

Speeding up SM Amplitude Calculations with Chirality Flow

FYSIKDAGARNA 15 JUNE 2022 - ANDREW LIFSON

BASED ON HEP-PH:2003.05877 (EPJC), HEP-PH:2011.10075 (EPJC), AND HEP-PH:2203.13618 (ACCEPTED BY EPJC)

IN COLLABORATION WITH JOAKIM ALNEFJORD, CHRISTIAN REUSCHLE, MALIN SJÖDAHL, AND ZENNY WETTERSTEN



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Chirality Flow

Massless QED Examples

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 - Flow Rules
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- 3 Automation
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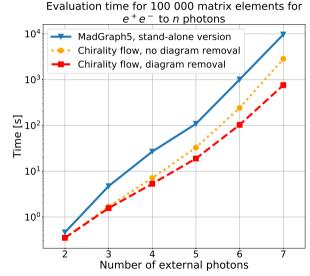
Our Main Result (hep-ph:2203.13618)

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How to Calculate a Process

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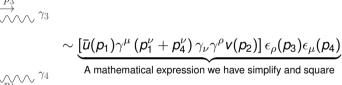
Automatic

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Conclusion

Sum all Feynman diagrams, square, and integrate

Often spin structure is non-trivial



Most common method: use helicity basis

Each diagram is a complex number, easy to square Can use algebra to simplify first, or brute force matrix multiplication





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$\begin{array}{c} e^{+} & \xrightarrow{p_{2}} & \xrightarrow{p_{3}} & \gamma_{3}^{-} \\ + & & & \downarrow^{+} \\ \downarrow^{+} & \downarrow^{+} \\ e^{-} & & \downarrow^{p_{1}} & \gamma_{4}^{+} \end{array}$

- $|p\rangle \equiv$ right-chiral spinor
- $|p| \equiv |eft\text{-chiral spinor}|$
- lacksquare $au^{\mu}, ar{ au}^{\mu} \equiv ext{Pauli matrices}$
- lacksquare $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

Spinor helicity: analytic

$$\sim \langle p_{1}|\bar{\tau}^{\mu}\underbrace{(|p_{1}]\langle p_{1}|+|p_{4}]\langle p_{4}|)}_{p_{1}+p_{4}}\bar{\tau}^{\nu}|p_{2}\underbrace{\frac{\langle r_{3}|\bar{\tau}_{\nu}|p_{3}]}{\langle r_{3}3\rangle}}_{\epsilon_{3}^{-}}\underbrace{\frac{[r_{4}|\tau_{\mu}|p_{4})}{[4r_{4}]}}_{[4r_{4}]}$$

$$= \frac{(\langle p_{1}|\bar{\tau}^{\mu}|p_{1}]+\langle p_{1}|\bar{\tau}^{\mu}|p_{4}])[r_{4}|\tau_{\mu}|p_{4}\rangle}{\langle r_{3}3\rangle[4r_{4}]}$$

$$= \frac{\langle 1r_{4}\rangle([41]\langle 13\rangle+[44]\langle 43\rangle)[r_{3}2]}{\langle r_{3}3\rangle[4r_{4}]} = \underbrace{\frac{\langle 1r_{4}\rangle([41]\langle 13\rangle[r_{3}2]}{\langle r_{3}3\rangle[4r_{4}]}}_{\text{Fierz identities like }\langle i|\bar{\tau}^{\mu}|j|[k|\tau_{\mu}|l\rangle=\langle il\rangle[kl]}_{[il]} = \underbrace{\frac{\langle 1r_{4}\rangle([41]\langle 13\rangle[r_{3}2]}{\langle r_{3}3\rangle[4r_{4}]}}_{[il]=0}$$



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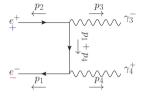
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- lacksquare $au^{\mu}, ar{ au}^{\mu} \equiv ext{Pauli matrices}$
- lacksquare $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

Spinor helicity: explicit matrix multiplication

$$\sim \left[ar{u}^-(p_1)\gamma^\mu\epsilon_\mu^+(p_4)\left(p_1^
u+p_4^
u
ight)\gamma_
u\gamma^
ho\epsilon_
ho^-(p_3)v^+(p_2)
ight]$$

Most common numerical method





Scattering Amplitudes Recap

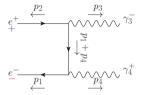
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- $|p\rangle\equiv$ right-chiral spinor
- $|p| \equiv \text{left-chiral spinor}$
- lacksquare $au^{\mu}, ar{ au}^{\mu} \equiv ext{Pauli matrices}$
- lacksquare $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

Spinor helicity: explicit matrix multiplication

$$\sim \left[ar{u}^-(
ho_1)\gamma^\mu\epsilon^+_\mu(
ho_4)\left(
ho_1^
u+
ho_4^
u
ight)\gamma_
u\gamma^
ho\epsilon^-_
ho(
ho_3)v^+(
ho_2)
ight]$$

Most common numerical method

Can we systematically remove need for algebra or matrix multiplication?



