Local dark matter density from the Milky Way's rotation curve using Gaia DR2 data

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Partikeldagarna – Linköping – 2nd October 2019

The presence of dark matter (DM)

Galaxy clusters

(Fritz Zwicky 1933)

Coma cluster

Image Credit: Russ Carroll, Robert Gendler, &
Bob Franke; Dan Zowada Memorial Observatory

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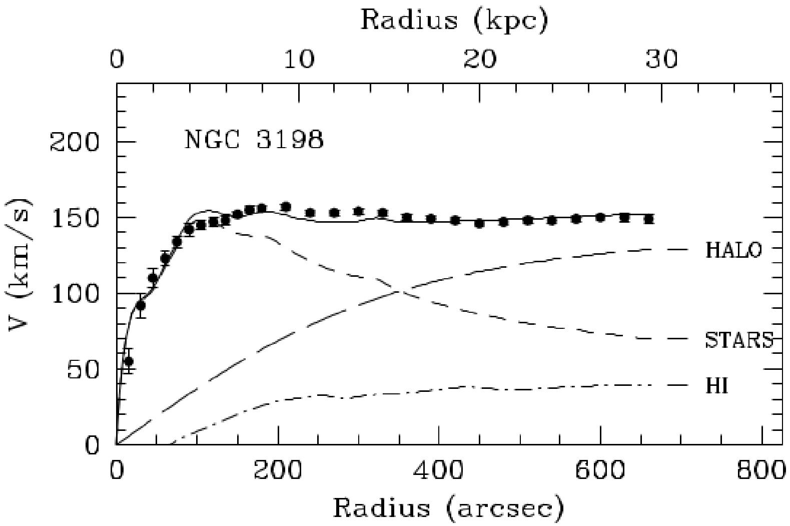
Rotation curves

(Vera Rubin)



S. Blais-Ouellette et al., Astron. J. 118 (1999) 2123

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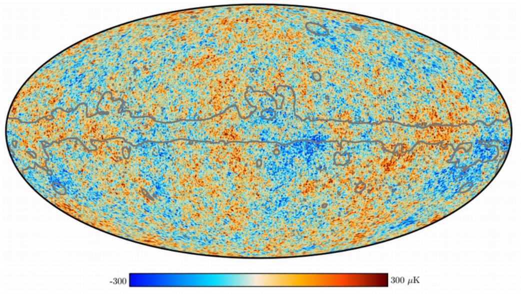
CMB anisotropies



Planck satellite 2018

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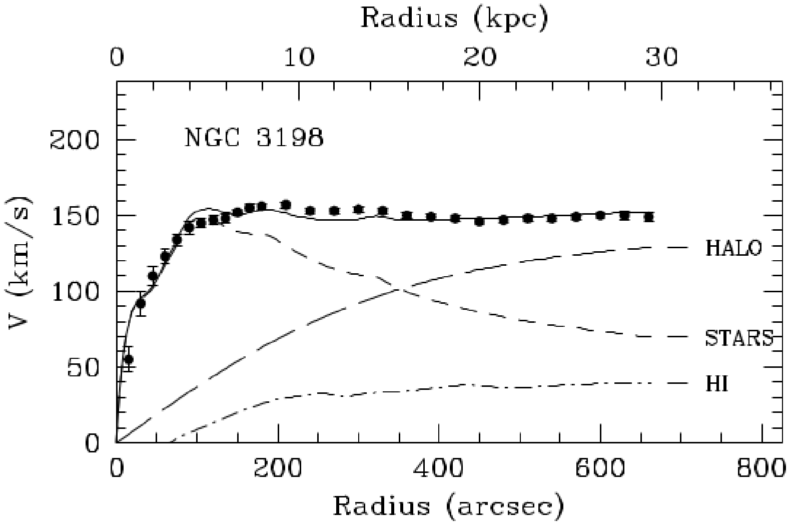

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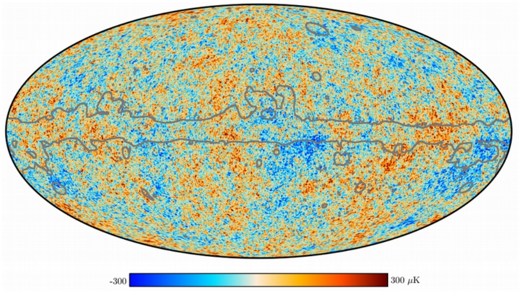

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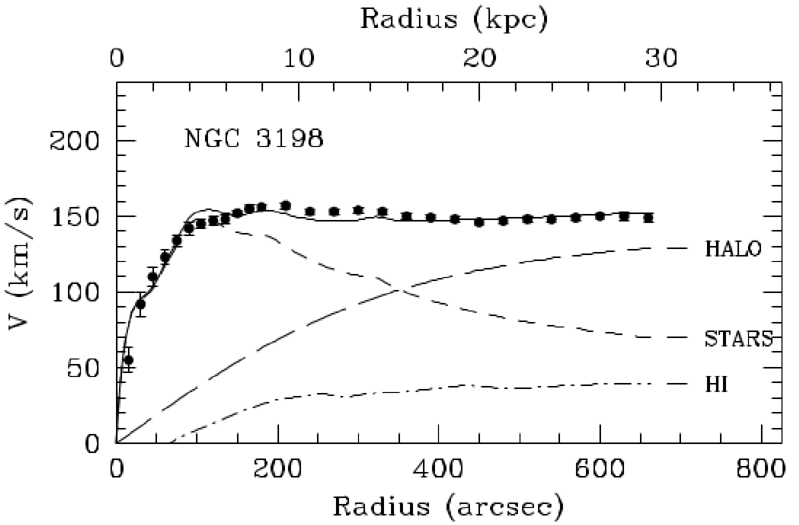

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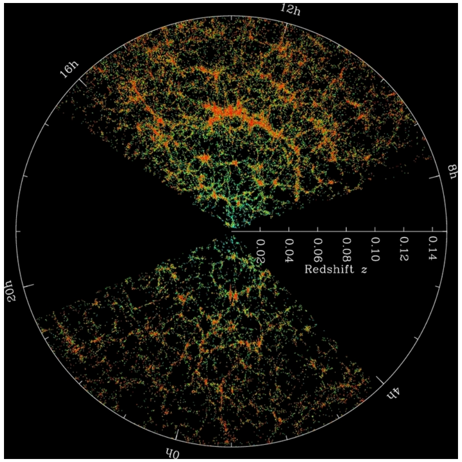

Large-Scale Structures

Image Credit:
Sloan Digital Sky Survey

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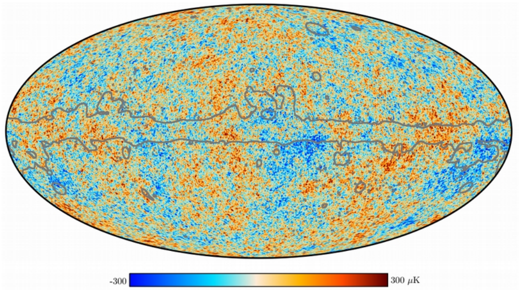

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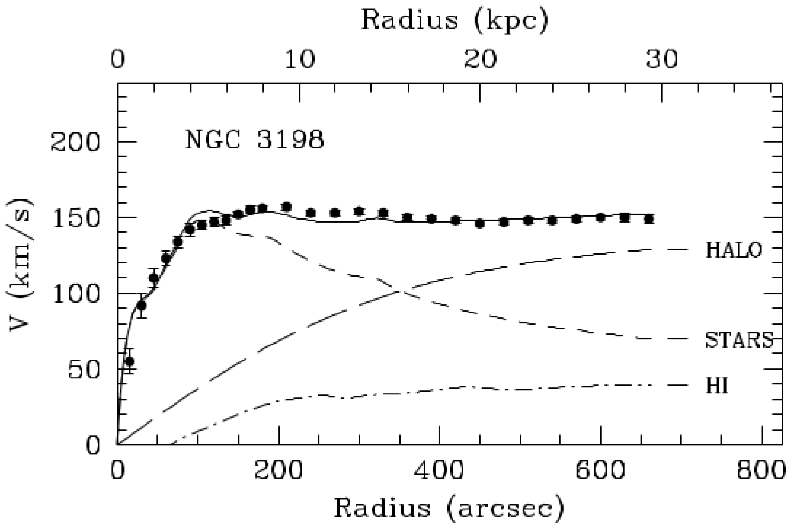


S. Blais-Ouellette et al., Astron. J. 118 (1999) 2123

Bullet cluster

Image Credit: X-ray: NASA/CXC/CfA/ M. Markevitch et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al. Optical: NASA/STScI; Magellan/ U.Arizona/ D.Clowe et al.

Large-Scale Structures

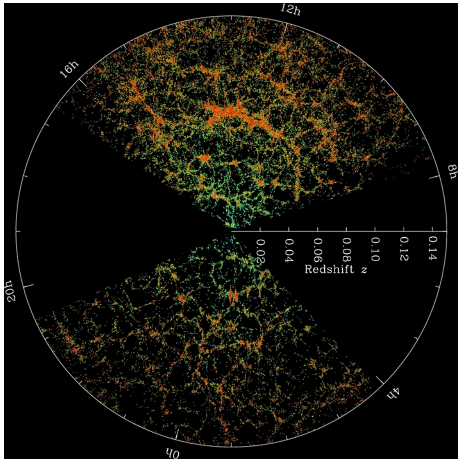
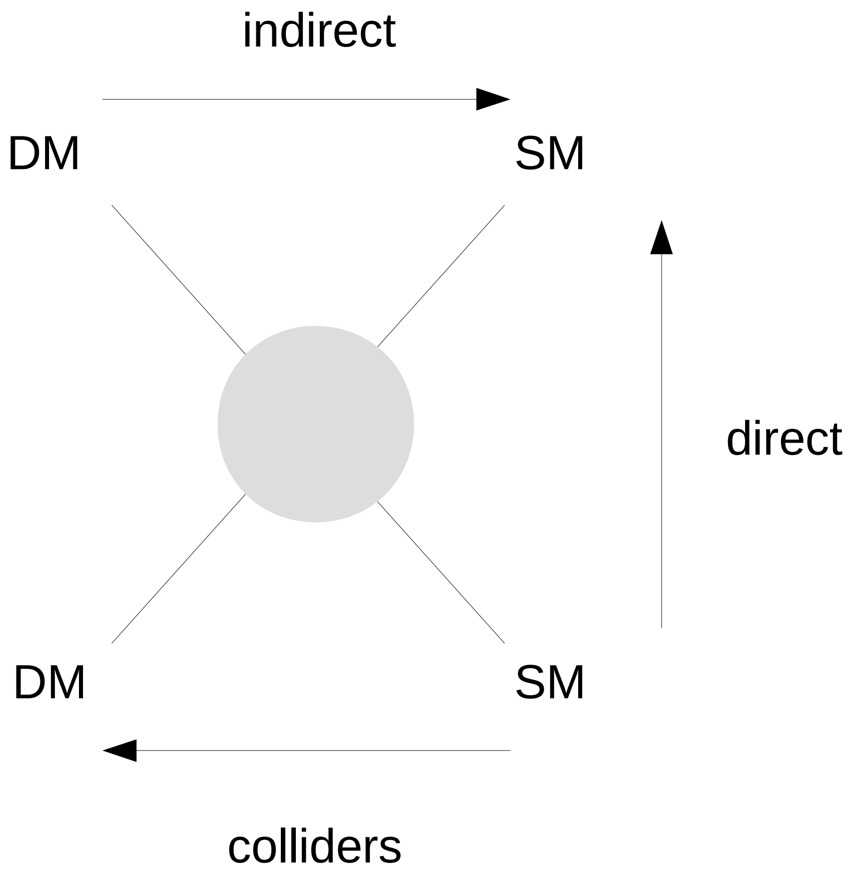


Image Credit: Sloan Digital Sky Survey

In order to detect dark matter...

- **Direct detection**
  - spin (in)dependent
  - annual modulation
- **Production at colliders**
  - Mono-X (missing E_T)
  - Resonances
- **Indirect detection**

  (astroparticle excesses)
  - gamma rays
  - positrons
  - neutrinos
  - ...



...we must know how much DM is there to be detected

- **Direct detection**
  - spin (in)dependent
  - annual modulation

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{\boldsymbol{\rho}_{\mathbf{DM},\odot}}{m_{\mathrm{DM}}} \frac{\sigma_{\mathrm{SI}} A^2}{2\mu^2} \int_{v>\sqrt{m_N E_R/(2\mu^2)}}^{v_{\max}} \frac{f(\mathbf{v},\mathbf{t})}{v} \, \mathrm{d}^3\mathbf{v}$$

- **Indirect detection**
  (astroparticle excesses)
  - gamma rays
  - positrons
  - neutrinos
  - ...

$$\left.\frac{\mathrm{d}N}{\mathrm{d}E}\right|_{\mathrm{annih.}} \propto \left(\frac{\boldsymbol{\rho}_{\mathbf{DM}}}{m_{\mathrm{DM}}}\right)^2$$

$$\left.\frac{\mathrm{d}N}{\mathrm{d}E}\right|_{\mathrm{decay}} \propto \frac{\boldsymbol{\rho}_{\mathbf{DM}}}{m_{\mathrm{DM}}}$$

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  (astroparticle excesses)
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  - ...

$$\left.\frac{\mathrm{d}N}{\mathrm{d}E}\right|_{\mathrm{annih.}} \propto \left(\frac{\boldsymbol{\rho_{\mathrm{DM}}}}{m_{\mathrm{DM}}}\right)^2$$

$$\left.\frac{\mathrm{d}N}{\mathrm{d}E}\right|_{\mathrm{decay}} \propto \frac{\boldsymbol{\rho_{\mathrm{DM}}}}{m_{\mathrm{DM}}}$$

Common methods to estimate $\rho_{DM,\odot}$

- **Local** methods
  - **Vertical z-Jeans equation**
  - Distribution function fitting

  - Small volume around the Solar neighbourhood
  - Less dependence on a specific DM profile

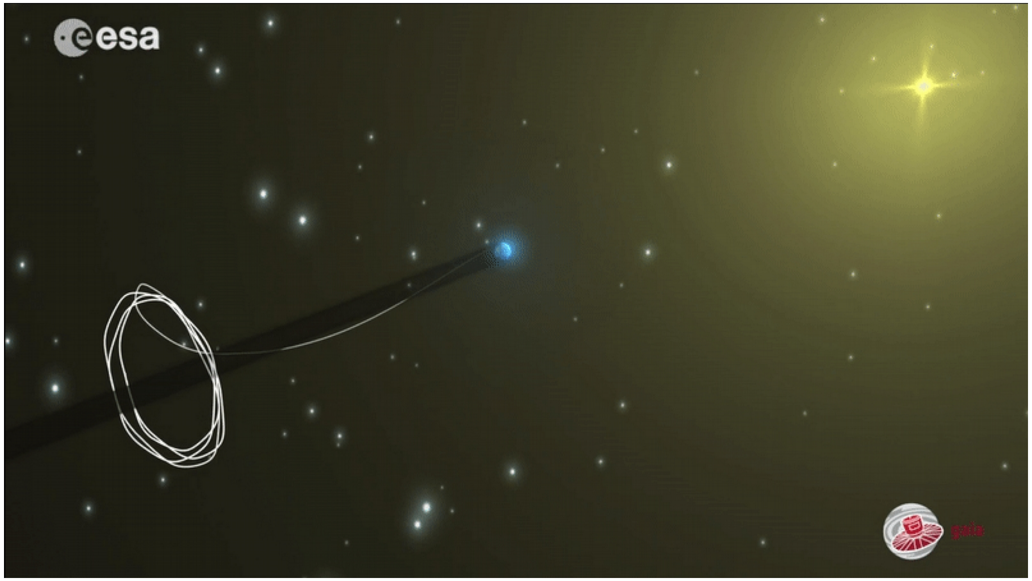
- **Global** methods
  - **Rotation curve**
  - Distribution function fitting

  - Large volume beyond the Solar neighbourhood

ESA/Gaia satellite mission

Mission timeline

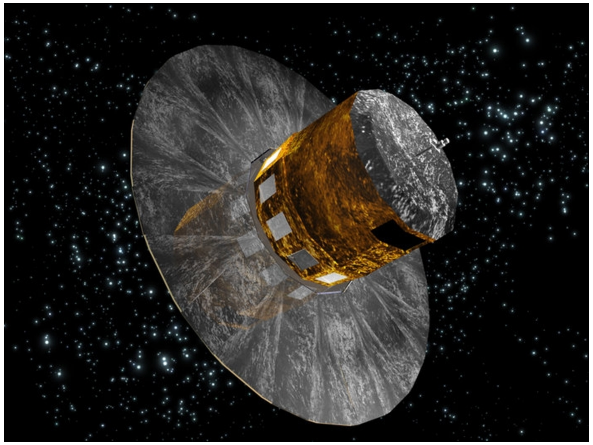
- Launch 19 December 2013
- Operation since 25 July 2014
- Nominal mission (5 years) July 2019
- Mission extended to 31 December 2022



Data Release

Gaia DR1: A.G.A. Brown et al., A&A 595 (2016) A2
Gaia DR2: A.G.A. Brown et al., A&A 616 (2018) A1

DR1	(14 months)	14 September 2016
DR2	(22 months)	25 April 2018
EDR3		third quarter 2020
DR3	(34 months)	second half 2021
Full Data Release		TBD

