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A CAREFUL LOOK AT PHASE TRANSITIONS IN THE STANDARD MODEL EFFECTIVE FIELD THEORY

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WHY (A FIRST ORDER) PHASE TRANSITION?

Transition from one vacuum (+symmetries) to another broken phase (+other symmetries).

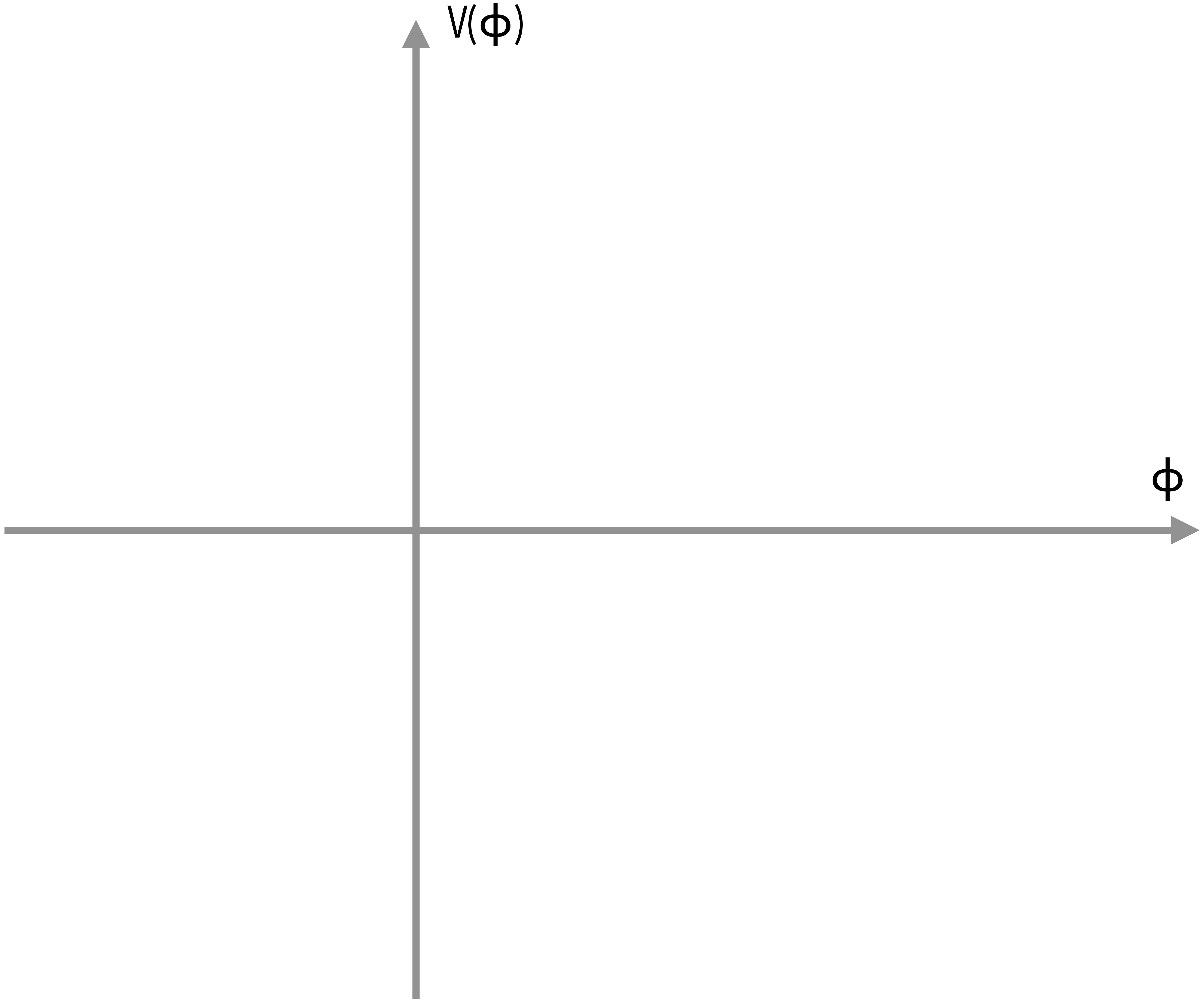
First order > STRONG

Second order > SOFT

Two reasons:

- Necessary for generating baryon asymmetry:
    1) Baryon number violation
    2) Charge and Charge-Parity violation
    3) Departure from thermal equilibrium

- First order phase transitions give strong gravitational wave signals



GAUGE INVARIANCE AND THE EFF. POTENTIAL

IS IT A PROBLEM?

