Finite-size effects on the leading electromagnetic corrections to the hadronic vaccuum polarisation

Nils Hermansson Truedsson

Lund University

Department of Astronomy and Theoretical Physics

J. Bijnens (supervisor), T. Janowski, A. Portelli, P. A. Boyle, J. Harrison, A. Jüttner

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Overview

1. Why is the $(g-2)_\mu$ interesting?
2. Results and implications
3. Conclusions

The anomalous magnetic moment of the muon

- Gyromagnetic ratio g_μ defined through $\mathbf{M} = g_\mu (e/2m_\mu)\mathbf{S}$
- Anomalous magnetic moment a_μ

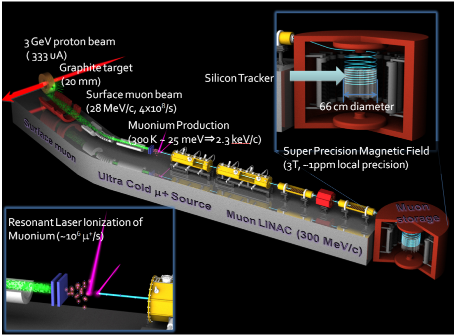
$$a_\mu = \frac{g_\mu - 2}{2}$$

- Deviation from Dirac prediction $g_\mu = 2$
- Latest experimental value is (BNL)

$$a_\mu = 11\,659\,208.9(5.4)_{\mathrm{stat}}(3.3)_{\mathrm{sys}} \cdot 10^{-10}\ [0.54\,\mathrm{ppm}]$$

- Roughly 3σ deviation between experiment and theory
- Aim to improve error by factor of four to 0.14 ppm

The anomalous magnetic moment of the muon



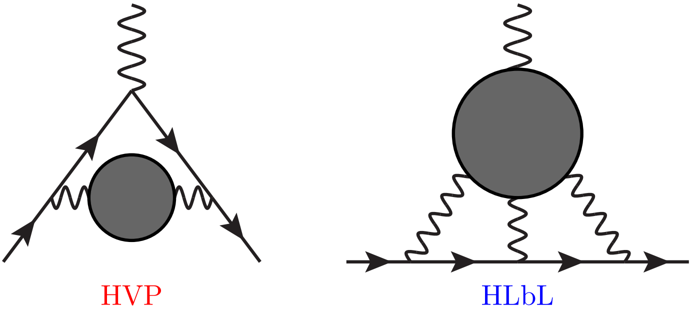
J-PARC

Fermilab

On the theoretical side

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}},$$

$$a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$



- The hadronic vacuum polarisation (HVP) can be calculated in various ways

HVP

- Vector 2-point function

$$\Pi^{\mu\nu}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|\, T\left(j^\mu(x) j^{\nu\,\dagger}(0)\right)|0\rangle$$

$$\Pi^{\mu\nu}(q^2) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi(q^2)$$

$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

- Dispersion relations: $\mathrm{Im}\,\hat{\Pi} \sim \sigma\left(e^+e^- \to \mathrm{hadrons}\right)$
- Lattice is competitive at the same accuracy

Finite volume corrections

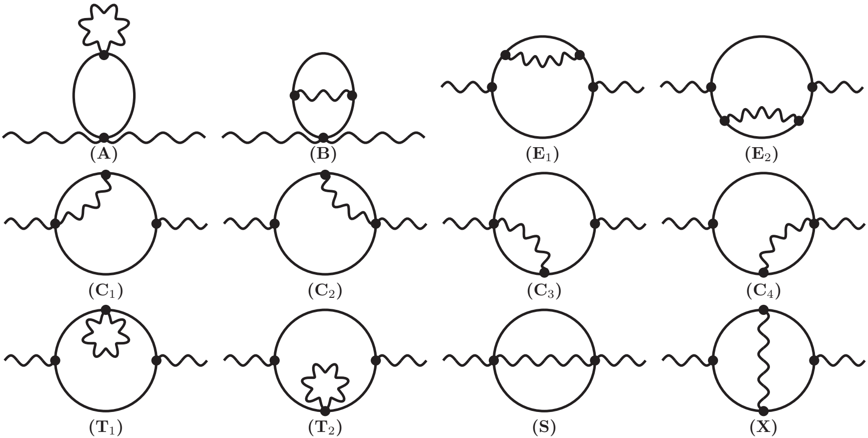
- Lattice is done in finite volume
- Produces finite volume effects depending on lattice size L
- Depend on particle properties

$$\text{Massive particles:} \quad e^{-mL}$$

$$\text{Massless particles:} \quad \frac{1}{L^a}$$

- Electromagnetic corrections to the HVP potentially dangerous
- How big are the effects? $a \geq 2$ but what is it really?

