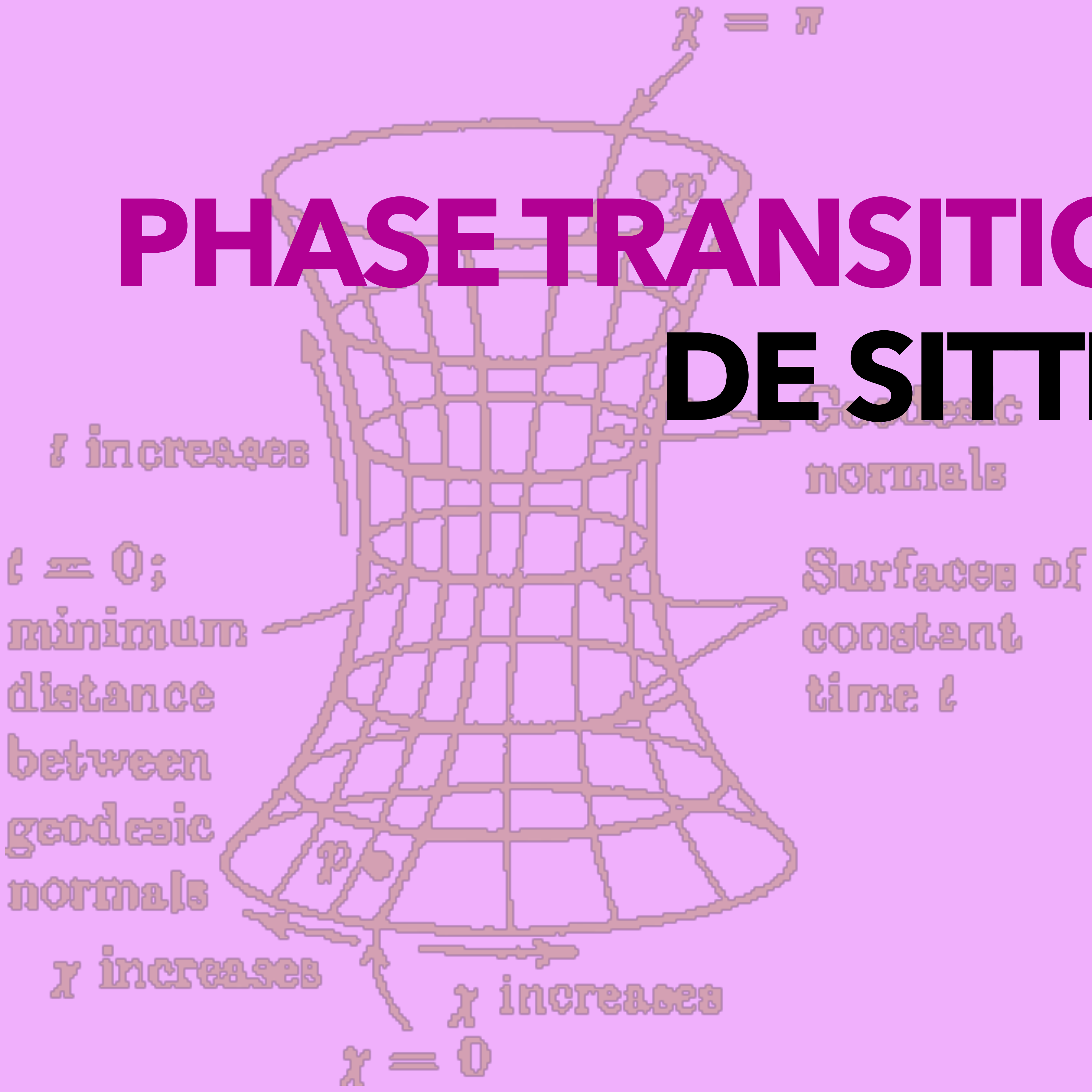
UPPSALA UNIVERSITET

Imperial College London

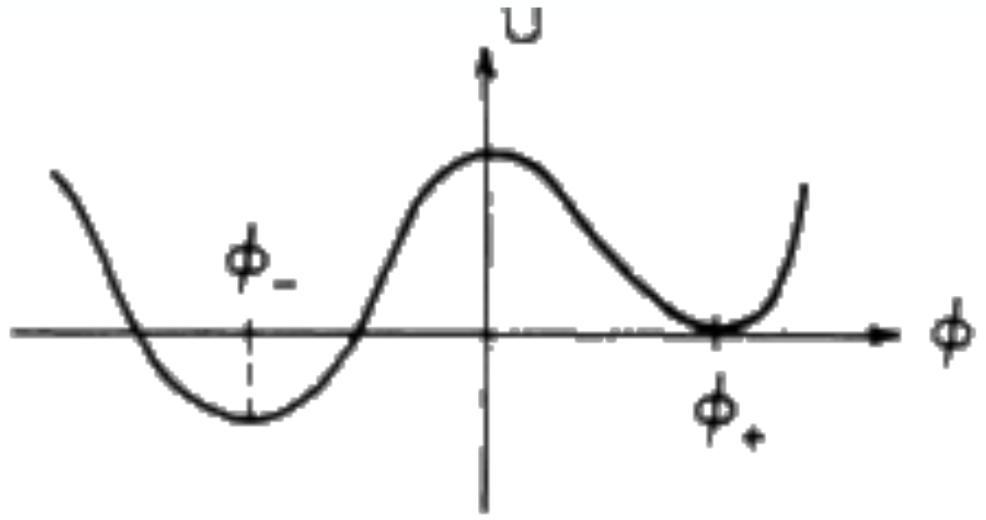
PHASE TRANSITION CONFIGURATIONS IN DE SITTER ARE O(4) SYMMETRIC



Eliel Camargo-Molina
Marco Guaraco

FALSE VACUUM QUICK RECAP

$$\langle x_f | e^{-H\tau_0} | x_i \rangle = N \int [Dx] e^{-S_E}$$



In QFT, transition processes are possible from higher to lower vacua of the potential energy. The