Chirality Flow Building Blocks

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Key idea (hep-ph:2003.05877)

Draw & connect lines to directly obtain inner products $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$ Removes need to do algebra or matrix multiplication

Define spinors as lines

$$\bar{u}_{i}^{-} = \bar{v}_{i}^{+} = \langle i | \alpha = \bigcirc \cdots i , \quad u_{j}^{+} = v_{j}^{-} = |j\rangle_{\alpha} = \bigcirc \cdots j$$

$$\bar{u}_{i}^{+} = \bar{v}_{i}^{-} = [i|_{\dot{\beta}} = \bigcirc \cdots \cdots i , \quad u_{j}^{-} = v_{j}^{+} = |j]^{\dot{\beta}} = \bigcirc \cdots \cdots j$$

Spinor inner products follow

$$\langle i|^{\alpha}|j\rangle_{\alpha} \equiv \langle ij\rangle = -\langle ji\rangle = i$$

$$[i|_{\dot{\beta}}|j]^{\dot{\beta}} \equiv [ij] = -[ji] = i$$
 j

Define slashed momentum as dot

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The Massless QED Flow Rules: External Particles

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Species	Feynman	Flow
$\bar{u}^-(p_i)$	<u>i</u>	i
$v^-(p_j)$	$\frac{j}{j}$	j
$v^+(p_j)$		j
$\bar{u}^+(p_i)$		i
$\epsilon^\mu(p_i,r)$		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\epsilon_+^\mu(p_i,r)$	$\bigcirc \sim \sim \sim^i_+$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Left-chiral \equiv dotted lines

right-chiral ≡ solid lines

The QED Flow Rules: Vertices and Propagators

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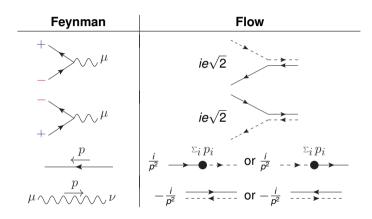
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Left-chiral ≡ dotted lines

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The Non-abelian Massless QCD Flow Vertices

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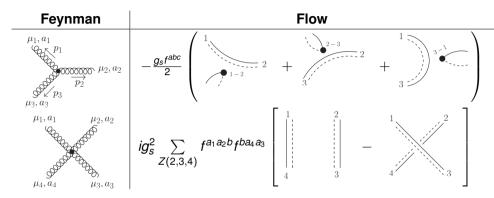
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Arrow directions only consistently set within full diagram Double line $\equiv g_{\mu\nu}$, momentum dot $\equiv p_{\mu}$ Rules for rest of SM also known (hep-ph:2011.10075)

The Non-abelian Massless QCD Flow Vertices

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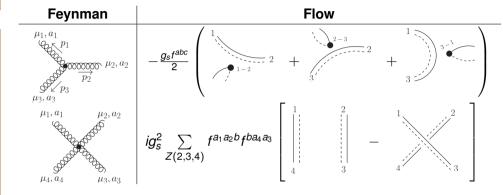
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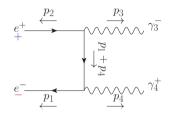
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Spinor helicity:
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$$= \frac{(\langle p_{1}|\bar{\tau}^{\mu}|p_{1}]+\langle p_{1}|\bar{\tau}^{\mu}|p_{4}])[r_{4}|\tau_{\mu}|p_{4}\rangle}{\langle r_{3}3\rangle[4r_{4}]} (\langle p_{1}|\bar{\tau}^{\nu}|p_{2}]+\langle p_{4}|\bar{\tau}^{\nu}|p_{2}])[p_{3}|\tau_{\nu}|r_{3}\rangle}_{\langle r_{3}3\rangle[4r_{4}]}$$

$$= \frac{\langle 1r_{4}\rangle([41]\langle 13\rangle+[44]\langle 43\rangle)[r_{3}2]}{\langle r_{3}3\rangle[4r_{4}]} = \frac{\langle 1r_{4}\rangle[41]\langle 13\rangle[r_{3}2]}{\langle r_{3}3\rangle[4r_{4}]}$$

