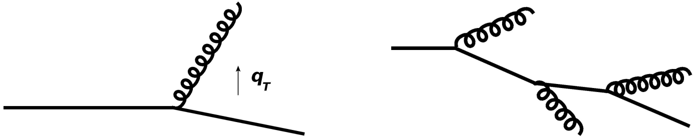



[Becher, TL, Neubert, Wilhelm]

# Large logarithms

Momenta of emitted particle soft, collinear ($q_T^2 \ll q^2$)

$\Rightarrow$ up to $\log^2 \frac{q_T^2}{q^2}$ for each power $\alpha_s$.




Specific structure $\Rightarrow$ factorize

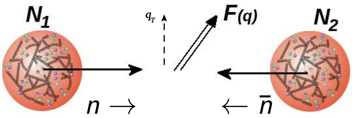
$\Rightarrow$ resum $\sim$ reorder expansion ($\alpha_s L \sim 1$)

# Factorization, pictorially

Consider:

$N_1 + N_2 \to F(q) + X$

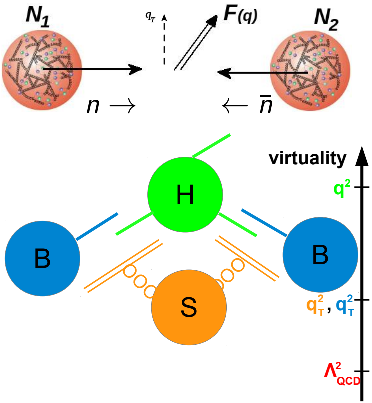
- $F = l^+ l^-, Z, W, H, Z', \ldots$



# Factorization, pictorially

Consider:
$N_1 + N_2 \to F(q) + X$

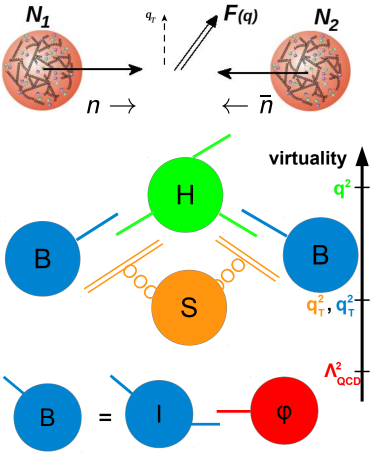
- $F = l^+ l^-, Z, W, H, Z', \dots$
- Enhanced contributions from collinear and soft regions.
- Characteristic structure $\Rightarrow$ factorization $d\sigma = H \otimes S \otimes B \otimes B$.
- Here $\lambda^2 = q_T^2 / q^2 \ll 1$.
- Explicit operator definitions. Determined process independent $S \otimes I \otimes I$ to NNLO [Gehrmann,TL,Yang].



# Factorization, pictorially

Consider:
$N_1 + N_2 \to F(q) + X$

- $F = l^+ l^-, Z, W, H, Z', \ldots$
- Enhanced contributions from collinear and soft regions.
- Characteristic structure $\Rightarrow$ factorization $d\sigma = H \otimes S \otimes B \otimes B$.
- Here $\lambda^2 = q_T^2/q^2 \ll 1$.
- Explicit operator definitions. Determined process independent $S \otimes I \otimes I$ to NNLO [Gehrmann,TL,Yang].



# $\frac{d\sigma}{dq_T dy}$ - counting $a_s$ and $L$

$\lambda = q_T/M$ , $L = \log \lambda$ , $a_s = \frac{\alpha_s}{4\pi}$ , PC= power correction.

$$\frac{d\sigma}{dq_T dy} = C(a_s) \exp\left[L g_1(a_s L) + g_2(a_s L) + a_s g_3(a_s L) + a_s^2 g_4(a_s L) + \ldots\right] + \mathcal{O}(\lambda)\,.$$

When expand:

| FO╲RES | | LL | NLL | NLL' | $\mathrm{N}^2$LL | $\mathrm{N}^2$LL' | $\mathrm{N}^3$LL | ... | PC | |
|---|---|---|---|---|---|---|---|---|---|---|
| LO | $a_s^0$[ | 1 | | | | | | | | ] |
| NLO | $a_s^1$[ | $L^2$ | $L^1$ | 1 | | | | | $\mathcal{O}(\lambda)$ | ] |
| $\mathrm{N}^2$LO | $a_s^2$[ | $L^4$ | $L^3$ | $L^2$ | $L^1$ | 1 | | | $\mathcal{O}(\lambda)$ | ] |
| $\mathrm{N}^3$LO | $a_s^3$[ | $L^6$ | $L^5$ | $L^4$ | $L^3$ | $L^2$ | $L^1$ | 1 | $\mathcal{O}(\lambda)$ | ] |
| | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | |

In this talk will discuss **CuTe**'s

- $\mathrm{N}^3\mathrm{LL}_p$ * resummation and
- $\mathrm{N}^2$LO matching ($\Rightarrow$ recover PC).

* p=partial: lacking values of $\Gamma_3^i$ and $F_i^{(3,0)}$.

# Factorization formula

- [Collins, Soper, Sterman], ..., [Becher, Neubert] for $\Lambda^2_{\rm QCD} \ll q_T^2 \ll M^2$ :

$$\frac{d^2\sigma_c}{dq_T^2 dy} = \sigma_c^{(0)} \sum_{k,j} \int d^2x_T \, e^{iq_T \cdot x_T} \, \widetilde{C}_{c\leftarrow kj}(z_1, z_2, x_T^2, M^2, \mu) \otimes \phi_{k/N_1}(z_1, \mu) \otimes \phi_{j/N_2}(z_2, \mu) \,,$$

- in impact parameter ($x_T$) space
- with **perturbative**

$$\widetilde{C}_{c\leftarrow ij}(z_1, z_2, x_T^2, M^2, \mu) = |C_c(-M^2, \mu)|^2 \bar{I}_{i\leftarrow k}(z_1, L_\perp, a_s) \bar{I}_{\bar{i}\leftarrow j}(z_2, L_\perp, a_s) e^{g_i(M, x_T, \mu)} \,,$$

- where $i(c) = q, \, g$ ; $L_\perp = \log \frac{x_T^2 \mu^2}{b_0^2}$, $\quad a_s = \alpha_s(\mu)/4\pi \quad$ and

$$g_i(M, x_T, \mu) = 2h_i(L_\perp, a_s) - F_i(L_\perp, a_s) \cdot \log \frac{x_T^2 M^2}{b_0^2} \,.$$

In $\widetilde{C}$ suppressed sum over tensor components ($i(c) = g$) or quark charges ($i(c) = q$).

