Color corrections in parton showers

LUND UNIVERSITY

Malin Sjödahl

In collaboration with Simon Plätzer (Vienna)
and Johan Thorén (Lund)

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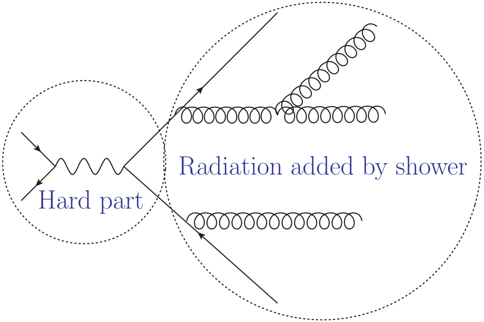
Section 1

Introduction

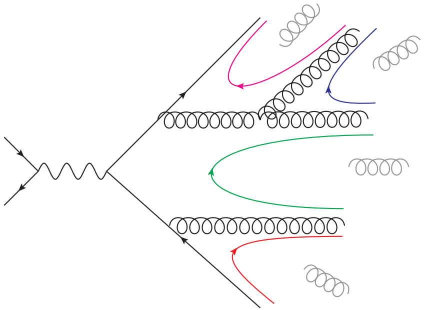
Parton shower basics

- A parton shower starts from a hard matrix element for some scattering process and dresses it up with additional radiation (mostly gluons)



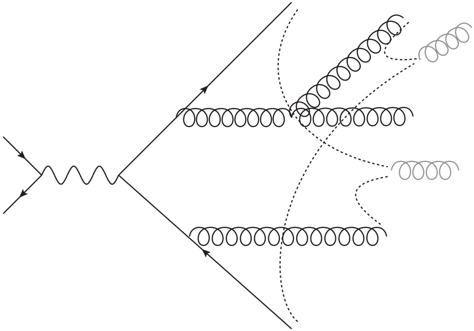
In a leading color parton shower things are easy ...

- In standard parton showers where QCD is treated as if it had infinitely many colors, the colors are described by color lines and only color connected partons (sharing a line) can radiate coherently $\rightarrow \sim N_{\text{parton}}$ dipoles



In real QCD with three colors

- For $N_c = 3$ radiation from almost any pair of partons can interfere $\rightarrow\sim N^2_{\text{partons}}$ possibilities, suppressed by $(1/N_c)$, $1/N_c^2$,...



Why investigate $N_c = 3$ color corrections?

- Expect that color suppressed terms become very important for *many* partons
- The colored initial state and the higher energy at the LHC gives rise to many colored partons and hence many color suppressed terms
- Needed for exact matching of matrix elements to parton showers
- Needed for $N_c = 3$ hadronization

Section 2

Dipole Showers

Dipole Factorization

- Parton showers work under the approximation that the next parton to be emitted is soft or collinear to one of the existing partons
- Dipole factorization gives, whenever i and j become collinear or one of them soft:

$$|\mathcal{M}_{n+1}(\ldots,p_i,\ldots,p_j,\ldots,p_k,\ldots)|^2 =$$
$$\sum_{k\neq i,j}\frac{1}{2p_i\cdot p_j}\langle\mathcal{M}_n(p_{\tilde{ij}},p_{\tilde{k}},\ldots)\,|\mathbf{V}_{ij,k}(p_i,p_j,p_k)|\,\mathcal{M}_n(p_{\tilde{ij}},p_{\tilde{k}},\ldots)\rangle$$

An emitter $\tilde{ij}$ splits into two partons i and j, with the spectator $\tilde{k}$ absorbing the momentum to keep all partons (before and after) on-shell. (Catani, Seymour hep-ph/9605323)

The spin averaged splitting kernel is

$$\mathbf{V}_{ij,k}(p_i,p_j,p_k) = -8\pi\alpha_s V_{ij,k}(p_i,p_j,p_k)\frac{\mathbf{T}_{\tilde{ij}}\cdot\mathbf{T}_k}{\mathbf{T}_{\tilde{ij}}^2}$$

Where, for example, for a final-final dipole configuration, we have

$$V_{q\to qg,k}(p_i,p_j,p_k) = C_F\left(\frac{2(1-z)}{(1-z)^2+p_\perp^2/s_{ijk}} - (1+z)\right)$$

