Scaling Relations between the Luminous and Dark Matter Density Parameters of Milky Way Dwarf Spheroidal Galaxies

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OUTLINE

Sample, Surface brightness (SB), & velocity dispersion (VD) Analysis

- Exclude MW dSphs due to sparse VD data and/or tidal disruption
- Establish MW dSph sample for this study
- Overview of SB profile analysis
  - 3D stellar density → $\nu(\nu_0, \gamma_*, n, R_*, r)$
- Overview of Spherical Jeans Equation (SJE)
- A Dark Matter (DM) Density Profile spanning Core to Cusp Models
  - DZDM: Dekel and Zhao (DZ) Model (Freundlich et al., 2020)
  - Tunable, inner logarithmic slope γ with integrable mass, etc.
- Virial Shape Parameters (VSPs)
- VD profile analysis

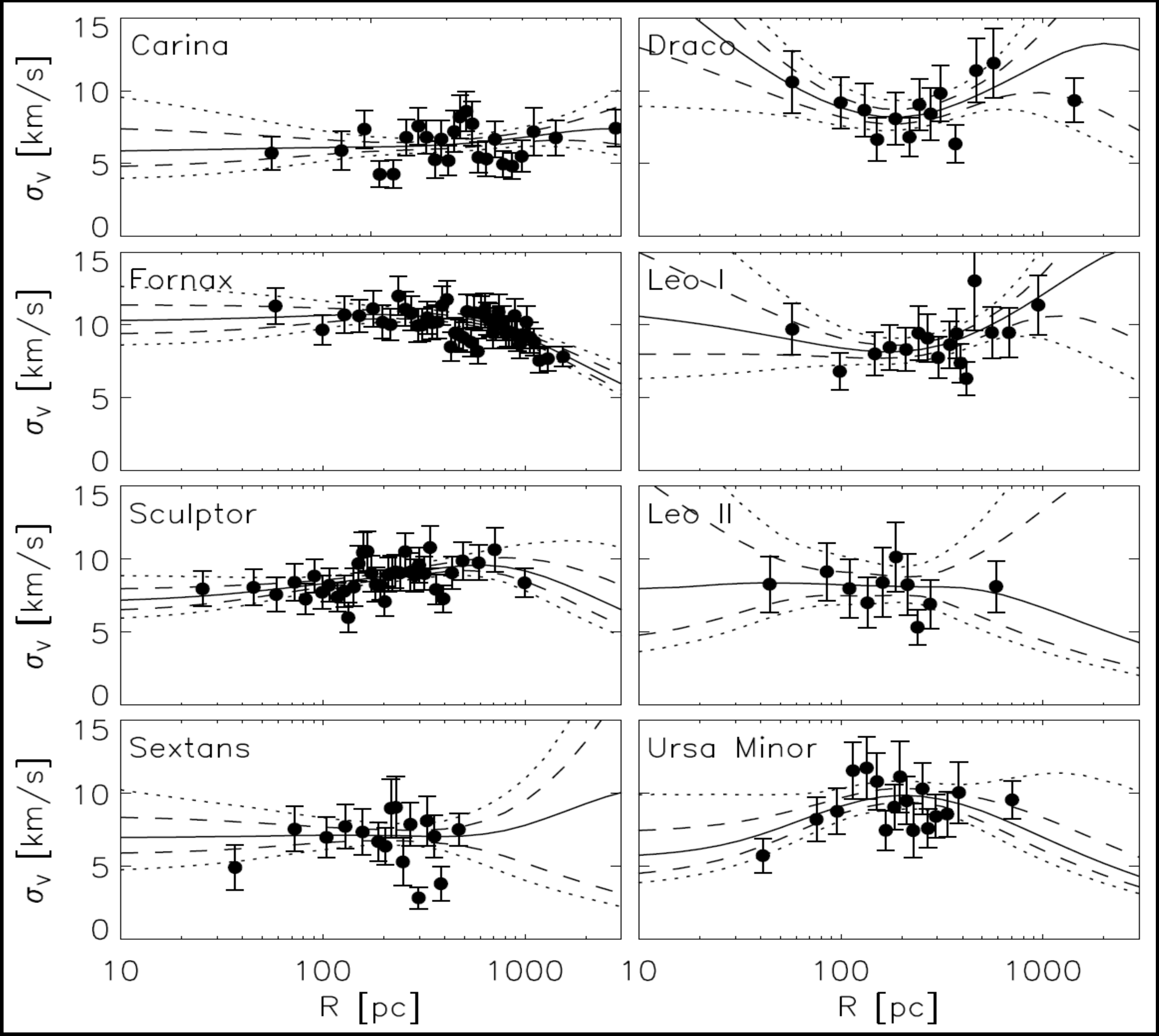
Results

- Scaling Relations between DZDM parameters (γ, r_{DM}, ρ_{DM}) and
  - γ_*
  - optical effective radius (R_{eff})
- Comparisons to Previous Studies
  - $M(R_{eff})$ (Walker et al., 2009); $M(1.67\ R_{eff})$
  - DDs and Spirals
  - $M(r_2)$ and $M(r_3)$ (Comparison to Wolf et al., 2010 and Lazar and Bullock, 2020)
  - γ_{150} and $\rho_{DM,\ 150}$ (Comparison to Read, Steger, and Walker, 2018)

