Radiative corrections for the decay $\Sigma^0 \to \Lambda e^+ e^-$

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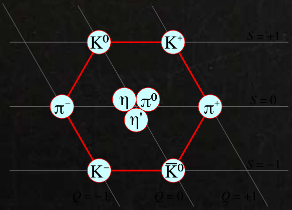
Introduction

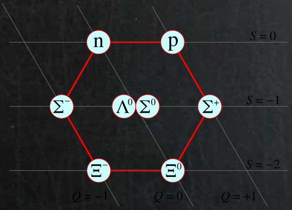
How are the building blocks distributed inside of the composite objects (nucleons)?

- electron–nucleon scattering
    - electromagnetic form factors
      $\hookrightarrow$ used to explore intrinsic structure of nucleons
    - low-energy quantities
      $\hookrightarrow$ electric charge, magnetic moment, electric and magnetic radii

Replacement of down quarks (of a nucleon/Δ) by strange quark(s)?

- played role in revealing quarks as building blocks of nucleons and hadrons in general
- close relation among the intrinsic structures of hyperons and nucleons
    - hyperon electromagnetic and transition form factors contain complementary information to the nucleon (and Δ) ones





Experimental knowledge of hyperons rather limited

- electric charges and magnetic moments known
- unstable $\longrightarrow$ electron scattering rather difficult

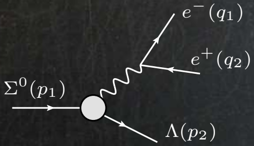
Form factors

- high energies
  - e^+e^- scattering to hyperon + antihyperon
    $\hookrightarrow$ both direct and transition form factors accessible
- low energies
  - Dalitz decay $Y \to Y' e^+ e^-$
  - possibly high statistics in future at FAIR (PANDA: $p\bar{p}$, HADES: pp)

Dalitz decays in baryon-octet sector?

Dalitz decay $\Sigma^0 \to \Lambda e^+ e^-$

- e^+e^- invariant mass only up to $M_\Sigma - M_\Lambda \simeq 77\,\text{MeV}$
- provides electric and magnetic transition form factors of $\Sigma^0 \to \Lambda$ transition
- extracting transition radii challenging
  - high-precision measurement required
  - competing with QED radiative corrections



Predictions of electric and magnetic radii

- *Kubis and Meissner*, EPJC 18 (2001)
- *Granados, Leupold and Perotti*, EPJA 53 (2017)

$\Sigma^0\Lambda\gamma$ vertex: $\langle 0|j^\mu|\Sigma^0\bar{\Lambda}\rangle = e\bar{v}_\Lambda(\vec{p}_2)\, G^\mu(p_1+p_2)\, u_\Sigma(\vec{p}_1)$, with

$$G^\mu(q) \equiv \left[\gamma^\mu - (M_\Sigma - M_\Lambda)\frac{q^\mu}{q^2}\right]G_1(q^2) - \frac{i\sigma^{\mu\nu}q_\nu}{M_\Sigma + M_\Lambda}G_2(q^2)$$

Define magnetic and electric form factors

$$G_\mathsf{M}(q^2) \equiv G_1(q^2) + G_2(q^2) = \kappa\left(1 + \frac{1}{6}\langle r_\mathsf{M}^2\rangle q^2 + \mathcal{O}(q^4)\right)$$

$$G_\mathsf{E}(q^2) \equiv G_1(q^2) + \frac{q^2}{(M_\Sigma + M_\Lambda)^2}G_2(q^2) = \frac{1}{6}\langle r_\mathsf{E}^2\rangle q^2 + \mathcal{O}(q^4)$$