Fierz identities like $\langle i|\bar{\tau}^{\mu}|j][k|\tau_{\mu}|I\rangle = \langle iI\rangle[kj]$

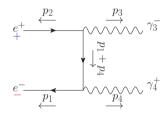
[ii]=0

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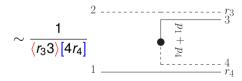
Massless QED Examples

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Chirality flow:



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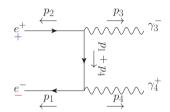
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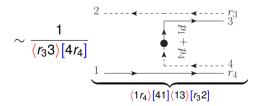
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Chirality flow:



A complicated QED Example

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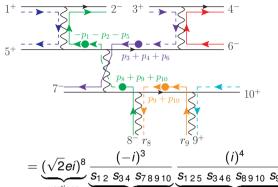
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Spinor-helicity analytic:

- 5 charge conjugation/Fierz+ rearranging
- Not possible to fit on single slide!

$$=\underbrace{(\sqrt{2}ei)^8}_{\text{vertices}}\underbrace{\frac{(-i)^3}{s_{1\,2}\ s_{3\,4}\ s_{7\,8\,9\,10}}}_{\text{photon propagators}}\underbrace{\frac{(i)^4}{s_{1\,2\,5}\ s_{3\,4\,6}\ s_{8\,9\,10}\ s_{9\,10}}}_{\text{fermion propagators}}\underbrace{\frac{1}{[8r_8]\langle r_99\rangle}}_{\text{polarization vectors}}$$

$$\frac{1}{[8r_8]\langle r_99\rangle} \quad [15]\langle 64\rangle [10 \ 9]$$

$$\left(\langle r_99\rangle[9r_8]+\langle r_910\rangle[10r_8]\right)\left(\underbrace{[33]}\langle 37\rangle+[34]\langle 47\rangle+[36]\langle 67\rangle\right)$$

$$\times \left(-\langle 89 \rangle [91]\langle 12 \rangle - \langle 89 \rangle [95]\langle 52 \rangle - \langle 8\,10 \rangle [10\,\,1]\langle 12 \rangle - \langle 8\,10 \rangle [10\,\,5]\langle 52 \rangle\right)$$

MadGraph and the Automation of Chirality Flow

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Summary

- Before: Quicker/easier to do explicit multiplication than spin algebra analytically
- We have made the analytical spin algebra trivial
- Can we use this to make even faster numerics?



MadGraph and the Automation of Chirality Flow

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Summary

- Before: Quicker/easier to do explicit multiplication than spin algebra analytically
- We have made the analytical spin algebra trivial
- Can we use this to make even faster numerics?

Use MadGraph5_aMC@NLO (MG5aMC) for proof of concept automation

- Make minimal changes to massless QED in MG5aMC
- Pro: any difference in speed from our changes ⇒ sound conclusions
- Con: MG5aMC not designed for chirality flow ⇒ not optimal implementation



Sources of Expect Speed Gains

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- Simplified vertices and propagators
 - We minimise matrix multiplication
 - Each component of a calculation is simpler



Sources of Expect Speed Gains

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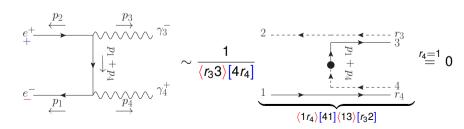
Massless QED Example

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Conclusion

- Simplified vertices and propagators
 - We minimise matrix multiplication
 - Each component of a calculation is simpler
- 2 Gauge-based diagram removal
 - Polarisation vectors contain arbitrary gauge-reference spinor of momentum *r*
 - Spinor inner products antisymmetric $\Rightarrow \langle ii \rangle = [jj] = 0$
 - Chirality-flow makes optimal choice of r obvious \Rightarrow remove diagrams!