# Factorization formula

- [Collins, Soper, Sterman], . . ., [Becher, Neubert] for $\Lambda^2_{\rm QCD} \ll q_T^2 \ll M^2$ :

$$\frac{d^2\sigma_c}{dq_T^2 dy} = \sigma_c^{(0)} \sum_{k,j} \int d^2x_T \, e^{iq_T \cdot x_T} \, \widetilde{C}_{c\leftarrow kj}(z_1,z_2,x_T^2,M^2,\mu) \otimes \phi_{k/N_1}(z_1,\mu) \otimes \phi_{j/N_2}(z_2,\mu) \,,$$

- in impact parameter ($x_T$) space
- with **perturbative**

$$\widetilde{C}_{c\leftarrow ij}(z_1,z_2,x_T^2,M^2,\mu) = |C_c(-M^2,\mu)|^2 \bar{I}_{i\leftarrow k}(z_1,L_\perp,a_s) \bar{I}_{\bar{i}\leftarrow j}(z_2,L_\perp,a_s) e^{g_i(M,x_T,\mu)} \,,$$

- where $i(c) = q, \, g$ ; $L_\perp = \log \frac{x_T^2 \mu^2}{b_0^2}$, $\quad a_s = \alpha_s(\mu)/4\pi \quad$ and $g_i(M, x_T, \mu) = 2h_i(L_\perp, a_s) - F_i(L_\perp, a_s) \cdot \log \frac{x_T^2 M^2}{b_0^2}$ .
- Each function depends on **single** physical scale $\Rightarrow$ Safely determine pert..
- Solving **RGEs** ($\mu$) $\Rightarrow$ **resummation** of $L = \log(x_T^2 M^2 / b_0^2)$.

In $\widetilde{C}$ suppressed sum over tensor components ($i(c) = g$) or quark charges ($i(c) = q$).

# Resummation

- RGEs $\Rightarrow$ Dependence on $\mu$ $\qquad i = i(c)$,

$$\frac{d}{d\log\mu}\log C_c(-M^2,\mu) = \Gamma^i(a_s)\log\frac{-M^2-i0^+}{\mu^2} + 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}F_i(L_\perp,a_s) = 2\,\Gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}h_i(L_\perp,a_s) = \Gamma^i(a_s)L_\perp - 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}\bar{I}_{i/j}(z,L_\perp,\mu) = -2\sum_k \bar{I}_{i/k}(z,L_\perp,a_s)\otimes P_{k/j}(z,a_s)\,,$$
$$\frac{d}{d\log\mu}\phi_{i/j}(z,\mu) = 2\sum_k P_{ik}(z,\mu)\otimes\phi_{k/j}(z,\mu)\,.$$

$\Rightarrow$ Resum logarithms. E.g.:

$$|C_c(-M^2,\mu)|^2 = |C_c(-M^2,\mu_h)|^2\exp\left\{2\mathrm{Re}[\mathrm{E}_{\mathrm{C_c}}(-\mathrm{M}^2,\mu,\mu_\mathrm{h})]\right\}\,,$$
$$E_{C_c}(-M^2,\mu,\mu_h) = \int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\left(\Gamma^i\log\frac{-M^2-i0^+}{\mu'^2} + 2\gamma^i\right)\,.$$

- Log indep. parts and anom. dims. from pert. calc..

# Towards N$^3$LL, required elements

Numbers refer to power $n$ in expansion $X = \sum_n a_s^n X^{(n)}$ .

| expression | needed to | known to | |
|---|---|---|---|
| $\Gamma^i$ | 4 | 3 | for RGEs |
| $\gamma_i$ | 3 | 3 | for RGEs |
| $P_{i/j}(z)$ | 3 | 3 | for RGEs |
| $\beta$ | 4 | 4 | for RGEs |
| $C_c(M^2)$ | 2 | 2 | at appr. $\mu$ |
| $F_i(L_\perp)$ | 3 | 2 | at appr. $\mu$ |
| $h_i(L_\perp)$ | 2 | 2 | at appr. $\mu$ |
| $\bar{I}_{i/j}(z, L_\perp)$ | 2 | 2 | at appr. $\mu$ |
| $\bar{I}'_{g/j}(z, L_\perp)$ | 1 (2)* | 1 | at appr. $\mu$ |

*: $I'$ starts at $\alpha_s^1$.

$\Rightarrow$ $n = 1$ sufficient if $C_{gg}$ does not mix $I'$ & $I$. (E.g. for Higgs.)

# $x_T$ integral and scale choice

Perform $\int d^2x_T\, e^{iq_T\cdot x_T}\, \widetilde{C}_{c\leftarrow ij} = \int_0^\infty dx_T\, x_T J_0(x_T q_T) \widetilde{C}_c(z_1, z_2, x_T^2, M^2, \mu)\,.$

- Essentially two kind of logs: $L_M = \log M^2/\mu^2$ and $L_\perp = \log x_T{}^2\mu^2/b_0^2$.

# $x_T$ integral and scale choice

Perform $\int d^2x_T\, e^{iq_T\cdot x_T}\,\widetilde{C}_{c\leftarrow ij} = \int_0^\infty dx_T\, x_T J_0(x_T q_T)\widetilde{C}_c(z_1,z_2,x_T^2,M^2,\mu)\,.$

- Essentially two kind of logs: $L_M = \log M^2/\mu^2$ and $L_\perp = \log x_T{}^2\mu^2/b_0^2$.
- Aim: small $L_\perp$.
- $\mu_x = b_0/x_T$: Run into Landau pole.
- Moreover, prefer physical choice $\mu(q_T, M)$.
- $\mu = \mu_* = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i/\exp\left(1/2\Gamma_0^i a_s\right) \Rightarrow \langle L_\perp\rangle$ small.
- Thanks to $x_T e^{g_i}$ →Gaussian peak: determines $q_*$, width $\sim 1\sqrt{a_s}$.