energy functional $U[\varphi(x)] = \int d^3x \left[\frac{1}{2}(\nabla\varphi)^2 + V(\varphi)\right]$ admits extrema that are not constant. They describe vacuum decay. They satisfy:

$$-\nabla_\mu \nabla^\mu \varphi + V'(\varphi) = 0.$$

$$\ddot{\varphi} + \frac{3}{r}\dot{\varphi} - V'(\varphi) = 0,$$

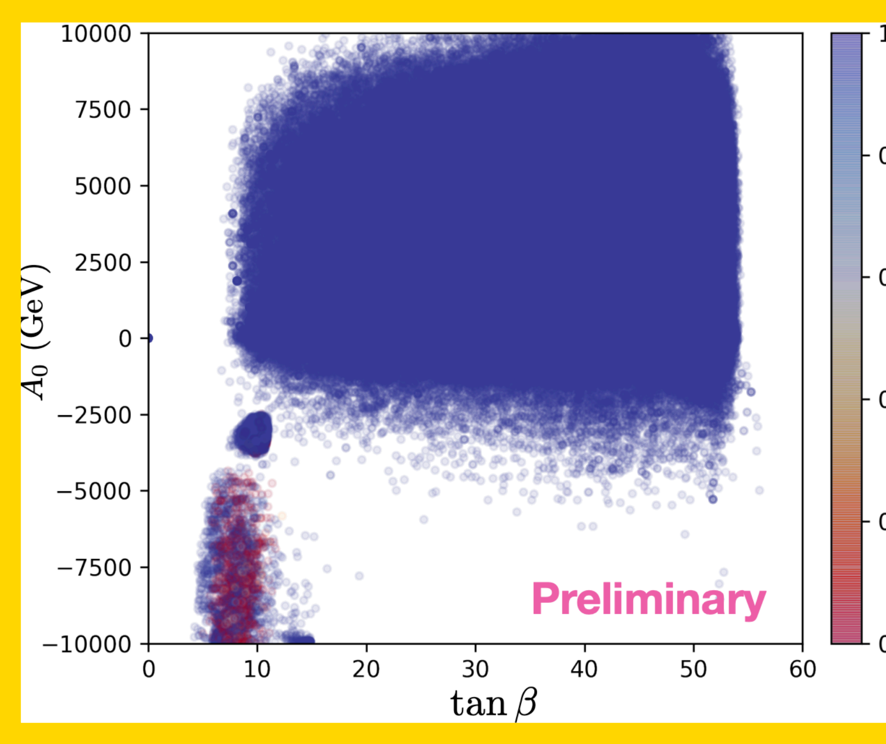
Action Minima among Solutions to a Class of Euclidean Scalar Field Equations