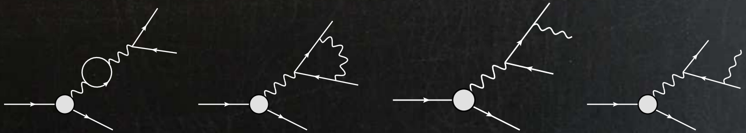
Matrix element squared dominated by the magnetic part

$$\overline{|\mathcal{M}^{\mathsf{LO}}(x,y)|^2} \simeq 2e^4|G_\mathsf{M}(\Delta_M^2 x)|^2\,\frac{(1-x)}{x}\left(1 + y^2 + \frac{\nu^2}{x}\right)$$

$$x \equiv \frac{(p_{e^+} + p_{e^-})^2}{(M_{\Sigma^0} - M_\Lambda)^2}\,, \quad y \equiv \frac{2\,p_{\Sigma^0}\cdot(p_{e^+} - p_{e^-})}{\lambda^{\frac{1}{2}}(p_{\Sigma^0}^2, p_\Lambda^2, (p_{e^+} + p_{e^-})^2)}$$

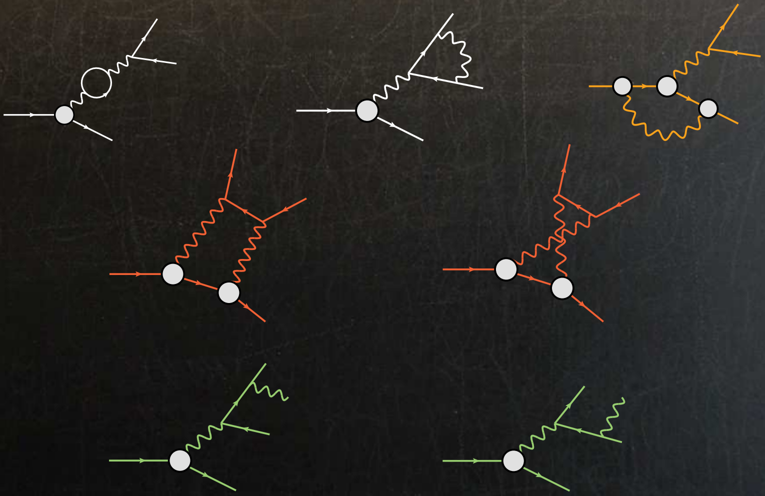
Radiative corrections to the differential decay width in soft-photon approximation

- *Sidhu and Smith*, PRD 4 3344 (1971)
  - no hard-photon corrections, low-x region not covered
  - resulting corrections negative all over the Dalitz plot
    $\hookrightarrow$ total correction known to be positive



TH and Leupold, EPJC 80 (2020), arXiv:1911.02571

- inclusive radiative corrections beyond the soft-photon approximation



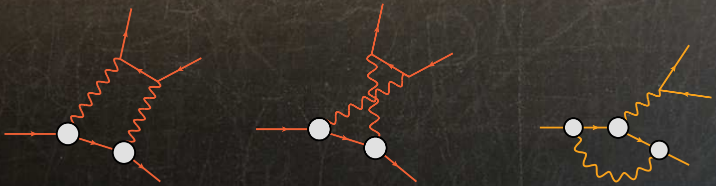
Low-energy expansion of the form factors:

$$G_{\mathsf{M}}\left((k+q_1+q_2)^2\right) \simeq G_{\mathsf{M}}\left((q_1+q_2)^2\right)\left\{1+\frac{1}{6}\langle r_{\mathsf{M}}^2\rangle[2k\cdot(q_1+q_2)]\right\}$$

$$G_{\mathsf{E}}\left((k+q_1+q_2)^2\right) \simeq G_{\mathsf{E}}\left((q_1+q_2)^2\right)\left\{1+\frac{2k\cdot(q_1+q_2)}{(q_1+q_2)^2}\right\}$$

Subsequently, integrate over the energy and emission angle of the bremsstrahlung photon

- radiative corrections for inclusive process



By loop-momenta-power counting, FFs required to regulate the UV region

1γIR: UV convergence already achieved in the simplest case with constant FFs

- $G_{\mathsf{E}}(q^2) = G_{\mathsf{E}}(0) = 0$ and $G_{\mathsf{M}}(q^2) = G_{\mathsf{M}}(0) = \kappa$

$$G_1(q^2) = \kappa \frac{q^2}{q^2 - M_V^2}\,, \quad G_2(q^2) = -\kappa \frac{M_V^2}{q^2 - M_V^2}$$