It is well known that the effective potential approach is gauge dependent to some extent.

- The shape of the potential might depend on the gauge choice
- The location of minima might also depend on gauge choice
- The depth of minima does not depend on gauge choice

GAUGE INVARIANCE AND THE EFF. POTENTIAL

IS IT A PROBLEM?

Things are (as usual) much simpler at lower orders, and one can say e.g. that:

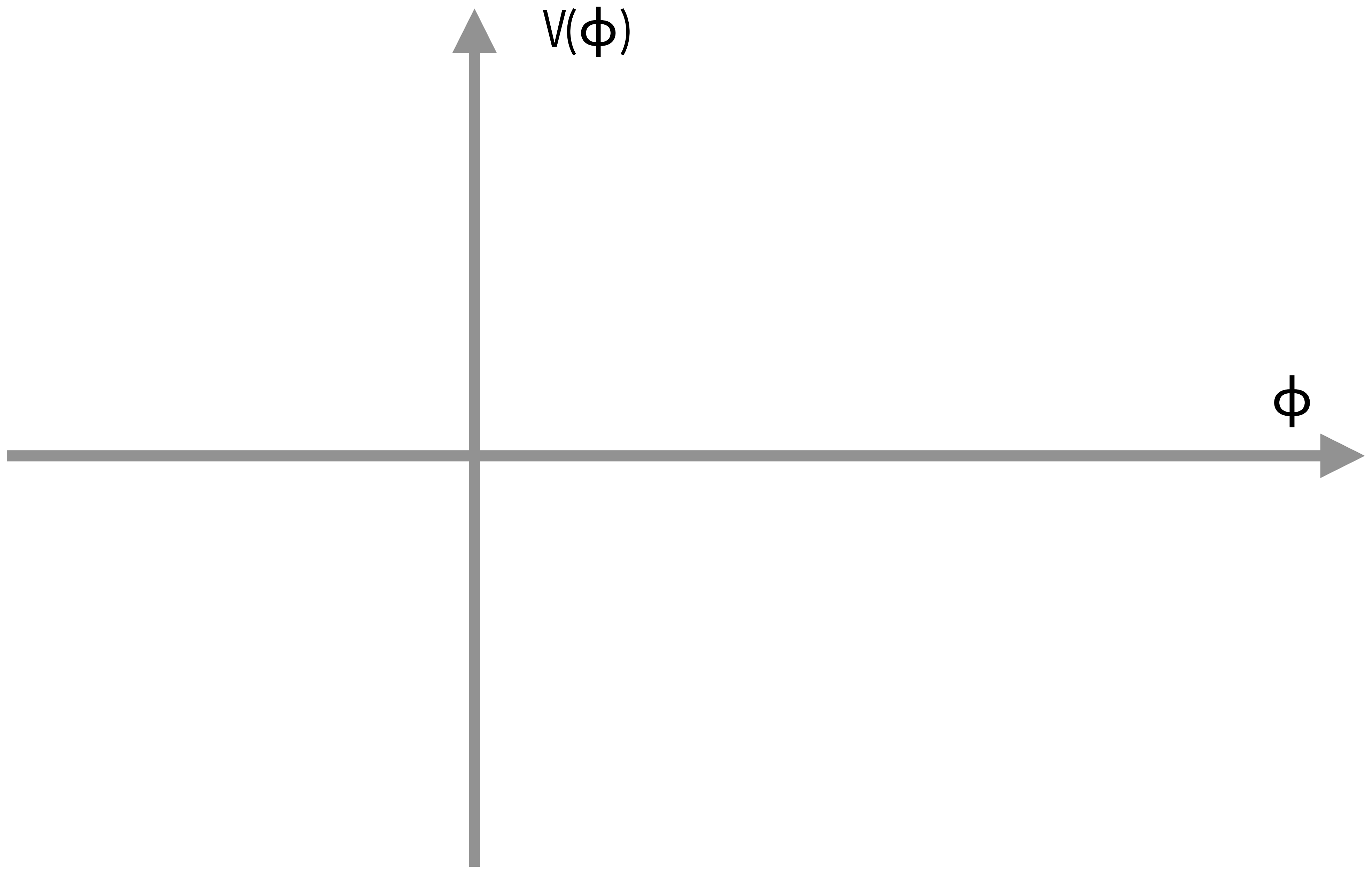
- At one-loop all gauge dependence manifests itself in the Goldstone boson masses!
- If one works at $T = 0$, then as long as you are minimally careful, all is well. Just be sure Goldstone masses vanish.
- At $T \neq 0$, resummations have to be included to improve the perturbative expansion. Then more care has to be taken with power counting to get the Goldstone masses to be 0.

