

UPPSALA **UNIVERSITET**

A CAREFUL LOOK AT PHASE TRANSITIONS IN THE STANDARD MODEL EFFECTIVE FIELD THEORY

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Two reasons:

• Necessary for generating baryon asymmetry:

> **1) Baryon number violation 2) Charge and Charge-Parity violation 3) Departure from thermal equilibrium**

• First order phase transitions give strong gravitational wave signals

WHY (A FIRST ORDER) PHASE TRANSITION? $V(\boldsymbol{\varphi})$

Transition from one vacuum (+symmetries) to another broken phase (+other symmetries).

First order > STRONG

Second order > SOFT

IS IT A PROBLEM? GAUGE INVARIANCE AND THE EFF. POTENTIAL

- **It is well known that the effective potential approach is gauge dependent to some extent.**
- **The shape of the potential might depend on the gauge choice**
- **The location of minima might also depend on gauge choice**
- **The depth of minima does not depend on gauge choice**

IS IT A PROBLEM? GAUGE INVARIANCE AND THE EFF. POTENTIAL

Things are (as usual) much simpler at lower orders, and one can say e.g. that:

- **At one-loop all gauge dependence manifests itself in the Goldstone boson masses! • If one works at T = 0, then as long as you are minimally careful, all is well. Just be sure**
- **Goldstone masses vanish.**
- **•At T ≠ 0, resummations have to be included to improve the perturbative expansion. Then more care has to be taken with power counting to get the Goldstone masses to be 0.**

Without careful power counting, phase transitions can even disappear for some gauge choices!

Power counting depends on the parameters of your theory! It is not a unique recipe

Incredibly relevant for > 1 loop

RESTORING GAUGE INVARIANCE: PROPER POWER COUNTING

We observe that the coefficient of the quadratic term is gauge-independent, as one expects based on the gauge-independence of thermal masses (see e.g., ref. [38]). In appendix C , we explicitly demonstrate this property for the general model. As we will discuss below, we take advantage of this property to define the high-temperature effective theory
used to skining sense independent selection scale. used to obtain a gauge-independent sphaleron scale.

Unfortunately, the coefficient E is not only gauge-dependent but strongly so. For example, by choosing $\xi = 3^{2/3}$ the E-coefficient can be made to vanish, and the barrier necessary for a first order phase transition is permanently absent. One might hope that example, by choosing $\xi = 3^{2/3}$ the E-coefficient can be made to vanish, and the barrier pendix C, we explicitly demonstrate this property for the general model. As we will discuss the general model.
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It is straightforward to see that na¨ıvely inverting these equations at one-loop order Daryon washout, creetroweak phase transition, and
norturbation thoory high-T approximation the ory in the function \mathbf{z} is the theory in the theory in the theory is the theory in the theory in the theory is the theory in the theory in the theory is the theory in the theory in the theory Julyon washout, perturbation theory \mathbf{r} ³/² [−] ¹ Baryon washout, electroweak phase transition, and

 $\overline{}$ l. Ramsey $\sum_{i=1}^{n} \frac{1}{i} \sum_{i=1}^{n} \frac{1}{i$ Hiren H. Patel^a and Michael J. Ramsey-Musolf^{a,b} \mathcal{L}

$$
V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\bar{\lambda}}{4}\phi^4 + \dots
$$

The coefficients D, T_0^2 , E and $\bar{\lambda}$ depend on the parameters of the underlying model. In \bullet ry! the SM, the coefficients are $[37]$ The coefficients D, T_0^2 , E and $\bar{\lambda}$ depend on the parameters of the underlying model. In