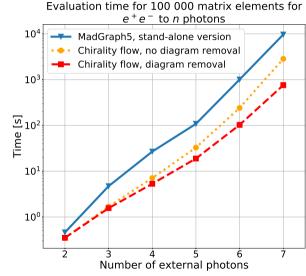


Our Main Result (hep-ph:2203.13618)

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Conclusions and Outlook

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Conclusions:

- Chirality flow is the shortest route from Feynman diagram to complex number
- We automised it for massless QED, found significant gains in MadGraph

Outlook and other work in this area:

- Malin Sjödahl and Simon Plätzer used chirality flow as basis for resummation (hep-ph:2204.03258)
- Use method analytically to calculate loop amplitudes
 - Ongoing work with Malin Sjödahl, Simon Plätzer and AL
- Automate for rest of (tree-level) Standard Model and tweak algorithm to use all possible features of chirality flow
 - Malin Sjödahl to supervise two master students to help achieve this

The Non-abelian Massless QCD Flow Vertices

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Massless QCD

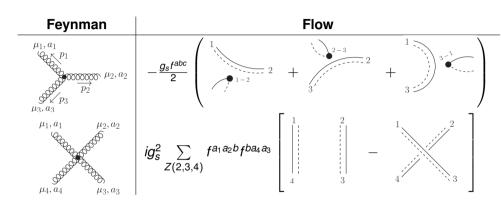
Massive Chirality Flow

Lorentz Group Details

Spinor-hel details

Chirality-Flow Motivation





Arrow directions only consistently set within full diagram Double line $\equiv g_{\mu\nu}$, momentum dot $\equiv p_{\mu}$

QCD Example: $q_1\bar{q}_1 \rightarrow q_2\bar{q}_2g$

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Massless QCD

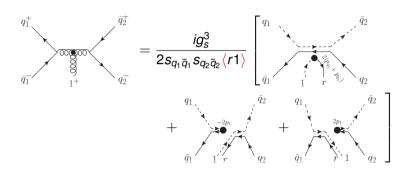
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Chirality-Flow Motivation





$$\begin{bmatrix} \cdots \end{bmatrix} \equiv \left\{ 2[q_1\bar{q}_2]\langle q_2\bar{q}_1\rangle ([1q_1]\langle q_1r\rangle + [1\bar{q}_1]\langle 1r\rangle) \\ -2[q_11]\langle 1\bar{q}_1\rangle \langle q_2r\rangle [1\bar{q}_2] + 2[q_11]\langle r\bar{q}_1\rangle \langle q_21\rangle [1q_2] \right\}$$

Incoming Massive Spinors in Chirality Flow

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Chirality-Flow Motivation



Spin operator $-\frac{\Sigma^{\mu}s_{\mu}}{2}=\frac{\gamma^{5}s^{\mu}\gamma_{\mu}}{2}, s^{\mu}=\frac{1}{m}(p^{\flat,\mu}-\alpha q^{\mu})$			
Spinor	Feynman	Flow	
$ar{v}^-(ho)$	<u>p</u> _ p	$\left(\bigcirc \cdots \blacktriangleleft \cdots p^{\flat} , \sqrt{\alpha} e^{i\varphi} \bigcirc \blacktriangleleft q \right)$	
$ar{v}^+(ho)$	p p +	$\left[\left(-\sqrt{\alpha} \mathbf{e}^{-i\varphi} \bigcirc \cdots \blacktriangleleft \cdots q \right) \right]$	
<i>u</i> ⁻ (<i>p</i>)		$\left(\sqrt{\alpha} e^{i\varphi} $	

 $p^{\mu}=p^{\flat,\mu}+\alpha q^{\mu}\;,\quad (p^{\flat})^2=q^2=0\;,\quad e^{i\varphi}\sqrt{\alpha}=rac{m}{(p^{\flat}q)}\;,\qquad e^{-i\varphi}\sqrt{\alpha}=rac{m}{(qp^{\flat})}$

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Some Fermion Flow Rules

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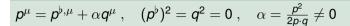
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Fermion-vector vertex

$$= ie(P_L C_L + P_R C_R) \gamma^{\mu} = ie\sqrt{2} \begin{pmatrix} 0 & C_R \\ C_L & 0 \end{pmatrix}$$

Fermion propagator

$$\frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \delta^{\dot{\alpha}}_{\ \dot{\beta}} & \sqrt{2} p^{\dot{\alpha}\beta} \\ \sqrt{2} \bar{p}_{\alpha\dot{\beta}} & m_f \delta_{\alpha}^{\ \beta} \end{pmatrix} = \frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \dot{\alpha} & \dot{\beta} & \ddots & \ddots \\ & \ddots & \ddots & \ddots & \ddots \\ & p_i & \ddots & \ddots & \ddots \\ & & p_i & \ddots & \ddots & \ddots \end{pmatrix}$$



Left and right chiral couplings may differ

A Massive *Illuminating* Example

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Massive Chirality Flow Massive Examples

