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NNLO Positivity Bounds on χPT for a General Number of Flavours

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Motivation

The curse of χPT (and other EFTs)

Many LECs — Limited observables

LO LECs (2):	high precision
NLO LECs (10):	%-level precision
NNLO LECs (90):	educated guesses
N^{3}LO LECs (1233):	unknown

(J. Bijnens and G. Ecker, *Ann. Rev. Nucl. Part. Sci.* 64 (2014) 149)

Whence bounds?

Guaranteed for "ordinary" QFT

- Analyticity
- Unitarity
- Crossing symmetry

- Perturbativity

Not necessarily consistent for EFT $\Rightarrow$ bounds!

Recent Developments

- Principles go back to the 60's; renewed interest in the 90's

  A. Martin, Springer-Verlag, 1 ed., 1969

  B. Ananthanarayan, D. Toublan and G. Wanders, *Phys. Rev. D* 51 (1995) 1093 [hep-ph/9410302]

  etc., etc.

- Our method is based on work by Manohar & Mateu

  A. V. Manohar and V. Mateu, *Phys. Rev. D* 77 (2008) 094019 [0801.3222]

  V. Mateu, *Phys. Rev. D* 77 (2008) 094020 [0801.3627]

- This talk is based on ongoing work by Alvarez, Bijnens & MS (extension of Benjamin Alvarez' 2018 master thesis)

  B. Alvarez, J. Bijnens and M. Sjö, [2111.XXXXX]

- Some recent (2020) extensions in similar directions

  Y.-J. Wang, F.-K. Guo, C. Zhang and S.-Y. Zhou, *JHEP* 07 (2020) 214 [2004.03992]

  A. J. Tolley, Z.-Y. Wang and S.-Y. Zhou, *JHEP* 05 (2021) 255 [2011.02400]

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Positivity Bounds

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Decomposition and Crossing

(Generalised) Isospin Decomposition

$$T(s,t) = \sum_{\mathcal{J}} a_{\mathcal{J}} T^{\mathcal{J}}(s,t)$$

$$\mathcal{J}: \quad \begin{cases} \text{isospin } 0,1,2 & \text{in 2-flavour,} \\ \text{representation } I, A, S, AS, SS, (AA) & \text{in 3(4+)-flavour} \end{cases}$$

Example: $\pi^\pm\pi^\pm \to \pi^\pm\pi^\pm$ is purely isospin 2 (or SS)

Bounds!

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Positivity bounds

Via “dark magic in the complex plane” (following Manohar & Mateu):

$$a_{\mathcal{J}} \frac{\mathrm{d}^k}{\mathrm{d}s^k} T^{\mathcal{J}}(s,t) \geq 0$$

Linear in all LECs up to NNLO

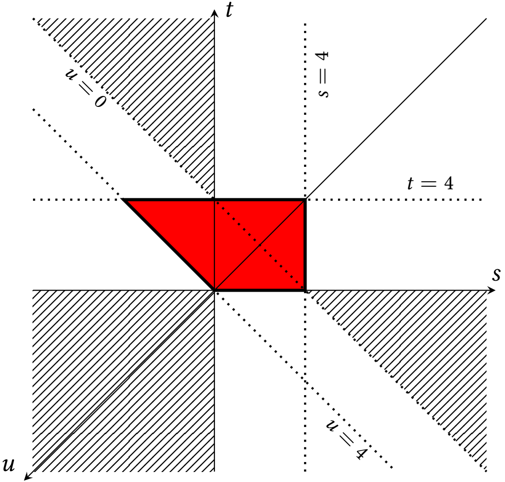
Conditions

$$k \geq 2 \text{ (convergence)}, \qquad k \text{ even in 2-flavour}$$

$$a_I \left\{ \delta^{I\mathcal{J}} \left[\frac{z-u}{z-s} \right]^{k+1} + (-1)^k C_u^{I\mathcal{J}} \right\} \geq 0 \quad \text{for all } z \geq 4$$

