SHORT DISTANCE CONTRIBUTIONS TO THE MUON $g-2$

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Vetenskapsrådet

Partikeldagarna 2020 Uppsala, Lund,... 23-25 November 2020

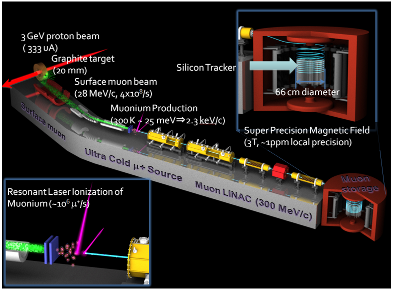
Introduction

- Magnetic moment: $\vec{\mu} = g\frac{q}{2m}\vec{S}$
- For angular momentum: $g = 1$
- Dirac equation: $g = 2$
- Structure and/or QFT give different values
- Anomaly: $a = \frac{g-2}{2}$
- QED (Schwinger): $a = 1 + \frac{\alpha}{2\pi} + \dots$

Jacob Bourjaily/Wikipedia

Why do we do this?

The muon $a_\mu = \frac{g_\mu - 2}{2}$ will be measured more precisely



J-PARC

Fermilab

Introduction

- Experiments done for electron and muon: very precise
- $a_e^{\exp} = 115965218.073(0.028)\ 10^{-11}$ Harvard
- Discrepancy 2.4σ: α from Cs-atom interferometry or a_e
  $\alpha^{-1} = 137.0359990460(270)(Cs)$
  $\alpha^{-1} = 137.0359991496(330)(a_e)$
- Both errors dominated by experiment
  (and largely systematic)
- Difference not relevant for *QED* part of a_μ

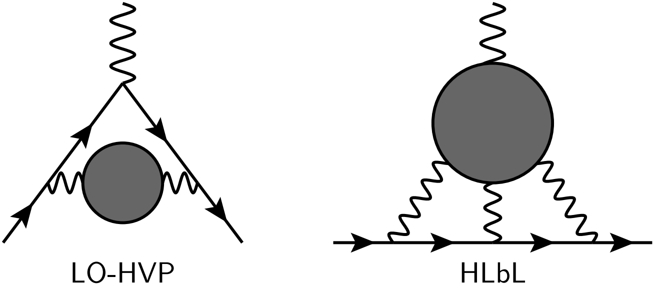
Introduction

- Experiments done for electron and muon: very precise
- $a_e^{\rm exp} = 115965218.073(0.028)\ 10^{-11}$ Harvard
- $a_\mu^{\rm exp} = 116592089(54)(33)\ 10^{-11}$ BNL
- $a_\mu^{\rm the} = 116591810(43)\ 10^{-11}$ White paper
- Discrepancy 3.7σ: $279(76)\ 10^{-11}$
- White paper: arXiv: 2006.04822 Phys. Rep. 887 (220) 1-166
  JB,N.Hermansson-Truedsson,S.Leupold,A.Rodríguez-Sánchez (132 authors)
  Large theory collaboration, error: consensus
  Numbers (except the new ones here) taken from there
- Impressive agreement theory experiment
  about 13 digits in a_e and about 9 in a_μ

Introduction

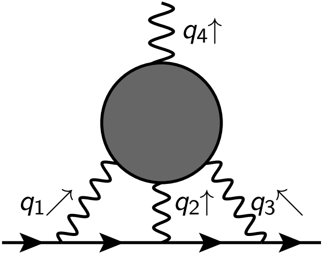
- $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$
- $a_\mu^{QED} = 116584718.931(104)\ 10^{-11}$
  up to 4 loops essentially analytically, 5-loops numerically, 6-loops estimate (main uncertainty)
- $a_\mu^{EW} = 153.6(1.0)\ 10^{-11}$
  Done to two-loop, main uncertainty from long-distance hadronic contributions (via anomaly)
- $a_\mu^{\exp} - a_\mu^{QED} - a_\mu^{EW} = 7216(63)\ 10^{-11}$

Hadronic contributions



- Muon and photon lines, representative diagrams
- The blobs are hadronic contributions
- There are higher order contributions of both types: known accurately enough
- $a_\mu^{HVP} = 6845(40)\ 10^{-11}$ (LO+NLO+NNLO)
- $a_\mu^{HLbL} = 92(18)\ 10^{-11}$ (LO+NLO)

HLbL: the main object to calculate

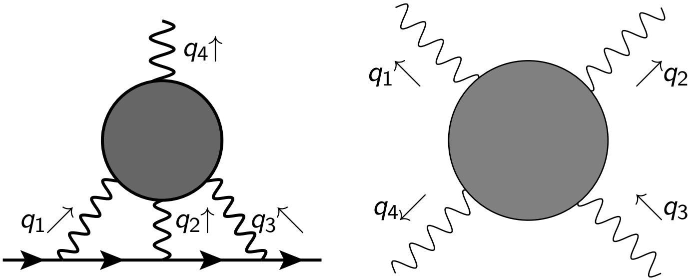


- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- q_4 always at zero
- Double counting needs to be avoided: hadron exchanges versus quarks

