QCD multiplet bases with arbitrary parton ordering

Johan Thorén

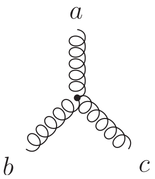
Coauthor: Malin Sjödahl
arXiv:1809.05002

October 17, 2018

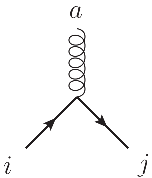
Section 1

Color structures in QCD

- The symmetry of QCD is $SU(3)$.
- The color structures parts of the Feynman rules are

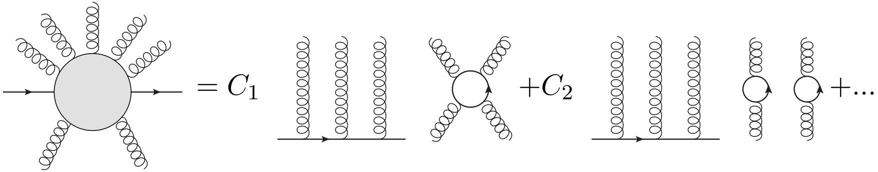


$$\propto i\, f^{abc}$$



$$\propto (t^a)^i_j$$

Trace bases



- Simple algorithm to reduce any color structure.
- Non-orthogonal and over-complete!
- Number of vectors grows factorially in the number of external gluons, N_g, plus external $\overline{q}q$-pairs, $N_{q\overline{q}}$: $\sim (N_g + N_{q\overline{q}})!$
- Squaring the amplitude gives factorial square scaling due to the non-orthogonality.

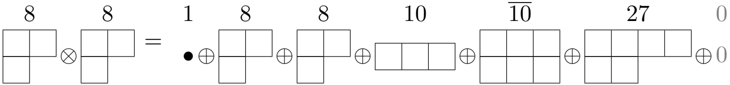
Section 2

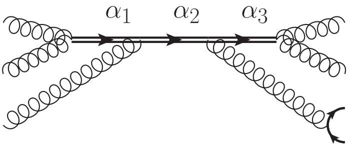
Multiplet bases

Existing multiplet bases

- Orthogonal and minimal basis (a general construction of this is a recently solved problem).

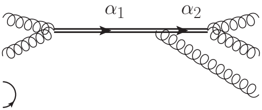
  Keppeler and Sjödahl, JHEP 1209, arXiv:1207.0609
- Number of vectors grow exponentially, not factorially.
- Orthogonality makes squaring easier.
- Downside is that decomposition and recursion are not as straightforward as with traditional bases.
- In multiplet bases, partons are combined into irreducible representations, for example, for $gg \rightarrow gg$ the irreducible representations are:

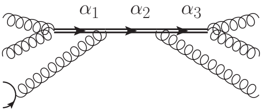




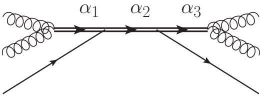
New multiplet bases

In the existing multiplet bases quarks are handled by combining the quarks and antiquarks into pairs, which can then be in either a singlet or an octet since $3 \otimes \overline{3} = 1 \oplus 8$,

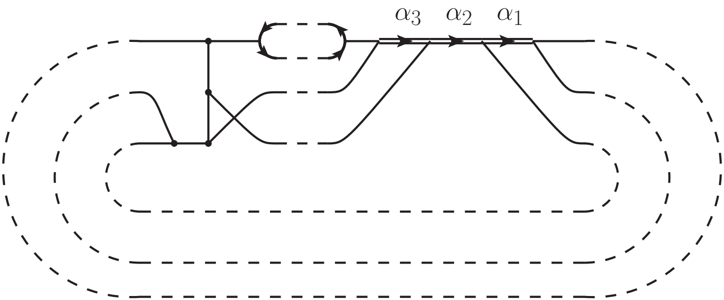




We want to construct multiplet bases where this is not required anymore,

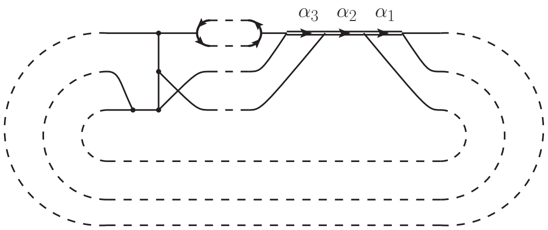


- In QCD we are interested in color summed/averaged quantities.
- Evaluating scalar products of color structures is equivalent to evaluating vacuum bubbles.
- Decomposition into the multiplet basis is equivalent to determining the scalar product between the basis vector and the color structure

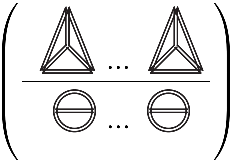


- Decompositions are evaluated by successively removing loops in exchange for sums over simpler color structures.

The result of such a procedure will be



$$= \sum_{\psi_1} \sum_{\psi_2} \cdots \sum_{\psi_n}$$



,

where the vacuum bubbles on the right are called Wigner $3j$ (numerator) and $6j$ (denominator) coefficients.

Section 3

Construction of projectors

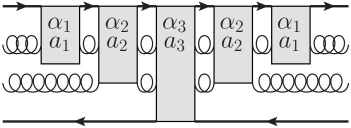
Projectors

The way we construct and use the multiplet basis is:

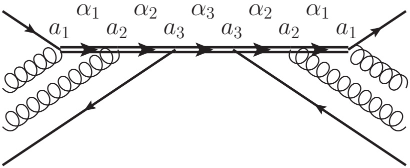
- Construct projectors onto the subspaces of the color space.
- Use the projectors to construct basis vectors.
- Use the basis vectors to calculate Wigner coefficients.
- Color structures are evaluated by using the Wigner coefficients.