$$
D = \frac{1}{32}(g_1^2 + 3g_2^2 + 4y_t^2 + 8\lambda),
$$

\n
$$
T_0^2 = \mu^2/2D,
$$

\n
$$
E = \frac{3 - \xi^{3/2}}{96\pi} (2g_2^3 + (g_1^2 + g_2^2)^{3/2}),
$$

\nand $\bar{\lambda} = \lambda + (\xi \text{-dep. log}),$

where y_t is the top yukawa coupling; g_1 and g_2 are the $U(1)$ and $SU(2)_L$ gauge coupling constants; and the scalar quartic self coupling $\bar{\lambda}$ picks up a logarithmic ξ -dependence. $\frac{1}{2}$ g; g_1 $\frac{1}{2}$ $\frac{1}{2}$ are the U(1) and SU(2)_{*r*} scalar quartic self coupling λ picks up
nat the coefficient of the quadratic term $\frac{S}{\sqrt{N}}$ is the top $\frac{S}{\sqrt{N}}$ can see $\frac{S}{\sqrt{N}}$. But $\frac{S}{\sqrt{N}}$ or $\frac{S}{\sqrt{N}}$ is $\frac{S}{\sqrt{N}}$ such the $\frac{S}{\sqrt{N}}$

⁹ 3.2 First-order transition 3.2 First-order transition zero for some temperature *Tc*. This requires a barrier to develop between the two minima. *T T* \overline{J} \overline{L} \overline{I} \overline{I} \overline{I} **T**2. (3.4)
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To be concrete, consider a high temperature expansion in the Abelian Higgs model. For high temperatures the potential is approximately *For concrete, consider a mgn* temperature expanding the meritial is approximately *T*₂. The *T*₂. (3.4)
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*T*₂. The *T*₁. C₁. The *T*₁. C₁. The *T*₁. The *T* \overline{C} _{)n} S_{α} does this scaling always work? No. It depends on the coupling on the couplings: vector bosons S_{α} of lambda pushed to higher orders); these terms scale as *e*0. Cracking on, NNLO is solely IO De concrete, consider a night temperature expansion in the Abenan riggs model. For
bigh temperatures the petential is approximately

Done for the SM. A better picture emerged but $V(\phi) \sim -m^2 \phi^2 + T^2 \phi^2 (e^2 + \lambda) - e^3 T \phi^3 +$ a power-counting.≥ **confirms that there is no first order PT in the SM with the right Higgs mass.**

potential is reshuffled with the scaling *T* ⇠ 1*/* The situation is alleviated with a proper power-counting. Consider the first-order transition scaling $\lambda \sim e^3$, $m_{\rm eff}^2(T) \sim e^3 T^2$, $T \sim \frac{1}{e}.$ A new minimum develops when the quartic $\frac{1}{2}$ er $e^2 \phi^2 \sim$ $\sim e^2T^2$ **r** while the photon mass is of order $e^2\phi^2 \thicksim e^2T^2.$ This means that the gauge dependent term competes with the mass term: $\phi \thicksim T.$ Now, the Goldstone mass is of order $\overline{G}\thicksim e^3T^2,$ terms (to leading order) cancel, leaving

$$
(\overline{G} + \xi e^2 \phi^2)^{3/2} T - \xi^{3/2} e^3 \phi^3 T = \frac{3}{2} T \sqrt{\xi} e \phi \overline{G} \sim e^4 T^4. \tag{3.8}
$$

a first-order transition, where the minimum above 3.1 Second-order transition s.i ^{SC} econd-order transition
product dependent terms are sub-activities are sub-activities are sub-activities are sub-activities are sub-activities

Consider first a second-order transition. With the scaling $T \sim 1/$ $\sqrt{ }$ \mathbf{e} can write things consider first a second-order transition. With the scaling $T \sim 1/\sqrt{\hbar}$ the energy is Consider first a second-order transition. With the scaling $T\thicksim 1/\sqrt{\hbar}$ the energy is

$$
V_{\min} = \left\{ \left(V_0 + T^2 V_1^2 \right) + \sqrt{\hbar} T \overline{V}_1^1 + \hbar \left(T^2 \overline{V}_2^2 + \overline{V}_1^0 - \sum_X \Pi_X \partial_X V_1^0 - T^2 \frac{(\phi_{1/2}(T))^2}{2} \left(\partial^2 V_0 + T^2 \partial^2 V_1^2 \right) \right) + \dots \right\} \Big|_{\phi_0(T)}.
$$
\n(3.2)