$$t \in [0,4], \qquad s \in [-t,4]$$

Conditions on s, t, u

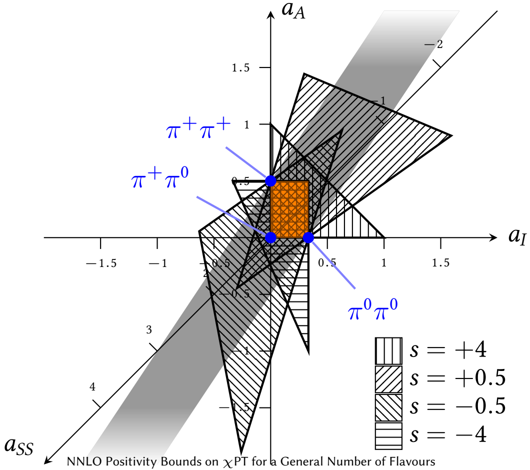


Conditions on $a_\mathcal{J}$

No need to restrict to **mass eigenstates**

No need to consider bounds valid **for all s, t** — specific values is enough

(also sufficient to check $z = 4$ and $z = \infty$)



Improving Bounds

- Move up to NNLO amplitude at equal meson masses

  J. Bijnens and J. Lu, *JHEP* 03 (2011) 028 [1102.0172]

- Most general choice of $a_{\mathcal{J}}$ (previous slide)
- Integration above threshold
  - Strengthens bounds
  - Reliant on low-energy approximation
  - Breakdown scale is $\lambda \sim 70/n$ at n flavours

    R. S. Chivukula, M. J. Dugan and M. Golden, *Phys. Rev. D* 47 (1993) 2930 [hep-ph/9206222]

  - We have performed analytic integral up to NNLO
- Mathematical framework for reducing sets of bounds

Linear Constraints

Definition of a Linear Constraint

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Linear Constraints

Parameters b_i, constants α_i, c

$$\alpha_1 b_2 + \alpha_2 b_2 + \ldots + \alpha_N b_N - c \geq 0 \quad \Leftrightarrow \quad \boldsymbol{\alpha} \cdot \boldsymbol{b} \geq c$$

Expressed as

"The (linear) constraint $\langle \boldsymbol{\alpha}, c \rangle$ is satisfied by $\boldsymbol{b}$"

Combination and Comparison

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Combined constraints

$\langle\boldsymbol{\alpha}, c\rangle + \langle\boldsymbol{\beta}, d\rangle$ satisfied if *both* $\langle\boldsymbol{\alpha}, c\rangle$ and $\langle\boldsymbol{\beta}, d\rangle$ satisfied.
Generally:

$$\Omega = \sum_i \langle\boldsymbol{\alpha}_i, c_i\rangle$$

Stronger and weaker constraints

If Ω' satisfied by all points satisfying Ω:

$$\Omega' \leq \Omega$$

Basic examples

$$\langle\boldsymbol{\alpha}, -1\rangle \leq \langle\boldsymbol{\alpha}, 0\rangle \leq \langle\boldsymbol{\alpha}, 1\rangle\,, \qquad \Omega \leq (\Omega + \Omega') \geq \Omega'$$

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Deep Results

Result 1

$\langle \boldsymbol{\alpha}, c\rangle$ weaker than $\Omega = \sum_i \langle \boldsymbol{\alpha}_i, c_i\rangle$ if and only if

$$\boldsymbol{\alpha} = \sum_i \lambda_i \boldsymbol{\alpha}_i, \qquad \sum_i \lambda_i c_i \geq c, \qquad \lambda_i \geq 0$$

Result 2

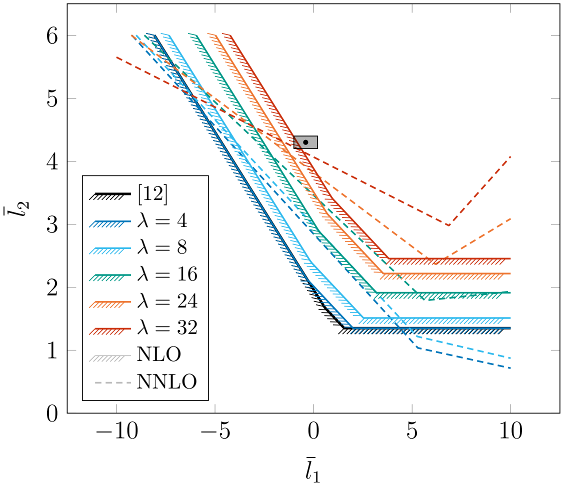
Equivalently, if and only if $\boldsymbol{\alpha}$ satisfies $\sum_j \langle \boldsymbol{n}_j, r_j\rangle$
Straightforward algorithm to generate $\langle \boldsymbol{n}_j, r_j\rangle$ based on convex hulls.

