On the relationship between *gauge* dependence and *IR* divergences in the $\hbar$-expansion of the effective potential

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Outline

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Introduction

Gauge Dependence

IR divergences

Effective Potential

Definition

$$V_{\text{eff}}(\phi) = V_0(\phi) + \hbar V_1(\phi) + \dots$$

1-Loop

$$V_1(\phi) \sim \frac{1}{2} \sum_{\text{d.o.f}} \int (\omega_B(\phi) - \omega_F(\phi)),$$

$$\omega = \sqrt{k^2 + m^2(\phi)}$$

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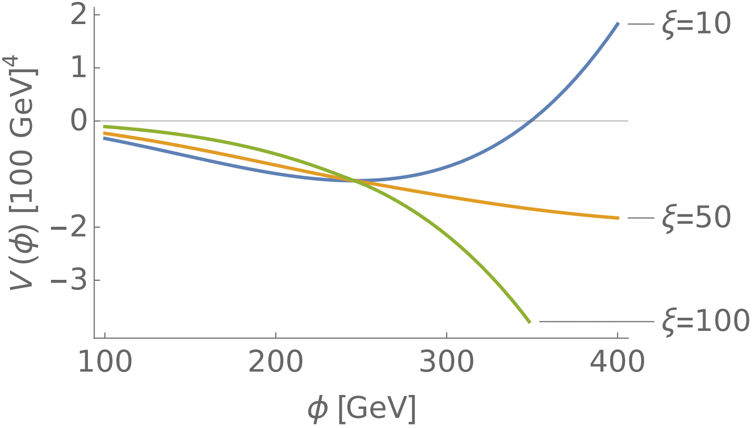
Gauge dependence



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IR divergences

IR divergences

$$V_3(\phi) \sim \log G(\phi),$$
$$V_4(\phi) \sim \frac{1}{G(\phi)}$$

At the L^{th} loop level

$$V_L(\phi) \sim G^{3-L}(\phi)$$

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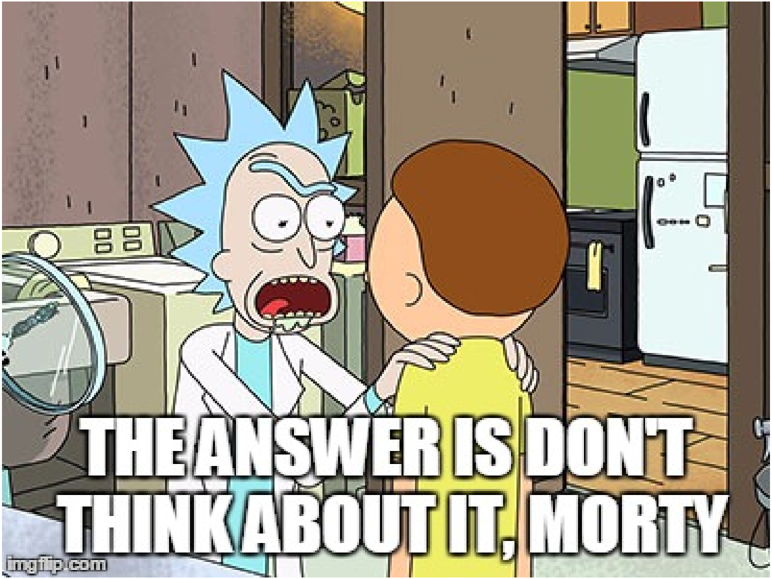

Outline

Nielsen identity

Gauge dependence

$$\left(\xi\frac{\partial}{\partial\xi}+C(\phi,\xi)\frac{\partial}{\partial\phi}\right)V_{\text{eff}}=0$$

$$\xi\frac{\partial}{\partial\xi}\phi^{\text{min}}(\xi)=C(\phi^{\text{min}},\xi)$$

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Perturbative expansion

$$\frac{\partial}{\partial \phi} V_{\text{eff}}(\phi, \xi)|_{\phi=\phi^{\text{min}}} = 0$$

$$V_{\text{eff}}(\phi, \xi) = V_0(\phi) + \hbar V_1(\phi, \xi) + \hbar^2 V_2(\phi, \xi) + \dots$$
$$\phi^{\text{min}} = \phi_0 + \hbar\, \phi_1 + \hbar^2\, \phi_2 + \dots$$
$$\frac{\partial}{\partial \phi} V_0|_{\phi=\phi_0} = 0$$

$\hbar$-expansion

Minimum

$$\left(V_0(\phi) + \hbar V_1(\phi, \xi) + \hbar^2 V_2(\phi, \xi) + \dots \right)_{\phi^{min} = \phi_0 + \hbar\phi_1 + \dots}$$

Potential

$$V_{\text{eff}}\big|_{\phi^{\text{min}}} = \left[V_0 + \hbar V_1 + \hbar^2 \left(V_2 + \phi_1 \partial V_1 + \frac{\phi_1^2}{2} \partial^2 V_0 \right) + \dots \right]_{\phi = \phi_0}$$

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Momenta split

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Hard & Soft

$$\text{Soft}: \; k^2 \sim G(\phi), \; \text{Hard}: \; k^2 \gg G(\phi)$$

$$V_{\text{eff}}(\phi,\xi) = V^S(\phi,\xi) + V^H(\phi,\xi)$$

Hard potential

$$\frac{\partial}{\partial\phi} V^H(\phi,\xi)|_{\phi=\phi^H} = 0$$

$$\xi \frac{\partial}{\partial\xi} \phi^H = 0$$

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IR divergences

IR Divergence Cancellation

Soft potential

$$V^S|_{\phi=\phi^{min}} = \ldots \hbar^L \left(V_L^S + \phi_1 \partial V_{L-1}^S + \frac{\phi_1^2}{2!} \partial^2 V_{L-2}^S + \ldots \right) + \ldots$$

$$V_L^S|_{\phi\approx\phi_0} \sim G^{3-L}(\phi)$$

$$\partial^n V_L^S|_{\phi\approx\phi_0} \sim G^{3-L-n}(\phi)$$

$$V^S(\phi,\xi)|_{\phi=\phi^{min}} = 0!$$

Conclusions

- *IR* & Gauge problems are solved by proper perturbative expansion
- Potential can be split into a hard & soft part
- Hard minima is **gauge invariant** & *IR* finite

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