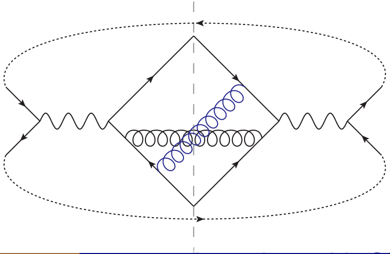
Emission probability

For a leading N_c shower, the emission probability is

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{\delta(\tilde{ij}, \tilde{k}\ \text{color connected})}{1+\delta_{\tilde{ij}\,g}}$$

Including subleading emissions, instead gives

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2}\frac{\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}}\cdot\mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle}{|\mathcal{M}|^2}$$



Overall Picture

Using Herwig's dipole shower

- Instead of only allowing color connected emitter-spectator pairs to radiate, all possible pairs can radiate
- All pairs may radiate in proportion to (for the first emission)

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle}{|\mathcal{M}|^2}$$

- Reweighting to encompass negative contributions
- The full color structure is evolved to be able to evaluate the above factor for the next emission
- Color structure is calculated using ColorFull (MS 1412.3967)
- $N_c = 3$ shower for a number of emissions, then standard leading N_c shower

Color structure

- A major challange is the SU(3) color structure of QCD
- The color structure can be decomposed in color bases

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha} |\alpha_n\rangle \leftrightarrow \mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T$$

  and for this project we use trace bases
- ... but these standard "bases" are non-orthogonal and overcomplete, with a dimension scaling $\sim (N_{\mathrm{g}} + N_{\mathrm{q\bar{q}}})! \rightarrow (N_{\mathrm{g}} + N_{\mathrm{q\bar{q}}})!^2$ terms when squaring
- See next talk by Johan Thorén for better bases

New Features

Compared to our previous e^+e^- results (SP, MS 1206.0180), we have added

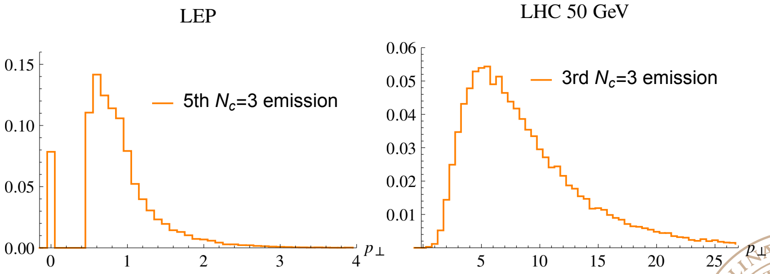
- The $g \to q\bar{q}$ splitting
- Hadronic initial state, meaning initial state radiation
- Full compatibility with all of the additional functionality in Herwig 7.1. (So we can run any process now, in particular LHC events)
- Subsequent standard leading N_c showering after the $N_c = 3$ shower

Section 3

Results

Full Color Shower Reaching Soft Scales

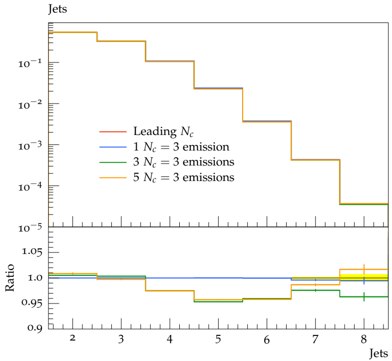
Since a limited number of $N_c = 3$ emissions are kept, up to 3 for LHC and 5 for LEP, we check the p_T of the last corrected emission

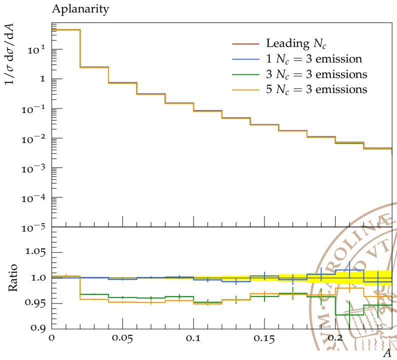


- $\rightarrow$ We go far down in p_T compared to relevant jet scales, at LEP close to the hadronization scale

LEP Preliminary Results

For most e^+e^- observables we find small corrections, at the percent level. However, some observables (thrust, out-of-plane $p_\perp$, hemisphere masses, aplanarity, jet multiplicities for many jets) are corrected by $\sim 5\%$.