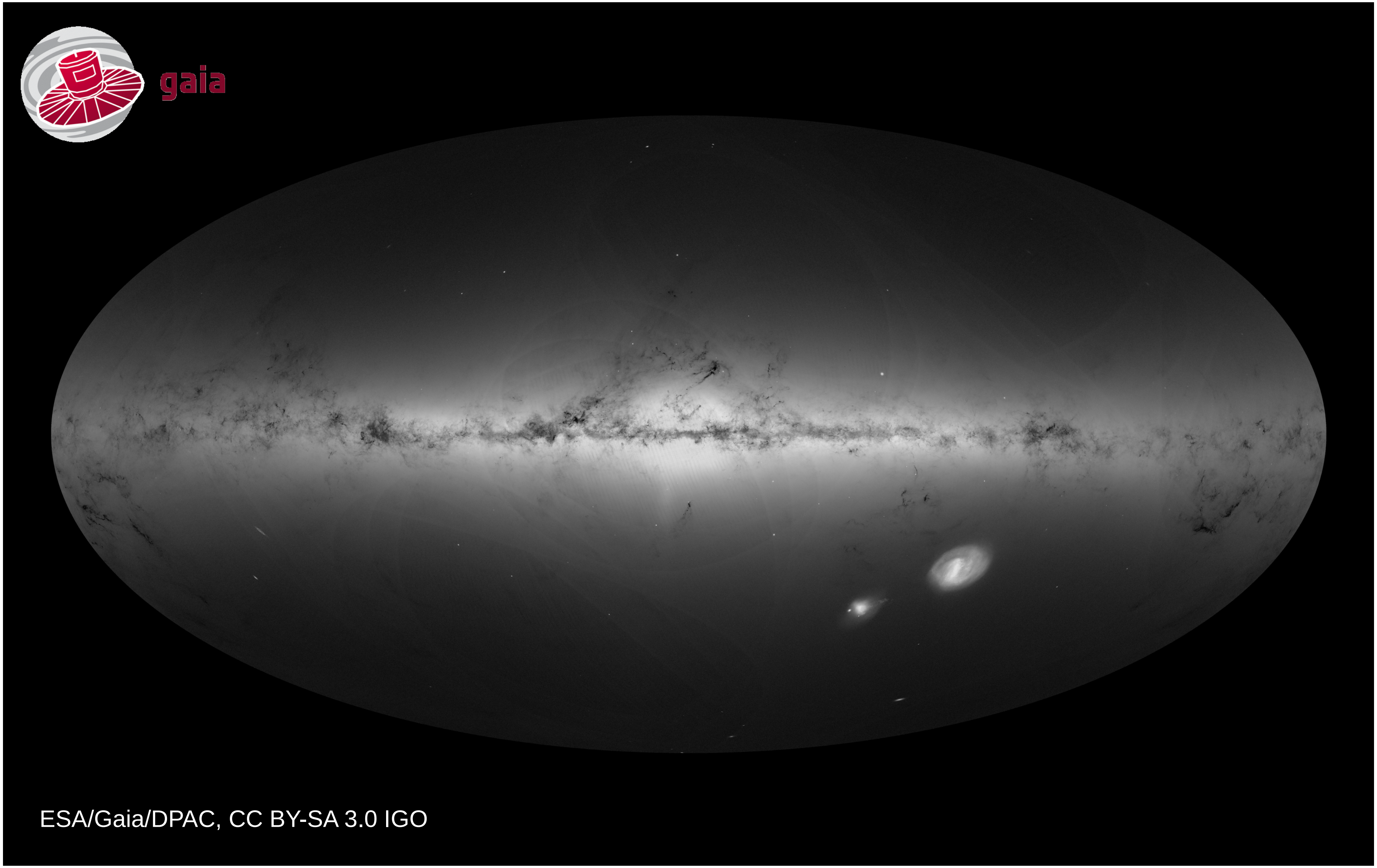


Credit for the images: ESA

Gaia Data Release (DR) overview

	# sources in Gaia DR2	# sources in Gaia DR1
Total number of sources	**1,692,919,135**	**1,142,679,769**
Number of 5-parameter sources	1,331,909,727	2,057,050 (TGAS)
Number of 2-parameter sources	361,009,408	1,140,622,719
Sources with mean G magnitude (3 < G < 21)	1,692,919,135	1,142,679,769
Sources with mean G_{BP}-band photometry	1,381,964,755	-
Sources with mean G_{RP}-band photometry	1,383,551,713	-
Sources with radial velocities	7,224,631	-
Variable sources	550,737	3,194
Known asteroids with epoch data	14,099	-
Gaia-CRF sources	556,869	2,191
Effective temperatures (T_{eff})	161,497,595	-
Extinction (A_G) and reddening ($E(G_{BP}$-$G_{RP})$)	87,733,672	-
Sources with radius and luminosity	76,956,778	-

Gaia DR2: Galactic density map



Local DM density from the rotation curve

- **Circular velocity**

Connection with theoretical ρ_{DM}

$$v_c^2(R) = R\left.\frac{\partial \phi}{\partial R}\right|_{z=0}$$

- ***R*-Jeans equation**

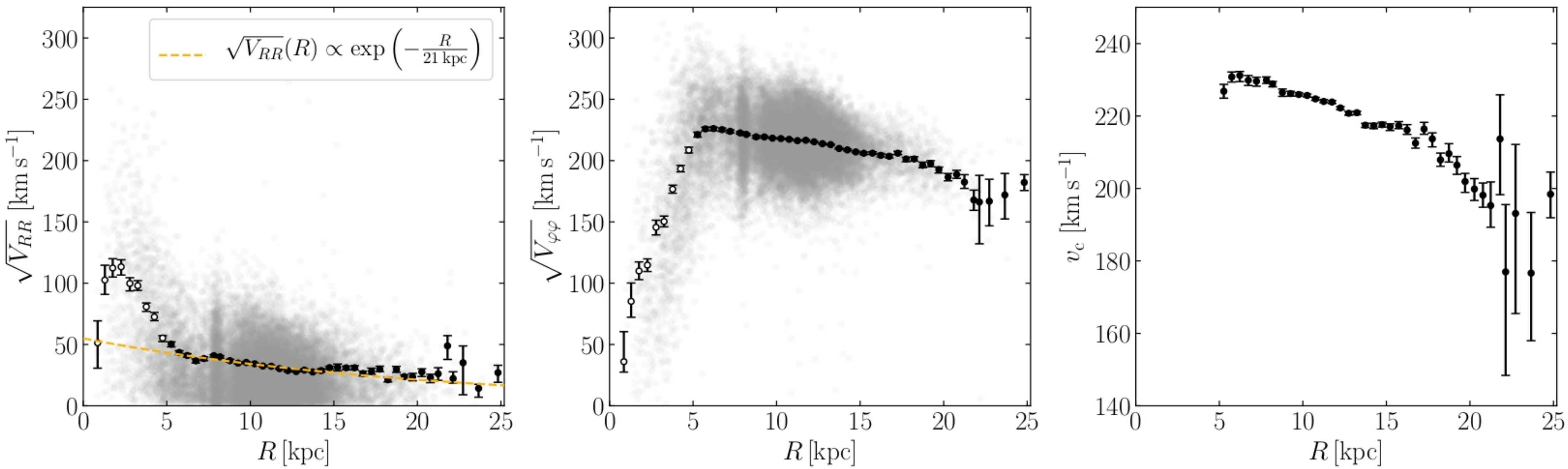
Connection with tracer's observations

$$v_c^2 = \overline{v_\varphi^2} - \overline{v_R^2} - \frac{R}{\nu}\frac{\partial\left(\nu\overline{v_R^2}\right)}{\partial R} - \frac{R}{\nu}\frac{\partial\left(\nu\overline{v_R v_z}\right)}{\partial z}$$

Local DM density from the rotation curve

[P.F. de Salas et al., arXiv:1906.06133]

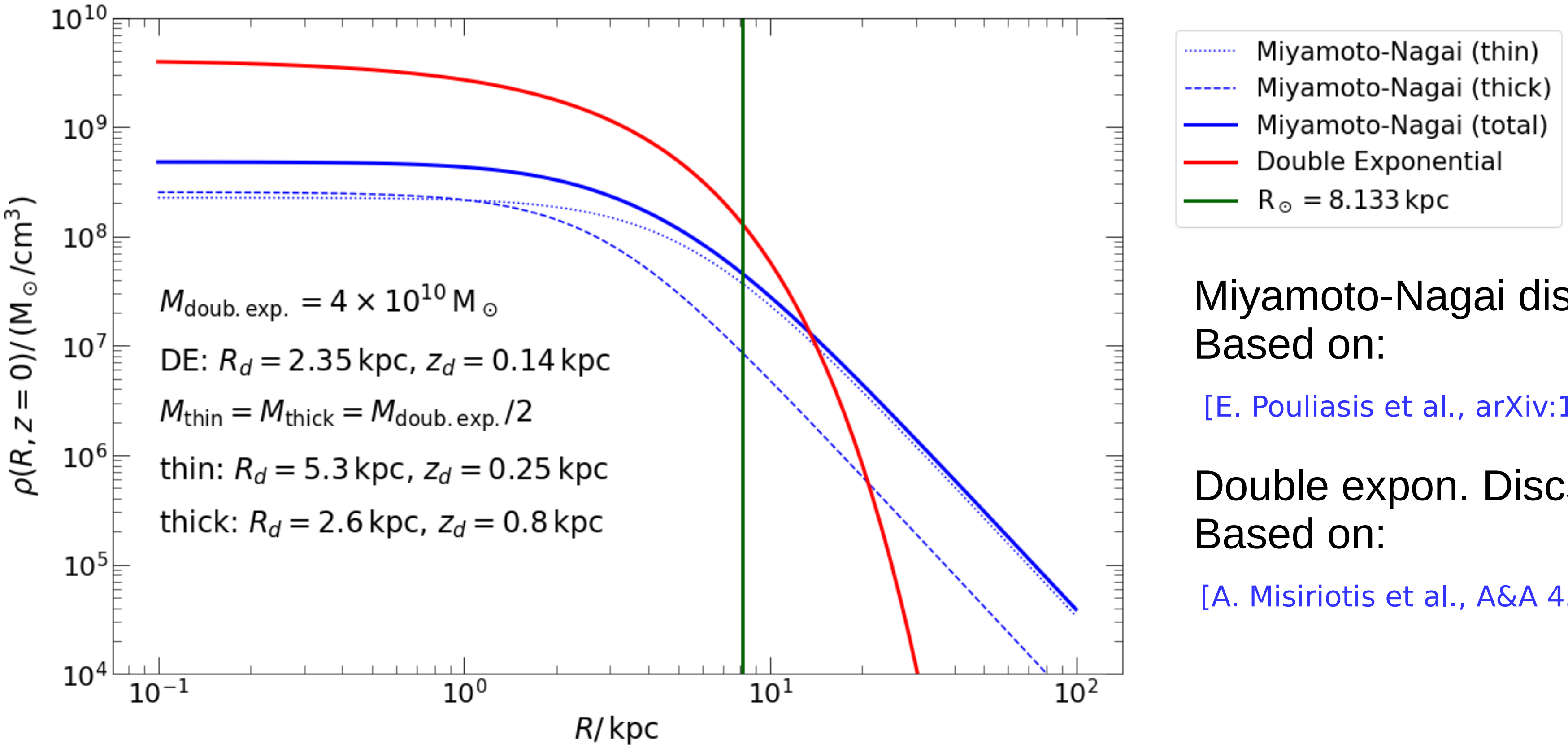
- **Survey**: Gaia DR2 + 2MASS + WISE + APOGEE [A.-C. Eilers et al., Astro. J. 871 (2019) 120]
- **Studied region**: $5\,\mathrm{kpc} \leq R \leq 25\,\mathrm{kpc}$
- **Tracer population**: Red-giant stars (23 129)



Local DM density from the rotation curve

[P.F. de Salas et al., arXiv:1906.06133]

- **Baryonic models**:



Miyamoto-Nagai discs **(B1)**
Based on:

[E. Pouliasis et al., arXiv:1611.07979]

Double expon. Discs **(B2)**
Based on:

[A. Misiriotis et al., A&A 459 (2006) 113]

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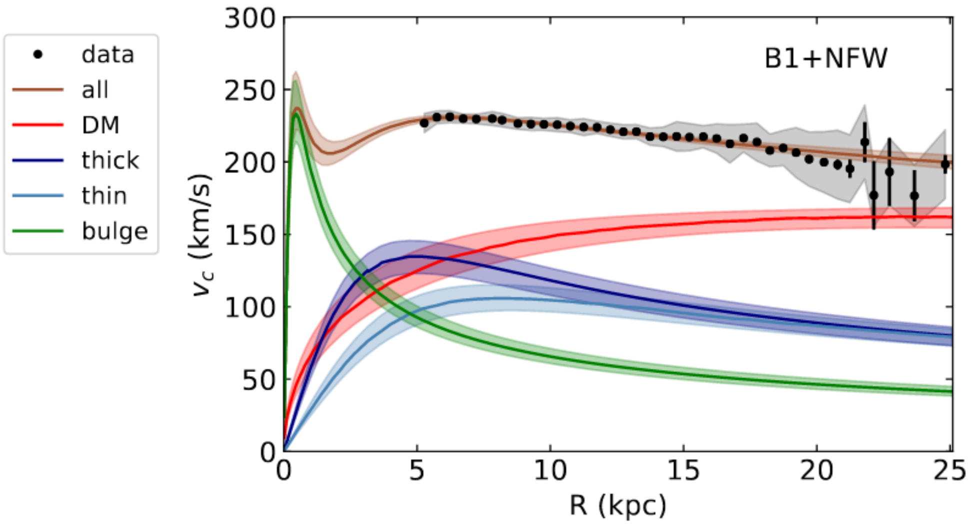
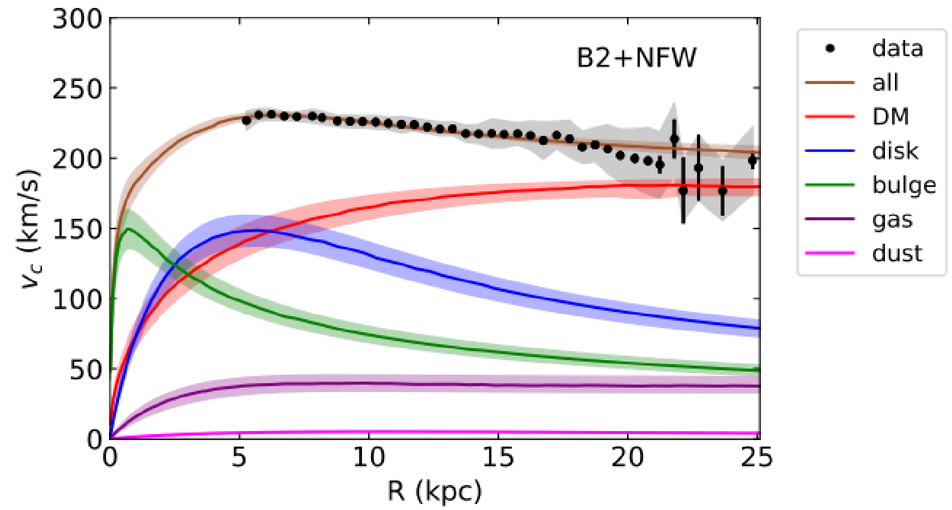
$\rho_{\mathrm{DM},\odot}/(\,\mathrm{GeV/cm^3})$	$\mathbf{0.30} \pm 0.03$	$\mathbf{0.38} \pm 0.03$