# $x_T$ integral and scale choice

Perform $\int d^2x_T \, e^{iq_T \cdot x_T} \widetilde{C}_{c \leftarrow ij} = \int_0^\infty dx_T \, x_T J_0(x_T q_T) \widetilde{C}_c(z_1, z_2, x_T^2, M^2, \mu)$.

- Essentially two kind of logs: $L_M = \log M^2/\mu^2$ and $L_\perp = \log x_T{}^2 \mu^2 / b_0^2$.
- Aim: small $L_\perp$.
- $\mu_x = b_0/x_T$: Run into Landau pole.
- Moreover, prefer physical choice $\mu(q_T, M)$.
- $\mu = \mu_* = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i / \exp\left(1/2\Gamma_0^i a_s\right) \Rightarrow \langle L_\perp \rangle$ small.
- Thanks to $x_T e^{g_i}$ →Gaussian peak: determines $q_*$, width $\sim 1\sqrt{a_s}$.
- Power counting for $\int dx_T$: $a_s \sim \epsilon^2$, $L_M \sim \epsilon^{-2}$, $L_\perp \sim \epsilon^{-1}$
  Do up to $\epsilon^5$.
- CuTe [Becher,Neubert,TL,Wilhelm]: Combine everything and numerically evaluate $\otimes\phi$, $x_T$, $y$ integrals.

# Phenomenology

## Apply CuTe

# CuTe: What is New

## CuTe 1.1

- Public C$^{++}$-code for $\gamma^*, W, Z$ and $H$. Integration via linked Cuba libary [Hahn].
- LHAPDF 5
- N$^2$LL resummation
- NLO matching
- $\mathcal{O}(\epsilon^2)$ for very small $q_T$

## CuTe 2.0

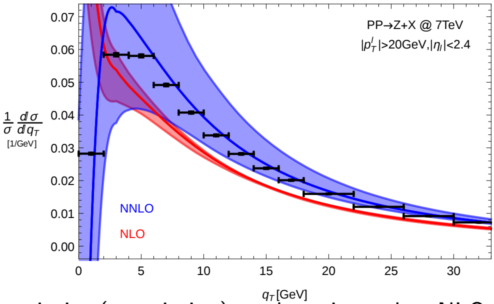
- Soon
    - All results are preliminary
- LHAPDF 6
- Full N$^3$LL implementation
    - 2-loop beam-functions
    - unknown $\Gamma_3^i$ and $F_i^{(3,0)}$ $\Rightarrow$ N$^3$LL$_p$ (partial)
- N$^2$LO matching
- $\mathcal{O}(\epsilon^5)$: Terms up to $\alpha_s^5 L_\perp^5$ in $\bar{I}$.
- Phase space cuts $y, p_T^l, \eta^l$

# Fixed order

- $\mu = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i/\exp\left(1/2\Gamma_0^i a_s\right)$.
- NNPDF 3.0

# Fixed order

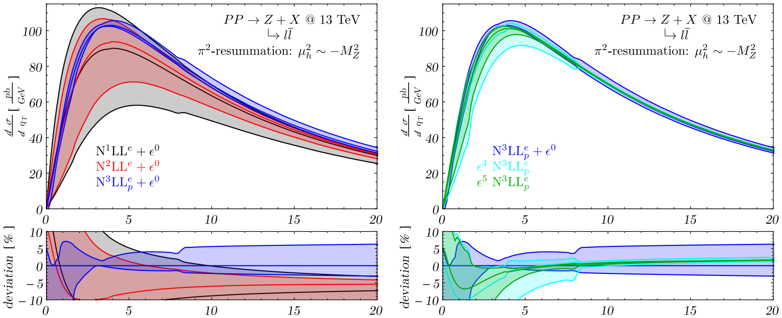
- $\mu = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i / \exp\left(1/2\Gamma_0^i a_s\right)$.
- NNPDF 3.0
- $Z$-production: NNLO results from [Gonsalves,Pawlowski,Wai]




At **peak**:

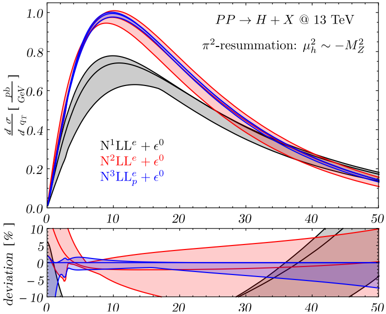
- Large uncertainties ($\mu$ variation), underestimated at NLO.
- Divergent at very small $q_T$.
- Requires resummation.

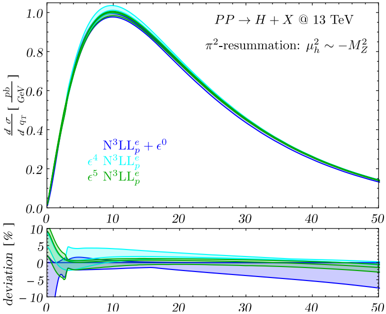
# Pure Resummed - different orders - Z




- $\mu(q_T) = q_T + q_* e^{-\frac{q_T}{q_*}}$
- Vary by factors 2, 1/2.
- $q_*^Z \sim 2GeV$.
- New channels $(q \to q', \bar{q})$ beyond $N^2LL + \epsilon^0$.

# Pure Resummed - different orders - H




- $\mu(q_T) = q_T + q_* e^{-\frac{q_T}{q_*}}$
- Vary by factors 2, 1/2.

- $q_*^H \sim 8 GeV$.
- Sizeable loop correction and new channels $(q \to g)$ at $N^2LL + \epsilon^0$.

# Matching schemes

Restoring power $(qT/M)$ suppressed contributions.