LHC Preliminary Results

For LHC observables, corrections are typically of order a few percent, but some observables show corrections of $10 - 20\%$

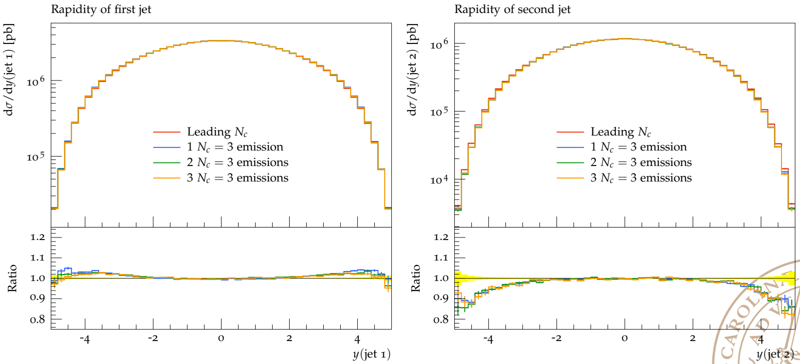


Figure: Rapidity of hardest and second hardest jet using a 50GeV analysis cut

If we could study quark-gluon scattering, we would find large corrections

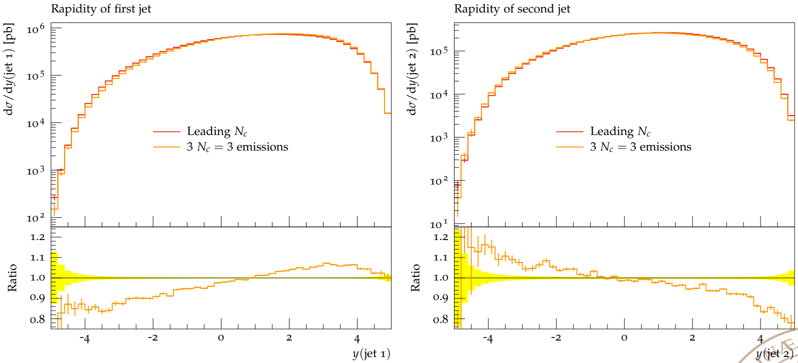


Figure: Rapidity distribution of the hardest and second hardest jet while considering only $qg \to qg$ scattering and a 50 GeV analysis cut.

... but we cannot

Requiring one forward (quark dominated) and one central (gluon dominated) jet we find sizable corrections for many observables

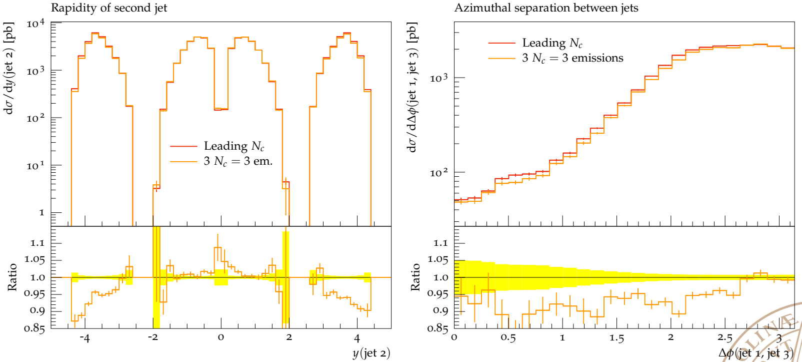


Figure: Rapidity and $\Delta\phi_{1,3}$ for the central/forward case

(400< M_{12} <600 GeV, $3.8 < |y_1 + y_2| < 5.2$, $1.5 < |y_2 - y_1| < 3.5$)

We have compared to LHC data for a wide range of observables. In general we find small corrections and no overall visible change in data description.