RESTORING GAUGE INVARIANCE: PROPER POWER COUNTING

Without careful power counting, phase transitions can even disappear for some gauge choices!

Power counting depends on the parameters of your theory! It is not a unique recipe

Incredibly relevant for > 1 loop



Baryon washout, electroweak phase transition, and perturbation theory

Hiren H. Patel[a] and Michael J. Ramsey-Musolf[a,b]

$$V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\bar{\lambda}}{4}\phi^4 + \ldots .$$

The coefficients D, T_0^2, E and $\bar{\lambda}$ depend on the parameters of the underlying model. In the SM, the coefficients are [37]

$$\begin{aligned} D &= \frac{1}{32}(g_1^2 + 3g_2^2 + 4y_t^2 + 8\lambda)\,, \\ T_0^2 &= \mu^2/2D\,, \\ E &= \frac{3 - \xi^{3/2}}{96\pi}\left(2g_2^3 + (g_1^2 + g_2^2)^{3/2}\right), \\ \text{and } \bar{\lambda} &= \lambda + (\xi\text{-dep. log})\,, \end{aligned} \tag{2.23}$$

where y_t is the top yukawa coupling; g_1 and g_2 are the U(1) and SU(2)$_L$ gauge coupling constants; and the scalar quartic self coupling $\bar{\lambda}$ picks up a logarithmic ξ-dependence.

We observe that the coefficient of the quadratic term is gauge-independent, as one expects based on the gauge-independence of thermal masses (see e.g., ref. [38]). In appendix C, we explicitly demonstrate this property for the general model. As we will discuss below, we take advantage of this property to define the high-temperature effective theory used to obtain a gauge-independent sphaleron scale.

Unfortunately, the coefficient E is not only gauge-dependent but strongly so. For example, by choosing $\xi = 3^{2/3}$ the E-coefficient can be made to vanish, and the barrier necessary for a first order phase transition is permanently absent. One might hope that

RESTORING GAUGE INVARIANCE: PROPER POWER COUNTING

With proper power counting, one can write things in a gauge invariant way.

Not every scaling leads to first order PT.

Combined with resummation of finite temperature corrections, it gives a good picture of PTs.

Done for the SM. A better picture emerged but confirms that there is no first order PT in the SM with the right Higgs mass.

λ small = tiny mass.

The situation is alleviated with a proper power-counting. Consider the first-order transition scaling $\lambda \sim e^3$, $m^2_{\text{eff}}(T) \sim e^3 T^2$, $T \sim \frac{1}{e}$. A new minimum develops when the quartic term competes with the mass term: $\phi \sim T$. Now, the Goldstone mass is of order $\overline{G} \sim e^3 T^2$, while the photon mass is of order $e^2\phi^2 \sim e^2 T^2$. This means that the gauge dependent terms (to leading order) cancel, leaving

$$(\overline{G} + \xi e^2\phi^2)^{3/2} T - \xi^{3/2} e^3 \phi^3 T = \frac{3}{2} T \sqrt{\xi} e \phi \overline{G} \sim e^4 T^4. \tag{3.8}$$

3.1 Second-order transition

Consider first a second-order transition. With the scaling $T \sim 1/\sqrt{\hbar}$ the energy is

$$V_{\text{min}} = \Big\{ \big(V_0 + T^2 V_1^2\big) + \sqrt{\hbar} T \overline{V}_1^1 + \hbar \Big(T^2 \overline{V}_2^2 + \overline{V}_1^0 - \sum_X \Pi_X \partial_X V_1^0 - T^2 \frac{(\phi_{1/2}(T))^2}{2} \big(\partial^2 V_0 + T^2 \partial^2 V_1^2\big) \Big) + \ldots \Big\} \Big|_{\phi_0(T)}. \tag{3.2}$$