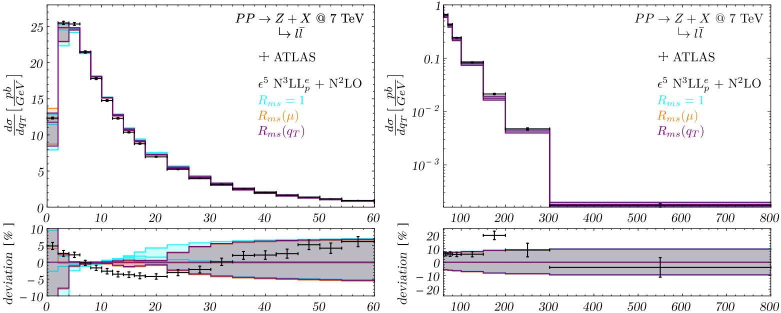
$$\frac{d\sigma^{\text{matched}}}{dq_T} = \frac{d\sigma^{\text{res}}}{dq_T} + \left.\frac{d\sigma^{\text{MC}}}{dq_T}\right|_{\text{MS}},$$
$$\left.\frac{d\sigma^{\text{MC}}}{dq_T}\right|_{\text{MS}} = R_{\text{MS}}\left(\frac{d\sigma^{\text{FO}}}{dq_T} - \frac{d\sigma^{\text{res}}}{dq_T}\right)\Bigg|_{\text{expanded to FO}}.$$

with $H(-M^2, \mu_h, \mu) = U_H(\mu, \mu_h) \cdot |C_i(-M^2, \mu_h)|^2$ and

$$\begin{aligned} R_{\text{add}} = R_{\text{ms}}(\mu_h) &= 1\,, \\ R_{\text{ms}}(\mu_*) &= H(-M^2, \mu_h, \mu_*) \cdot H^{-1}(-M^2, \mu_*, \mu_*)\,, \\ R_{\text{ms}}(q_T) &= H(-M^2, \mu_h, q_T) \cdot H^{-1}(-M^2, q_T, q_T)\,. \end{aligned}$$

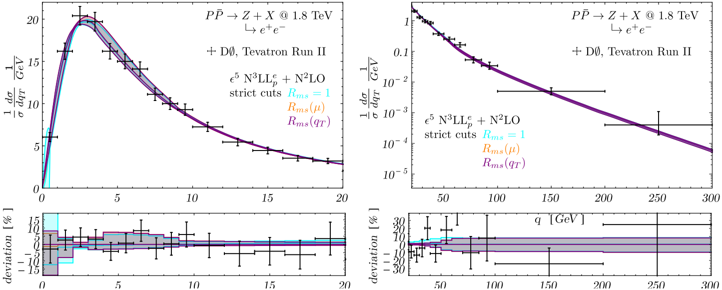
$H(-M^2, \mu, \mu)$ corresponds to the FO expansion of resummed $H(-M^2, \mu_h, \mu)$.
$R_{\text{MS}} = 1 + \mathcal{O}(a_s^3)$ but can supply Sudakov suppression to $d\sigma^{\text{MC}}$.

# Matched vs data - Z ATLAS



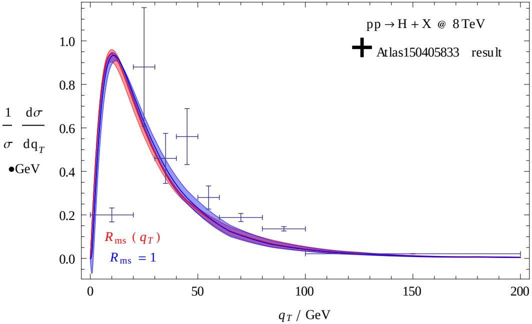

- Each MS with $\mu$ errors. [Becher,TL,Neubert,Wilhelm]*
- Very small matching scheme dependence.
- Good agreement with data:
  - ATLAS hep-ex/1406.3660 $Z/\gamma^*$ at 7TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $66 < M_{ll}/GeV < 116$,
  - $p_{T,l} > 20\text{GeV}$, $|\eta_l| < 2.4$, excluding $1.37 < |\eta_l| < 1.52$
- Lepton cuts included in theory prediction: Normalization and shift peak to the right.

# Matched vs data - Z D0



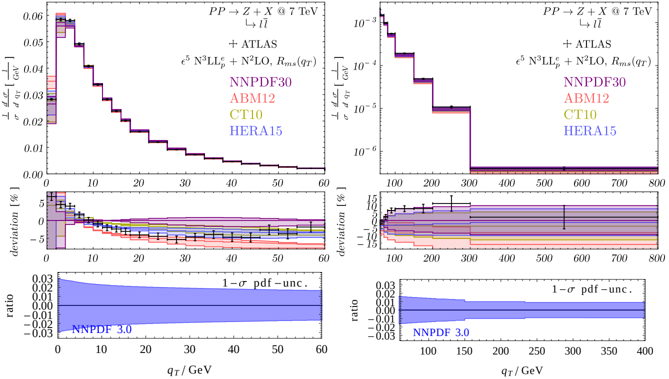

- Each MS with $\mu$ errors. Very small matching scheme dependence.
- Very good agreement with data: [Becher,TL,Neubert,Wilhelm]*
  - D0, hep-ex/9909020 $Z$ at 1.8TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $60 < M_{ll}/GeV < 120$,
  - $p_{T,l} > 25\text{GeV}$, $|\eta_{l,1}| < 1.1$, $|\eta_{l,2}| < 2.4$, excluding $1.11 < |\eta_{l,2}| < 1.5$
- Symmetrized lepton cuts included in theory prediction: Normalization and shift peak to the right.

# Matched vs data - H ATLAS




- Each MS with $\mu$ errors. Larger matching correction.
- Small matching scheme dependence: mainly error bands effected.
- Low statistics for data:
  - ATLAS, hep-ex/1504.05833 $H$ at 8TeV.
  - Cuts unfolded.

# PDF uncertainties




$\mu$ variation for each set.

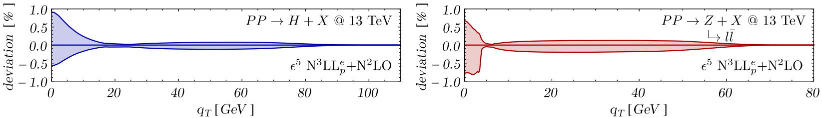
4 N$^2$LO pdf sets

Pdf-member variation of N$^2$LO NNPDF 3.0.