Diagrams at NLO



- Scalar QED: Pure pion loops
- Compare with lattice simulations and vegas MC evaluation of loop integrals in lattice perturbation theory

Actual calculation

- 2-loop integrals

$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4\ell}{(2\pi)^4}\tilde{\pi}\left(q^2,k,\ell\right)$$

- Do energy integrals analytically
- On the lattice momenta are discretised: $\mathbf{k}=(2\pi/L)\mathbf{n}$
- Taylow expand in $1/L$
- Want the finite volume corrections, so for each diagram U we need to calculate

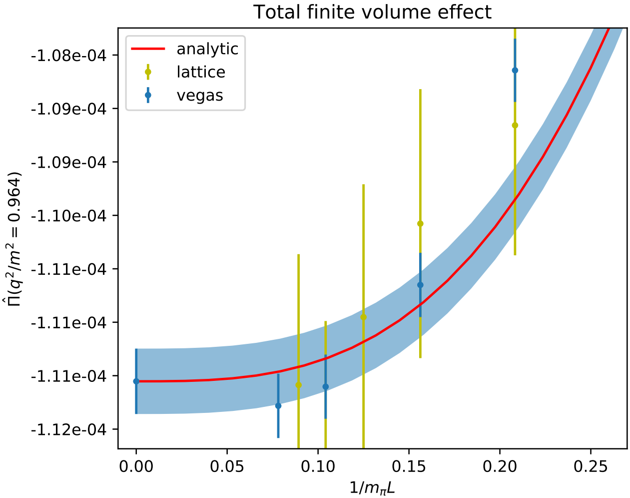
$$\Delta\hat{\Pi}_U(q^2)=\left(\sum_{\mathbf{n}}{}' - \int d^3\mathbf{n}\right)\int d^3\ell\,\hat{\pi}_U(q^2,\mathbf{n})+\mathcal{O}\left(e^{-m_\pi L}\right)$$

Finite volume corrections

$$\Delta\hat{\Pi}(q^2) = \frac{c_0}{m_\pi^3 L^3}\left(-\frac{16}{3}\Omega_{0,3} - \frac{5}{3}\Omega_{2,2} + \frac{40}{9}\Omega_{2,3} - \frac{3}{8}\Omega_{4,1} + \frac{7}{6}\Omega_{4,2} + \frac{8}{9}\Omega_{4,3}\right) + \mathcal{O}\left(\frac{1}{L^4}, e^{-m_\pi L}\right)$$

- Universality: adding form factors yields same result
- Leading order is $1/L^3$: Neutral current and photon far away sees no charge – dipole effect
- EM FV small for ordinary lattice sizes
- c_0 depends on lattice formulation of QED

Diagrams



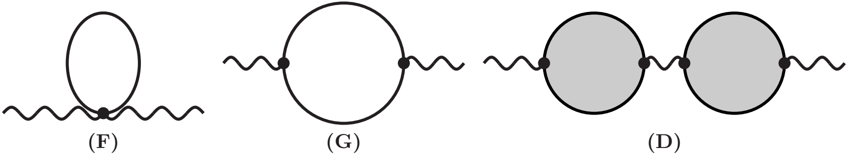
Total finite volume effect
analytic
lattice
vegas
-1.08e-04
-1.09e-04
-1.09e-04
-1.10e-04
-1.11e-04
-1.11e-04
-1.11e-04
-1.12e-04
0.00
0.05
0.10
0.15
0.20
0.25
$1/m_\pi L$
$\hat{\Pi}(q^2/m^2 = 0.964)$

Conclusions

- Finite volume corrections smaller than anticipated: $c_0/(m_\pi L)^3$
- Need not worry about these corrections at normal lattice sizes
- Hadronic vacuum polarisation can be evaluated entirely on the lattice

Back-up slides

LO and NLO disconnected diagrams



Charged currents

$$\Delta\hat{\Pi}_{\text{charged}}\left(q^2\right) = \frac{1}{m^3L^3}\left(-\frac{13}{24}\Omega_{2,2}+\frac{20}{9}\Omega_{2,3}-\frac{15}{64}\Omega_{4,1}+\frac{7}{24}\Omega_{4,2}+\frac{4}{9}\Omega_{4,3}\right)$$
$$+\frac{c_1}{m^2L^2\pi}\left(-\frac{8}{3}\Omega_{-1,3}+\Omega_{1,2}+\frac{8}{3}\Omega_{1,3}+\frac{1}{8}\Omega_{3,1}\right)$$
$$+\mathcal{O}\left(\frac{1}{L^4},e^{-mL}\right)$$

Lattice numbers

- Time extent $T = 128$
- Eight different volumes with L between 16 and 64
- $am_0 = 0.2$
- $q^2 = 0.964\, m_0^2$
- 40 000 photon field configurations for $L \leq 48$, 10 000 for $L = 56$ and $L = 64$