The leading-order term $\left(V_0 + T^2 V_1^2\right)$ determines the temperature dependent VeV 0(*T*). high temperatures the potential is approximately *^V*() ⇠ *m*2² ⁺ *^T*22(*e*² ⁺) *^e*3*T*³ ⁺ 4. (3.3) Terms in $T^2V_1^2$ are gauge invariant and are of the form $\sim e^2\phi^2T^2$ for some coupling e [5]. So all that changes for finite T is $m^2 \to m_{\text{eff}}^2(T)$. The transition occurs at the temperature **Tinite** where $m_{\text{eff}}^2(T)$ changes sign: $m_{\text{eff}}^2(T_{2nd}) = 0$. This is a second-order transition. $\mathcal{L}_{\mathbf{H}}$ for a moment power-counting about proper power-counting and just try to naively to naively try to nai Following $\frac{8}{3}$, these various terms have to balance each other for a barrier to develop. The set of a barrier to develop The leading-order term $\left(V_0+T^2V_1^2\right)$ determines the temperature dependent VeV $\phi_0(T)$. So all that changes for finite T is $m^2 \to m_{\text{eff}}^2(T)$. The transition occurs at the temperature

$$
V(\phi) \sim -m^2 \phi^2 + T^2 \phi^2 (e^2 + \lambda) - e^3 T \phi^3 + \lambda \phi^4.
$$
 (3.3)

 $\int \lambda \phi^2 \sim e^3 T \phi \sim (-m^2 + T^2 e^2 + \lambda T^2) \equiv m_{\text{eff}}^2(T)$, or Following [8], these various terms have to balance each other for a barrier to develop. The balance occurs if $\lambda \phi^2 \sim e^3 T \phi \sim (-m^2 + T^2 e^2 + \lambda T^2) \equiv m_{\rm eff}^2(T)$, or F_e Ilevina^[0] these verieve terme heve to belance each other fer a berrier to develop ⁿ a political power-
⊇ **P** in the SIVI balance occurs if $\lambda \phi^2 \sim e^3 T \phi \sim (-m^2 + T^2 e^2 + \lambda T^2) \equiv m_{\text{eff}}^2(T)$, or various terms have to balance each other for a barrier to develop. The

$$
\phi \sim \frac{e^3}{\lambda} T \quad \text{&} \quad m_{\text{eff}}^2(T) \sim \frac{e^6}{\lambda} T^2. \tag{3.4}
$$

This leaves only one option [8], shimilarly break the perturbative expansion. This leaves only one option [8], $\frac{1}{2}$ $\frac{1}{2}$

$$
\lambda \sim e^{3}: \qquad \phi \sim T \quad \& \quad m_{\text{eff}}^{2}(T) \sim e^{3}T^{2} \quad \& \quad T \sim \frac{1}{e}. \tag{3.5}
$$
\n
$$
\text{Andence} \quad \mathsf{Elctodt}^{*}, \text{d} \quad \text{and} \quad \mathsf{Iobon} \quad \mathsf{Efront}^{*}, \text{b}
$$

Andreas Ekstedt^{*, a} and Johan Löfgren^{t, b} $S_{\rm eff}$ does this scaling always work? No. It depends on the coupling on the coupling scalings: vector bosons Andreas Ekstedt τ and jonan Loigr *e* <u>) د</u>

high temperatures the potential is approximately **With proper power counting, one can write things in a gauge invariant way.**

3.2 First-order transition **RESTORING GAUGE INVARIANCE: PROPER POWER COUNTING**

Not every scaling leads to first order PT.

Combined with resummation of finite temperature corrections, it gives a good picture of PTs.

λ small = tiny mass.

First step D = 6 operators. perators.