At large $q_T > M$ might want to resum threshold logs.
e.g. [Del Duca, Gonsalves, Kidonakis, Sabio Vera], [Becher,Lorentzen,Schwartz],. . .

# Estimated impact of unknown $\Gamma_3$, $F^{(3,0)}$

- Vary unknown $\Gamma_3$, $F^{(3,0)}$ between $[-2, 2]$ Pade approx./$F^{(2,0)}$




- Slightly larger impact of $F^{(3,0)}$.
- Uncertainty small compared to e.g. pdf-unc..
- Largest impact at small $q_T$.

Further effects not discussed here:

- EW-corrections, mass/off-shell effects, non-pert. effects, ...

# Alternatives and extensions

- Monte Carlo generators.
- CSS: DYRes, HRes [Catani,Cieri,de Florian, Ferrera, Grazzini], ResBos [Balazs, Yuan] ...
- Our method can be applied to **all** other processes with color neutral final states: $V$, $H$, $VV'$, $HH$, $Z'$, ...
- Currently available in CuTe: $\gamma^*$, $Z$, $W$, $H$.
- For others to include F.O. part.
  - $VV'$ at fixed order see e.g. Mainz/Zürich/Karlsruhe-groups [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Manteuffel, Pozzorini, Rathlev, Tancredi, Torre, Weihs] [Caola, Henn, Melnikov, Smirnov, Smirnov].
- NNLO+N$^2$LL':
  - $\gamma\gamma$: [Cieri, Coradeschi, de Florian]
  - $ZZ$, $W^+W^-$ [Grazzini, Kallweit, Rathlev, Wiesemann]

# Conclusions

- **Resummation** essential for small $q_T/M$.
- Perform via **T**PDFs (determined to NNLO).
- **Generic** framework to obtain precise $d\sigma/dq_T(/dy)$ for large class of processes at hadron colliders.
- Determine these to $\mathbf{N^2LO{+}N^3LL}_p$ precision for $\gamma^*$, $Z$, $W$, $H$ with **CuTe**.
- Obtained very accurate description of $q_T$ spectrum.
- Code will become public.

# Appendix

# Non-perturbative effects

- TPDFs must vanish rapidly at $x_T > r_{\text{proton}}$. Ansatz:

$$B_{i/N}(z, x_T^2, \mu) = f_{\text{hadr}}(x_T \Lambda_{\text{NP}})\, B_{i/N}^{\text{pert}}(z, x_T^2, \mu) \;,$$

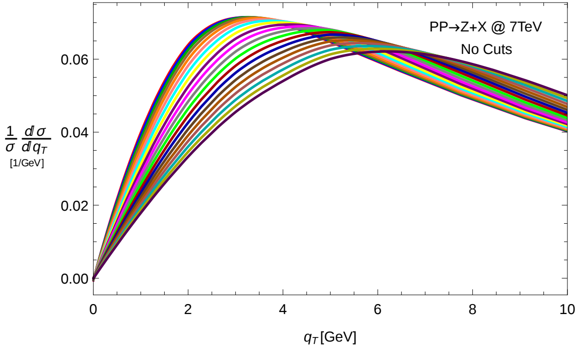
- with

$$f_{\text{hadr}}^{\text{gauss}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2\right] ,$$
$$f_{\text{hadr}}^{\text{dipole}}(x_T \Lambda_{\text{NP}}) = \frac{1}{1 + \Lambda_{\text{NP}}^2\, x_T^2} ,$$
$$f_{\text{hadr}}^{\text{enhanced}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2 \log(x_T^2 M^2 / b_0^2)\right] \;.$$

- Last see [Becher, Bell].
- $\Lambda_{\text{NP}} = 0\text{GeV}$: no correction.

# Non-perturbative effects

- Results for Gauss and Dipole basically equivalent.
- Gauss: $\Lambda_{\mathrm{NP}} = 0\,\mathrm{GeV} \longrightarrow 2\,\mathrm{Gev}$: **Shifts right** & damps.




- Enhanced for 'enhanced'. Form different, though.

# Relation to framework by Collins, Soper, Sterman

$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu)$$

$$= \left|C_V(-q^2,\mu)\right|^2 \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu)\, I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\,,$$

compare this to [Collins, Soper, Sterman]:

$$= \exp\left\{ -\int_{\mu_b}^{q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\log\frac{q^2}{\bar{\mu}^2}\, A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right]\right\}$$
$$\times\, C_{qi}(z_1,\alpha_s(\mu_b))\, C_{\bar{q}j}(z_2,\alpha_s(\mu_b))\,,$$

$x_T$ dependence via $\mu_b = b_0 {x_T}^{-1}$.

**Relations:**

- $C_{qi}(z,\alpha_s(\mu_b)) = \left|C_V(-\mu_b^2,\mu_b)\right| I_{q/i}(z,x_T^2,\mu_b)\,,$
- $A$ & $B$ related to $F$ and anomalous dimensions.

# Dictionary CSS vs BN

Using $b_0 = 2e^{-\gamma_e}$, $\mu_b = b_0 x_T^{-1}$ and $\bar{x}_T = b_0 \bar{\mu}^{-1}$:

$$C_{qi}(z, \alpha_{s(\mu_b)}) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/i}(z, \bar{x}_T^2, \mu_b)\,,$$

$$A(\alpha_{s(\bar{\mu})}) = \Gamma^q(\alpha_s) - \bar{\mu}^2 \frac{dF_{q\bar{q}}(\bar{x}_T, \bar{\mu})}{d\bar{\mu}^2}\,,$$