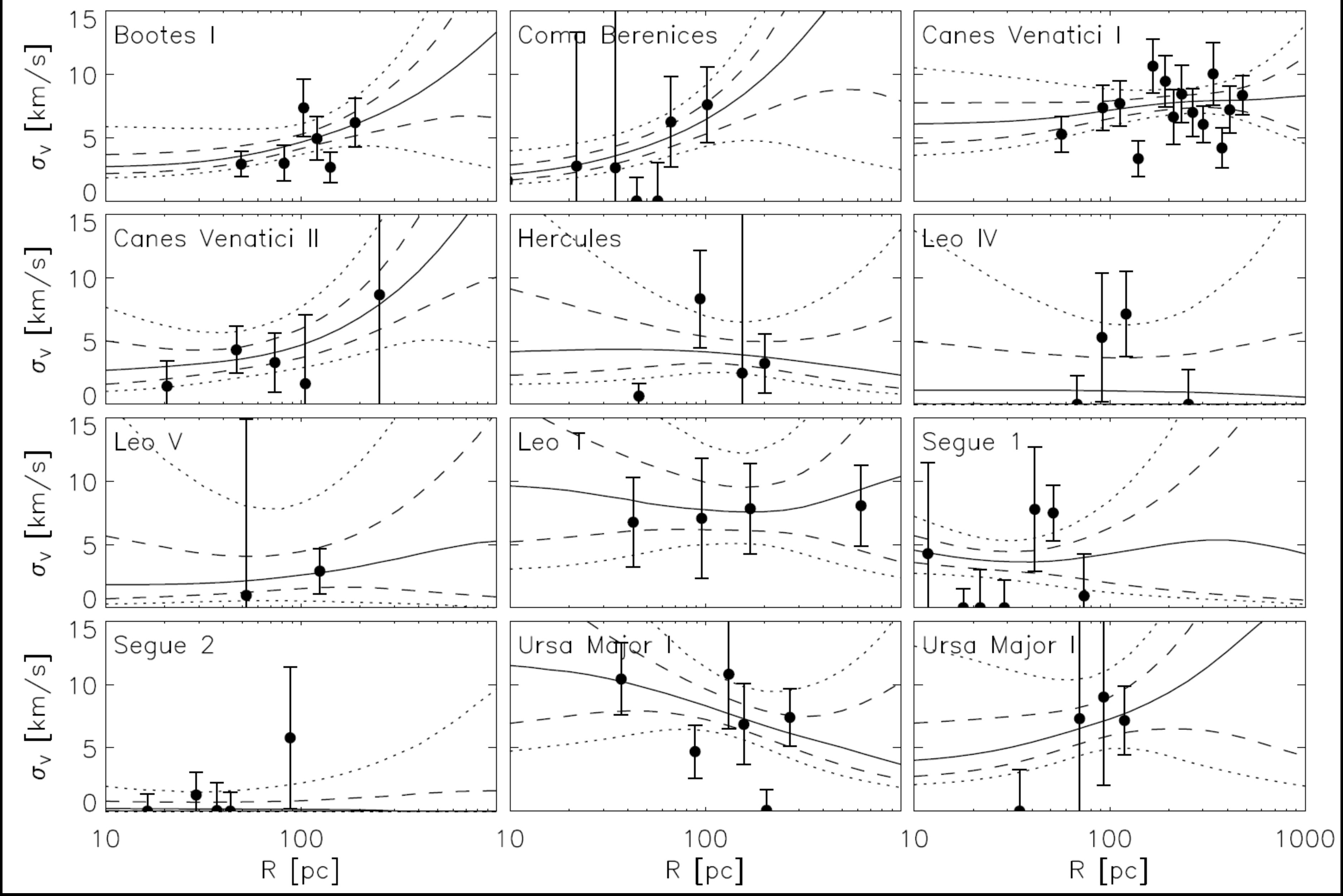
Summary and Conclusions

Establish MW dSph Sample