We can then get right Higgs mass with small λ and *^G*⁰ ! *^ZG*⁰ ⁰ (the pseudoscalar Goldstone mode), we get a Lagrangian mass term for the Higgs mass $\mathsf S$

 $M_h^2 = \lambda v^2$

 $\mathcal{L}_\mathrm{H} = (D_\mu \phi)^\dagger (D^\mu \phi) + m^2 (\phi^\dagger \phi) - \frac{\lambda}{2}$ 2 $(\phi^\dagger \phi)^2$ $\mathcal{L}_{\mathrm{H}} - (\mathcal{L} \mu \varphi)$ ($\mathcal{L} \varphi$) ind ($\varphi \varphi$) and $2^{(\varphi \varphi)}$ $\mathsf{NDORDER?}$ $\mathcal{C}_{xx} = (\mathbf{D} \phi)^\dagger (\mathbf{D}^\mu \phi) + m^2 (\phi^\dagger \phi) - \frac{\lambda}{2} (\phi^\dagger \phi)^2$

SMEFT FIRST OR SECOND ORDER? one obtains the physical Higgs field $\mathcal{H}_{\mathcal{B}}$ field h and $G_{\mathcal{B}}$ with canonically normalized has $\mathcal{H}_{\mathcal{B}}$

We accept the hierarchy between **the trace-level square-level squared mass of the normalized Higgs field h** now reads, \mathbf{r} **EW scale and new physics. SM can be thought as an EFT and all possible higher dimensional operators can be included.** and after minimization of $M_h^2 = \lambda v^2$

Let's start by considering the dimension 6 operators (in the Warsaw basis) relevant for the Higgs mechanism, To ensure that the Higgs field has a canonically normalized kinetic term, we have to rescale the fields (in the fields (in the following α all calculation will be kept to first order in the EFT expansion). After the rescaling *^h* ! (1+1*/*2*C^DvevC*⇤² $\overline{}$

Think of them as contributions that appear à la G_{Fermi,} when we **integrate out particles.** sterm, when the contract of the gauge sector of the se 3.2 The gauge sector

Z^h ≡ 1 +

ZG⁰ ≡ 1 +

- **First order phase transition is not a rare situation, but it wants tiny λ**
- **Points with right Higgs mass can be found.**
- **What about other pheno.?**

PHASE TRANSITIONS IN THE SMEFT 1.4 $m_H \approx 125$ GeV 1.2 1.0 ϕ_0/T_c 0.8 0.6 0.4 0.010 0.015 0.020 0.025 0.030 0.035 0.040 0.045 0.050 λ

- **With the power counting that allows a first order phase transition, it is still possible to stay within reasonable pheno. in other sectors.**
- **We looked (so far) at Higgs physics and EWPT.**
- **•We "turned on" the minimal set of WCs!**

 $\times 10^{-7}$

 0.0

 -0.5

 -1.0

 -1.5

 -2.0

 -2.5

 $\left(\right)$

 $d\phi^{\prime}$

PHASE TRANSITIONS IN THE SMEFT

 $C_{\phi \Box}$

OUTLOOK: DI-HIGGS AS A PROBE?

- **• In the SMEFT not only the Higgs mass is affected. The Higgs self-couplings (among many other things) change as well.**
- **• With the power counting (and thus values for WCs and couplings) we need for first order PT, what happens to e.g. Di-Higgs production?**

$$
\begin{array}{ccc}\n & | & h \\
& | & -3i\lambda v + 15iv^3C^{\varphi} \\
& | & -ivC^{\varphi}\cap(3p_1\cdot p_1 + 2) \\
& h & -1 & -1 \\
& + \frac{iv}{4}C^{\varphi D}\left(9\lambda v^2 - 4(3p_1\cdot p_1 + 2) + 2(3p_1\cdot p_1 +
$$

 $(2p_1\cdot p_2+2p_1\cdot p_3+3p_2\cdot p_2+2p_2\cdot p_3+3p_3\cdot p_3+9\lambda v^2)$ $(p_1\cdot p_2+p_1\cdot p_3+p_2\cdot p_3))$

We are thinking about it…

CONCLUSIONS

- **• First order phase transitions are interesting and important for pheno.**
- **• In the SM alone, not possible to have this.**
- **• In the SMEFT? It seems like it is. After proper power counting and resummation, that is.**
- **• The scenario can be probed with other experiments, not just Cosmo.**
- ina th **• Could be that EWSB comes as a step in more complicated symmetry breaking though!**