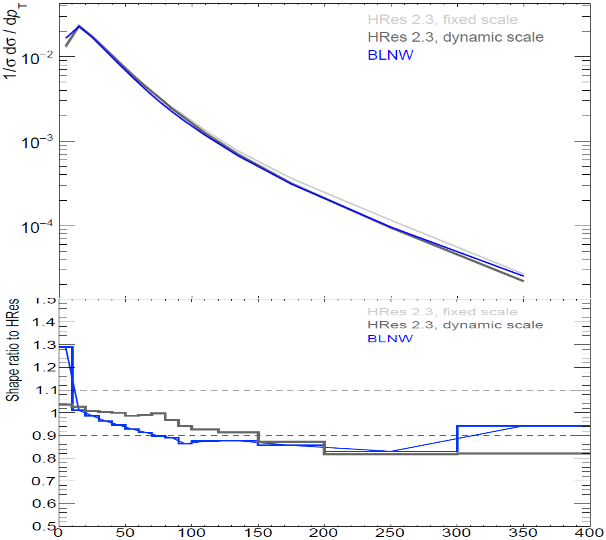
$$B(\alpha_{s(\bar{\mu})}) = 2\gamma_q(\alpha_s) + F_{q\bar{q}}(\bar{x}_T, \bar{\mu}) - \bar{\mu}^2 \frac{d \log |C_V(-\bar{\mu}^2, \bar{\mu})|^2}{d\bar{\mu}^2}\,.$$

$\Rightarrow$ Apply results in preferred resummation framework.

Can reconstruct $\mathcal{H}^{DY}_{q\bar{q}\leftarrow i\bar{j}}$, $\mathcal{H}^{H}_{gg\leftarrow i\bar{j}}$, ... in [Catani, Cieri, de Florian, Ferrera, Grazzini].

# H: Our vs HRes (from HXWG, G. Petrucciani)

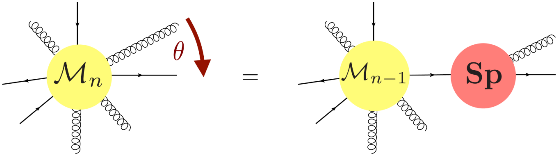
Normalized comparison of HRes, BLNW



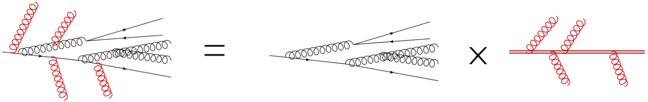
# QCD simplified

Soft and collinear emissions: Important contributions to high-energy processes; have characteristic structure.
In both limits interactions simplify:

- Collinear limit, multiple particles move in similar direction




- Soft limit, particles with small energy and momentum are emitted ⇒ Eikonal interactions.



# Soft-collinear effective theory

EFT of QCD.

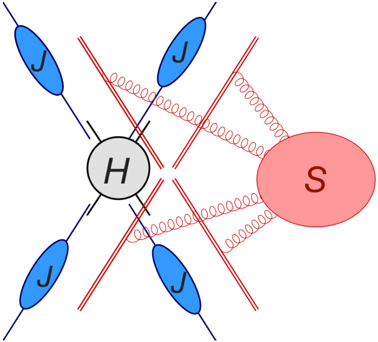
- Structure of soft and collinear interactions implemented on Lagrangian level.

⇒ Soft and collinear fields with definite interactions and power counting.

- Efficient formalism to (re-)derive factorization theorems for multi scale problems.

+: Gauge invariant operator definitions for the soft and collinear contributions.

- Resummation via renormalization group.

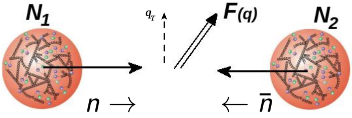



$$d\sigma \sim H(s_{ij}^2) S(\Lambda_{ij}^2) \otimes \prod_i J_i(M_i^2)$$

[Bauer, Fleming, Pirjol, Stewart, ...]

# Notation

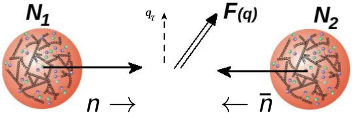
- LC vectors $n$, $\bar{n}$ ,
  $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2$
- LC basis:




$$v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu = (v_-, v_+, v_\perp)^\mu$$

# Notation

- LC vectors $n$, $\bar{n}$ ,
  $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2$
- LC basis:




$$v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu = (v_-, v_+, v_\perp)^\mu$$

- Distinguishing fields with different momentum scaling

$$\begin{array}{lll} \text{hard } (h): & p_h & \sim \sqrt{q^2}(1,1,1) \\ n\text{−collinear } (n): & p_n & \sim \sqrt{q^2}(1,\lambda^2,\lambda) \\ \bar{n}\text{−collinear } (\bar{n}): & p_{\bar{n}} & \sim \sqrt{q^2}(\lambda^2,1,\lambda) \\ \text{soft } (s): & p_s & \sim \sqrt{q^2}(\lambda,\lambda,\lambda) \end{array}$$

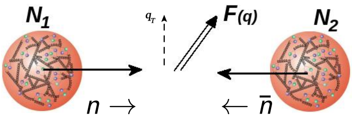
$$p_h^2 \sim q^2$$

$$p^2 \sim q_T^2$$

- For $\lambda^2 = \frac{q_T^2}{q^2}$

# Notation

- LC vectors $n$, $\bar{n}$ ,
  $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2$
- LC basis:




$$v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu = (v_-, v_+, v_\perp)^\mu$$

- Distinguishing fields with different momentum scaling

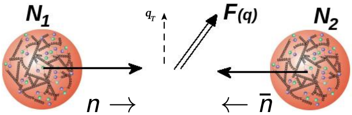
| | | | |
|---|---|---|---|
| ( hard ($h$): | $p_h$ | $\sim \sqrt{q^2}(1,1,1)$ ) | $p_h^2 \sim q^2$ |
| $n-$collinear ($n$): | $p_n$ | $\sim \sqrt{q^2}(1,\lambda^2,\lambda)$ | $p^2 \sim q_T^2$ |
| $\bar{n}-$collinear ($\bar{n}$): | $p_{\bar{n}}$ | $\sim \sqrt{q^2}(\lambda^2,1,\lambda)$ | |
| soft ($s$): | $p_s$ | $\sim \sqrt{q^2}(\lambda,\lambda,\lambda)$ | |

- For $\lambda^2 = \frac{q_T^2}{q^2} \ll 1$:

$$\mathcal{L}_{QCD} \xrightarrow{\int DA_h} \mathcal{L}_{SCET} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_s + \mathcal{O}(\lambda^2)$$