Contributions

- "Long distance": under good control
  - Dispersive method: Berne group around G. Colangelo
  - π^0 (and η, η') pole: 93.8(4.0) 10^{-11} S.Leupold
  - Pion and kaon box (pure): $-16.4(2)$ 10^{-11}
  - $\pi\pi$-rescattering (include scalars below 1 GeV): $-8(1)$ 10^{-11}
- Charm (beauty, top) loop: 3(1) 10^{-11}
- "Short and medium distance"
  - Axial vector: 6(6) 10^{-11}
  - Short-distance: 15(10) 10^{-11}
- Clearly the last item needs improvement

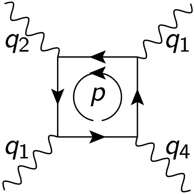
Short-distance



- $=\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$
- Actually we really need $\left.\dfrac{\delta\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)}{\delta q_{4\rho}}\right|_{q_4=0}$
- Mixed short-distance: q_4 at zero, q_1^2, q_2^2, q_3^2 large
- $q_i^2 = -Q_i^2$

Quarkloop

- Use (constituent) quark loop
- Used for full estimates since the beginning (1970s)
- Used for short-distance estimates with mass as a cut-off
  JB, Pallante, Prades, 1996
- We recalculated:



- In agreement with quarkloop formulae from
  Hoferichter, Stoffer, private communication
- In agreement with known numerics

Quarkloop: u, d, s

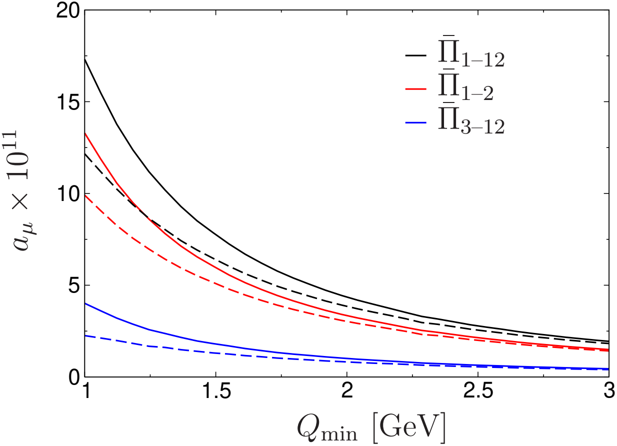


figure: Hoferichter

$Q_1, Q_2, Q_3 > Q_{\min}$

$M_Q = 0$: full

$M_Q = 0.3$ GeV dashed

$a_\mu^{\mathrm{HLbLQ}} = 54 \times 10^{-11}$

- M_Q provides an infrared cut-off, $M_Q \to 0$ divergent
- About 12×10^{-11} from above 1 GeV for $M_Q = 0.3$ GeV
- About 17×10^{-11} from above 1 GeV for $M_Q = 0$

Quarkloop

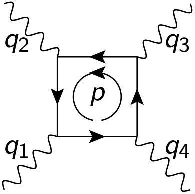
- Is it a first term in a systematic OPE?
- OPE has been used as constraints on specific contributions
  - $\pi^0\gamma^*\gamma^*$ asymptotic behaviour
  - Constraints on many other hadronic formfactors
  - $Q_1^2 \approx Q_2^2 \gg Q_3^2$ Melnikhov, Vainshtein 2003
- JB, N. Hermansson-Tuedsson, A. Rodríguez-Sánchez,
  Phys.Lett. B798 (2019) 134994[arxiv:1908.03331]
  +Laub, JHEP 10 (2020) 203 [arxiv:2008.13487]
  +to be published (soon?)

Short-distance: first attempt

$$\Pi^{\mu\nu\lambda\sigma} = -i \int d^4x d^4y d^4z e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)} \left\langle T\left(j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\right)\right\rangle$$

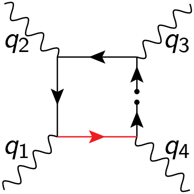
- Usual OPE: x, y, z all small
- First term in the expansion is the quark-loop
  no problem with $\partial/\partial q_4^\rho$ and $q_4 \to 0$

  p in loop $\Rightarrow$ no singular propagators:



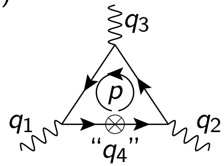
- Next term problems: no loop momentum;

  $q_4 \to 0$ propagator diverges:



Short-distance: correctly

- Similar problem in QCD sum rules for electromagnetic radii and magnetic moments
- Ioffe, Smilga, Balitsky, Yung, 1983
- For the q_4-leg use a constant background field and do the OPE in the presence of that constant background field
- Use radial gauge: $A_4^\lambda(w) = \frac{1}{2} w_\mu F^{\mu\lambda}$
  whole calculation is immediately with $q_4 = 0$.
- First term is exactly the usual quark loop
  (quark masses: next order)



Short-distance: next term(s)

- Do the usual QCD sum rule expansion in terms of vacuum condensates
- There are new condensates, induced by the constant magnetic field: $\langle \bar{q}\sigma_{\alpha\beta}q\rangle \equiv e_q F_{\alpha\beta} X_q$
- Lattice QCD Bali et al., arXiv:1206.4205
  $X_u = 40.7 \pm 1.3$ MeV,
- Only starts at $1/Q^2$ via $m_q X_q$ corrections to the leading quark-loop result
- X_q and m_q are very small, only a very small correction
- X_q: contain IR divergent perturbative parts, combine with the m_q^2 corrections from the quark-loop consistently
- Next order: very many condensates contribute, lots of IR mixing and redefinitions.