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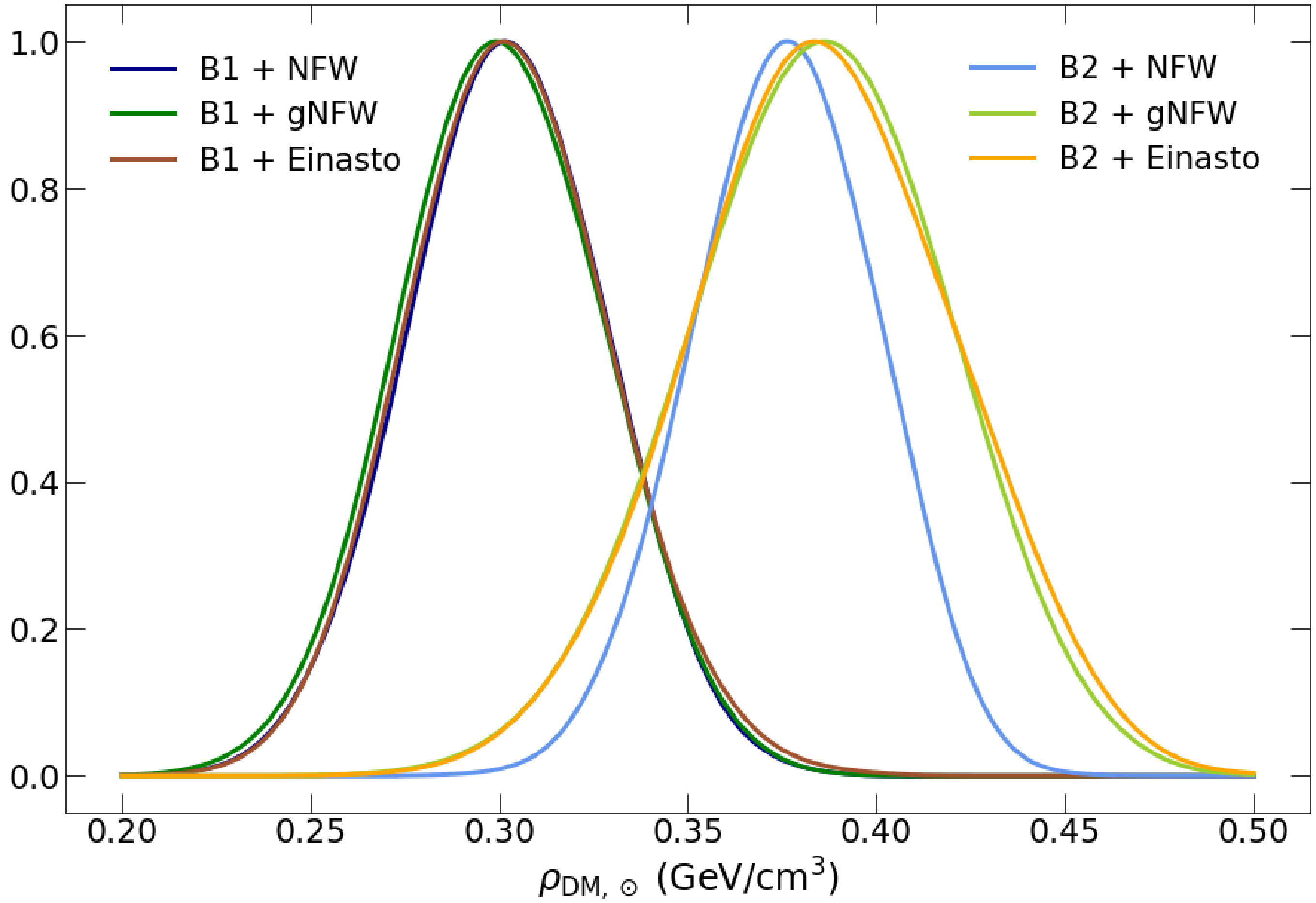
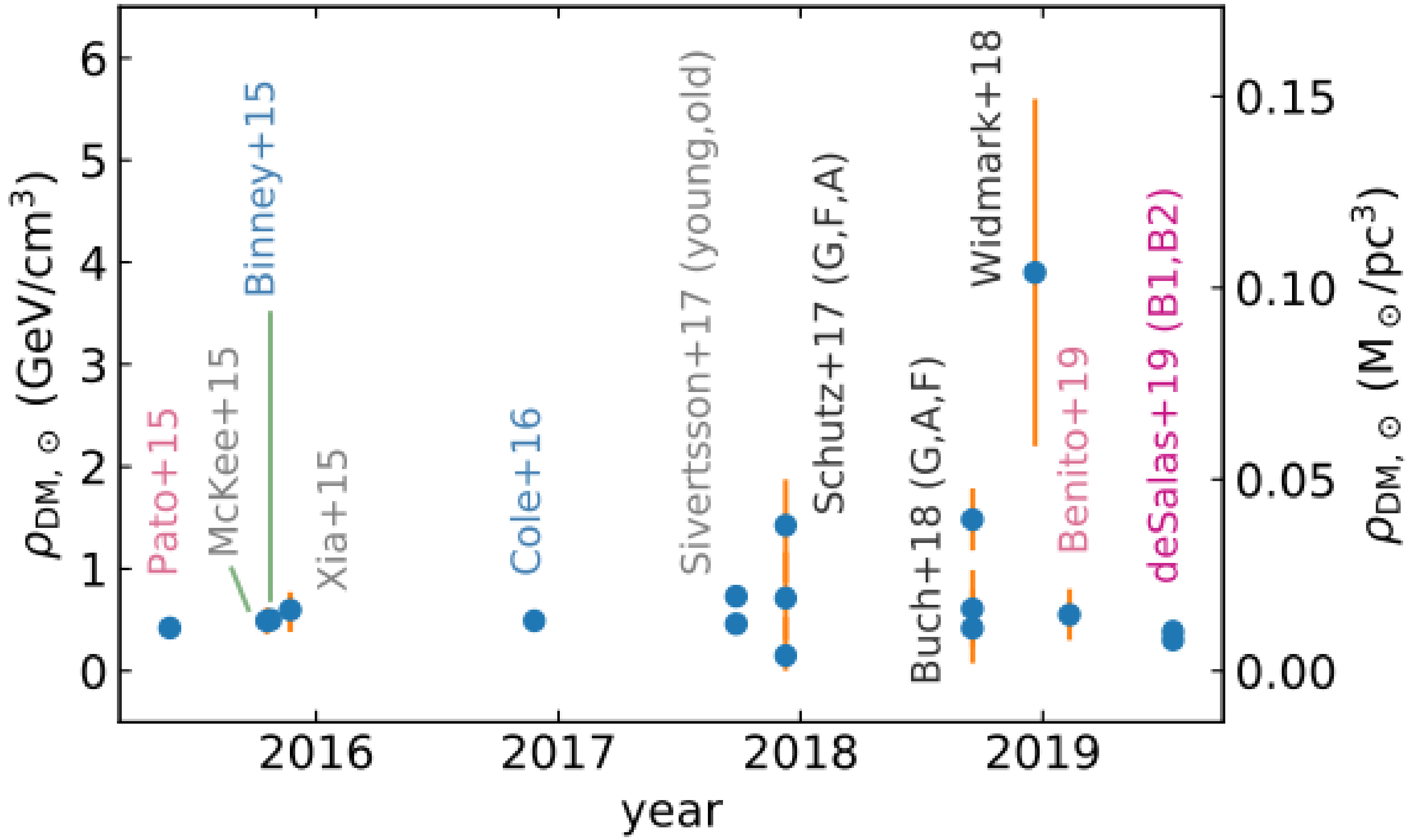


Recent estimates of $\rho_{DM,\odot}$



Method:

- Rotation curve
- Distribution Function
- Vertical Jeans eq.

(dark colors: Gaia data)

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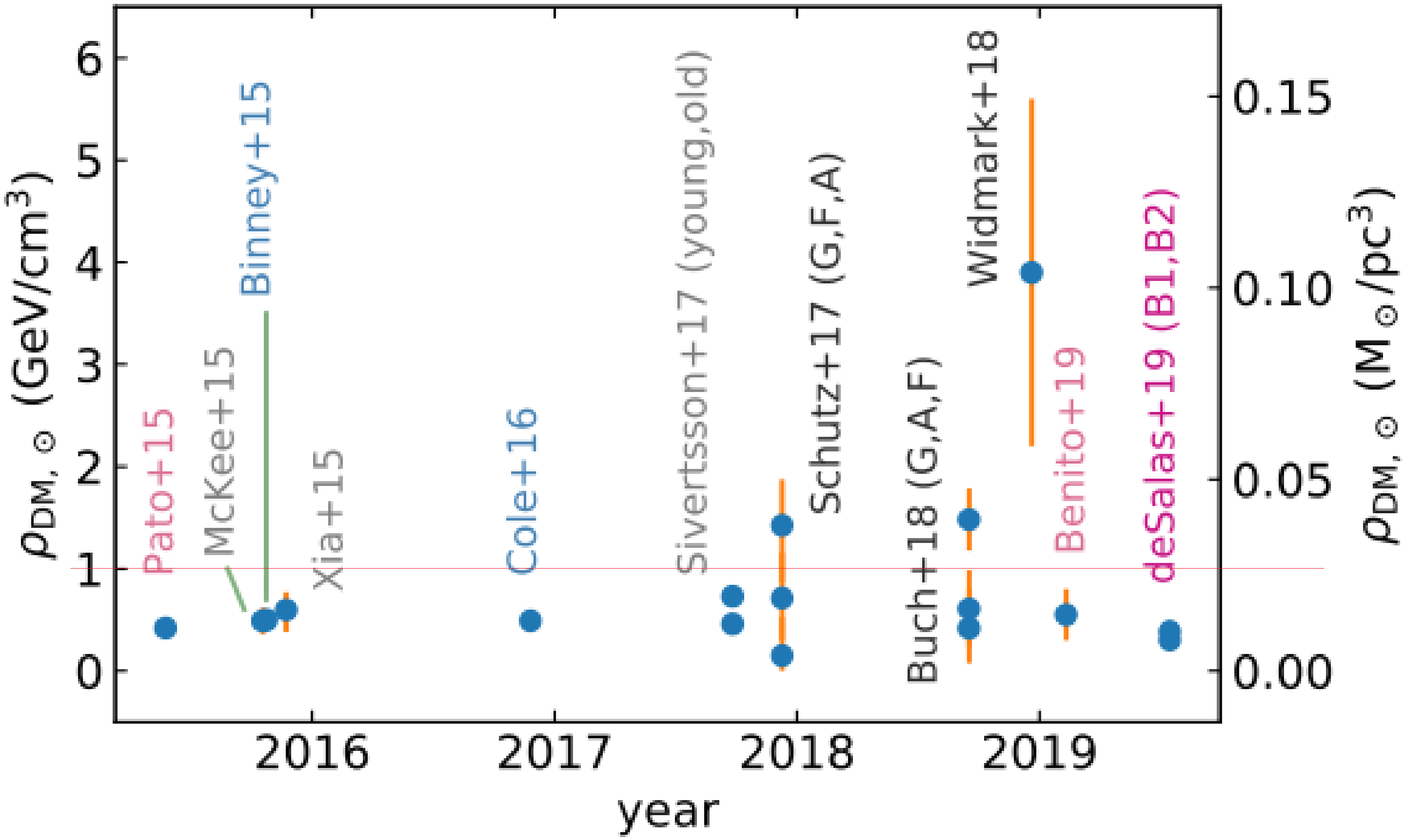

Method:

- Rotation curve
- Distribution Function
- Vertical Jeans eq.

(dark colors: Gaia data)

$\rho_{\text{baryons},\odot} = (3.3 \pm 0.3)\,\text{GeV/cm}^3$

[Schutz et al., arXiv:1711.03103]

Why so different?

- **Differences in the data?** Differences found when same survey is used

- **Differences in the methods?** Different methods cover different regions (The Galaxy is neither in equilibrium nor axisymmetric)

- **Disequilibria effects?**
  Two population HRD [e.g. A. Helmi+ arXiv:1806.06038]
  Phase-space spirals [e.g. T. Antoja+ arXiv:1804.10196]

- **New physics?** Dark disk [e.g. J.I. Read, arXiv:0803.2714, C.W. Purcell, arXiv:0906.5348, J. Fan, arXiv:1303.1521]

- **Uncertainties in baryonic data?** Underestimated cold gas? [A. Widmark, arXiv:1811.07911]

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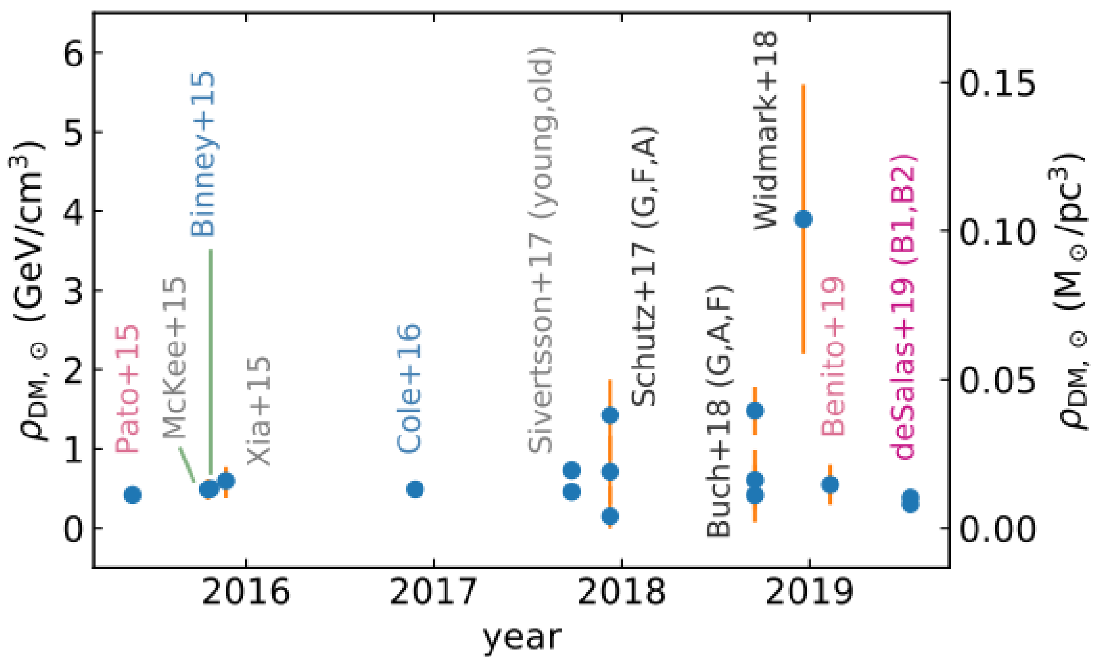


- Different populations:
  - Different age
  - Can be affected differently by disequilibria

Stellar populations:

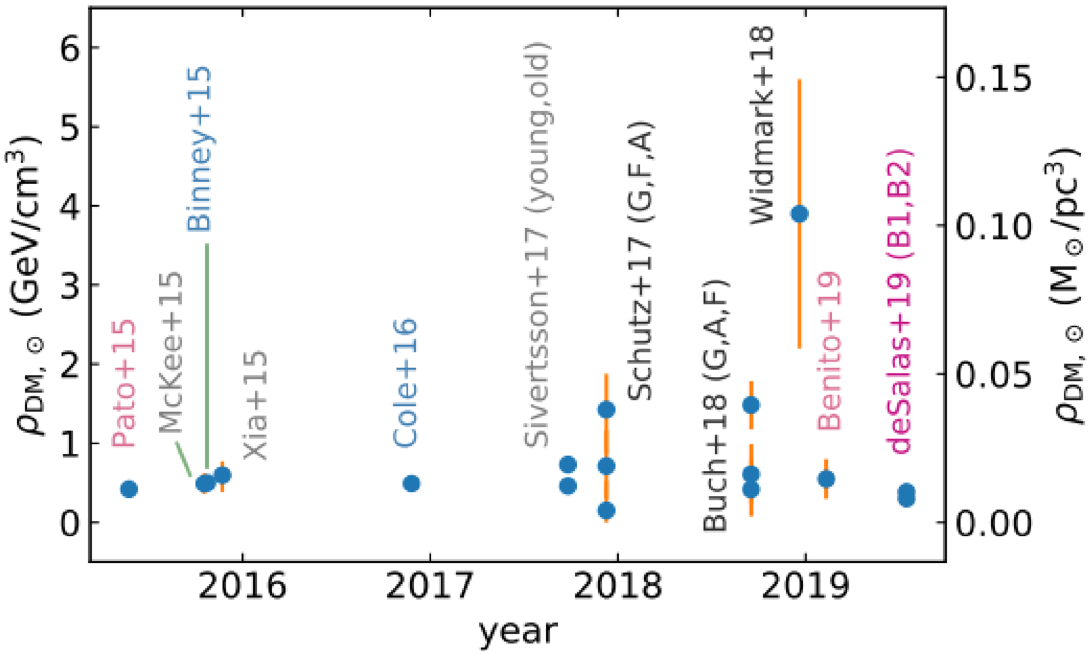
A stars	F stars	G stars
$\mathbf{0.608}^{+0.380}_{-0.380}$	$\mathbf{1.482}^{+0.304}_{-0.304}$	$\mathbf{0.418}^{+0.380}_{-0.342}$

[J. Buch et al., JCAP 04 (2019) 026]

Why so different?

- **Differences in the methods?**

Different methods cover different regions
(The Galaxy is neither in equilibrium nor axisymmetric)



- Different methods:
  - Different assumptions
  - Different volume coverage
  - Can be affected differently by disequilibria

Method:

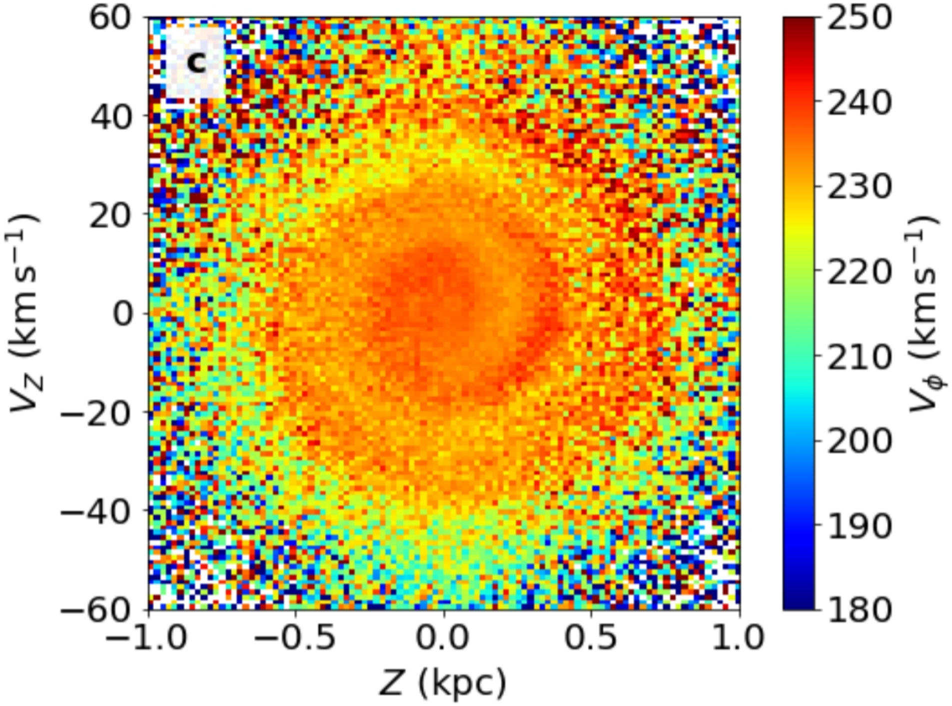
- Rotation curve
- Distribution Function
- Vertical Jeans eq.