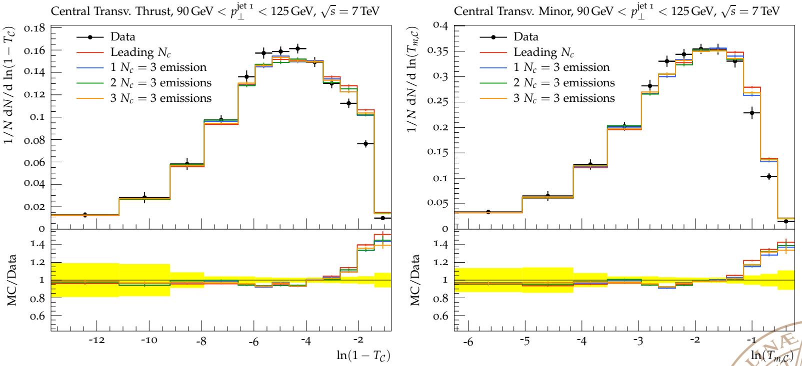


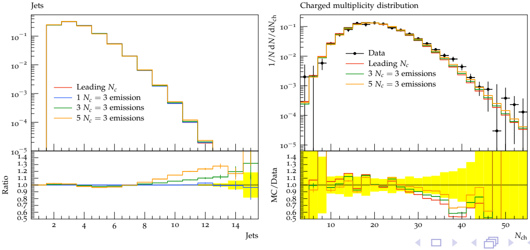
Figure: Central transverse thrust and thrust minor for $\sqrt{s} = 7$GeV, CMS 1102.0068, $T_C = \max_{\hat{n}_T} \frac{\sum_i |\bar{p}_{\perp,i} \cdot \hat{n}_T|}{\sum_i p_{\perp,i}}, T_{m,C} = \frac{\sum_i |\bar{p}_{\perp,i} \times \hat{n}_T|}{\sum_i p_{\perp,i}}$ for jet i, with $\eta < 1.3$

Conclusion, Hard Perturbative Region

- We have considered a wide range of observables at LEP and LHC and compared to data
- Overall the data description does not change
- As long as soft scales/observables with very many jets are not considered, the matrix element correction type of corrections are accurately described by correcting the first few emissions
- In general, percent level corrections are found at LEP, for some observables (thrust, out-of-plane $p_\perp$, hemisphere masses, aplanarity, jet multiplicities for many jets) effects of around 5%
- At the LHC, corrections are often a few percent, for some observables (mostly rapidity) corrections around 10-20%

Going Soft/Very Many Colored Partons

For soft QCD, where we cannot expect reliable results due to the need of more color suppressed terms, resummation, hadronization and MPI, we find larger corrections in many cases, (jet resolution scales, cluster masses in Herwig, number of very soft jets at LEP, charged multiplicity distribution, individual hadron multiplicities), indicating that subleading N_c effects probably play an important role for soft(ish) QCD



Conclusion, Soft Region

In the soft region/region of many colored partons:

- In this region, we cannot claim accurate results, however,
- we often find large corrections of several ten percent
- This affects the state going into the hadronization
- meaning that we can expect a significant effect on the tune
- Subleading N_c effects can therefore be hidden in the tune
- Need to retune

Section 4

Current Status and Future Work

Current Status and Future Work

- We have a fully functional $N_c = 3$ parton shower for any LEP or LHC process
- Tuning should be performed before a reliable comparison to standard showers can be done
- We still miss virtual corrections, which rearrange the color structure without any real emissions. These are important for gap-survival observables
- In the more distant future, an update of hadronization models to an $N_c = 3$ final state would be and interesting research task

Thank you!

Section 5

Backup Slides

Weight distribution

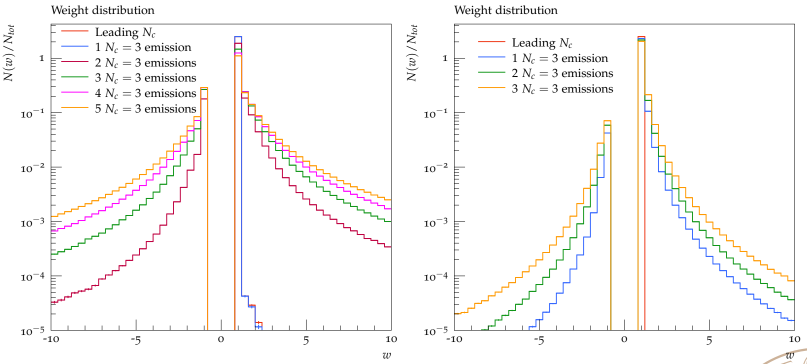
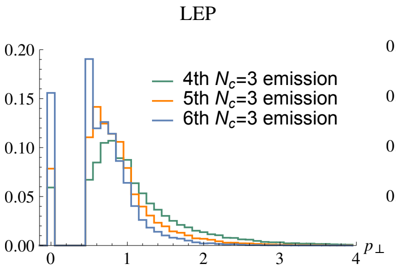
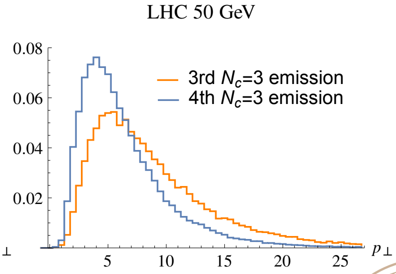