The leading-order term $\big(V_0 + T^2 V_1^2\big)$ determines the temperature dependent VeV $\phi_0(T)$. Terms in $T^2 V_1^2$ are gauge invariant and are of the form $\sim e^2\phi^2 T^2$ for some coupling e [5]. So all that changes for finite T is $m^2 \to m^2_{\text{eff}}(T)$. The transition occurs at the temperature where $m^2_{\text{eff}}(T)$ changes sign: $m^2_{\text{eff}}(T_{2\text{nd}}) = 0$. This is a second-order transition.

3.2 First-order transition

To be concrete, consider a high temperature expansion in the Abelian Higgs model. For high temperatures the potential is approximately

$$V(\phi) \sim -m^2\phi^2 + T^2\phi^2(e^2 + \lambda) - e^3 T \phi^3 + \lambda \phi^4. \tag{3.3}$$

Following [8], these various terms have to balance each other for a barrier to develop. The balance occurs if $\lambda\phi^2 \sim e^3 T \phi \sim (-m^2 + T^2 e^2 + \lambda T^2) \equiv m^2_{\text{eff}}(T)$, or

$$\phi \sim \frac{e^3}{\lambda} T \quad \& \quad m^2_{\text{eff}}(T) \sim \frac{e^6}{\lambda} T^2. \tag{3.4}$$

This leaves only one option [8],

$$\lambda \sim e^3: \qquad \phi \sim T \quad \& \quad m^2_{\text{eff}}(T) \sim e^3 T^2 \quad \& \quad T \sim \frac{1}{e}. \tag{3.5}$$

Andreas Ekstedt[*,a] and Johan Löfgren[†,b]

SMEFT

FIRST OR SECOND ORDER?

We accept the hierarchy between EW scale and new physics. SM can be thought as an EFT and all possible higher dimensional operators can be included.

Think of them as contributions that appear à la G_{Fermi}, when we integrate out particles.

First step D = 6 operators.

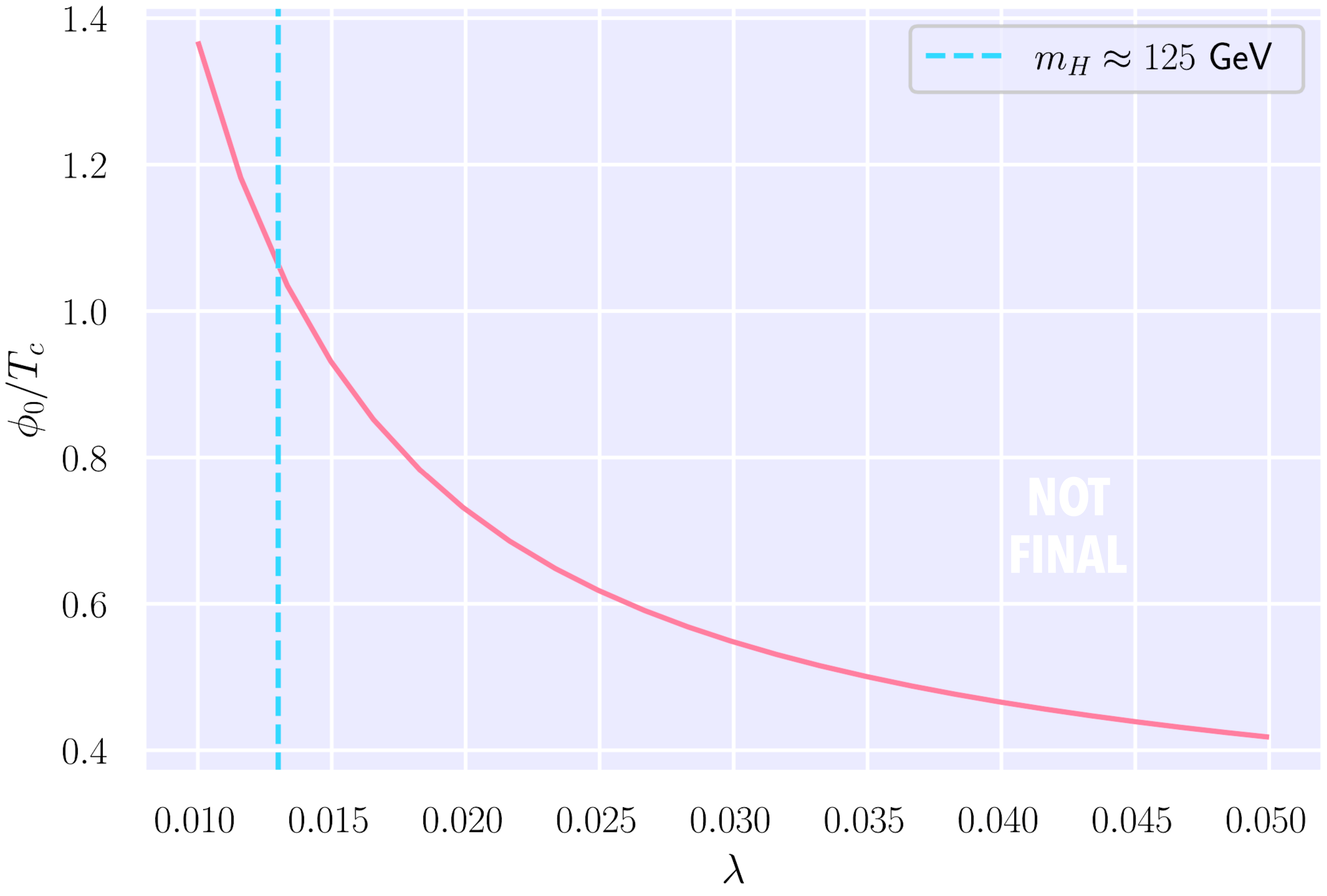
We can then get right Higgs mass with small λ