(dark colors: Gaia data)

- **McKee, Xia, Sivertsson:** larger $z \sim$ kpc
- **Schutz, Buch, Widmark:** smaller $z < 200$ pc

Disequilibria effects

- **Phase-space spirals**



[T. Antoja, Nature 561 (2018) 360]

Possible source:

- Sagittarius dwarf passage [Laporte+, arXiv:1808.00451]
- Buckling of the bar [Khoperskov+, arXiv:1811.09205]

- **Two populations in HR diagram**

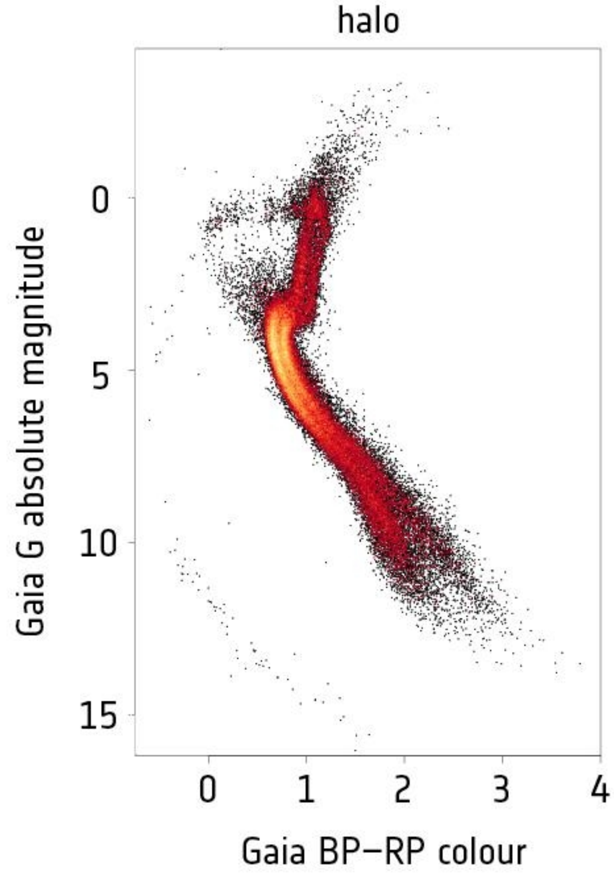


Image credits: ESA/Gaia/DPAC, CC BY-SA 3.0 IGO

Possible source:

- Gaia-Enceladus merger [Helmi+, arXiv:1806.06038]

Conclusions

- **Present:**
  - Recent precise information on hand (Gaia), but a good Galactic model missing
  - Under equilibrium, axisymmetry and typical baryonic models:

$$\rho_{DM,\odot} = 0.3\text{–}0.5\ \mathrm{GeV/cm^3}$$

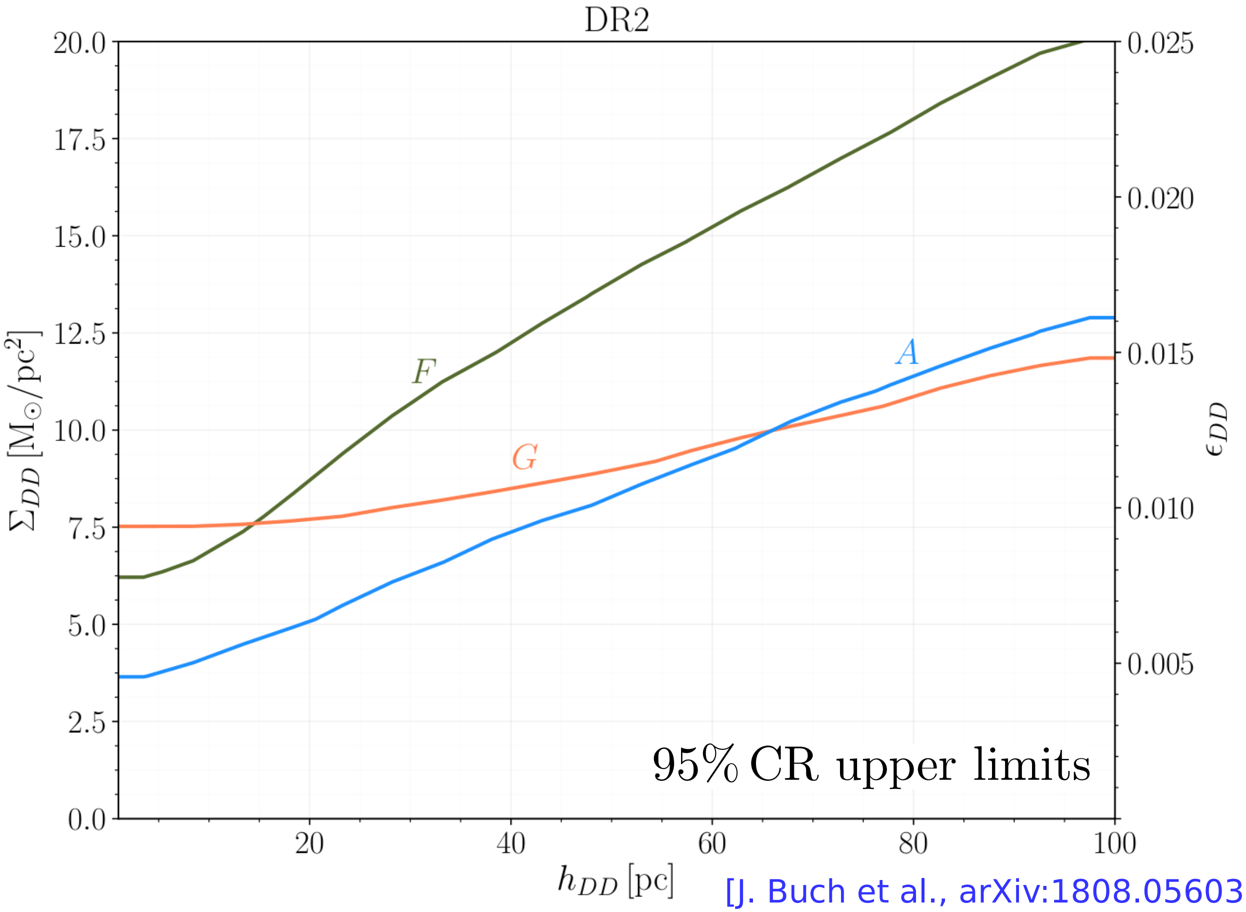
  - Larger measured values can be related to e.g. disequilibria in the Milky Way
  - Uncertainties dominated by underlying baryonic model
- **Future:**
  - Combine different (old and new) methods and data
  - Develop a better model for the Milky Way

Why so different?

- **New physics?**

Dark disk

[e.g. J.I. Read, arXiv:0803.2714, C.W. Purcell, arXiv:0906.5348, J. Fan, arXiv:1303.1521]



[J. Buch et al., arXiv:1808.05603]

- Dark disk:
  - Cannot explain alone differences in populations
  - Can explain differences between local and rotation curve methods

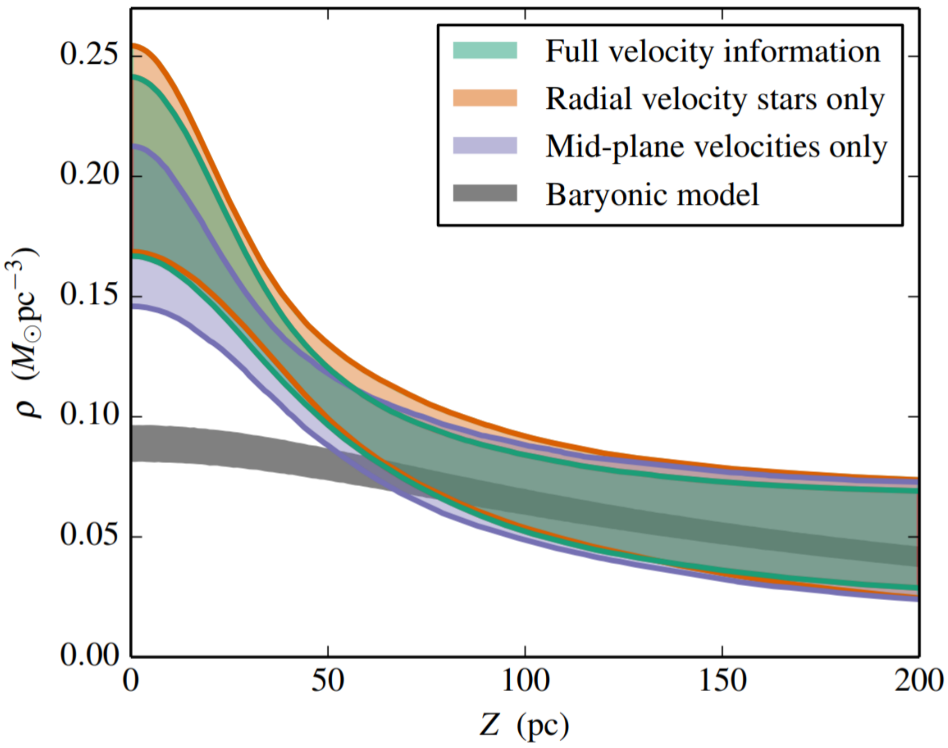
$$\Sigma_{DD}(R_\odot) = \frac{\epsilon_{DD} M_{\rm DM}^{\rm gal}}{2\pi R_{DD}^2} \exp(-R_\odot/R_{DD})$$

$$R_{DD} = 2.15\,{\rm kpc}$$

$$M_{\rm DM}^{\rm gal} \sim 10^{12}\,{\rm M}_\odot$$

Why so different?

- **Uncertainties in baryonic data?** Underestimated cold gas? [A. Widmark, arXiv:1811.07911]



[A. Widmark, A&A 623 (2019) A30]

$$100\,\mathrm{pc} < r_{\mathrm{from\ Earth}} < 200\,\mathrm{pc}$$

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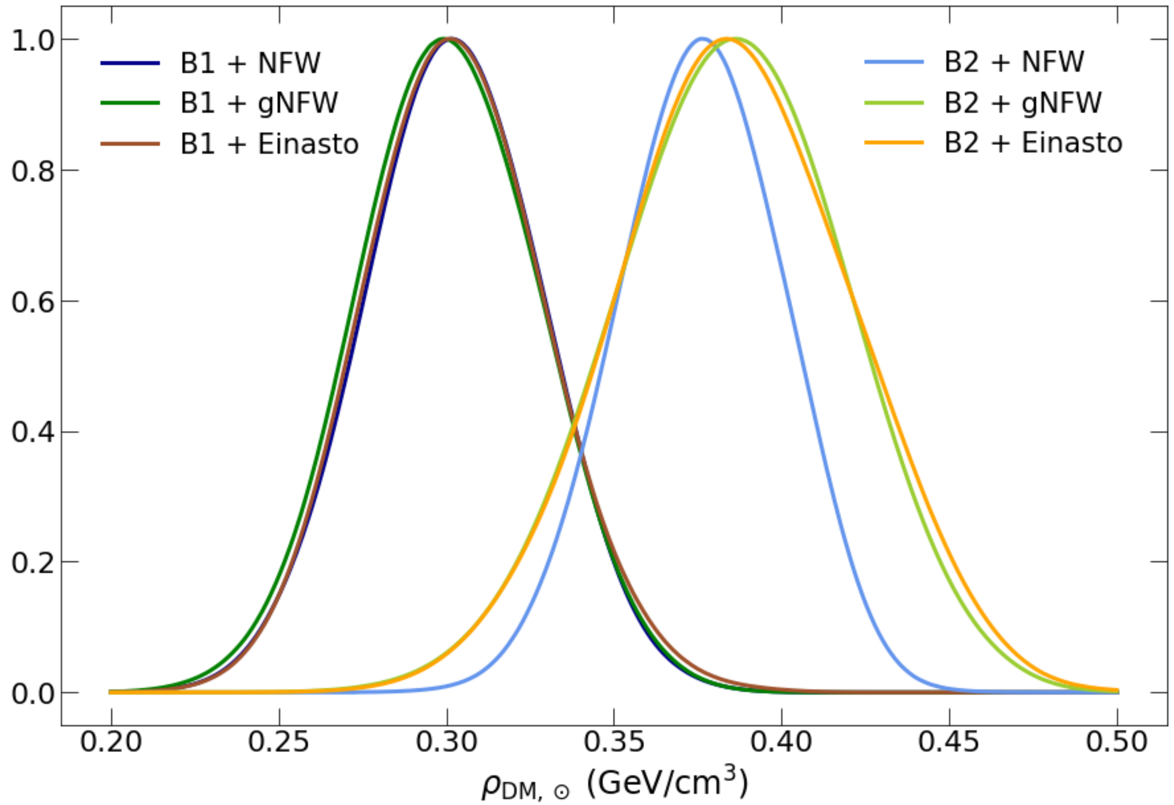

$\mathbf{0.30} \pm 0.03$ $\mathbf{0.38} \pm 0.03$

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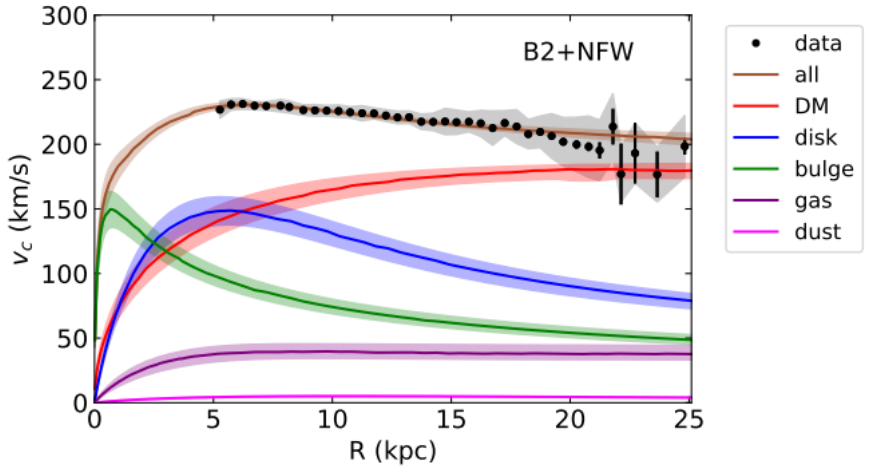


Data from:

[A.-C. Eilers et al., Astro. J. 871 (2019) 120]

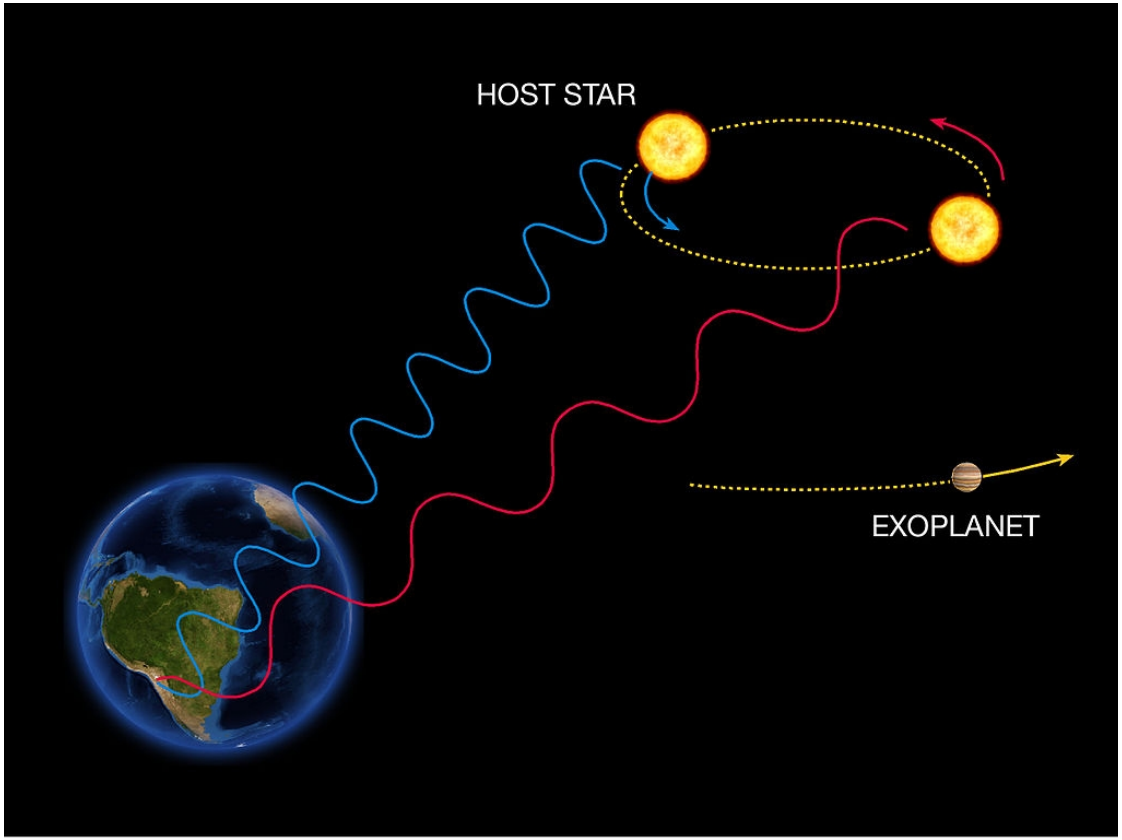
Baryonic model B1 from:

