Gauged FN

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Understanding fermion masses with a gauged Froggatt-Nielsen model

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Based on work together with Johan Rathsman

Introduction

Discovery of Higgs boson $\Rightarrow$ particle content of Standard Model complete

Still SM not complete description of nature:

Problems within SM

- no explanation of neutrino masses (no ν_R)
- many free parameters with huge hierarchies ($m_e/m_t \sim 10^{-6}$)
- fine-tuning of Higgs mass compared to Planck mass
- absence of CP-violation in strong interactions ($\theta_s < 10^{-11}$)
- Higgs too heavy for electroweak baryogenesis

Problems outside SM

- no dark matter in SM
- quantum theory of gravity

Need physics beyond the SM

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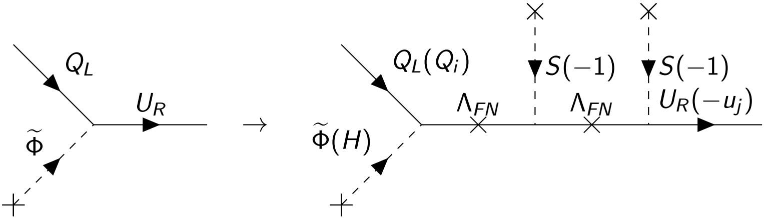
Froggatt-Nielsen model

New **flavon** symmetry (charge) to understand hierarchies of fermion masses and mixings

Yukawa interaction only an effective vertex



S scalar charged (-1) under new symmetry

vector-like Froggatt-Nielsen fermion: mass Λ_{FN}, same SM charges as corresponding SM fermions

each insertion gives a suppression $\dfrac{\langle S \rangle}{\Lambda_{FN}}$

flavon charge conservation $\Rightarrow$ Yukawa coupling

$$(Y^U)_{ij} \sim (g^U)_{ij} \left(\frac{\langle S \rangle}{\Lambda_{FN}} \right)^{|Q_i + u_j + H|}, \text{(FN)} \ \text{assume} \, (g^U)_{ij} = \mathcal{O}(1)$$

SM Yukawa Lagrangian with neutrino masses:

$$-\mathcal{L}_Y = \overline{Q}_L \tilde{\Phi} Y^U U_R + \overline{Q}_L \Phi Y^D D_R + \overline{L}_L \Phi Y^L E_R + \overline{L}_L \tilde{\Phi} Y^N N_R + \mathrm{H.c.}$$

Mass matrices:

$$M_{ij}^F = \langle \Phi \rangle_0 \, Y_{ij}^F = \langle \Phi \rangle_0 \, g_{ij}^F \left(\frac{\langle S \rangle}{\Lambda_{FN}} \right)^{|n_{ij}^F|}, \quad F = U, D, L, N$$

Left-handed fermion fields: $Q_L^i, (U_R^i)^c, (D_R^i)^c, L_L^i, (E_R^i)^c, (N_R^i)^c$
Higgs fields: Φ

Denote flavon charges by $Q_i, u_i, d_i, L_i, e_i, \nu_i$ and H

$$n_{ij}^U = Q_i + u_j + H, \qquad n_{ij}^N = L_i + \nu_j + H$$

$$n_{ij}^D = Q_i + d_j - H, \qquad n_{ij}^L = L_i + e_j - H$$

Bi-unitary transformations gives diagonal mass-matrices, $D^U, \ D^D$

$$\begin{aligned} Y^U &= (V_L^U)^\dagger D^U V_R^U \\ Y^D &= (V_L^D)^\dagger D^D V_R^D \end{aligned}$$

Froggatt-Nielsen procedure requires ordering of exponents

$$|Q_i + u_j + H| \geq |Q_{i+1} + u_j + H| \geq |Q_{i+2} + u_j + H| \, ,$$
$$|Q_i + d_j - H| \geq |Q_{i+1} + d_j - H| \geq |Q_{i+2} + d_j - H| \, .$$

then

$$(V_L^U)_{ij} \sim \epsilon^{|Q_i - Q_j|}, \qquad (V_R^U)_{ij} \sim \epsilon^{|u_i - u_j|}$$
$$(V_L^D)_{ij} \sim \epsilon^{|Q_i - Q_j|}, \qquad (V_R^D)_{ij} \sim \epsilon^{|d_i - d_j|}$$

diagonal elements of mass matrices same as diagonal entries of Y

$$(D^U)_{ii} \sim \epsilon^{|Q_i + u_i + H|} \quad (D^D)_{ii} \sim \epsilon^{|Q_i + d_i - H|}$$