Figure: Weight distribution for e^+e^- (left) and pp collisions (right) depending on the number of $N_c = 3$ emissions allowed. All generated events are used in these plots, i.e., no further analysis cut is applied.

$N_c = 3$ Shower Reaching Soft Scales





More LEP Observables

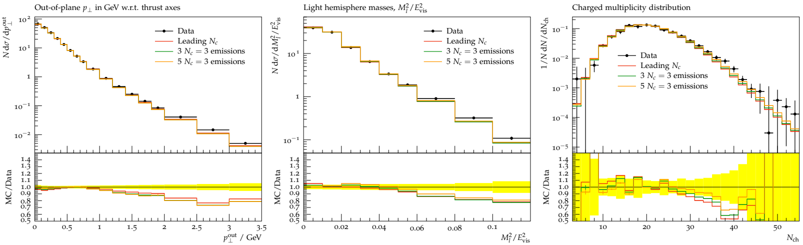


Figure: Out-of-plane $p_\perp$ w.r.t. the thrust and thrust major axes (left), light hemisphere mass (middle) and fraction of events containing N_{ch} charged particles. DELPHI, ALEPH

Top at LHC

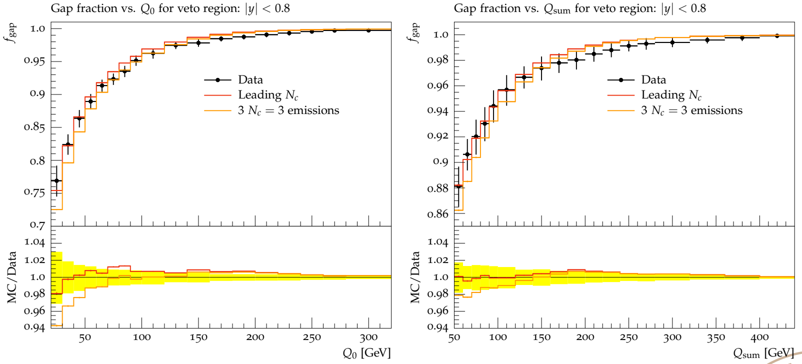


Figure: Fraction of events having no additional jet with $p_\perp$ above Q_0 within a rapidity interval $|y| < 0.8$ (left) and fraction of events where the scalar sum of transverse momenta within $|y| < 0.8$ does not exceed Q_{sum} (right) for $t\bar{t}$ events at $\sqrt{s} = 7$ TeV. ATLAS 1203.5015

QCD "Coherence" observable

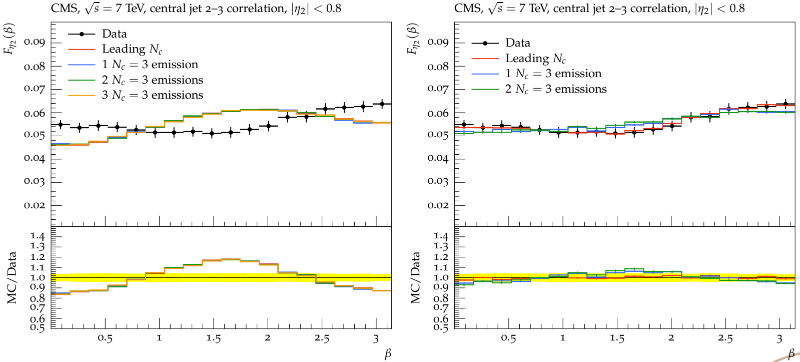


Figure: The angle β, $\tan\beta = \frac{|\phi_3 - \phi_2|}{\text{sign}(\eta_2)(\eta_3 - \eta_2)}$, using (left) an underlying $2 \to 2$ hard process and (right) an underlying $2 \to 3$ hard process.
CMS 1102.0068