S. Coleman*, V. Glaser, and A. Martin
CERN, CH-Geneva, Switzerland

1978

proved that in R^4 such solutions are bubbles.

JECM et al.



Then decay width is then easy to write down (though hard to calculate):

$$\Gamma = A \exp(-B), \quad A = \left(\frac{B}{2\pi}\right)^2 \left|\frac{\det'(S''[\varphi_B])}{\det(S''[\varphi_{fv}])}\right|^{-\frac{1}{2}}, \qquad B = S[\varphi_B] - S[\varphi_{fv}]$$

DE SITTER

de Sitter space is the maximally symmetric vacuum solution of Einstein's field equations with a positive cosmological constant

Same basic framework, now with curvature

$$ds^2 = d\chi^2 + a^2(\chi) d\Omega_3^2,$$

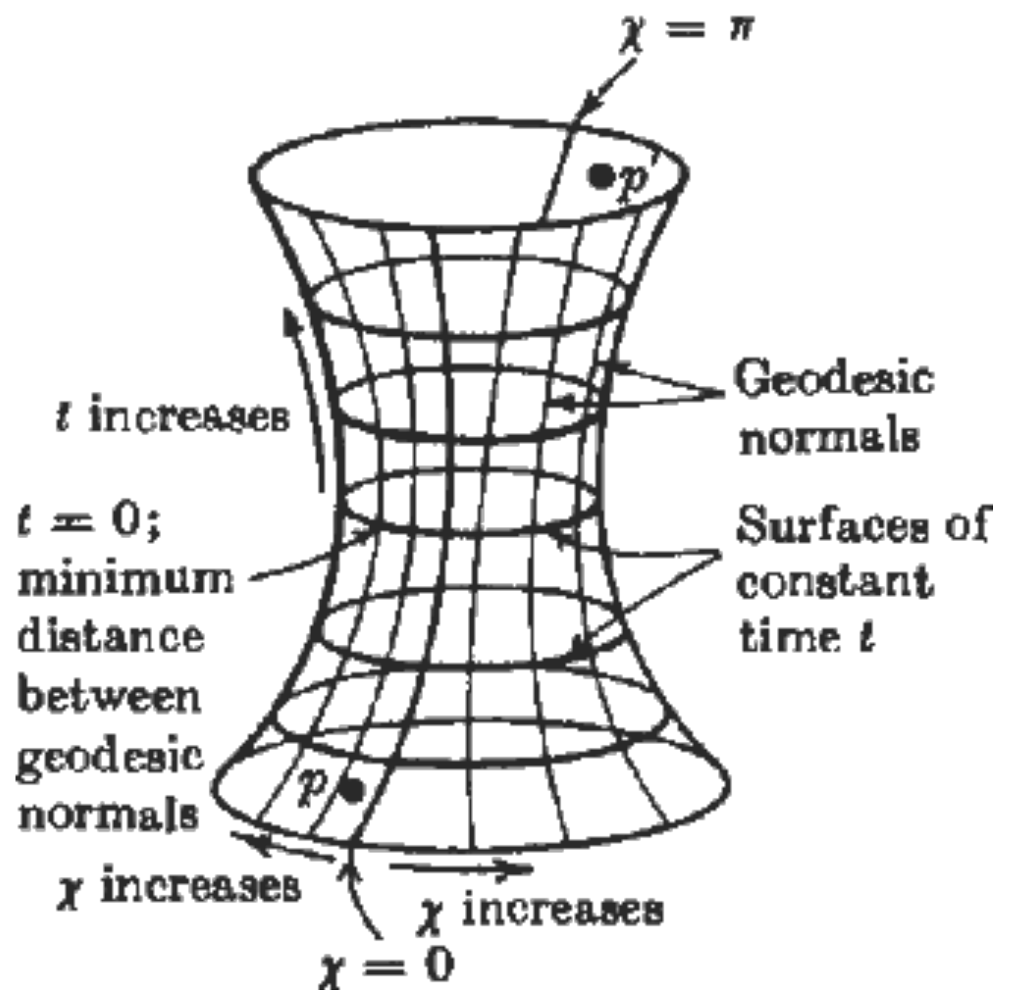
$$-\nabla_\mu \nabla^\mu \varphi + V'(\varphi) = 0.$$