Finally CKM-matrix given by

$$(V_{CKM})_{ij} = (V_L^U)_{ik} (V_L^{D \ \dagger})_{kj} \sim \epsilon^{|Q_i - Q_j|}$$

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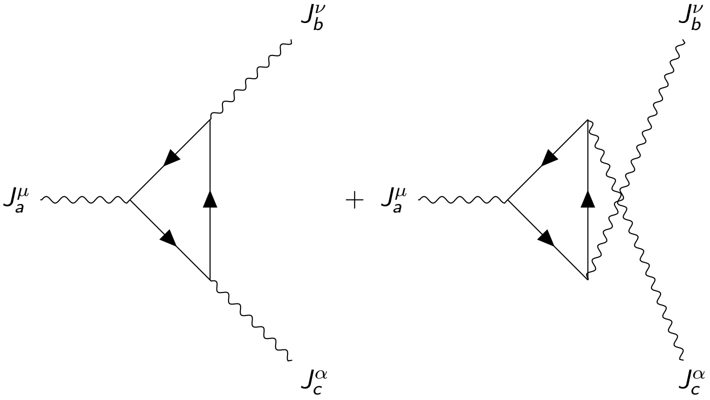
Anomalies

Breaks symmetries of classical Lagrangian at quantum level



Anomaly constraints $\mathcal{A}_{XYZ} = \frac{1}{2}\mathrm{tr}[T_X\{T_Y, T_Z\}]$

T_X generators of gauge group X in fundamental representation

hypercharge normalization $Y = 2(Q - T_3)$

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ plus gravity

non-trivial anomaly constraints involving the $U(1)'$-charges

$$
\begin{aligned}
\mathcal{A}_{11'1'} &= 2\sum_{j=1}^{3}\left(Q_j^2 - 2u_j^2 + d_j^2 - L_j^2 + e_j^2\right) = 0 \\
\mathcal{A}_{111'} &= \frac{2}{3}\sum_{j=1}^{3}\left(Q_j + 8u_j + 2d_j + 3L_j + 6e_j\right) = 0 \\
\mathcal{A}_{331'} &= \frac{1}{2}\sum_{j=1}^{3}\left(2Q_j + u_j + d_j\right) = 0 \\
\mathcal{A}_{221'} &= \frac{1}{2}\sum_{j=1}^{3}\left(3Q_j + L_j\right) = 0 \\
\mathcal{A}_{1'1'1'} &= \sum_{j=1}^{3}\left(6Q_j^3 + 3u_j^3 + 3d_j^3 + 2L_j^3 + e_j^3 + \nu_j^3\right) = 0 \\
\mathcal{A}_{gg1'} &= \sum_{j=1}^{3}\left(6Q_j + 3u_j + 3d_j + 2L_j + e_j + \nu_j\right) = 0
\end{aligned}
$$

Sum rules relate mass constraints and anomalies

$$
\begin{aligned}
n^D_{11} + n^D_{22} + n^D_{33} - n^L_{11} - n^L_{22} - n^L_{33} &= \sum_{j=1}^{3} (Q_j + d_j - L_j - e_j) = \\
&= \frac{8}{3}\mathcal{A}_{331'} - \frac{1}{4}\mathcal{A}_{111'} - \mathcal{A}_{221'} = 0
\end{aligned}
$$

$$
\begin{aligned}
n^U_{11} + n^U_{22} + n^U_{33} + n^L_{11} + n^L_{22} + n^L_{33} &= \sum_{j=1}^{3} (Q_j + u_j + L_j + e_j) = \\
&= -\frac{2}{3}\mathcal{A}_{331'} + \frac{1}{4}\mathcal{A}_{111'} + \mathcal{A}_{221'} = 0
\end{aligned}
$$

$$
\begin{aligned}
n^U_{11} + n^U_{22} + n^U_{33} + n^D_{11} + n^D_{22} + n^D_{33} &= \sum_{j=1}^{3} (2Q_j + u_j + d_j) = \\
&= 2\mathcal{A}_{331'} = 0
\end{aligned}
$$

$$\sum_{i=1}^{3}(n_{ii}^{U}+n_{ii}^{D})=2\mathcal{A}_{33F}=0$$
$$\sum_{i=1}^{3}(n_{ii}^{D}-n_{ii}^{L})=\frac{8}{3}\mathcal{A}_{33F}-\frac{1}{4}\mathcal{A}_{11F}-\mathcal{A}_{22F}=0$$
$$\sum_{i=1}^{3}(n_{ii}^{L}+n_{ii}^{N})=\mathcal{A}_{ggF}-6\mathcal{A}_{33F}=0$$