# Factorization

Consider e.g.:
$N_1 + N_2 \to V(q) + X$




$$d\sigma \sim \sum_X \delta^4(\sum \hat{p})(-g_{\mu\nu}) \langle N_1 N_2 | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | N_1 N_2 \rangle \ .$$

Match to SCET. Current:

$$J^\mu(x) = \bar{q}(x)\gamma^\mu q(x) \to C_V(-q^2)(\bar{\xi}_{\bar{n}} W_{\bar{n}})(x)\gamma_\perp^\mu (S_{\bar{n}}^\dagger S_n)(x)(W_n^\dagger \xi_n)(x)$$

hard — $n$-col. — soft — $\bar{n}$-col

Wilson lines: $W_{\bar{n}}(x) = P \exp\left[ig \int_{-\infty}^0 ds\, n\cdot A_{\bar{n}}(x+sn)\right]$,
correspondingly for $W_n$, $S_{\bar{n}}$, $S_n$.

$$|X\rangle \to |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\rangle \ , \ |N_1\, N_2\rangle \to |N_1\rangle \otimes |N_2\rangle \otimes |0\rangle \ .$$

Multipole expand in $x = (x_-, x_+, x_\perp)$.

# 'Factorized' differential cross section

From this obtain [Becher, Neubert]:

$$\frac{d^2\sigma}{dq_T^2 dy} - \mathcal{O}\left(\frac{q_T^2}{q^2}\right) = \frac{\pi\alpha_{ew}^2}{N_c s} \left|C_V(-q^2)\right|^2 \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp} \sum_q e_q^2$$

$$\times \left[\mathcal{S}(x_T^2)\mathcal{B}_{q/N_1}(z_1, x_T^2)\bar{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2) + (q \leftrightarrow \bar{q})\right]_{q^2},$$

with

**TPDF (quark, $n$ collinear, gauge invariant)**

$$\mathcal{B}_{q/N_1}(z, x_T^2) = \frac{1}{2\pi}\int \mathrm{d}t \; e^{-izt\bar{n}\cdot p} \sum_{X_n} \frac{\not{\bar{n}}_{\alpha\beta}}{2}$$

$$\times \langle N_1(p)| \, (\bar{\xi}_n W_n)_\alpha(t\bar{n} + x_\perp) \, |X_n\rangle \langle X_n| \, (W_n^\dagger \xi_n)_\beta(0) \, |N_1(p)\rangle \,.$$

- $\bar{n}$-collinear $\bar{\mathcal{B}}_{\bar{q}/N_2}$ correspondingly with $p \sim n \leftrightarrow \bar{n} \sim \bar{p}$ & $q \leftrightarrow \bar{q}$.
- Soft function $\mathcal{S}(x_\perp) = \dfrac{1}{N_c} \sum_{X_s} Tr \, \langle 0| \, (S_n^\dagger S_{\bar{n}})(x_\perp) \, |X_s\rangle \langle X_s| \, (S_{\bar{n}}^\dagger S_n)(0) \, |0\rangle \,.$

# Transverse PDFs

## TPDF (quark, $n$ collinear, gauge invariant)

$$\mathcal{B}_{q/N_1}(z, x_T^2) = \frac{1}{2\pi}\int \mathrm{d}t\; e^{-izt\bar{n}\cdot p} \sum_{X_n} \frac{\bar{\not{n}}_{\alpha\beta}}{2} \times \langle N_1(p)|\, \bar{\chi}_\alpha(t\bar{n} + x_\perp)\, |X_n\rangle \langle X_n|\, \chi_\beta(0)\, |N_1(p)\rangle \,.$$

- $\chi_n = (\bar{\xi}_n W_n)$
- TPDF: Generalization of usual PDF $\phi_{i/N}(z)$.
- $k$ = momentum of $X_n$:
  $k_-$ and (after F.T.) $k_\perp$ fixed:
- $\int dte^{-iztp_-}\bar{\chi}(t\bar{n} + x_\perp) \Rightarrow \delta(k_- - (1-z)p_-)\bar{\chi}$ ,
- $\bar{\chi}(t\bar{n} + x_\perp) \Rightarrow e^{-ik_\perp x_\perp} \overset{\text{F.T.}}{\rightarrow} \delta^{(2)}(k_\perp + q_\perp)\bar{\chi}$ .

# Gluons

- Similar fact. theorems for other $q\bar{q}$ and $gg$ initiated processes.
- If $gg$ initiated: $C$, $\mathcal{B}$ & $\bar{\mathcal{B}}$ become Lorentz tensors.

**gluon TPDF ($n$ collinear) [Becher, Neubert, Wilhelm]**

$$\mathcal{B}^{\mu\nu}_{g/N}(z, x_\perp) = \frac{-z\bar{n}\cdot p}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \sum_X \langle N| \mathcal{A}^{\mu a}_{n,\perp}(t\bar{n} + x_\perp) |X\rangle \langle X| \mathcal{A}^{\nu a}_{n,\perp}(0) |N\rangle$$

- with gauge invariant gluon field $\mathcal{A}^\mu_{n\perp}(x) = \left(W_n^{\mathrm{adj}} A^\mu_{n\perp}\right)(x)$.
- Decompose tensor as

$$\mathcal{B}^{\mu\nu}_{g/N}(z, x_\perp) = \frac{g_\perp^{\mu\nu}}{d-2} \mathcal{B}_{g/N}(z, x_T^2) + \left[\frac{g_\perp^{\mu\nu}}{d-2} + \frac{x_\perp^\mu x_\perp^\nu}{x_T^2}\right] \mathcal{B}'_{g/N}(z, x_T^2) .$$

- Discussion below for $\mathcal{B}$ only. $\mathcal{B}'$ analogous.