Ansatz satisfying high-energy constraints

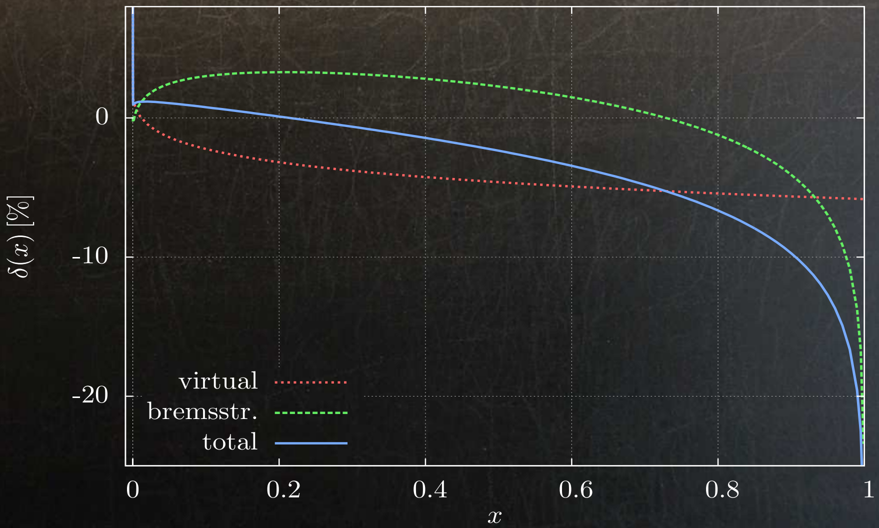
$$G_1(q^2) = \kappa \left(3 - \frac{M_V^2 \langle r_{\mathsf{M}}^2 \rangle}{6}\right) \frac{q^2 M_V^4}{(q^2 - M_V^2)^3}\,, \quad G_2(q^2) = -\kappa \frac{M_V^6}{(q^2 - M_V^2)^3}$$