[E. Pouliasis et al., arXiv:1611.07979]

Baryonic model B2 from:

[A. Misiriotis et al., A&A 459 (2006) 113]

Stellar acceleration: Radial Velocity Method



The Radial Velocity Method

ESO Press Photo 22e/07 (25 April 2007)

- Same technique as exoplanet searches
- Doppler spectroscopy
- Less modelling assumptions

$$4\pi G\rho = -\nabla \cdot \vec{a}$$

- Since the Sun is also accelerating, we need to move out from $R_\odot$
- Local acceleration:

$$a_\odot = 2 \cdot 10^{-8}\,\mathrm{cm/s^2}$$

- Needed sensitivity in 10 years:

$$\Delta v_r \approx 5\,\mathrm{cm/s}$$

- Other Doppler shift sources are stronger (best scenario lonely stars)
- Disentangle DM contribution as complex as in other methods

[A. Ravi et al., arXiv:1812.07578]
[H. Silverwood et al., arXiv:1812.07581]

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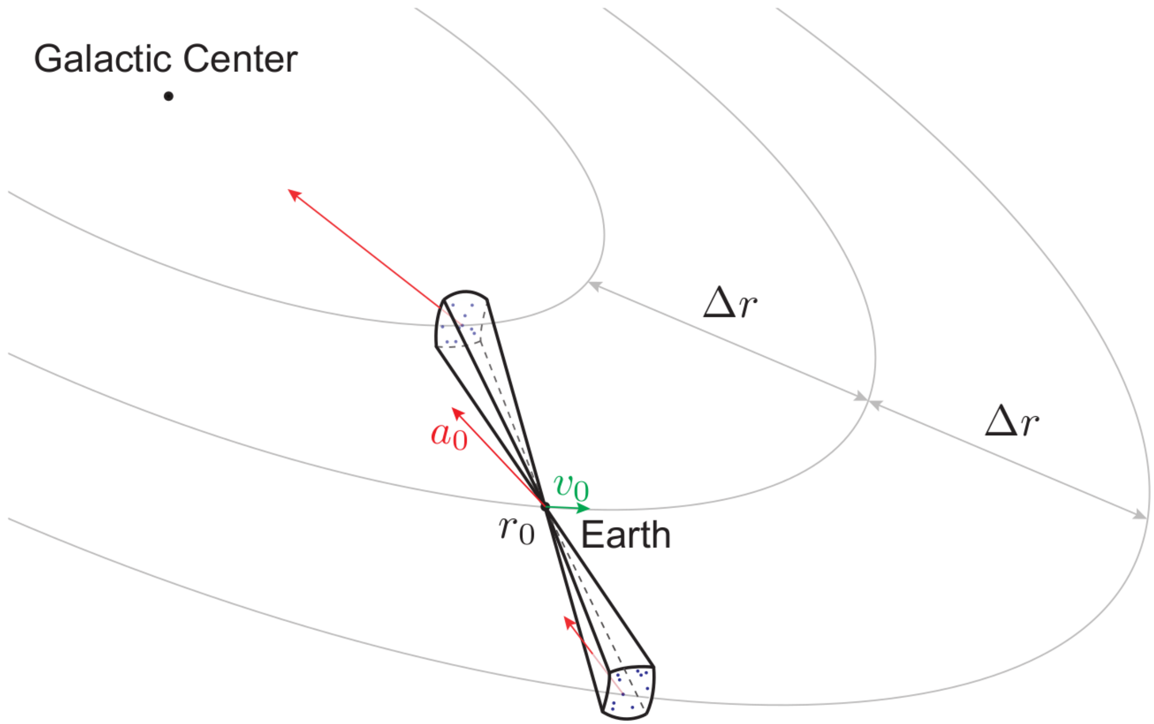

[Figure from A. Ravi et al., arXiv:1812.07578]

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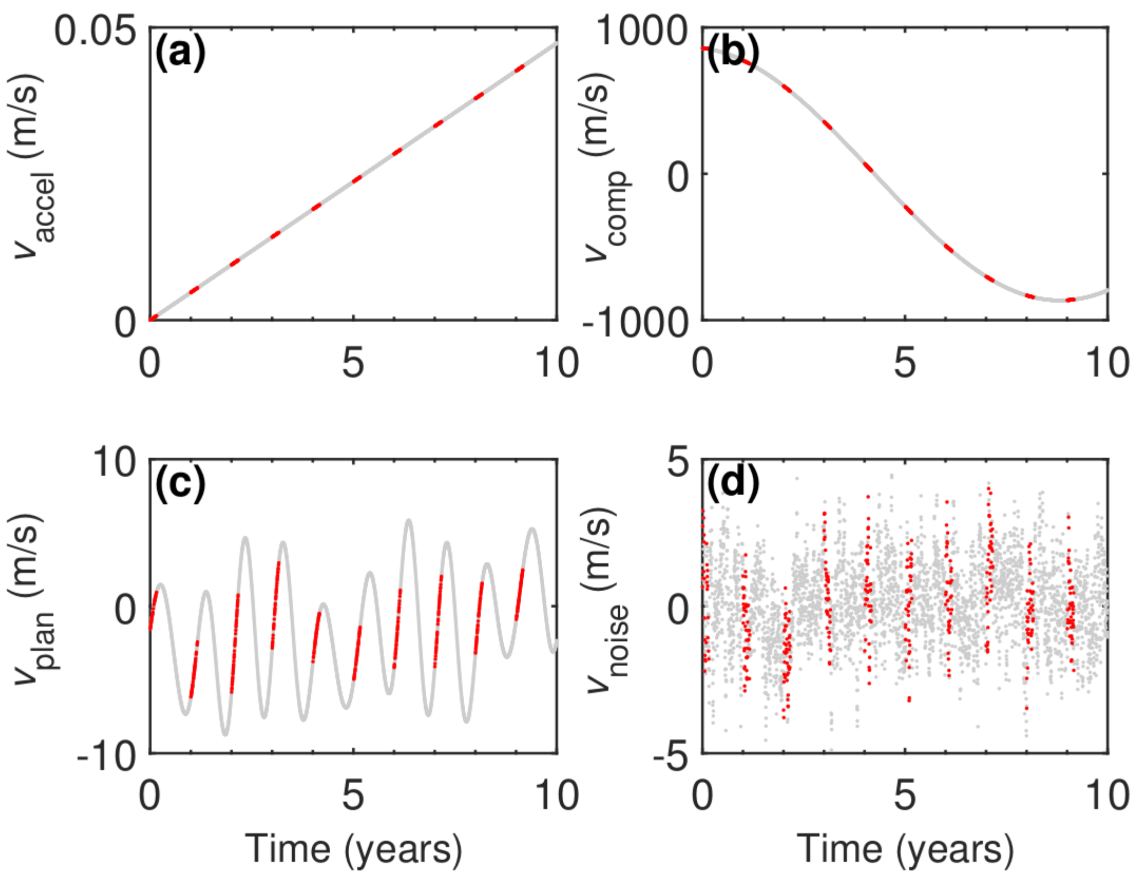

[Figure from A. Ravi et al., arXiv:1812.07578]

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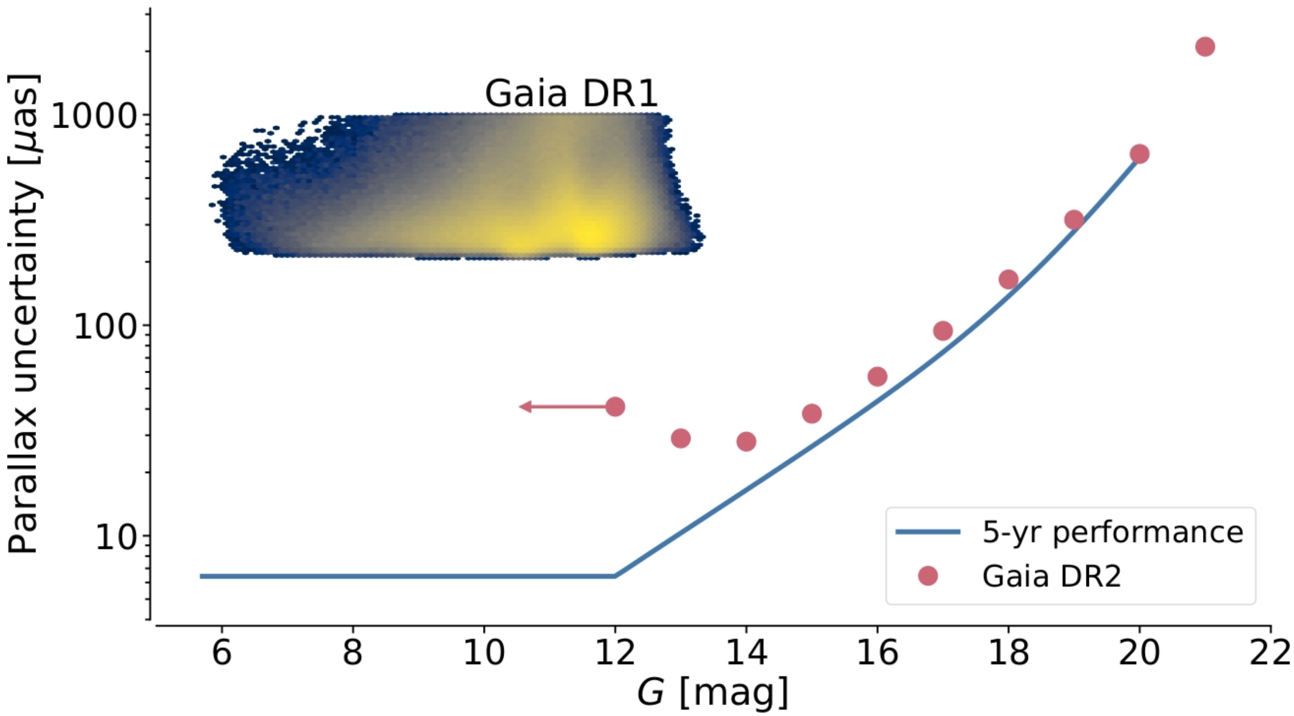
Voyage 2050 white paper

[J. Bergé et al., arXiv:1909.00834]

Science goal #2– *Improve our knowledge of the local dark matter and baryon densities* It can be fulfilled by monitoring the dynamics of a spacecraft in the Solar System neighborhood, the spacecraft carrying a clock and an accelerometer. The clock will be sensitive to local dark matter inhomogeneities. The combination of ranging and accelerometric data will also be sensitive to local gravitational disturbances, such as those that could be created by a massive enough clump. Finally, accelerometric data will be sensitive to the friction of any baryonic matter (dust and gas) on the spacecraft, allowing for a direct measurement of the baryonic matter density along the spacecraft trajectory. This will allow us to perform the first truly local measurement of the dark matter halo density ρ_0 and to improve the characterization of the dark matter constraints from direct detection experiments.

- It can probe a very local environment (~ 150 AU) 1 AU = 4.85e-6 pc
- It requires new propulsion methods: Breakthrough Starshot laser project
- Many technological challenges (propulsion, tracking, power...)

Gaia DR2 astrometric precision



A.G.A. Brown et al., A&A. 616 (2018) A1

Proper motion uncertainties:

0.06 mas/yr (for G < 15 mag)

0.2 mas/yr (for G = 17 mag)

1.2 mas/yr (for G = 20 mag)

Common methods to estimate $\rho_{DM,\odot}$

Common assumptions:

- Equilibrium (steady state)
- Axisymmetry

From visible tracers to DM:

- Collisionless Boltzmann equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \phi = 0$$

- Poisson equation

$$\nabla_x^2 \phi = 4\pi G \rho$$

Common methods to estimate $\rho_{DM,\odot}$

Common assumptions:

- Equilibrium (steady state)
- Axisymmetry

From visible tracers to DM:

$$\begin{cases} f: & \text{tracer's phase-space distribution} \\ \phi: & \text{gravitational potential} \\ \rho: & \text{matter energy density} \end{cases}$$

- Collisionless Boltzmann equation

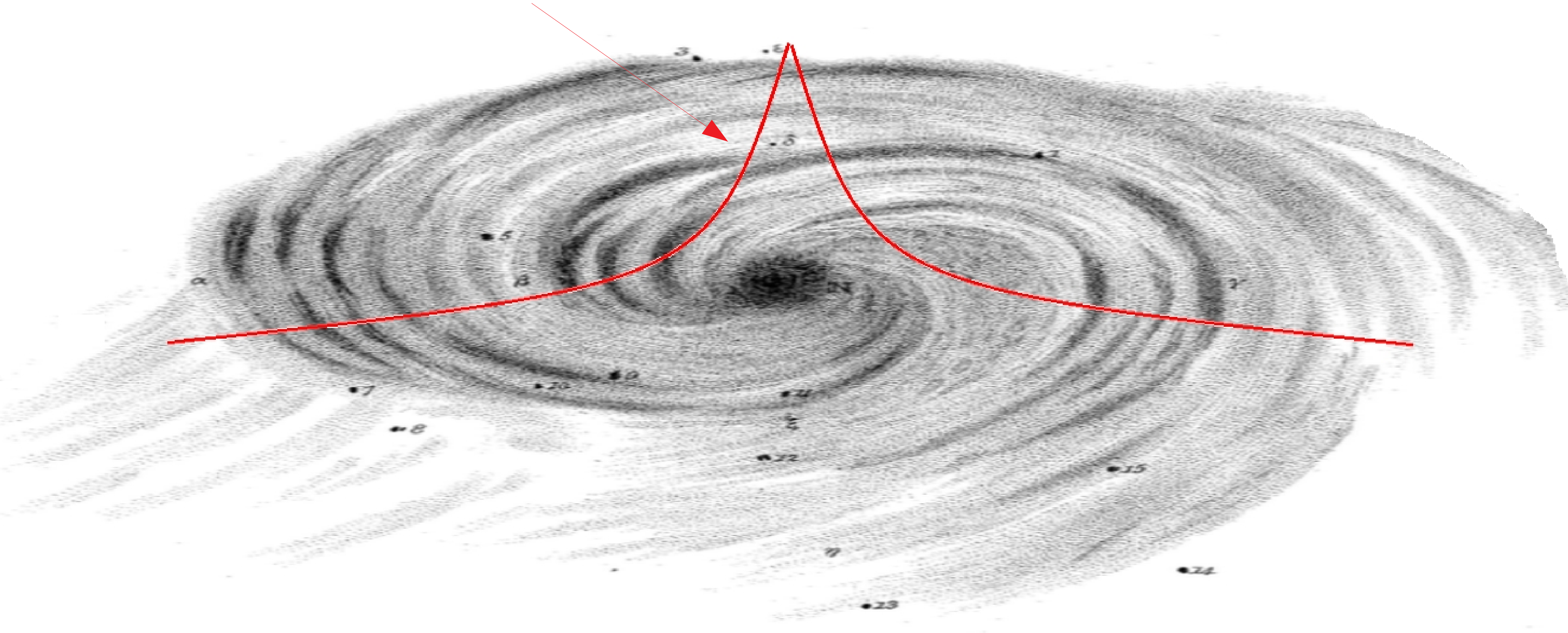
$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \phi = 0$$