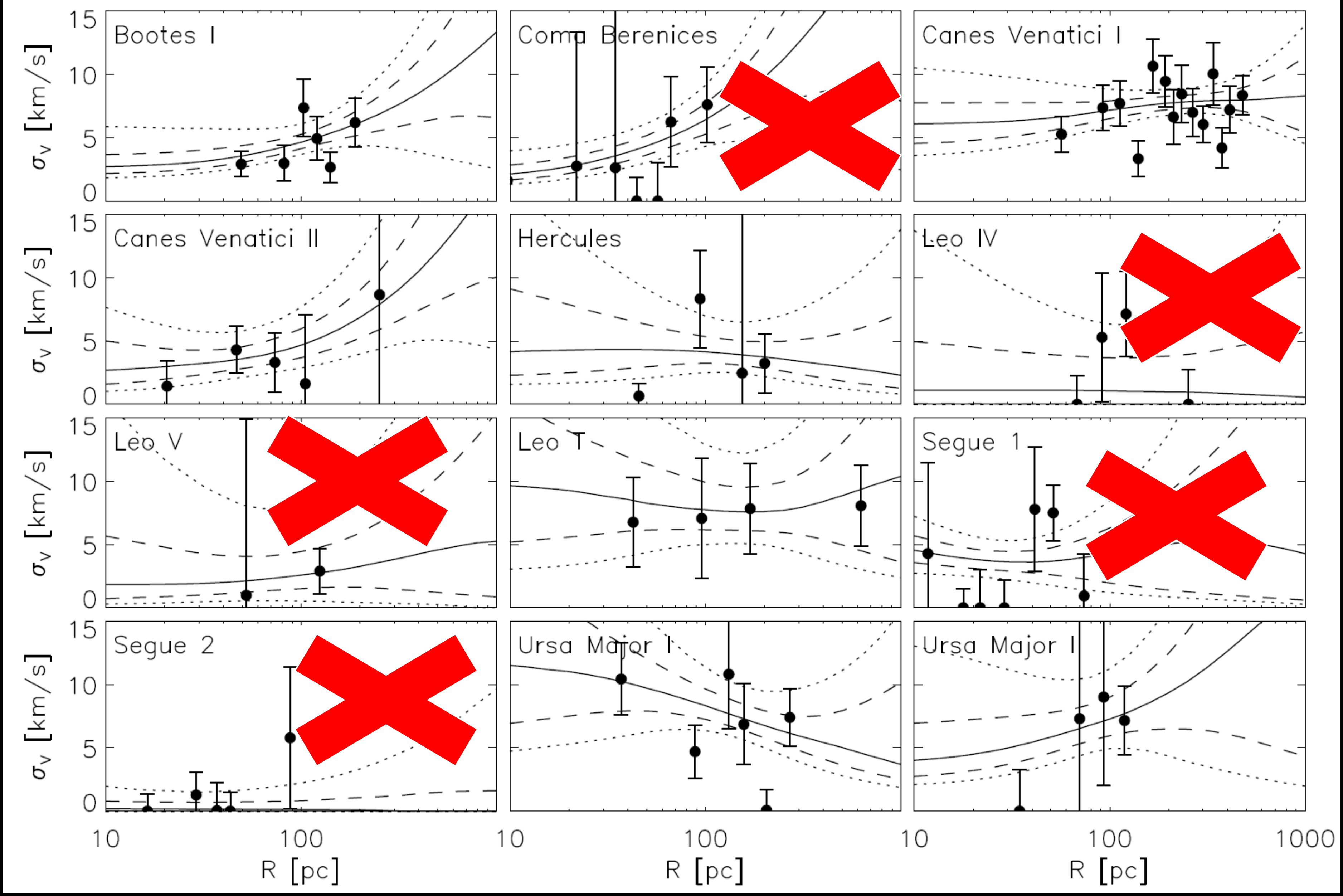
MW Classical (Cl.) dSphs VD Profiles (Geringer-Sameth et al. 2015)