Result 3

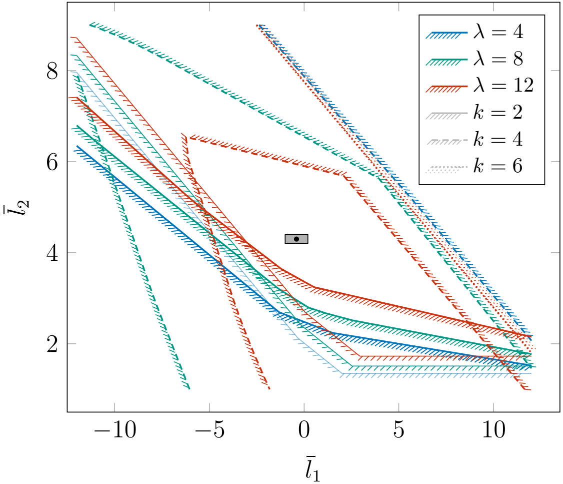
Of all possible sets $\mathcal{S}$ such that $\Omega = \sum_{\langle \boldsymbol{\alpha}, c\rangle \in \mathcal{S}} \langle \boldsymbol{\alpha}, c\rangle$,
there is a (nearly) unique *smallest* such set.
Obtained as side-effect of above algorithm.

New Bounds on χPT

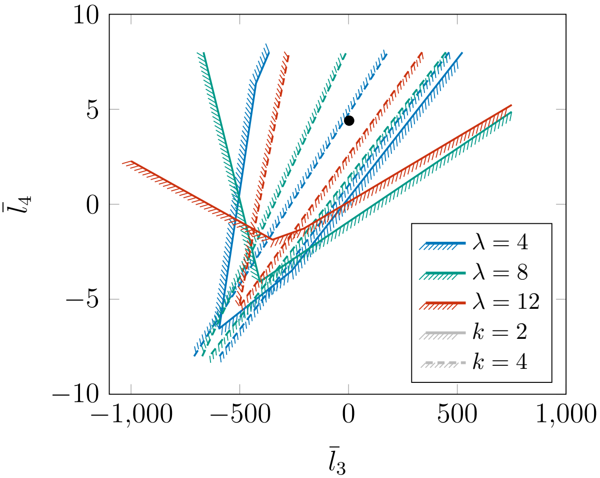
2-flavour NLO



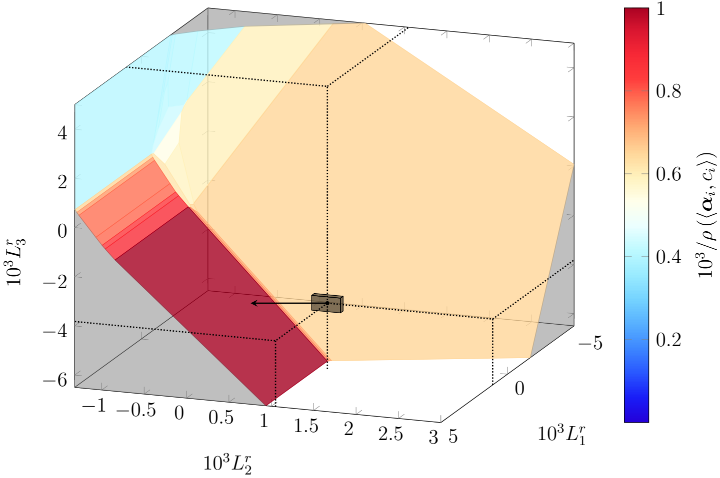
2-flavour NNLO



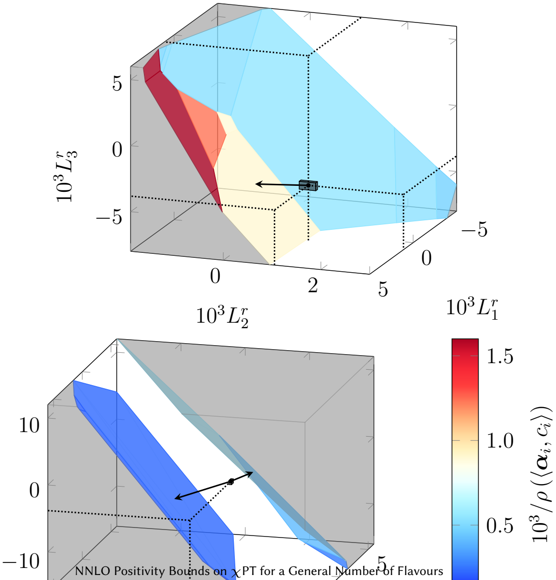
2-flavour NNLO



3-flavour NLO

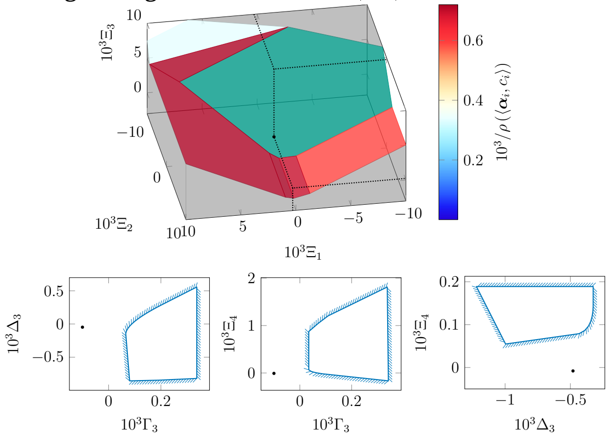


3-flavour NNLO



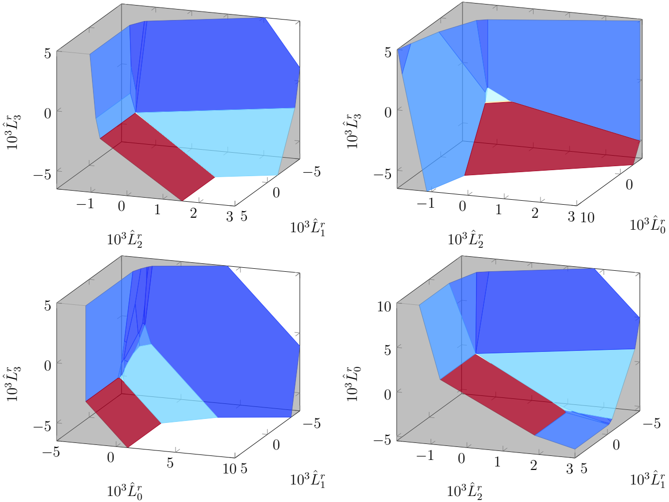
3-flavour NNLO

NNLO LECs appearing through combinations $\Xi_i, \Gamma_i, \Delta_i$



Higher Flavour

4-flavour NLO, no great difference at even higher flavour



Summary & Outlook

Summary

- First full NNLO bounds
- Stronger and more general bounds produced
- Powerful mathematical framework