These contributions to the NLO decay width are found to be negligible

Results

Corrections to two-fold differential decay width $\delta(x, y)$ given in %

x \ y	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
0.01	2.49	2.43	2.30	2.16	2.02	1.85	1.63	1.32	0.80	-0.24	-8.33
0.02	2.65	2.59	2.47	2.32	2.16	1.97	1.72	1.37	0.80	-0.33	-6.00
0.03	2.69	2.64	2.52	2.38	2.21	2.01	1.75	1.37	0.77	-0.42	-5.96
0.04	2.68	2.64	2.53	2.39	2.22	2.01	1.74	1.35	0.73	-0.51	-6.08
0.05	2.65	2.61	2.51	2.37	2.21	2.00	1.72	1.32	0.68	-0.59	-6.24
0.06	2.61	2.57	2.48	2.35	2.19	1.98	1.70	1.29	0.63	-0.67	-6.41
0.07	2.56	2.53	2.44	2.31	2.15	1.95	1.66	1.25	0.58	-0.75	-6.58
0.08	2.50	2.47	2.39	2.27	2.12	1.91	1.63	1.21	0.53	-0.83	-6.75
0.09	2.44	2.42	2.34	2.22	2.07	1.87	1.59	1.16	0.47	-0.90	-6.91
0.10	2.38	2.35	2.28	2.17	2.03	1.83	1.54	1.12	0.42	-0.98	-7.07
0.15	2.04	2.02	1.97	1.89	1.76	1.57	1.30	0.86	0.14	-1.35	-7.81
0.20	1.67	1.66	1.63	1.56	1.45	1.29	1.02	0.58	-0.17	-1.71	-8.49
0.25	1.29	1.28	1.26	1.21	1.12	0.97	0.71	0.28	-0.48	-2.09	-9.13
0.30	0.89	0.89	0.88	0.84	0.77	0.64	0.39	-0.04	-0.82	-2.47	-9.76
0.35	0.47	0.47	0.47	0.45	0.40	0.28	0.04	-0.39	-1.17	-2.88	-10.4
0.40	0.03	0.03	0.04	0.04	0.00	-0.11	-0.34	-0.76	-1.56	-3.30	-11.0
0.45	-0.44	-0.43	-0.42	-0.41	-0.43	-0.53	-0.74	-1.16	-1.97	-3.75	-11.6
0.50	-0.94	-0.93	-0.90	-0.88	-0.89	-0.97	-1.18	-1.59	-2.41	-4.24	-12.3
0.55	-1.48	-1.47	-1.43	-1.40	-1.40	-1.47	-1.66	-2.07	-2.90	-4.77	-12.9
0.60	-2.07	-2.06	-2.02	-1.97	-1.95	-2.01	-2.19	-2.60	-3.44	-5.35	-13.6
0.65	-2.73	-2.71	-2.66	-2.61	-2.58	-2.62	-2.79	-3.20	-4.05	-6.00	-14.4
0.70	-3.48	-3.46	-3.40	-3.33	-3.29	-3.31	-3.48	-3.89	-4.75	-6.73	-15.3
0.75	-4.34	-4.32	-4.25	-4.17	-4.11	-4.13	-4.29	-4.69	-5.57	-7.59	-16.2
0.80	-5.38	-5.35	-5.28	-5.19	-5.11	-5.12	-5.27	-5.67	-6.56	-8.62	-17.4
0.85	-6.69	-6.66	-6.58	-6.47	-6.39	-6.38	-6.52	-6.93	-7.83	-9.92	-18.8
0.90	-8.50	-8.47	-8.38	-8.26	-8.16	-8.14	-8.27	-8.68	-9.60	-11.7	-20.7
0.95	-11.5	-11.5	-11.4	-11.3	-11.1	-11.1	-11.2	-11.7	-12.6	-14.7	-23.8
0.99	-18.4	-18.4	-18.2	-18.1	-18.0	-17.9	-18.1	-18.5	-19.4	-21.6	-30.7


0
-10
-20
0
0.2
0.4
0.6
0.8
1
x
$\delta(x)$ [%]
virtual
bremsstr.
total

Integrate over the Dalitz plot (values from *Kubis and Meissner*, EPJC 18 (2001))

$$R \equiv \frac{\Gamma(\Sigma^0 \to \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \to \Lambda \gamma)} = 5.541(2) \times 10^{-3}$$

$\hookrightarrow$ neglect electric form factor $\to$ expansion in magnetic form-factor slope $a \equiv \frac{1}{6}\langle r_{\mathrm{M}}^2 \rangle \Delta_M^2$

$$R = R_0 + a R_1 + \mathcal{O}(a^2)$$

$+$ higher-order corrections as additional uncertainty

$$R = [5.530(3) + 0.626(2)a] \times 10^{-3}$$

$\hookrightarrow$ consistent with the NLO result in *Sidhu and Smith*, PRD 4 3344 (1971)

$$R_{\mathrm{S\&S}} = (5.532 + 0.627a) \times 10^{-3} \,[\approx 5.544 \times 10^{-3}\,]$$

$$\Downarrow$$

Using present values for physical constants (taking NLO expressions from *Sidhu and Smith* (1971))

$$R_{\mathrm{S\&S}}^{\mathrm{new}} = (5.52975 + 0.62640a) \times 10^{-3}$$

Compares well with our result $R = (5.52974 + 0.62640a) \times 10^{-3}$ (only electron loop in VP)

	LO	virt	BS|C	BS|D	total
$R_0\,[10^{-3}]$	5.4838	−0.01667	−0.06443	0.12713	5.5298
$R_1\,[10^{-3}]$	0.6189	−0.02006	0.00010	0.02747	0.6264
$\delta\,[\%]$	—	−0.310	−1.173	2.322	0.839