?

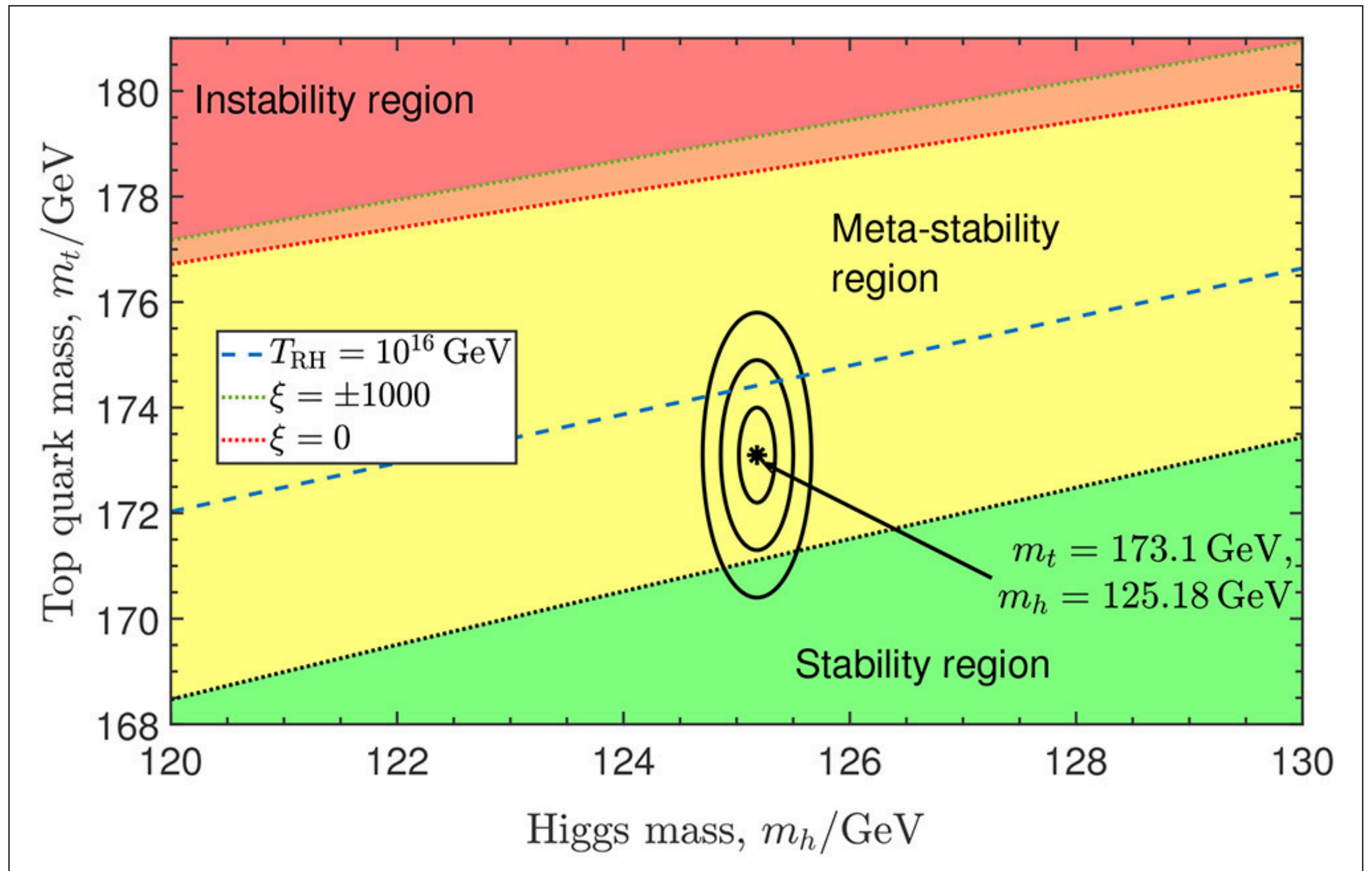
O(4) symmetry assumed

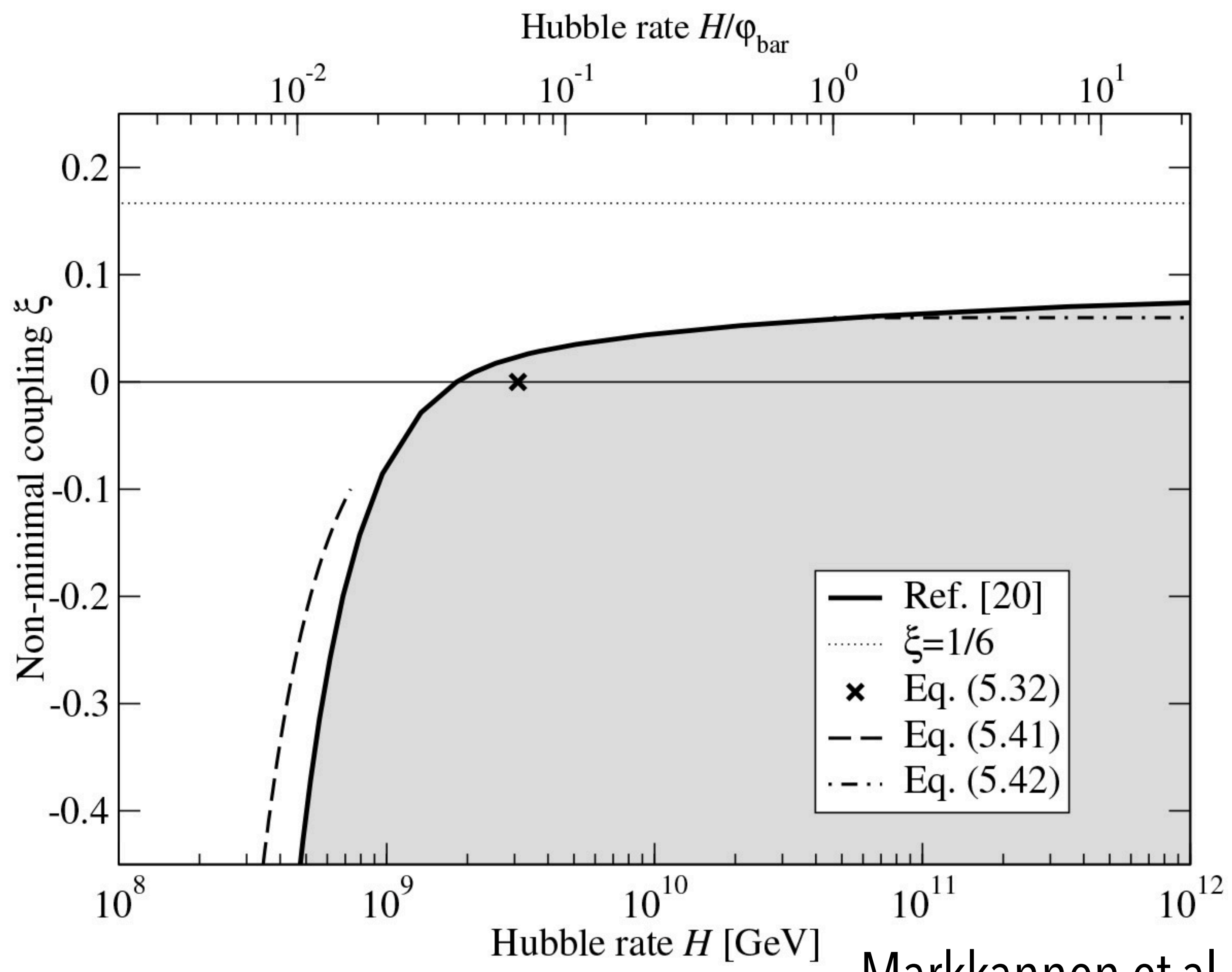
$$\ddot{\varphi} + \frac{3\dot{a}}{a}\dot{\varphi} - V'(\varphi) = 0$$