SM at 100 TeV

$$\begin{array}{lll} |n_{11}^{U}|=8, & |n_{22}^{U}|=4, & |n_{33}^{U}|=0, \\ |n_{11}^{D}|=7, & |n_{22}^{D}|=5, & |n_{33}^{D}|=3, \\ |n_{11}^{L}|=8, & |n_{22}^{L}|=5, & |n_{33}^{L}|=3 \end{array}$$

The sum rules are NOT satisfied!

One way to satisfy the sum rules is to choose $|n_{22}^{L}|=4$, thus $g_{22}^{L}\approx=0.378$ and $|n_{11}^{U}|=19$. However, it is still impossible to find rational flavon charges, this can be proved by triangularization.

Model Example

2HDM, exact type-II $\mathbb{Z}_2$ symmetry, ALL fermions (also neutrinos) have Dirac masses

2HDM at 100 TeV:

$$Y^U \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}, \; Y^D \sim \begin{pmatrix} \epsilon^7 & \epsilon^6 & \epsilon^5 \\ \epsilon^6 & \epsilon^5 & \epsilon^4 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \end{pmatrix}, \; Y^L \sim \begin{pmatrix} \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^4 & \epsilon^2 \end{pmatrix}$$

$$Y^N \sim \begin{pmatrix} \epsilon^a & \epsilon^{19} & \epsilon^{18} \\ \epsilon^a & \epsilon^{19} & \epsilon^{18} \\ \epsilon^a & \epsilon^{19} & \epsilon^{18} \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \; U_{PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Assume normal neutrino hierarchy with the lightest neutrino mass left undetermined

$$(Q_1 - Q_2)^2 = 1, \quad (Q_2 - Q_3)^2 = 4, \quad (Q_1 - Q_3)^2 = 9,$$
$$(Q_1 + u_1 + H_2)^2 = 8^2, \quad (Q_2 + u_2 + H_2)^2 = 4^2, \quad Q_3 + u_3 + H_2 = 0,$$
$$(Q_1 + d_1 - H_1)^2 = 7^2, \quad (Q_2 + d_2 - H_1)^2 = 5^2, \quad (Q_3 + d_3 - H_1)^2 = 2^2,$$
$$(L_1 + e_1 - H_1)^2 = 8^2, \quad (L_2 + e_2 - H_1)^2 = 4^2, \quad (L_3 + e_3 - H_1)^2 = 2^2,$$
$$(L_1 + \nu_1 + H_2)^2 = a^2, \quad (L_2 + \nu_2 + H_2)^2 = 19^2, \quad (L_3 + \nu_3 + H_2)^2 = 18^2,$$
$$L_2 - L_3 = 0, \qquad L_1 - L_2 = 0,$$
$$\mathcal{A}_{1FF} = 0, \quad \mathcal{A}_{11F} = 0, \quad \mathcal{A}_{33F} = 0, \quad \mathcal{A}_{22F} = 0, \quad \mathcal{A}_{FFF} = 0, \quad \mathcal{A}_{ggF} = 0,$$
$$(Q_1 + u_3 + H_2)^2 \geq (Q_2 + u_3 + H_2)^2, \quad (Q_2 + u_3 + H_2)^2 \geq (Q_3 + u_3 + H_2)^2,$$
$$(Q_1 + u_2 + H_2)^2 \geq (Q_2 + u_2 + H_2)^2, \quad (Q_2 + u_2 + H_2)^2 \geq (Q_3 + u_2 + H_2)^2,$$
$$(Q_1 + d_3 - H_1)^2 \geq (Q_2 + d_3 - H_1)^2, \quad (Q_2 + d_3 - H_1)^2 \geq (Q_3 + d_3 - H_1)^2,$$
$$(Q_1 + d_2 - H_1)^2 \geq (Q_2 + d_2 - H_1)^2, \quad (Q_2 + d_2 - H_1)^2 \geq (Q_3 + d_2 - H_1)^2,$$
$$(L_1 + e_2 - H_1)^2 \geq (L_2 + e_2 - H_1)^2, \quad (L_1 + e_3 - H_1)^2 \geq (L_2 + e_3 - H_1)^2$$

This semi-algebraic system can be solved with real triangularization.