Short-distance: numerical results

Order	Contribution	$Q_{\min} = 1\,\mathrm{GeV}$	$Q_{\min} = 2\,\mathrm{GeV}$
$1/Q^2_{\min}$	quark-loop	$1.73 \cdot 10^{-10}$	$4.35 \cdot 10^{-11}$
$1/Q^4_{\min}$	quark-loop, m_q^2	$-5.7 \cdot 10^{-14}$	$-3.6 \cdot 10^{-15}$
	$X_{2,m}$	$-1.2 \cdot 10^{-12}$	$-7.3 \cdot 10^{-14}$
$1/Q^6_{\min}$	X_{2,m^3}	$6.4 \cdot 10^{-15}$	$1.0 \cdot 10^{-16}$
	X_3	$-3.0 \cdot 10^{-14}$	$-4.7 \cdot 10^{-16}$
	X_4	$3.3 \cdot 10^{-14}$	$5.3 \cdot 10^{-16}$
	X_5	$-1.8 \cdot 10^{-13}$	$-2.8 \cdot 10^{-15}$
	X_6	$1.3 \cdot 10^{-13}$	$2.0 \cdot 10^{-15}$
	X_7	$9.2 \cdot 10^{-13}$	$1.5 \cdot 10^{-14}$
	$X_{8,1}$	$3.0 \cdot 10^{-13}$	$4.7 \cdot 10^{-15}$
	$X_{8,2}$	$-1.3 \cdot 10^{-13}$	$-2.0 \cdot 10^{-15}$

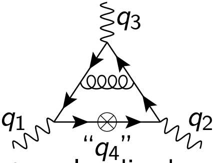
- $Q_1, Q_2, Q_3 \geq Q_{\min}$
- Nonperturbative short-distance corrections are small
- Suppression by small quark masses or small condensates
- Nonperturbative short-distance corrections are small

Short-distance: $1/Q^2_{\min}$

- Can we understand scaling with $Q_{\min}$?
- $a_\mu = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 dQ_2 Q_1^3 Q_2^3 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1,12} \hat{T}_i \overline{\Pi}_i$
- Do $Q_i \to \lambda Q_i$
- overall factor goes as λ^8
- Quark loop has no scale thus $\hat{\Pi}_i$ scale with their dimension $\hat{\Pi}_1, \hat{\Pi}_4 \sim \lambda^{-4}, \qquad \hat{\Pi}_7, \hat{\Pi}_{17}, \hat{\Pi}_{39}, \hat{\Pi}_{54} \sim \lambda^{-6}$
- $\Longrightarrow \overline{\Pi}_{1,\dots,4} \sim \lambda^{-4} \qquad \overline{\Pi}_{5,\dots,12} \sim \lambda^{-6}$
- Expand the T_i for $Q_i \gg m_\mu$: $T_1 \sim m_\mu^4$, $T_{i\neq 1} \sim m_\mu^2$ $T_1 \sim \lambda^{-8}$, $T_{2,3,4} \sim \lambda^{-6}$, $T_{5,\dots,12} \sim \lambda^{-4}$
- Put all together: quark-loop scales as $a_\mu^{\mathrm{SD\ ql}} \sim \lambda^{-2}$
- $m_q X_q$ adds an overall factor $\Longrightarrow a_\mu^{\mathrm{SD}X_q} \sim \lambda^{-4}$

Perturbative corrections

- Representative diagram:



- Use method of master integrals: disadvantage: large numerical cancellations between integrals
- All integrals are known
- Infrared and UV divergences in individual diagrams
- Dimensional regularization: $d = 4 - 2\epsilon$
- All $1/\epsilon^3, 1/\epsilon^2, 1/\epsilon^2$ cancel
- Several independent calculations that agree
- Find some typos in integral papers (I hate signs)

Perturbative corrections

- Preliminary results
- $a_\mu^{\text{HLbL SD gluonic}} = -1.7\ 10^{-11}$
- $Q_{\text{min}} = 1$ GeV, $\alpha_S = 0.33$
- Main uncertainty: how do we handle α_S
- No sign that it is very large (about -10%)

Conclusions

- We have shown that the massless quarkloop really is the first term of a proper OPE expansion for the HLbL
- We have shown how to properly go to higher orders
- We have calculated the next two terms in the OPE
  - NLO: suppressed by quark masses and a small X_q
  - NNLO: large number of induced condensates but all small
  - Numerically not relevant at the present precision
- Gluonic corrections about -10%
- Why do this: matching of the sum over hadronic contributions to the expected short distance domain
- Finding the onset of the asymptotic domain