Construction

A projector is similar to a basis vector



$\Leftrightarrow$



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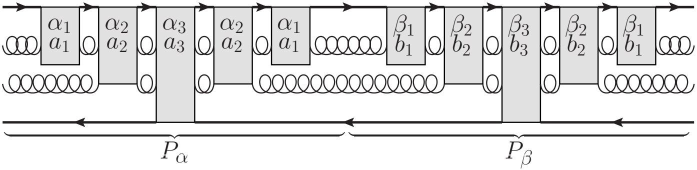
Projectors are idempotent, i.e. $P_\alpha P_\alpha = P_\alpha$, transversal

$P_\alpha P_\beta =$

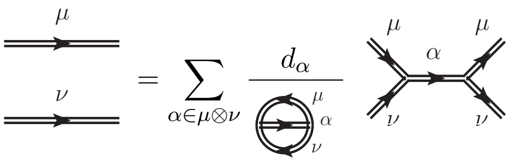


$= \delta_{\alpha\beta} P_\alpha,$

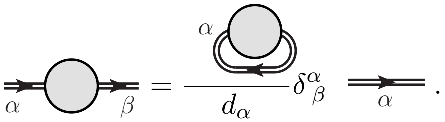
and their trace is $\mathrm{Tr}(P_\alpha) = d_\alpha$.

Necessary relations

We need two group theoretical relations, the completeness relation



and Schur's lemma,

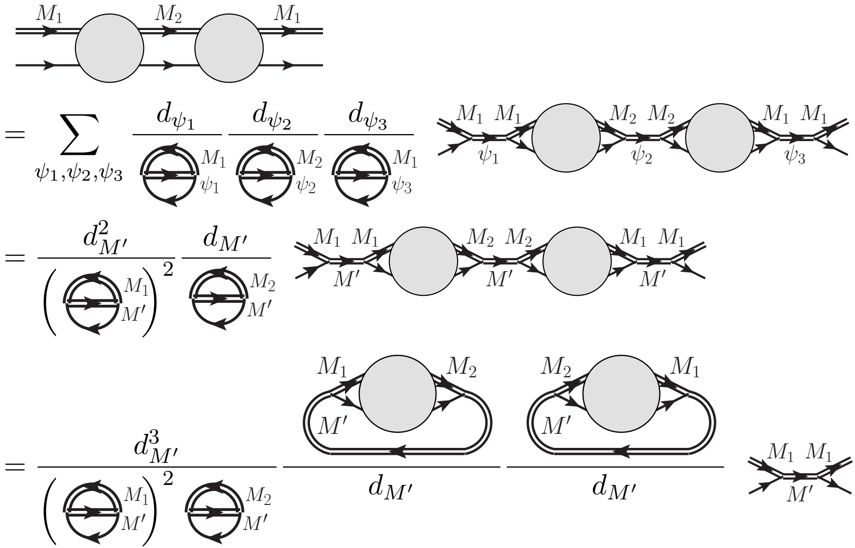


Constructing projectors

- Our construction is recursive. We assume that the projectors for N_p partons have been constructed and we want to construct the projectors for N_p partons and one quark (antiquark).
- The key observation was that the tensor products of $q \otimes M_1$ and $q \otimes M_2$ can share at most one representation if $M_1 \neq M_2$.

$$q \otimes M_1 = M' \oplus \dots$$
$$q \otimes M_2 = M' \oplus \dots$$

Constructing projectors



$= cP_{M'}$.

Section 4

Conclusions

Conclusions

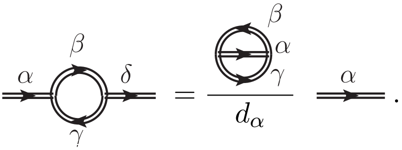
- The multiplet bases are minimal and orthogonal.
- We can construct multiplet basis vectors for any ordering of the quarks, antiquarks and gluons.
- This is useful whenever the order matters, for example:
  - Decomposing color structures with quarks and antiquarks into a multiplet basis.
  - Recursion relations with quarks and antiquarks.
  - Parton showers (as in Malin Sjödahls talk).
- We have constructed the basis vectors for up to 8 quarks and antiquarks, and Wigner $6j$ coefficients that can be calculated from them.

Section 5

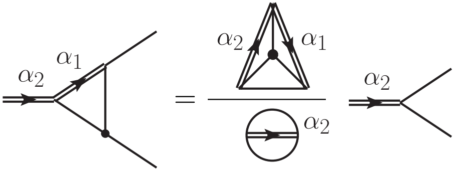
Extra slides

Contracting loops

Two-vertex loops gives a factor



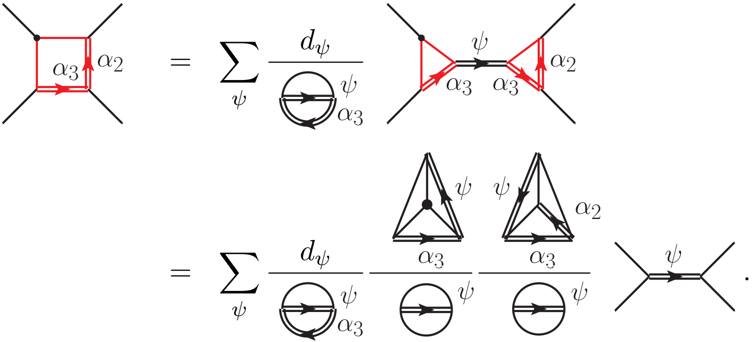
Vertex corrections also only gives a factor



with a possible sum over vertices.

Contracting loops

A 4-vertex loop is less trivial than a vertex correction



The pattern continues, with n-vertex loops having $n-3$ sums over representations. Hence, contracting shorter loops is more efficient.

Contracting loops

If a quark was present, the existing multiplet basis would not allow certain contractions,