$$\mathcal{L}_{\rm H} = (D_\mu\phi)^\dagger(D^\mu\phi) + m^2(\phi^\dagger\phi) - \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$M_h^2 \quad = \lambda v^2$$

PHASE TRANSITIONS IN THE SMEFT

- First order phase transition is not a rare situation, but it wants tiny λ
- Points with right Higgs mass can be found.
- What about other pheno.?

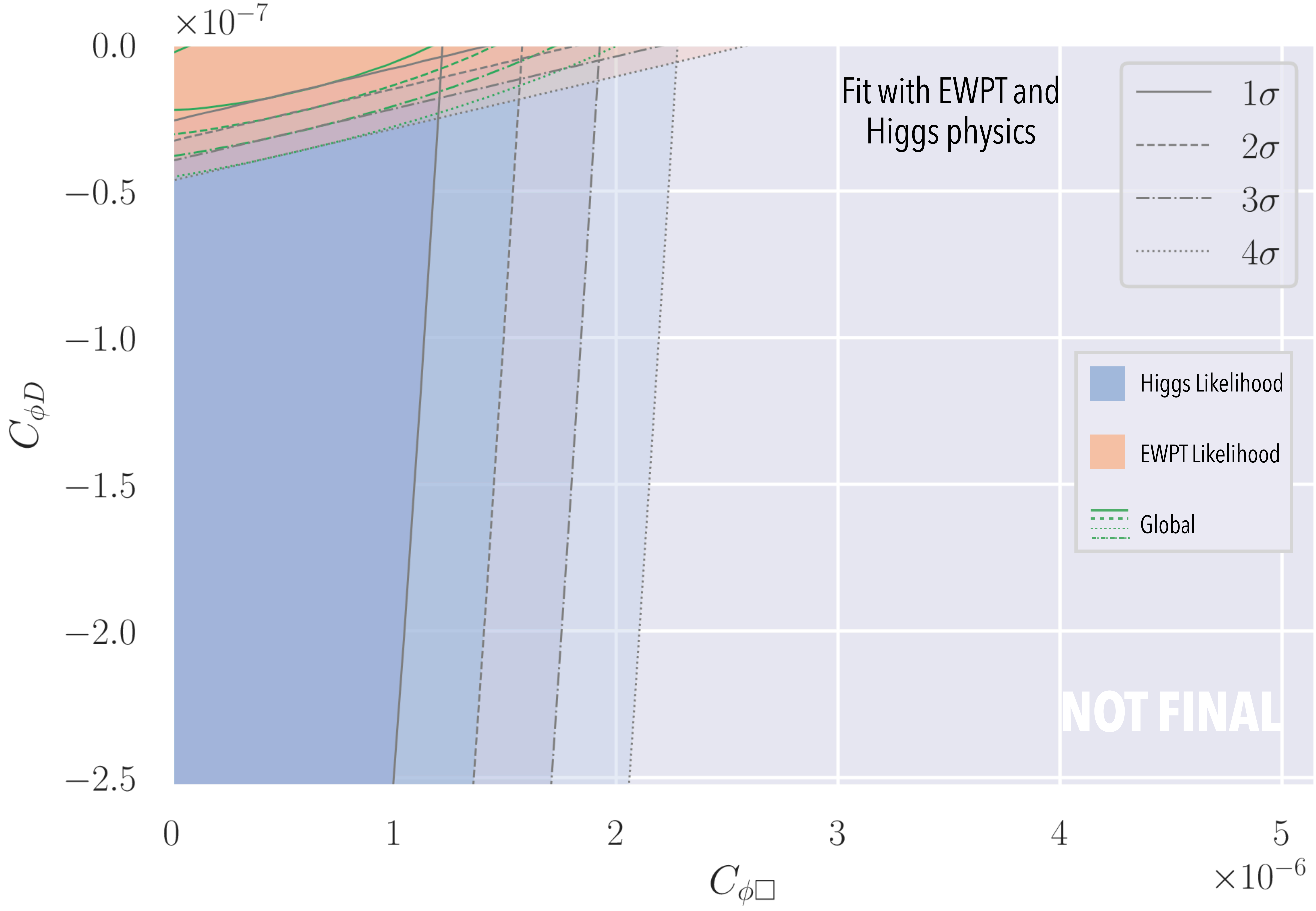


PHASE TRANSITIONS IN THE SMEFT

- With the power counting that allows a first order phase transition, it is still possible to stay within reasonable pheno. in other sectors.

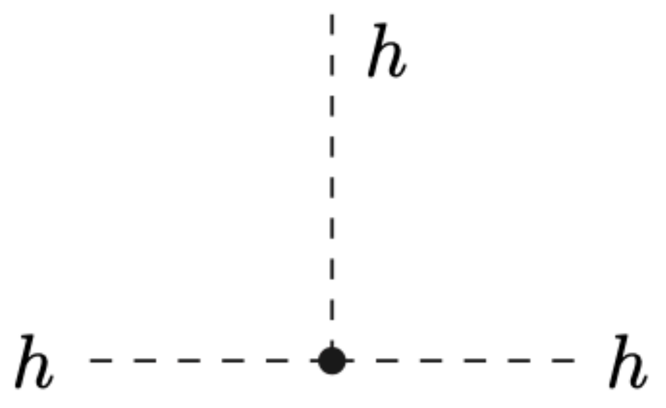
- We looked (so far) at Higgs physics and EWPT.

- We "turned on" the minimal set of WCs!



OUTLOOK: DI-HIGGS AS A PROBE?

- In the SMEFT not only the Higgs mass is affected. The Higgs self-couplings (among many other things) change as well.
- With the power counting (and thus values for WCs and couplings) we need for first order PT, what happens to e.g. Di-Higgs production?



$$
\begin{aligned}
&-3i\lambda v + 15 i v^3 C^{\varphi} \\
&\quad - ivC^{\varphi\Box}\left(3p_1\cdot p_1 + 2p_1\cdot p_2 + 2p_1\cdot p_3 + 3p_2\cdot p_2 + 2p_2\cdot p_3 + 3p_3\cdot p_3 + 9\lambda v^2\right) \\
&\quad + \frac{iv}{4}C^{\varphi D}\left(9\lambda v^2 - 4(p_1\cdot p_2 + p_1\cdot p_3 + p_2\cdot p_3)\right)
\end{aligned}
$$

We are thinking about it…

CONCLUSIONS

- First order phase transitions are interesting and important for pheno.
- In the SM alone, not possible to have this.
- In the SMEFT? It seems like it is. After proper power counting and resummation, that is.
- The scenario can be probed with other experiments, not just Cosmo.
- Could be that EWSB comes as a step in more complicated symmetry breaking though!