- Poisson equation

$$\nabla_x^2 \phi = 4\pi G \rho$$

Methods to estimate $\rho_{DM,\odot}$

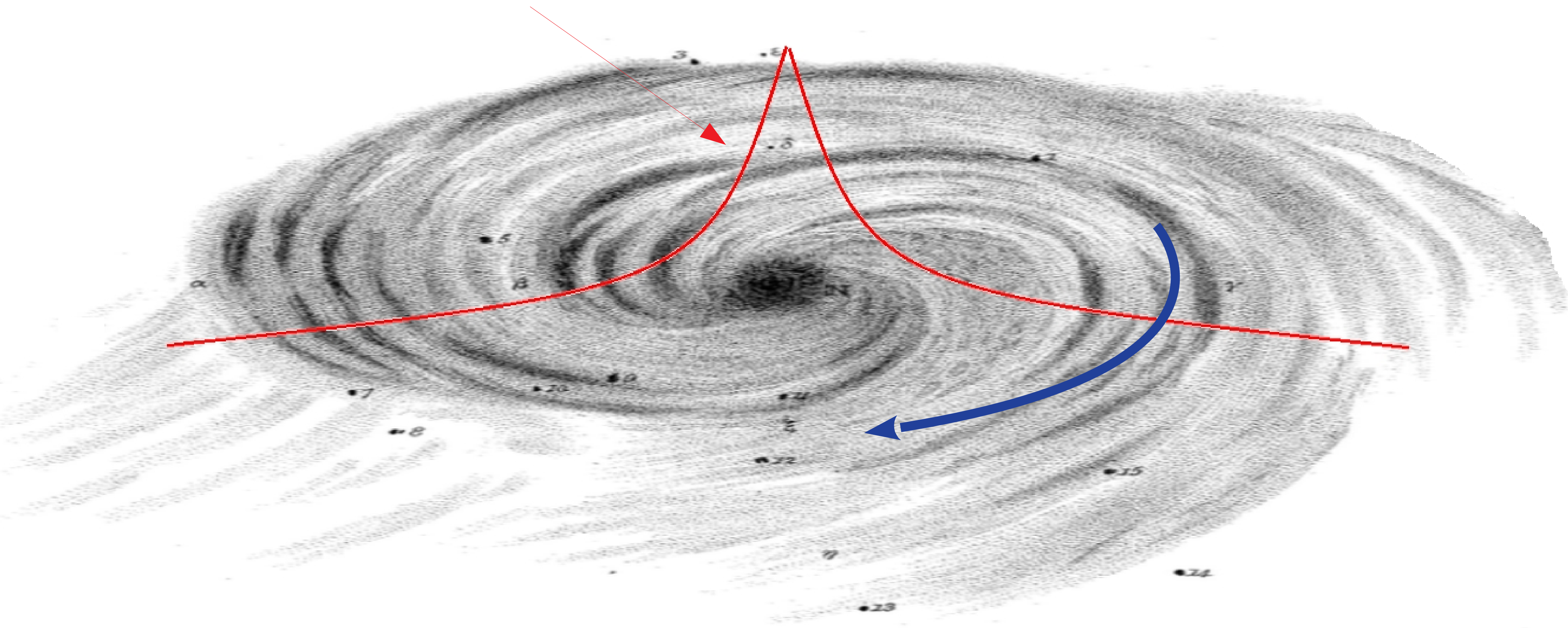
Galactic matter density



Methods to estimate $\rho_{DM,\odot}$

- Rotation curve method

Galactic matter density



Model construction

$$v_c^2(R) = R \left.\frac{\partial \phi}{\partial R}\right|_{z=0}$$

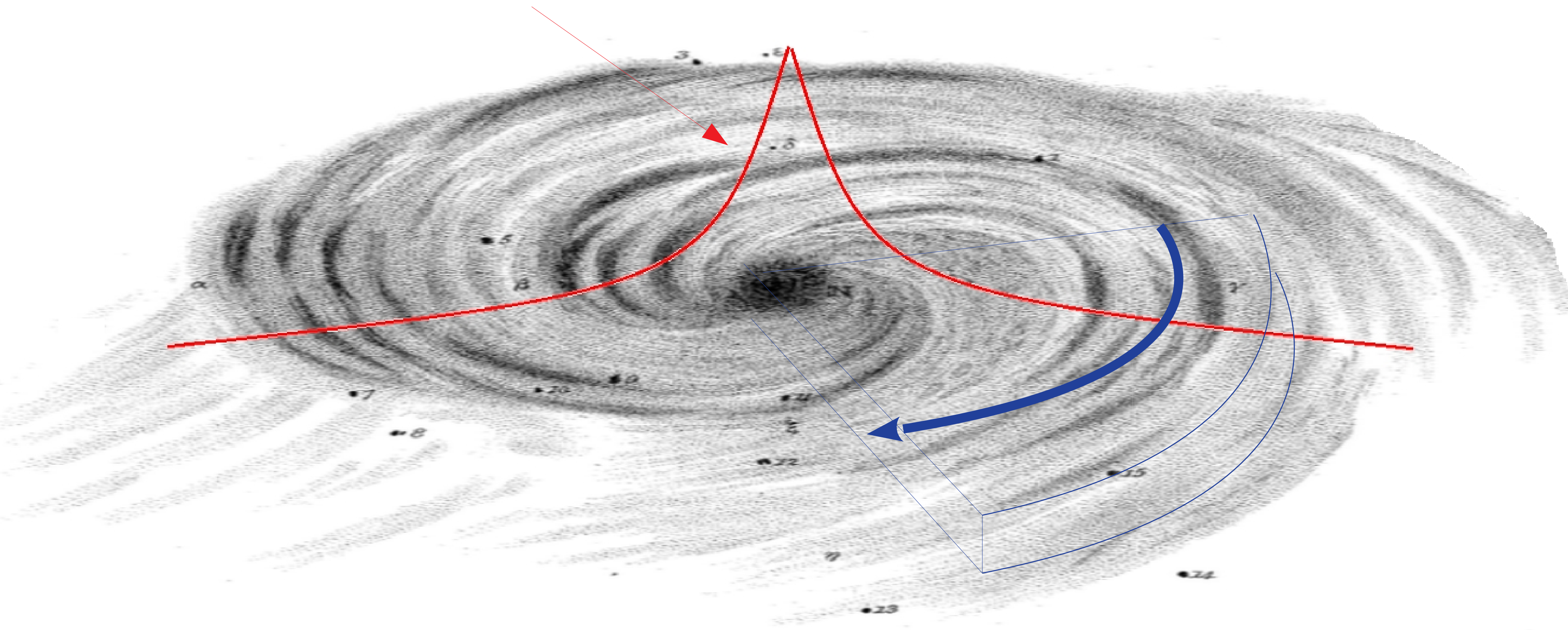
Observational estimate

$$v_c^2 = \overline{v_\varphi^2} - \overline{v_R^2} - \frac{R}{\nu}\frac{\partial\left(\nu\overline{v_R^2}\right)}{\partial R}$$

Methods to estimate $\rho_{DM,\odot}$

- Rotation curve method

Galactic matter density



Model construction

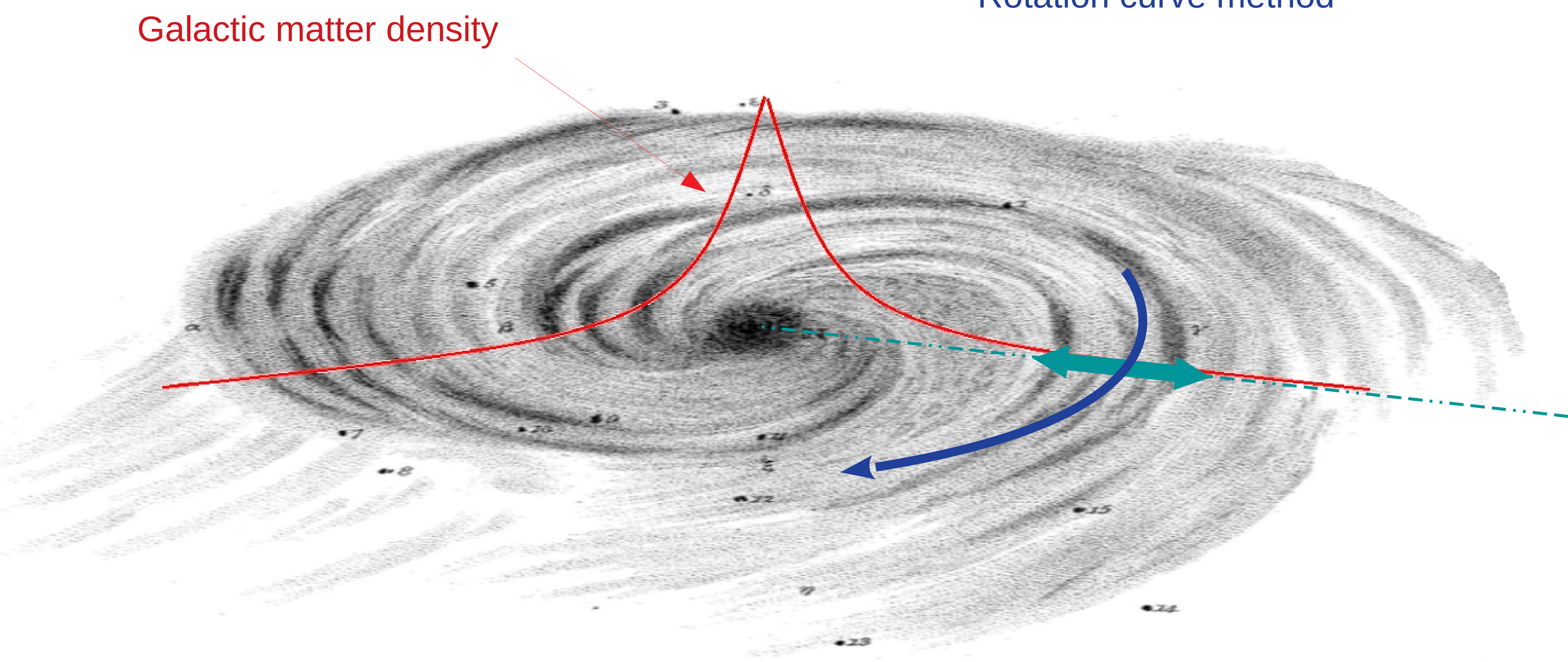
$$v_c^2(R) = R\,\frac{\partial \phi}{\partial R}\bigg|_{z=0}$$

Observational estimate

$$v_c^2 = \overline{v_\varphi^2} - \overline{v_R^2} - \frac{R}{\nu}\frac{\partial\left(\nu\overline{v_R^2}\right)}{\partial R}$$

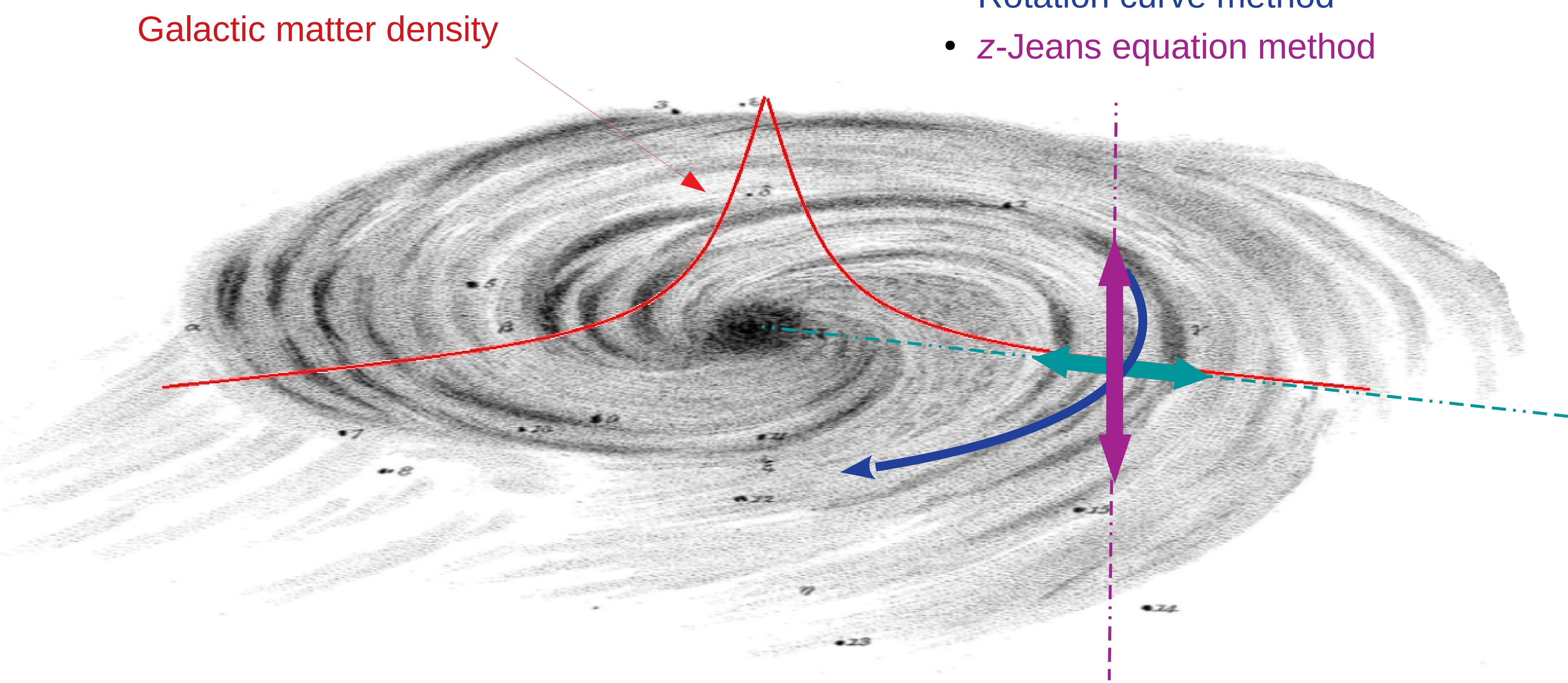
Methods to estimate $\rho_{DM,\odot}$

- Rotation curve method



Methods to estimate $\rho_{DM,\odot}$

- Rotation curve method
- *z*-Jeans equation method



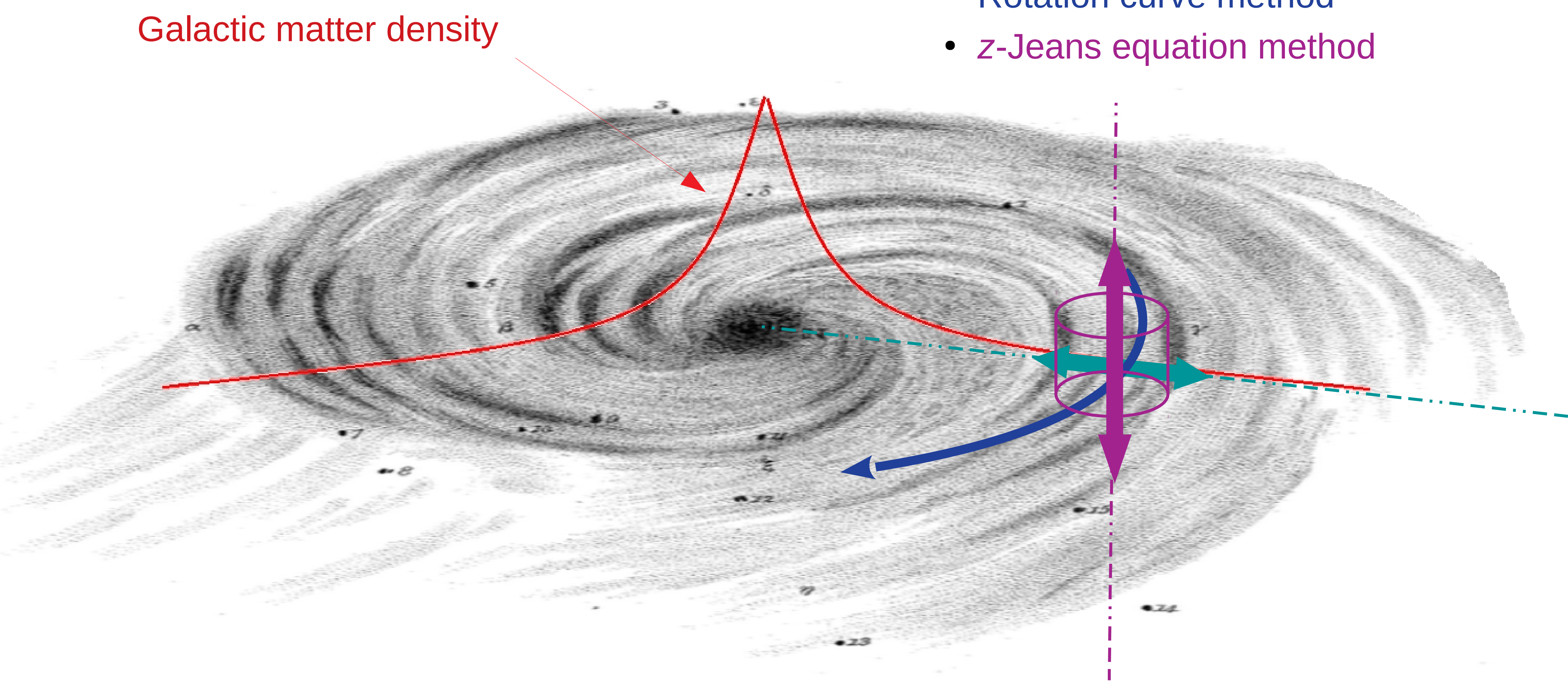
$$\frac{\partial\left(\nu\overline{v_z^2}\right)}{\partial z}+\nu\frac{\partial\phi}{\partial z}=0$$

1D *z*-Jeans equation method

$$\frac{\partial^2\phi}{\partial z^2}=4\pi G\rho$$

Methods to estimate $\rho_{DM,\odot}$

- Rotation curve method
- *z*-Jeans equation method



$$\frac{\partial\left(\nu\overline{v_z^2}\right)}{\partial z} + \nu\frac{\partial\phi}{\partial z} = 0$$