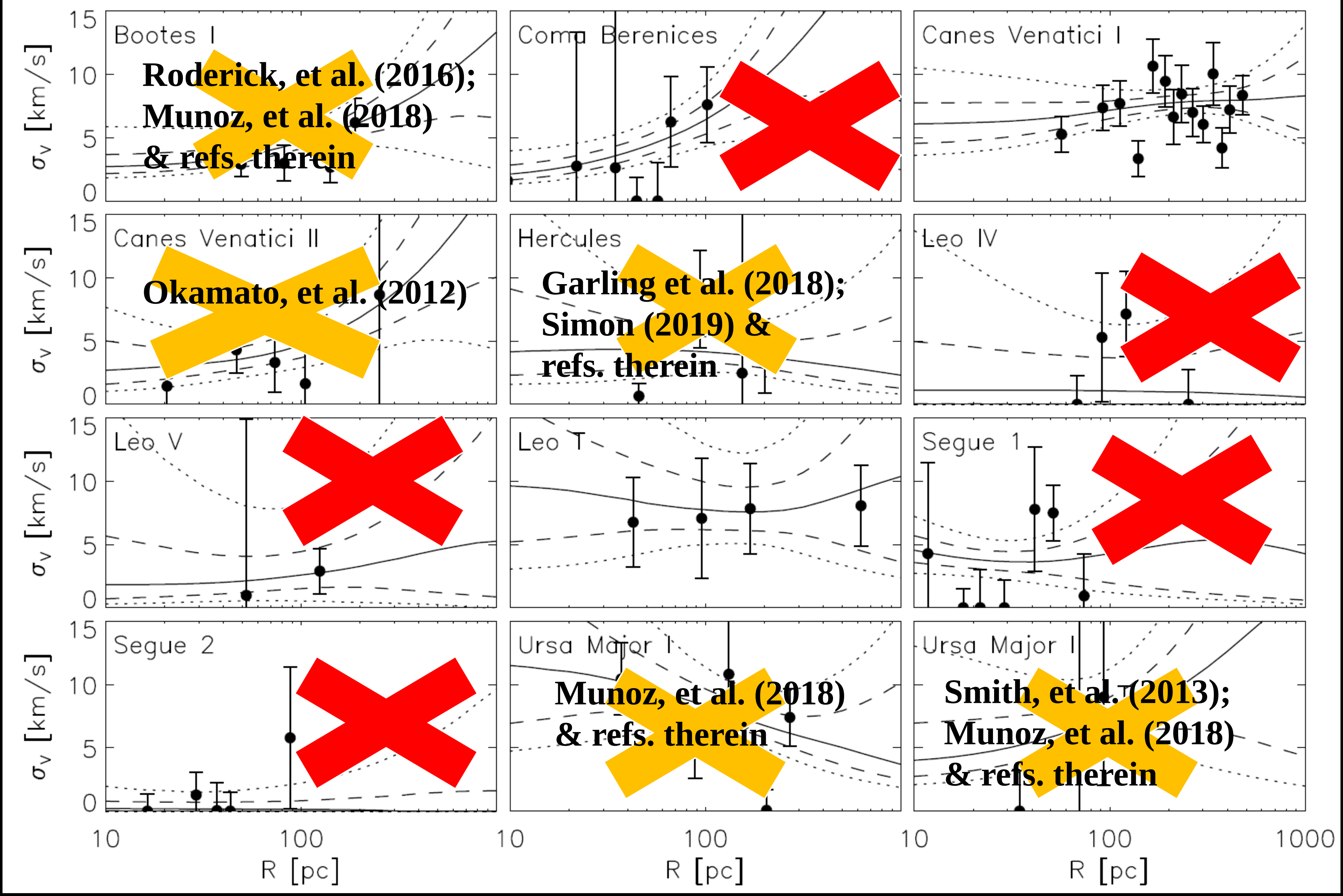
MW Ultra-faint (UF) dSphs VD Profiles (Geringer-Sameth et al. 2015)