Lorentz Group Details

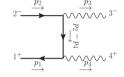
Spinor-hel detail

Chirality-Flow Motivation



Consider the same diagram of $f_1^+ \bar{f}_2^- \to \gamma_3^+ \gamma_4^-$ as before but include mass m_f

- Obtain 3 new terms
- Simplify with choices of q_1, q_2, r_3, r_4
- $lackbox{e}^{iarphi_i}\sqrt{lpha_i}=rac{m_i}{\langle
 ho_i^\flat q_i
 angle}\;,\quad lackbox{e}^{-iarphi_i}\sqrt{lpha_i}=rac{m_i}{[q_i
 ho_i^\flat]}$



$$=\frac{-2ie^{2}}{(s_{23}-m_{f}^{2})\langle r_{3}3\rangle[4r_{4}]}\left\{\begin{array}{c} p_{2}^{\frac{1}{2}} & r_{3}^{\frac{1}{3}} \\ p_{4}-p_{1}^{\frac{1}{2}}-q_{1} \\ p_{1}^{\frac{1}{2}} & r_{4}^{\frac{1}{4}} \end{array}\right. -\sqrt{\alpha_{1}\alpha_{2}}e^{i(\varphi_{2}-\varphi_{1})} \left(\begin{array}{c} q_{2} & r_{3}^{\frac{3}{2}} \\ p_{4}-p_{1}^{\frac{1}{2}}-q_{1} \\ q_{1} & r_{4}^{\frac{1}{4}} \end{array}\right)$$

$$+ m_{f} \left(\sqrt{\alpha_{2}} e^{i\varphi_{2}} \right)^{q_{2}} - \sqrt{\alpha_{1}} e^{-i\varphi_{2}} \left(\sqrt{\alpha_{1}} e^{-i\varphi_{2}} \right)^{q_{2}} \left(\sqrt{\alpha_{1}} e^{-i\varphi_{2}} \right)^{q_{1}} + m_{f} \left(\sqrt{\alpha_{1}} e^{-i\varphi_{2}} \right)^{q_{2}} \left(\sqrt{\alpha_{1}} e^{-i\varphi_{2}} \right)^{q_{1}} + m_{f} \left(\sqrt{\alpha_{1}} e^{-i\varphi_{2}} \right)^{q_{2}} \left($$

A Second Massive Example: $f_1\bar{f}_2 \rightarrow W \rightarrow f_3\bar{f}_4h_5$

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Massive Chirality Flow Massive Examples

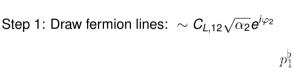
Lorentz Group Details

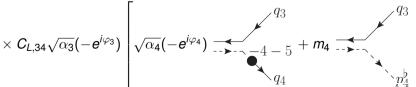
Spinor-hel detail

Chirality-Flow Motivation

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- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \dots q_5$
- $\bullet e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle \rho_i^\flat q_i \rangle} \ , \quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i \rho_i^\flat]}$
- Scalar has no flow line





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A Second Massive Example: $f_1\bar{f}_2 \rightarrow W \rightarrow f_3\bar{f}_4h_5$

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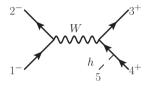
Spinor-hel details

Chirality-Flow Motivation

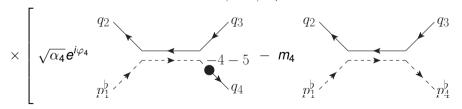
- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \dots q_5$

$$\bullet e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle \rho_i^\flat q_i \rangle} , \quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i \rho_i^\flat]}$$

Scalar has no flow line



Step 2: Flip arrows and connect: $C_{L,12}C_{L,34}\sqrt{\alpha_2\alpha_3}e^{i(\varphi_2+\varphi_3)}$





Lorentz Group Representations

Lorentz Group Details



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Lorentz group elements: $e^{i(\theta_i J_i + \eta_i K_i)}$ $J_i \equiv \text{rotations}, \quad K_i \equiv \text{boosts}$

- Lorentz group generators \simeq 2 copies of su(2) generators
 - $so(3,1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

Group algebra defined by commutator relations

$$\begin{split} [J_i,J_j] &= i\epsilon_{ijk}J_k, \quad [J_i,K_j] = i\epsilon_{ijk}K_k, \quad [K_i,K_j] = -i\epsilon_{ijk}J_k \\ N_i^\pm &= \frac{1}{2}(J_i\pm iK_i) \;, \quad [N_i^-,N_j^+] = 0 \;, \\ [N_i^-,N_j^-] &= i\epsilon_{ijk}N_k^- \;, \qquad [N_i^+,N_j^+] = i\epsilon_{ijk}N_k^+ \end{split}$$
 Representations (i.e. realisations of N_i^\pm)