1D *z*-Jeans equation method

$$\frac{\partial^2\phi}{\partial z^2} = 4\pi G\rho$$

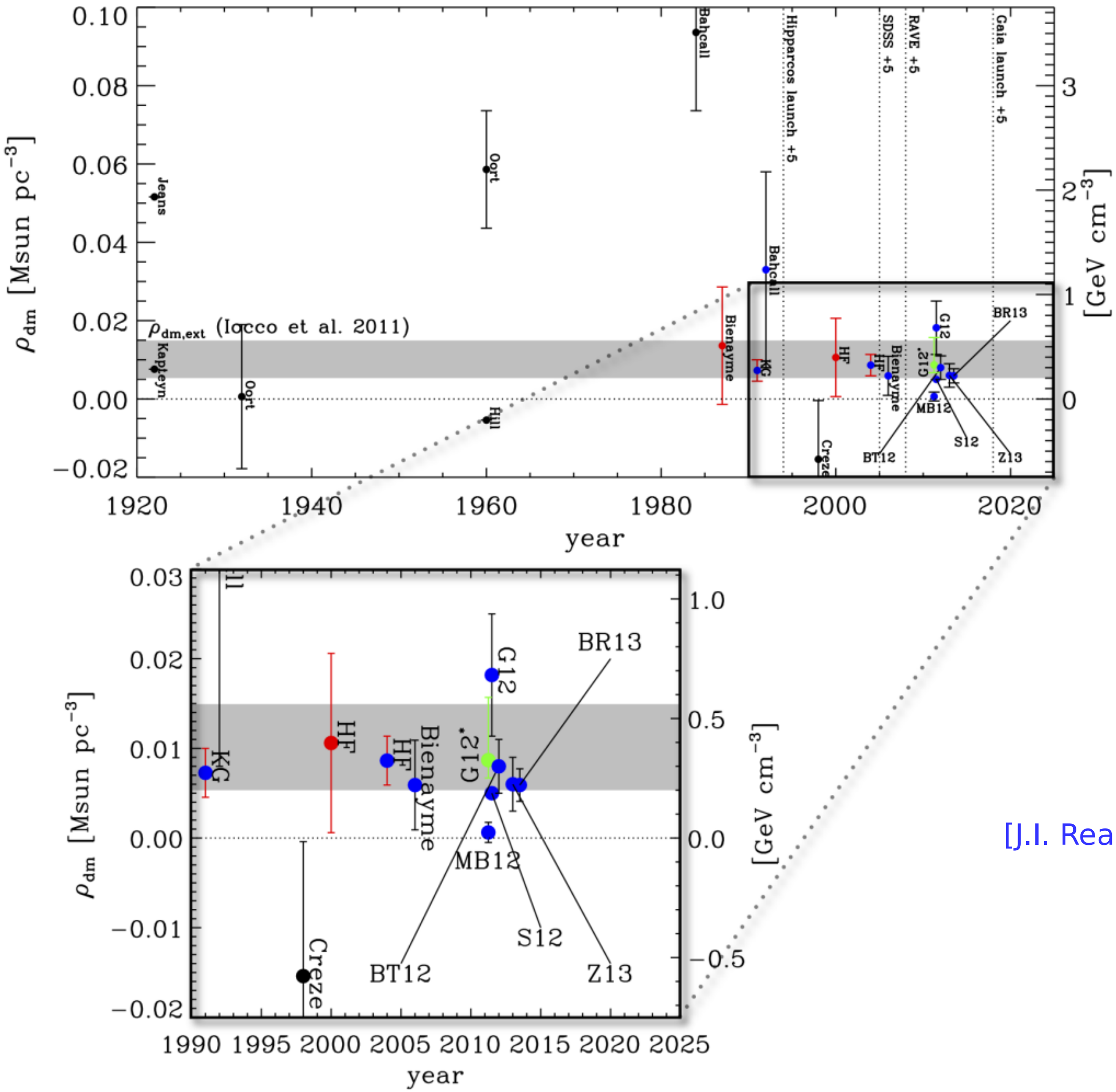
Methods to estimate $\rho_{DM,\odot}$

1) Choose one or more tracer populations ν_i

2) Relate ν_i to the gravitational potential Φ

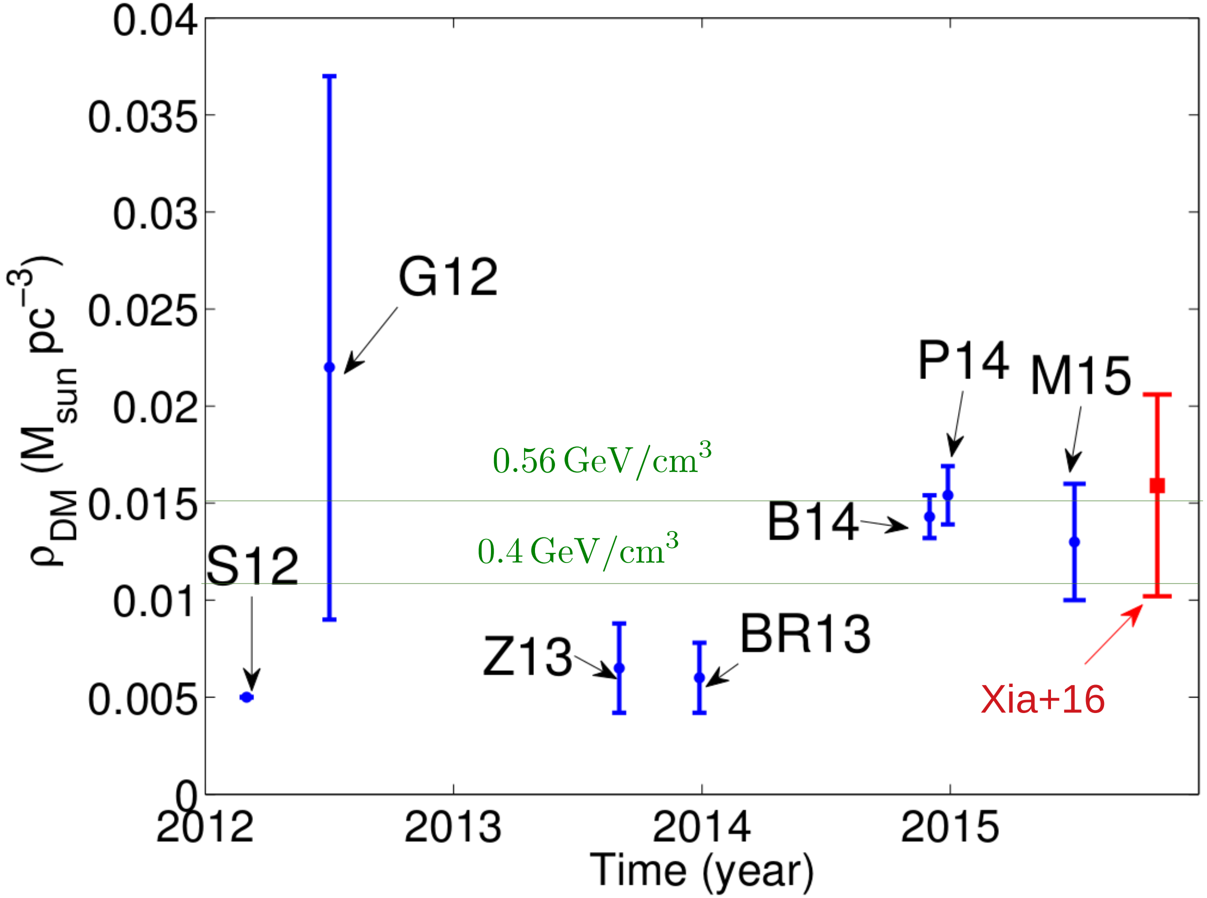
3) Connect Φ with ρ_{DM} $\rightarrow$ connect ν with ρ_{DM}

Previous estimates of $\rho_{DM,\odot}$



[J.I. Read, J.Phys G41 (2014) 063101]

Previous estimates of $\rho_{DM,\odot}$



[Plot from Q. Xia et al., MNRAS 458 (2016) 3839]

[Smith et al., arXiv:1111.6920] LJ
[Garbari et al., arXiv:1206.0015] LJ
[Zhang et al., arXiv:1209.0256] LJ
[Bovy & Rix, arXiv:1309.0809] DF
[Bienaymé et al., arXiv:1406.6896] LJ
[Piffl et al., arXiv:1406.4130] DF
[McKee et al., arXiv:1509.05334] LJ
[Q. Xia et al., MNRAS 458 (2016) 3839]

Vertical Jeans equation method

[J. Buch et al., JCAP 04 (2019) 026]

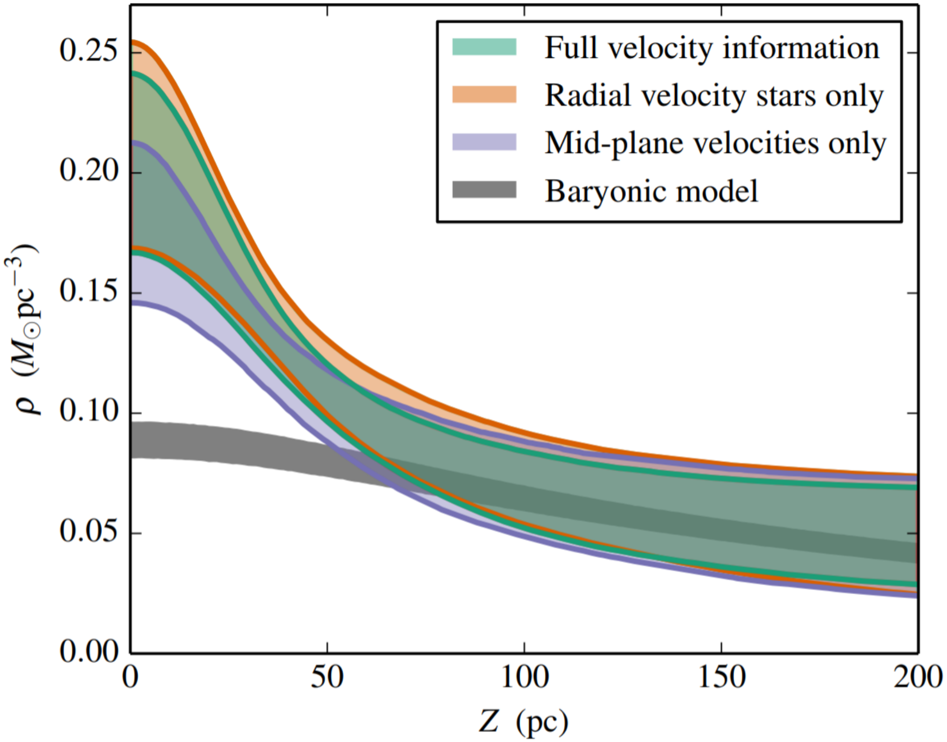
• **Survey**: Gaia DR2 + 2MASS	4445 stars	37707 stars	43332 stars
• **Studied region**:	$R \sim 0.15\,\mathrm{kpc}$	$\lvert z \rvert < 200\,\mathrm{pc}$	
• **Stellar populations**:	A stars	F stars	G stars
• $\rho_{\mathrm{DM},\odot}/(\,\mathrm{GeV}/\mathrm{cm}^3)$	$\mathbf{0.608}^{+0.380}_{-0.380}$	$\mathbf{1.482}^{+0.304}_{-0.304}$	$\mathbf{0.418}^{+0.380}_{-0.342}$

Vertical Jeans equation method

[A. Widmark, A&A 623 (2019) A30]

- **Survey**: Gaia DR2 ~ 8 x 23 000 stars
- **Studied region**: $100\,\mathrm{pc} < r_{\mathrm{from\ Earth}} < 200\,\mathrm{pc}$
- **Stellar populations**: 8 samples with M_G from 3.0–6.3
- If **excess** interpreted in terms of DM, at z=0

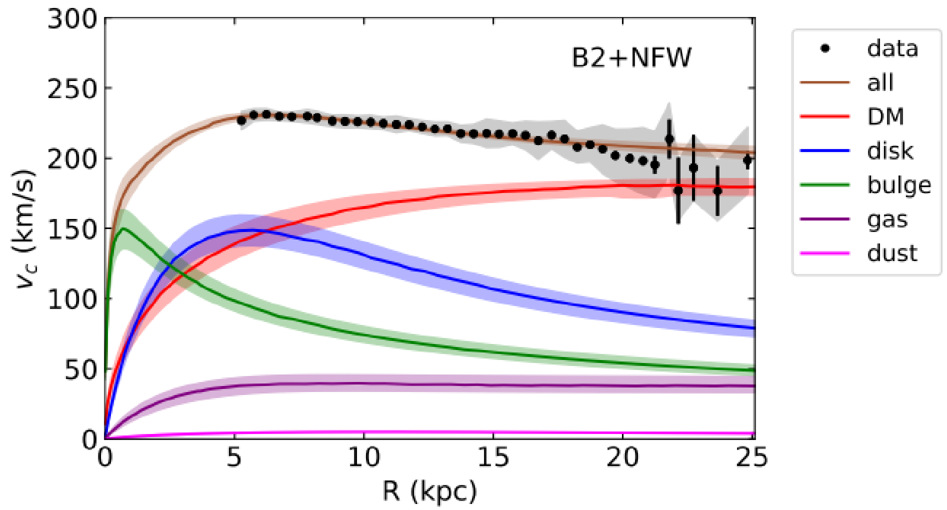
$$\rho_{\mathrm{DM},\odot} \sim 2.2\text{–}5.6\,\mathrm{GeV/cm^3}$$



Rotation curve method

[P.F. de Salas et al., arXiv:1906.06133]

- **Survey**: Gaia DR2 + 2MASS + WISE + APOGEE
  [A.-C. Eilers et al., Astro. J. 871 (2019) 120]
- **Studied region**: $5\,\mathrm{kpc} \leq R \leq 25\,\mathrm{kpc}$
- **Tracer population**: Red-giant stars



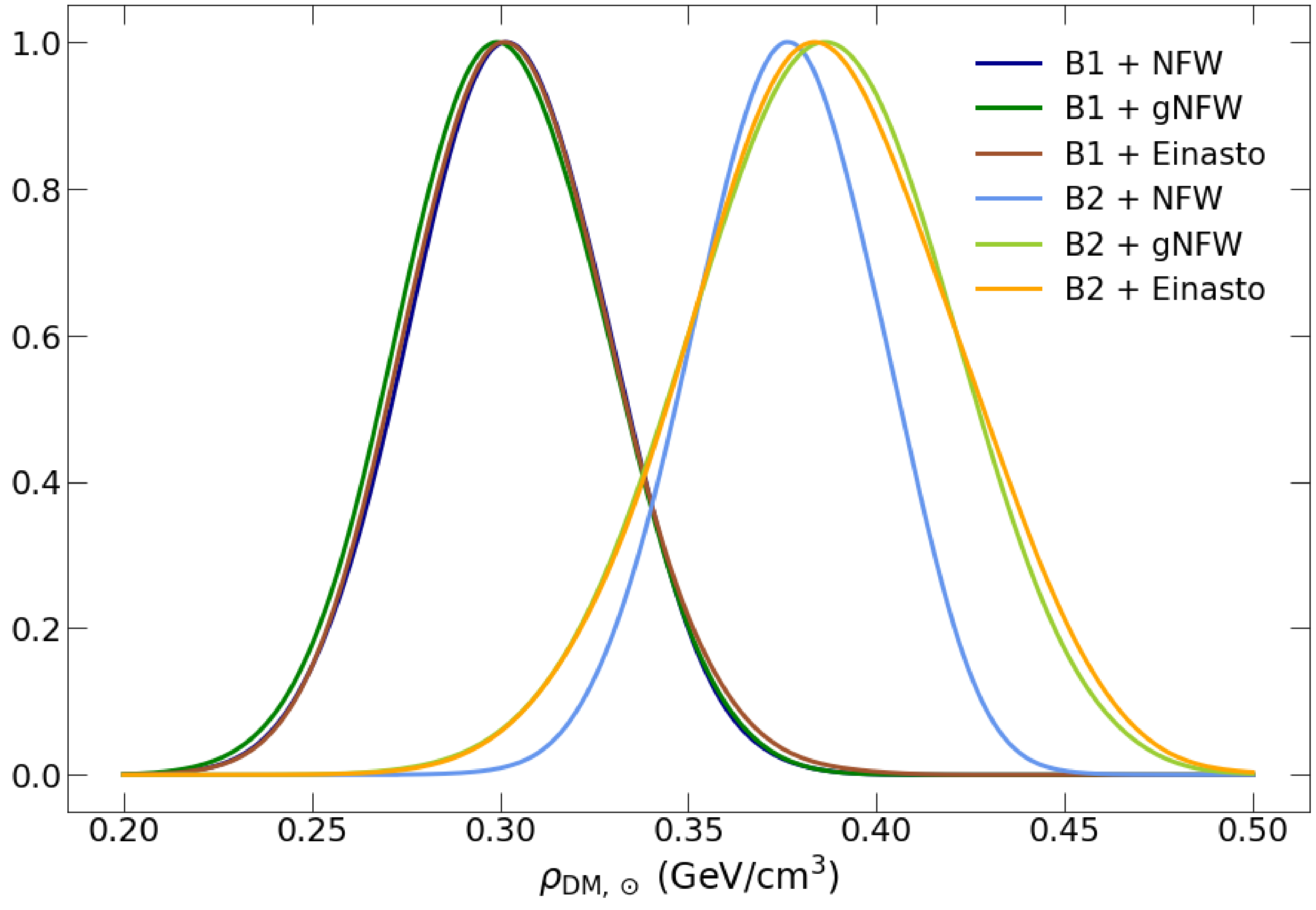
- **Baryonic models**:

Miyamoto-Nagai discs (B1)	Double expon. Discs (B2)
Based on: [E. Pouliasis et al., arXiv:1611.07979]	Based on: [A. Misiriotis et al., A&A 459 (2006) 113]