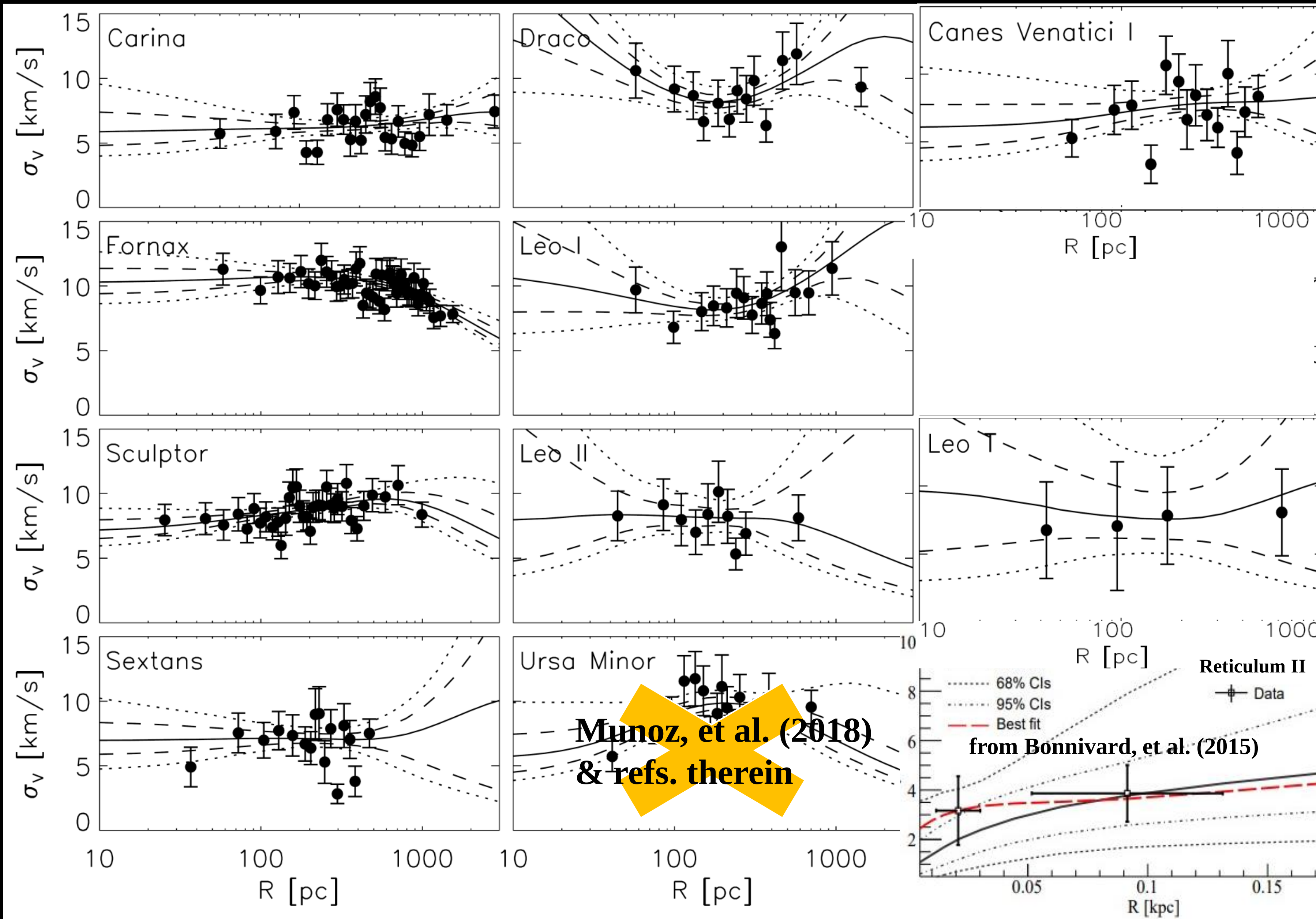
Eliminated UF dSphs: Data



Eliminated UF dSphs: Data & Tidal Disruption (TD)