Two values of $|a|$: 25 and 49, i.e.

$$m_{\nu_1} \sim \epsilon^{25} \cdot 174 \text{ GeV} \approx 5.8 \cdot 10^{-7} \text{ eV}$$

$$m_{\nu_1} \sim \epsilon^{49} \cdot 174 \text{ GeV} \approx 9.8 \cdot 10^{-24} \text{ eV}.$$

For $|a| = 25$:

$$Q_1 = \frac{82111}{51651} + \frac{1}{3}H_2, \quad Q_2 = \frac{30460}{51651} + \frac{1}{3}H_2, \quad Q_3 = -\frac{72842}{51651} + \frac{1}{3}H_2,$$

$$u_1 = \frac{331097}{51651} - \frac{4}{3}H_2, \quad u_2 = \frac{176144}{51651} - \frac{4}{3}H_2, \quad u_3 = \frac{72842}{51651} - \frac{4}{3}H_2,$$

$$d_1 = -\frac{168196}{51651} + \frac{2}{3}H_2, \quad d_2 = -\frac{219847}{51651} + \frac{2}{3}H_2, \quad d_3 = -\frac{271498}{51651} + \frac{2}{3}H_2$$

$$L_1 = L_3, \quad L_2 = L_3, \quad L_3 = -\frac{13243}{17217} - H_2,$$

$$e_1 = \frac{1765}{17217} + 2H_2, \quad e_2 = -\frac{67103}{17217} + 2H_2, \quad e_3 = -\frac{101537}{17217} + 2H_2,$$

$$\nu_1 = -\frac{417182}{17217}, \quad \nu_2 = \frac{340366}{17217}, \quad \nu_3 = \frac{323149}{17217},$$

$$H_1 = H_2 - \frac{26}{3}, \quad H_2 \in \mathbb{Q}$$

H_2 free parameter

We may fix H_2 by e.g. removing mixing between $U(1)_Y$ and $U(1)'$

$$\sum_{j=1}^{3}(2Q_j - 4u_j + 2d_j - 2L_j + 2e_j) = 0$$

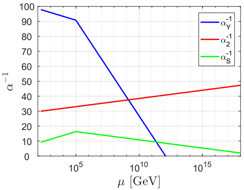
This gives the final solution

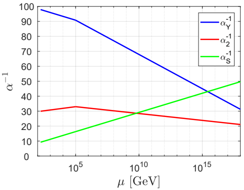
$$\begin{aligned}
&Q_1 = 196891/86085, && Q_2 = 110806/86085, && Q_3 = \tfrac{-61364}{86085},\\
&u_1 = 311671/86085, && u_2 = 53416/86085, && u_3 = \tfrac{-118754}{86085},\\
&d_1 = -53416/28695, && d_2 = -82111/28695, && d_3 = \tfrac{-110806}{28695},\\
&L_1 = -82111/28695, && L_2 = -82111/28695, && L_3 = \tfrac{-82111}{28695},\\
&e_1 = 369061/86085, && e_2 = 24721/86085, && e_3 = \tfrac{-147449}{86085},\\
&\nu_1 = -417182/17217, && \nu_2 = 340366/17217, && \nu_3 = \tfrac{323149}{17217},\\
&H_1 = -565952/86085, && H_2 = 180118/86085.
\end{aligned}$$

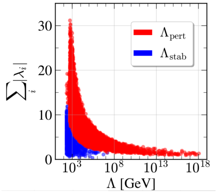
$$H_2 - H_1 = \frac{26}{3}$$

the $\mathbb{Z}_2$ symmetry is a residual effect from the $U(1)'$ symmetry

UV completion







Left: vector-like fermions produces a Landau pole

Midle: scalar completion from Bijnens and Wetterich (click me)

Right: Summed magnitudes of quartic couplings and break-down scales for 2HDM (click me)

Conclusions

- Fermions masses and mixings can be understood in gauged Froggatt-Nielsen model
- Flavon charges constrained by masses and mixings as well as anomalies
- Real triangularization powerful way to find the charges
- All masses and mixings are explained in a 2HDM with Dirac neutrinos
- UV may still be difficult, even with scalar completion, more work is needed