- $\rho_{\mathrm{DM},\odot}/(\,\mathrm{GeV/cm^3})$ $\mathbf{0.30} \pm 0.03$ $\mathbf{0.38} \pm 0.03$

Rotation curve method

[P.F. de Salas et al., arXiv:1906.06133]



Distribution Function fitting method

- **Jeans' theorem**: The DF of an equilibrium stellar system depends on (**x**,**v**) only through integrals of motion I_i(**x**,**v**)
- Computationally demanding
- *Axisymmetry is not required*

Notice!
-equilibrium approximation

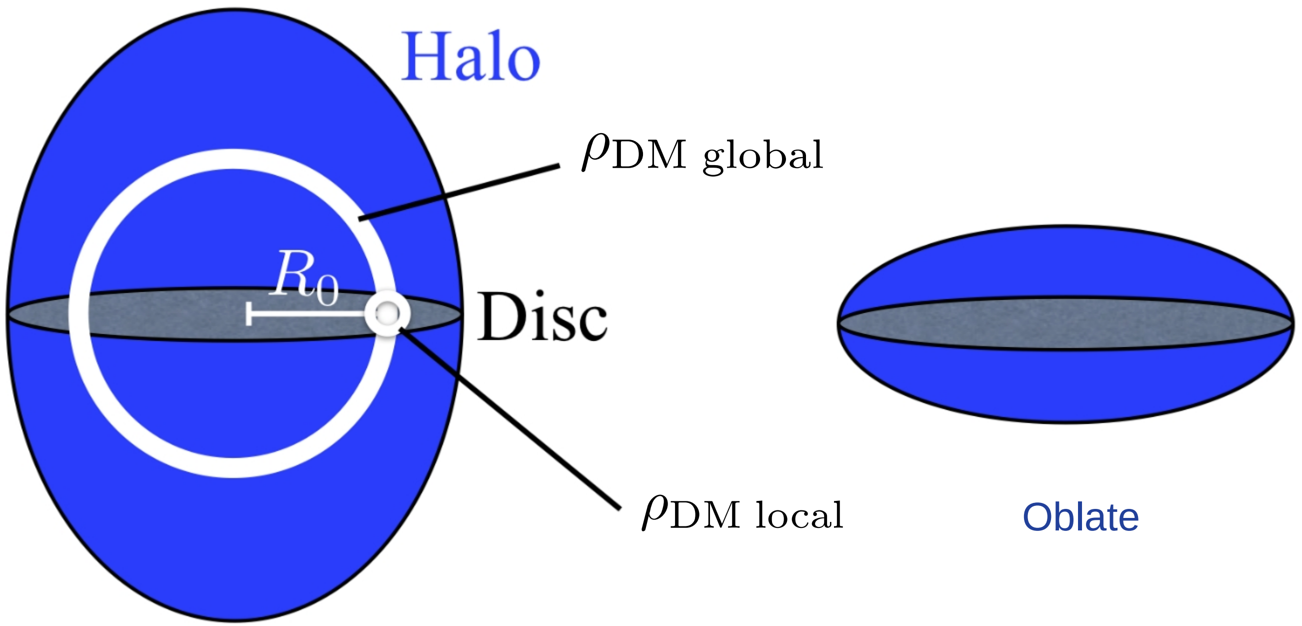
- Choose a multicomponent Galactic potential Φ
- Built a DF *f*(**J**) in terms of convenient constants of motion (actions J_i) for different components
- Fit parameters of Φ and *f*(**J**) to observations

Selected list of recent works on the subject

[J. Binney, arXiv:1207.4910]
[P.J. McMillan et al., arXiv:1303.5660]
[J. Bovy et al., arXiv:1309.0809]
[T. Piffl et al., arXiv:1406.4130]
[J. Binney et al., arXiv:1509.06877]
[D.R. Cole et al., arXiv:1610.07818]
[J. L. Sanders et al., arXiv:1511.08213]
[J. Binney, arXiv:1706.01374]

Methods to estimate $\rho_{DM,\odot}$

- Local and global methods can be complementary
  - Different methods are affected by different systematics and disequilibria effects



[J.I. Read, J. Phys G41 (2014) 063101]

Prolate

$\rho_{\rm DM\ local} < \rho_{\rm DM\ global}$

$\rho_{\rm DM\ local} > \rho_{\rm DM\ global}$

Moment method: Jeans equations

Start from the steady-state collisionless Boltzmann equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \phi = 0$$

Moment method: Jeans equations

Start from the steady-state collisionless Boltzmann equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \cancel{\frac{\partial f}{\partial t}} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \phi = 0$$

Write it in cylindrical coordinates $\{R, \varphi, z\}$

$$v_R \frac{\partial f}{\partial R} + \frac{v_\varphi}{R}\frac{\partial f}{\partial \varphi} + v_z \frac{\partial f}{\partial z} - \left(\frac{\partial \phi}{\partial R} - \frac{v_\varphi^2}{R}\right)\frac{\partial f}{\partial v_R} - \frac{1}{R}\left(v_R v_\varphi + \frac{\partial \phi}{\partial \varphi}\right)\frac{\partial f}{\partial v_\varphi} - \frac{\partial \phi}{\partial z}\frac{\partial f}{\partial v_z} = 0$$

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Multiply by $v_{R,\varphi,z}$ and integrate over all velocities (axisymmetry assumed)

$$\frac{\partial\left(\nu \overline{v_R^2}\right)}{\partial R} + \frac{\partial\left(\nu \overline{v_R v_z}\right)}{\partial z} + \nu\left(\frac{\overline{v_R^2} - \overline{v_\varphi^2}}{R} + \frac{\partial \phi}{\partial R}\right) = 0 \qquad R - \mathrm{Jeans}$$

$$\frac{1}{R^2}\frac{\partial\left(R^2 \nu \overline{v_R v_\varphi}\right)}{\partial R} + \frac{\partial\left(\nu \overline{v_\varphi v_z}\right)}{\partial z} = 0 \qquad \varphi - \mathrm{Jeans}$$

$$\frac{1}{R}\frac{\partial\left(R \nu \overline{v_R v_z}\right)}{\partial R} + \frac{\partial\left(\nu \overline{v_z^2}\right)}{\partial z} + \nu \frac{\partial \phi}{\partial z} = 0 \qquad z - \mathrm{Jeans}$$

Moment method: Jeans equations

Start from the steady-state collisionless Boltzmann equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \phi = 0$$

Write it in cylindrical coordinates $\{R, \varphi, z\}$

$$v_R \frac{\partial f}{\partial R} + \frac{v_\varphi}{R}\frac{\partial f}{\partial \varphi} + v_z \frac{\partial f}{\partial z} - \left(\frac{\partial \phi}{\partial R} - \frac{v_\varphi^2}{R}\right)\frac{\partial f}{\partial v_R} - \frac{1}{R}\left(v_R v_\varphi + \frac{\partial \phi}{\partial \varphi}\right)\frac{\partial f}{\partial v_\varphi} - \frac{\partial \phi}{\partial z}\frac{\partial f}{\partial v_z} = 0$$

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$$\frac{1}{R^2}\frac{\partial\left(R^2\nu\overline{v_R v_\varphi}\right)}{\partial R} + \frac{\partial\left(\nu\overline{v_\varphi v_z}\right)}{\partial z} = 0 \qquad \varphi - \text{Jeans}$$

$$\frac{1}{R}\frac{\partial\left(R\nu\overline{v_R v_z}\right)}{\partial R} + \frac{\partial\left(\nu\overline{v_z^2}\right)}{\partial z} + \nu\frac{\partial \phi}{\partial z} = 0 \qquad z - \text{Jeans}$$

Rotation curve method

- **Circular velocity**

Connection with theoretical ρ_{DM}

$$v_c^2(R) = R \left. \frac{\partial \phi}{\partial R} \right|_{z=0}$$

- ***R*-Jeans equation**

Connection with tracer's observations

$$v_c^2 = \overline{v_\varphi^2} - \overline{v_R^2} - \frac{R}{\nu} \frac{\partial \left(\nu \overline{v_R^2} \right)}{\partial R} - \frac{R}{\nu} \frac{\partial \left(\nu \overline{v_R v_z} \right)}{\partial z}$$

Vertical Jeans equation method

- ***z*-Jeans equation**

$$\underbrace{\frac{1}{R}\frac{\partial\left(R\nu\overline{v_R v_z}\right)}{\partial R}}_{\text{tilt term } \mathcal{T}} + \frac{\partial\left(\nu\overline{v_z^2}\right)}{\partial z} + \nu\frac{\partial\phi}{\partial z} = 0$$

Notice!
-equilibrium approximation
-axisymmetry

- **Poisson equation**

$$\underbrace{\frac{1}{R}\frac{\partial v_c^2(R,z)}{\partial R}}_{\text{rotation curve term } \mathcal{R}} + \frac{\partial^2\phi}{\partial z^2} = 4\pi G\rho$$

Vertical Jeans equation method

- ***z*-Jeans equation**

$$\underbrace{\frac{1}{R}\frac{\partial\left(R\nu\overline{v_R v_z}\right)}{\partial R}}_{\text{tilt term } \mathcal{T}} + \frac{\partial\left(\nu\overline{v_z^2}\right)}{\partial z} + \nu\frac{\partial\phi}{\partial z} = 0$$

Notice!
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Ignoring $\mathcal{T}$ induces a < 10% error [J.I. Read, J. Phys G41 (2014) 063101]

- **Poisson equation**

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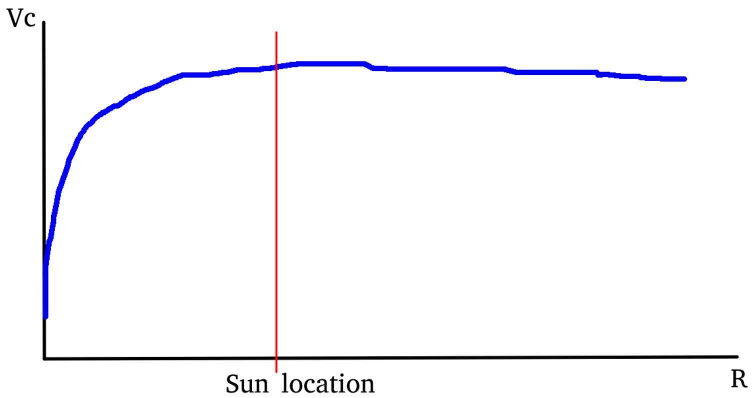
Assuming flat rotation curve at $R_\odot$



Vertical Jeans equation method

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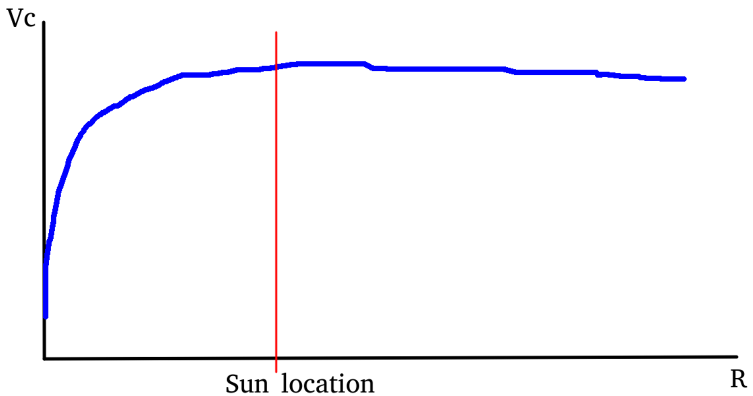


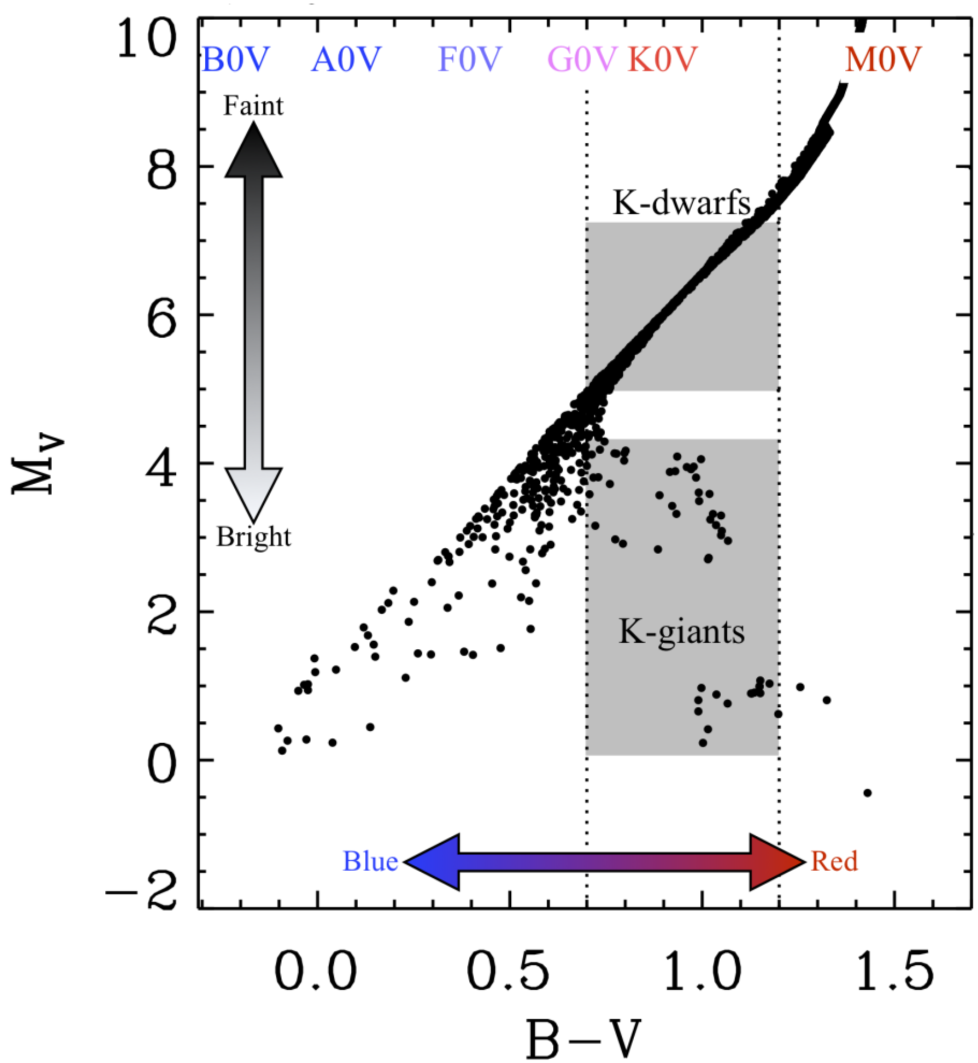
1D *z*-Jeans equation method

[S. Sivertsson et al., MNRAS 478 (2018) 1677]

$[X/Y] \equiv \log_{10} \frac{X}{Y}$ in units of the Solar System

	α-young	α-old
• **Survey**: SDSS-SEGUE G-dwarf		
• **Studied region**: $R \sim 1\,\mathrm{kpc}$	$515\,\mathrm{pc} < z < 1247\,\mathrm{pc}$	$634\,\mathrm{pc} < z < 2266\,\mathrm{pc}$
• **Stellar populations**:	α-young $[\alpha/\mathrm{Fe}] < 0.2$ $-0.5 < [\mathrm{Fe/H}]$	α-old $0.3 < [\alpha/\mathrm{Fe}]$ $-1.2 < [\mathrm{Fe/H}] < -0.3$
• $\rho_{\mathrm{DM},\odot}$	$\mathbf{0.46}^{+0.07}_{-0.09}\ \mathrm{GeV/cm^3}$	$\mathbf{0.73}^{+0.06}_{-0.05}\ \mathrm{GeV/cm^3}$

From bright to not so bright jargon



[J.I. Read, J. Phys G41 (2014) 063101]

$$M_V \equiv -2.5\log_{10}\left(L_V/L_\odot\right) + 4.83$$

V waveband centred on $\lambda = 550\,\mathrm{nm}$

Photometric system

Filter Letter	Effective Wavelength Midpoint λ_{eff} for Standard Filter[2]	Full Width Half Maximum[2] (Bandwidth $\Delta\lambda$)	Variant(s)	Description
Ultraviolet				
U	365 nm	66 nm	u, u', u*	"U" stands for ultraviolet.
Visible				
B	445 nm	94 nm	b	"B" stands for blue.
V	551 nm	88 nm	v, v'	"V" stands for visual.
G[3]	464 nm	128 nm	g'	"G" stands for green.
R	658 nm	138 nm	r, r', R', R_c, R_e, R_j	"R" stands for red.
Near-Infrared				
I	806 nm	149 nm	i, i', I_c, I_e, I_j	"I" stands for infrared.
Z	900 nm[4]		z, z'	
Y	1020 nm	120 nm	y	
J	1220 nm	213 nm	J', J_s	
H	1630 nm	307 nm		
K	2190 nm	390 nm	K Continuum, K', K_s, K_{long}, K^8, nbK	
L	3450 nm	472 nm	L', nbL'	
Mid-Infrared				
M	4750 nm	460 nm	M', nbM	
N	10500 nm	2500 nm		
Q	21000 nm[5]	5800 nm[5]	Q'	

Source: Wikipedia