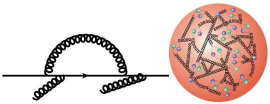
# Matching kernel

If $x_T^{-2} \gg \Lambda^2_{\mathrm{QCD}}$, refactorize these scales:

## Matching kernel $\mathcal{I}_{i/k}$

$$\mathcal{B}_{i/N}(z, x_T^2) = \sum_k \int_z^1 \frac{d\rho}{\rho} \mathcal{I}_{i/k}(\rho, x_T^2) \phi_{k/N}(z/\rho) + \mathcal{O}(\Lambda^2 x_T^2)$$

$$= \sum_k \mathcal{I}_{i/k}(z, x_T^2) \otimes \phi_{k/N}(z) \, .$$

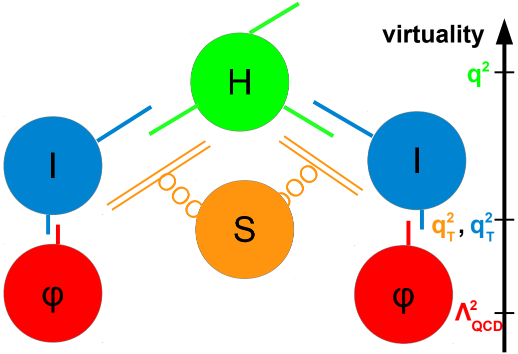


- $\mathcal{I}_{i/k}$ perturbative: Extract from perturbative $\mathcal{B}_{i/j}$ and $\phi_{k/j}$.
- Determine to NNLO: [Gehrmann,TL,Yang].

# Factorization, pictorially

$N_1 + N_2 \to F(q) + X$
at $\Lambda^2_{\mathrm{QCD}} \ll q_T^2 \ll q^2$:

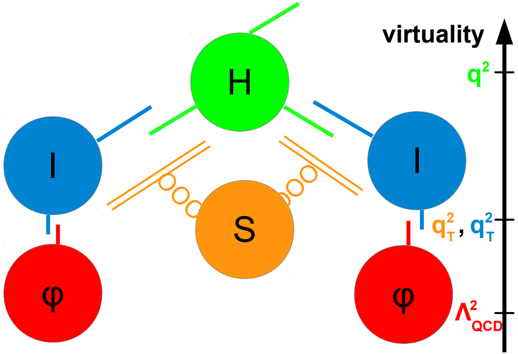
$$\frac{d\sigma}{dq_T^2} = H \otimes S \otimes \mathcal{I} \otimes \bar{\mathcal{I}} \otimes \phi \otimes \phi.$$



# Factorization, pictorially

$N_1 + N_2 \to F(q) + X$
at $\Lambda^2_{\mathrm{QCD}} \ll q_T^2 \ll q^2$:

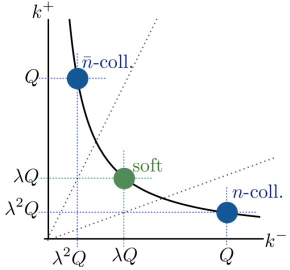
$$\frac{d\sigma}{dq_T^2} = H \otimes S \otimes \mathcal{I} \otimes \bar{\mathcal{I}} \otimes \phi \otimes \phi.$$




Focus on $S$, $\mathcal{I}$, $\bar{\mathcal{I}}$.

Same virtuality
$k^2 \sim q_T^2$.

Differ in rapidity
$y(k) = \frac{1}{2} \log \frac{k_+}{k_-}$



# Divergences

- Besides usual UV/IR divergences regulated in $d = 4 - 2\epsilon$, encounter 'rapidity' divergences: Unregulated by $\epsilon$; from extreme rapidities $y(k) = \frac{1}{2} \log \frac{k_+}{k_-}$ .
- Arise in integrals along LC-direction: $\int d^d k = \frac{1}{2} \int dk_+ dk_- d^{d-2} k_\perp$ .
- $\delta^{(d-2)}(k_\perp + q_\perp) \;\Rightarrow\;$ kills $\int d^{d-2} k_\perp$ without putting $\epsilon$ to $k_+$ and $k_-$.

# Divergences

- Besides usual UV/IR divergences regulated in $d = 4 - 2\epsilon$, encounter 'rapidity' divergences: Unregulated by $\epsilon$; from extreme rapidities $y(k) = \frac{1}{2} \log \frac{k_+}{k_-}$ .
- Arise in integrals along LC-direction: $\int d^d k = \frac{1}{2} \int dk_+ dk_- d^{d-2} k_\perp$ .
- $\delta^{(d-2)}(k_\perp + q_\perp) \;\Rightarrow$ kills $\int d^{d-2} k_\perp$ without putting $\epsilon$ to $k_+$ and $k_-$.

$\Rightarrow$ Need additional regulator. Various possibilities:

  - Avoid divergent $k_\pm$ denominators: Use $W$ away from $n$, $\bar{n}$; Introduce 'LC-mass', ....
  - Regulate $k_\pm$ integrals: Analytic regulator in phase space, ....

- For each various ways.
- Combination $\mathcal{S}_i \mathcal{B}_{i/j} \bar{\mathcal{B}}_{\bar{\imath}/k}$ must not dependent on this regularization.

# Analytic regulator

- We use analytic regulator $\alpha$ [Becher, Bell]: $\times\left(\frac{\nu}{n\cdot l_i}\right)^\alpha$ for each external parton.
- Same LC vector $n$ for all functions. Breaks symmetry $(n \leftrightarrow \bar{n})$.
- Simple soft function $\mathcal{S} = 1$.
- $\alpha$ poles cancel in product:

## Refactorization

$$\left[\mathcal{S}(x_T^2)\mathcal{B}_{i/j}(z_1, x_T^2)\bar{\mathcal{B}}_{\bar{i}/k}(z_2, x_T^2)\right]_{q^2} \overset{\alpha=0}{=} \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{i\bar{i}}^b(x_T^2)} B_{i/j}^b(z_1, x_T^2) B_{\bar{i}/k}^b(z_2, x_T^2)\,,$$

$\nwarrow$ $b_0 = 2e^{-\gamma_e}$

- On refactorized RHS no $\alpha$ and $\nu$ dependence left.
- Hard scale $q^2$ generated.
- 'Collinear anomaly' [Becher, Neubert]. From RRG [Chiu, Jain, Neill, Rothstein]. $F_i = \gamma_\nu^{B_i}$.