Density Operator

We can write the amplitude as a vector in some basis (trace, multiplet, etc.),

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \leftrightarrow \mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T \qquad (1)$$

and construct a "density operator" $M_n = \mathcal{M}_n\mathcal{M}_n^\dagger$, that we evolve by

$$M_{n+1} = -\sum_{i\neq j}\sum_{k\neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger \qquad (2)$$

where

$$V_{ij,k} = \mathbf{T}_{\tilde{ij}}^2 \frac{p_i \cdot p_k}{p_j \cdot p_k}. \qquad (3)$$

This allows us to calculate the color matrix element corrections.

Color Matrix Element Corrections

Evolving the density operator, we can calculate the color matrix element corrections for any number of emissions

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\text{Tr}\left(S_{n+1} \times T_{\tilde{k},n} M_n T_{\tilde{ij},n}^{\dagger}\right)}{\text{Tr}\left(S_n \times M_n\right)} \tag{4}$$

- Note that ω_{ik}^{n} can be negative, this is included through the weighted Sudakov algorithm (Bellm, SP, Richardson, Siodmok, Webster, 1605.08256)
- This initially resulted in very large weights. Modifications to the weighted Sudakov veto algorithm drastically reduced the weights.

Standard veto algorithm

Standard veto algorithm: we want to generate a scale q and additional splitting variables x (e.g. z and ϕ) according to a distribution $\mathrm{d}S_P$.

$$\mathrm{d}S_P(\mu, x_\mu | q, x | Q) = \mathrm{d}q\mathrm{d}^d x \left(\Delta_P(\mu|Q)\delta(q-\mu)\delta(x-x_\mu) + P(q,x)\theta(Q-q)\theta(q-\mu)\Delta_P(q|Q)\right)$$

Where Δ_P is the Sudakov form factor,

$$\Delta_P(q|Q) = \exp\left(-\int_q^Q \mathrm{d}k \int \mathrm{d}^d z P(k,z)\right)$$

To do this we use an overestimate of the distribution (with nicer analytical properties) dS_R (change $P \to R$ in the above eqs.). Where we require $R(q,x) \geq P(q,x)$ for all q, x.

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Standard veto algorithm

$P(q,x) > 0$ and $R(q,x) \geq P(q,x)$. Set $k = Q$

1. Generate q and x according to $S_R(\mu, x_\mu | q, x | k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Else, accept the emission with the probability

$$\frac{P(q,x)}{R(q,x)}.$$

4. If the emission was vetoed, set $k = q$ and go back to 1.

Weighted veto algorithm

Introduce an acceptance probability $0 \leq \epsilon(q, x|k, y) < 1$ and a weight ω. Set $k = Q$, $\omega = 1$.

1. Generate q and x according to $S_R(\mu, x_\mu|q, x|k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Accept the emission with the probability $\epsilon(q, x|k, y)$, update the weight

$$\omega \to \omega \times \frac{1}{\epsilon} \times \frac{P}{R}$$

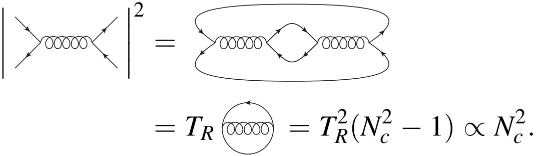
4. Otherwise update the weight to

$$\omega \to \omega \times \frac{1}{1-\epsilon} \times \left(1 - \frac{P}{R}\right)$$

and start over at 1 with $k = q$.

Example of $1/N_c$ suppressed terms

Leading color structure:



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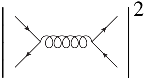
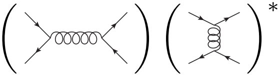
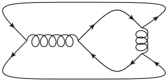


$$\propto N_c^2.$$

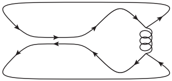
Interference term:



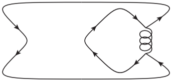
$$=$$



$$= T_R$$



$$- \frac{T_R}{N_c}$$



$$= 0 - T_R^2 \frac{N_c^2 - 1}{N_c} \propto N_c.$$

Example of $1/N_c$ suppressed terms