- - (0,0) scalar particles
 - $(\frac{1}{2},0)$ left-chiral and $(0,\frac{1}{2})$ right-chiral Weyl (2-component) spinors.
 - $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$, Dirac (4-component) spinors.
 - $\left(\frac{1}{2},\frac{1}{2}\right)$ vectors, e.g. gauge bosons

How to Calculate? Spinor-Helicity

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Spinor-hel details

Chirality-Flow Motivation

Spinors (in chiral basis): $u^{+}(p) = v^{-}(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \qquad u^{-}(p) = v^{+}(p) = \begin{pmatrix} |p| \\ 0 \end{pmatrix}$ $\bar{u}^{+}(p) = \bar{v}^{-}(p) = ([p| \quad 0) \qquad \bar{u}^{-}(p) = \bar{v}^{+}(p) = (0 \quad \langle p|)$

Give each particle a defined helicity ⇒ amplitude now a number!

 $\gamma^{\mu} = egin{pmatrix} 0 & \sqrt{2} au^{\mu} \ \sqrt{2}ar{ au}^{\mu} & 0 \end{pmatrix} \qquad \sqrt{2} au^{\mu} \ = (1,ec{\sigma}), \ \sqrt{2}ar{ au}^{\mu} = (1,-ec{\sigma}),$

■ Amplitude written in terms of Lorentz-invariant spinor inner products

$$\langle ij \rangle = -\langle ji \rangle \equiv \langle i||j \rangle$$
 and $[ij] = -[ji] \equiv [i||j]$

- These are well known complex numbers, $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_i}$
- Remove $\tau/\bar{\tau}$ matrices in amplitude with

$$\langle i|\bar{\tau}^{\mu}|j][k|\tau_{\mu}|I\rangle = \langle il\rangle[kj], \qquad \langle i|\bar{\tau}^{\mu}|j] = [j|\tau^{\mu}|i\rangle$$



Define Problem

Massless QCD

Massive Chirality Flow

Massive Examples

Lorentz Group Details

Spinor-hel detail

Chirality-Flow Motivation

LUND

Kinematic part of amplitude slowed by spin and vector structures

- Can we still improve on this?
 - Deriving spinor inner products $\langle ij \rangle$, [kl] requires at least 2 steps
 - Re-write every object as spinors
 - Use Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau^{\dot{\alpha}\beta}_{\mu} = \delta^{\beta}_{\alpha}\delta^{\dot{\alpha}}_{\dot{\beta}}$
 - Not intuitive which inner products we obtain
- In SU(N) use graphical reps for calculations
 - E.g. using the colour-flow method
 - (Also birdtracks etc.)
- Spinor-helicity $\equiv su(2) \oplus su(2)$
 - Can we use graphical reps?

Creating Chirality Flow: Building Blocks

Massless QCD

Massive Chirality Flow Massive Examples

Lorentz Group Details

Spinor-hel detail

Chirality-Flow Motivation A flow is a directed line from one object to another su(2) objects have dotted indices and su(2) objects undotted indices

First step: Ansatz for spinor inner products (only possible Lorentz invariant)

$$\langle i|^{\alpha}|j\rangle_{\alpha} \equiv \langle ij\rangle = -\langle ji\rangle = i \longrightarrow j$$

 $[i|_{\dot{\beta}}|j]^{\dot{\beta}} \equiv [ij] = -[ji] = i \longrightarrow j$

Spinors and Kronecker deltas follow

$$\begin{aligned} \langle i|^{\alpha} &= \bigcirc \qquad \quad i \quad , & |j\rangle_{\alpha} &= \bigcirc \qquad \quad j \\ [i|_{\dot{\beta}} &= \bigcirc \cdots \qquad \quad i \quad , & |j]^{\dot{\beta}} &= \bigcirc \cdots \qquad \quad j \\ \delta_{\alpha}^{\ \beta} &\equiv \mathbb{1}_{\alpha}^{\ \beta} &= \stackrel{\alpha}{\longrightarrow} \qquad \stackrel{\beta}{\longrightarrow} \quad , & \delta_{\dot{\alpha}}^{\dot{\beta}} &\equiv \mathbb{1}_{\dot{\alpha}}^{\dot{\beta}} &= \stackrel{\dot{\beta}}{\longrightarrow} \cdots \stackrel{\dot{\alpha}}{\longrightarrow} \end{aligned}$$