Outlook

- Multiple imporvements possible (general masses, etc.)
- Mostly plug-and-play for new amplitudes
  - N^{3}LO, higher-point, etc.
  - Other EFTs

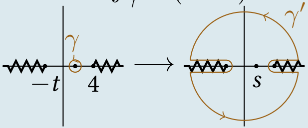
Thank you!

Extra Slides

Deriving Bounds

Dispersion relations (following Manohar & Mateu)

$$a_{\mathcal{J}} \frac{\mathrm{d}^k}{\mathrm{d}s^k} T^{\mathcal{J}}(s,t) = \frac{k!}{2\pi i} \oint_\gamma \mathrm{d}z \frac{a_{\mathcal{J}} T^{\mathcal{J}}(z,t)}{(z-s)^{k+1}} \xrightarrow{\text{Contour manipulation}}$$



$$= \frac{k!}{\pi} \int_4^\infty \mathrm{d}z \left(\frac{a_{\mathcal{J}}}{(z-s)^{k+1}} + \frac{(-1)^k a_I C_u^{I\mathcal{J}}}{(z-u)^{k+1}} \right) \mathrm{Im}\, T^{\mathcal{J}}(z+\varepsilon i, t)$$

Partial-wave expansion, optical theorem

$$\mathrm{Im}\, T^{\mathcal{J}}(s,t) = \sum_{\ell=0}^\infty (2\ell+1) s \beta(s) \sigma_\ell^I P_\ell \left(1 + \frac{2t}{s-4}\right) \geq 0$$