$$\dot{a}^2 = 1 - \frac{a^2}{3M_{\rm P}^2}\left(-\frac{\dot{\varphi}^2}{2} + V(\varphi)\right).$$



Interesting formally, but also to understand the vacuum dynamics of the SM (and BSM) in the early universe.





Markkannen et al.

THE PROOF IN ONE MINUTE

Turns out mathematicians have been thinking about those solutions too (in the context of minimal surfaces):

We study solutions $u : \Omega \to \mathbb{R}$ to the semilinear elliptic equation

$$\begin{cases} \Delta u - f(u) = 0 & \text{in} \quad \Omega \\ \partial_\nu u = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{1}$$

Solutions to equation (1) are critical points of the *energy functional*

$$E(u) = \int_\Omega \frac{|\nabla u|^2}{2} + F(u), \quad u \in W^{1,2}(\Omega). \tag{2}$$

GROUND STATES OF SEMILINEAR ELLIPTIC EQUATIONS

RAYSSA CAJU, PEDRO GASPAR, MARCO A. M. GUARACO AND HENRIK MATTHIESEN

ABSTRACT. We study solutions of $\Delta u - F'(u) = 0$, where the potential F can have an arbitrary number of wells at arbitrary heights, including bottomless wells with subcritical decay. In our setting, *ground state* solutions correspond to unstable solutions of least energy. We show that in convex domains of $\mathbb{R}^N$ and manifolds with Ric ≥ 0, ground states are always of mountain-pass type and have Morse index 1. In addition, we prove symmetry of the ground states if the domain is either an Euclidean ball or the entire sphere S^N. For the Allen-Cahn equation $\varepsilon^2\Delta u - W'(u) = 0$ on S^N, we prove the ground state is unique up to rotations and corresponds to the equator as a minimal hypersurface. We also study bifurcation at the energy level of the ground state as $\varepsilon \to 0$, showing that the first $N+1$ min-max Allen-Cahn widths of S^N are ground states, and we prove a gap theorem for the corresponding $(N+2)$-th min-max solution.

Their ideas live in the classical part of the physics calc.

$$\Gamma = A \exp(-B), \quad A = \left(\frac{B}{2\pi}\right)^2 \left|\frac{\det'(S''[\varphi_B])}{\det(S''[\varphi_{fv}])}\right|^{-\frac{1}{2}},$$

Using techniques from the theory of minimal surfaces (and some old theorems), one can show that:

A) Non-constant extremal configs of lowest action are saddle points of the action (not the potential) with one negative eigenvalue.

B) If a saddle point is not O(4) symmetric then it is possible to construct a saddle point with strictly lower action which is O(4) symmetric.

Thinking about this raised a lot of interesting questions about the quantum side of things.

Wick rotation? Euclidean path integral? WKB?

A lot to think about.

It looks like we'll end up proving that physical intuition was right. But it needs to be done, new insights might be hiding within!

Collaboration with pure math is not just useful in mathematical physics

Also useful to tackle phenomenology problems

The translation is not trivial, but it is also a lot of fun and you'll end up understanding the underlying physics under a different light