From the constraint $\mathcal{B}(\Sigma^0 \to \Lambda\gamma) + \mathcal{B}(\Sigma^0 \to \Lambda e^+e^-) + \mathcal{B}(\Sigma^0 \to \Lambda\gamma\gamma) \simeq 1$ we find

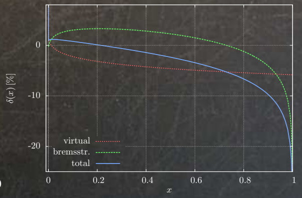
$$\mathcal{B}(\Sigma^0 \to \Lambda\gamma) \simeq \frac{1}{1+R} = [99.4501(3) - 0.0619(2)a]\,\%$$

$$\mathcal{B}(\Sigma^0 \to \Lambda e^+e^-) \simeq \frac{R}{1+R} = [0.5499(3) + 0.0619(2)a]\,\%$$

Using conservative $a = 0.02(2)$

$$\mathcal{B}(\Sigma^0 \to \Lambda\gamma) = 99.449(2)\,\%$$

$$\mathcal{B}(\Sigma^0 \to \Lambda e^+e^-) = 0.551(2)\,\%$$



Estimate of size of correction to the magnetic form-factor slope a

- take half of the slope of the curve in the low-x region
- farther from the threshold ($\nu^2 \ll x_0 \ll 1$):

$$\Delta a \equiv a_{(+\mathrm{QED})} - a \simeq \frac{1}{2}\frac{\mathrm{d}\delta(x)}{\mathrm{d}x}\bigg|_{x=x_0}$$

$a_{(+\mathrm{QED})}$: measured value implicitly containing the QED radiative correction

$$\frac{1}{2}\frac{\mathrm{d}\delta(x)}{\mathrm{d}x}\bigg|_{x=x_0} \approx -3.5\,\%$$

- bigger than the estimate on the slope $a \equiv \frac{1}{6}\langle r_\mathrm{M}^2\rangle\Delta_M^2$ itself ($a \approx 1.8(3)\,\%$)

Using no radiative corrections in the experimental analysis

- one expects "measured" radius $\langle r_\mathrm{M}^2\rangle_{(+\mathrm{QED})}$ to be negative

$$\langle r_\mathrm{M}^2\rangle_{(+\mathrm{QED})} = \langle r_\mathrm{M}^2\rangle + \frac{6}{\Delta_M^2}\Delta a\,, \quad \text{with } \frac{6}{\Delta_M^2}\Delta a \approx -35\,\mathrm{GeV}^{-2}$$

$(\chi\mathrm{PT}:\ \langle r_\mathrm{M}^2\rangle = 18.5(2.6)\,\mathrm{GeV}^{-2})$; in general for hadronic radii: $\langle r^2\rangle \leq (1\,\mathrm{fm})^2 \approx 25\,\mathrm{GeV}^{-2}$

Summary

We calculated complete inclusive NLO QED radiative correction to the differential decay width
$\hookrightarrow \Sigma^0 \to (\Lambda e^+ e^- +$ arbitrary many photons) relative to QED LO calculation of $\Sigma^0 \to \Lambda e^+ e^-$

In particular

- lepton bremsstrahlung beyond the soft-photon approximation
- explicit calculation of two-photon-exchange (1γIR) contrb. and correction to $\Sigma^0 \Lambda \gamma$ vertex
- radiative corrections that involve other hadronic form factors can be safely neglected
  $\hookrightarrow$ model-independent results in terms of a single hadronic parameter a

Precise and conservative predictions for $\mathcal{B}(\Sigma^0 \to \Lambda\gamma)$ and $\mathcal{B}(\Sigma^0 \to \Lambda e^+ e^-)$

Estimate on the correction to the magnetic form-factor slope: $\Delta a \approx -3.5\,\%$

TH and S. Leupold, EPJC 80 (2020), arXiv:1911.02571

Thank you for listening!