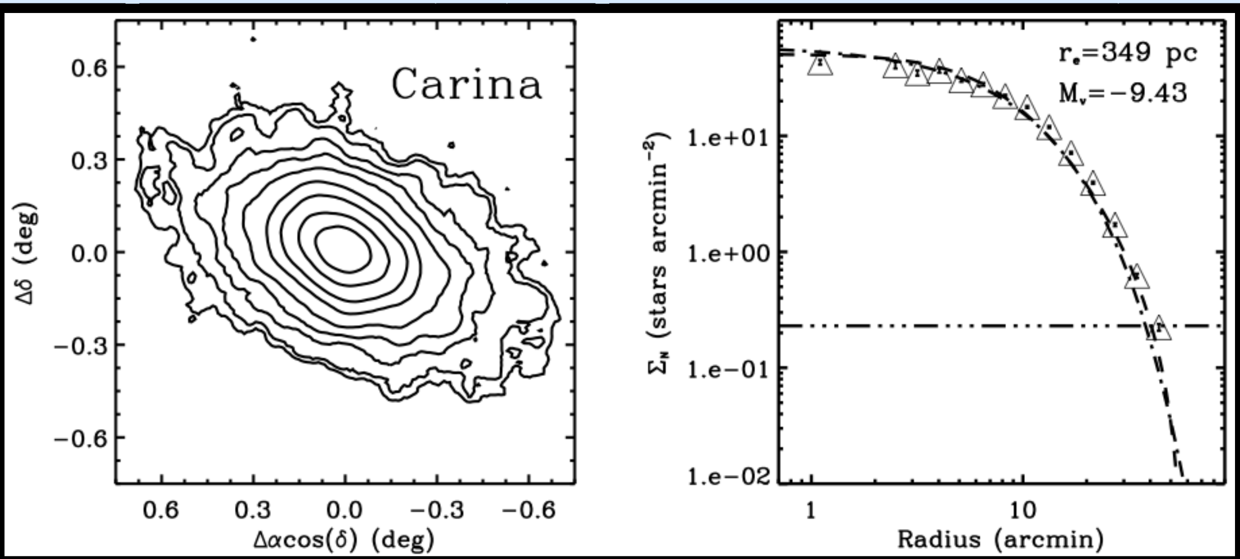
MW Cl. (minus U Minor due to TD) + UF dSphs Data Set



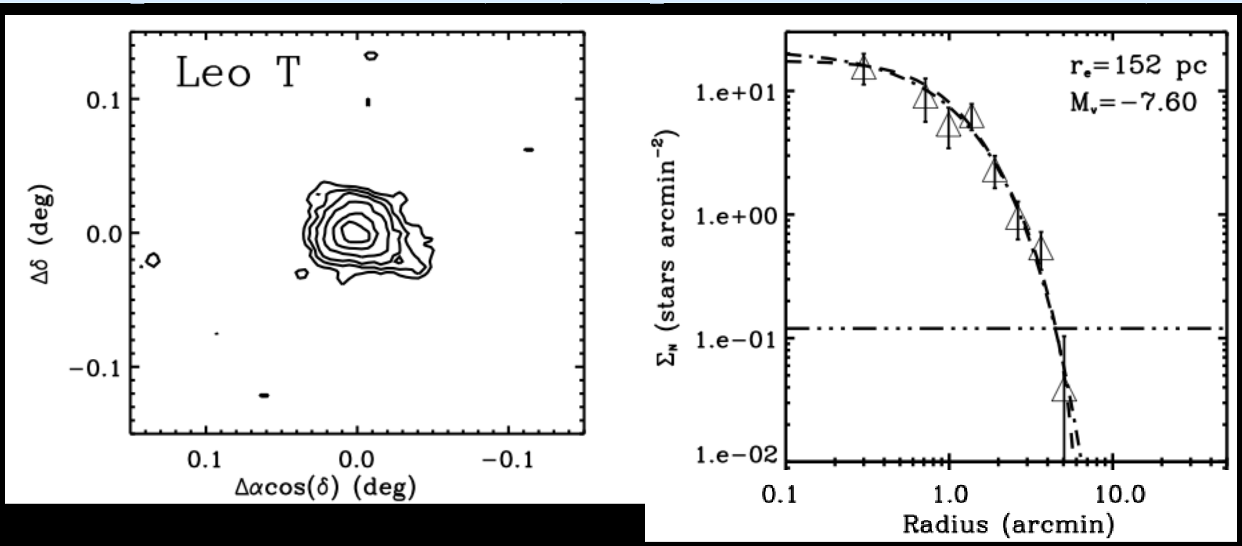
SB Profile Analysis

Sample Surface Brightness Profiles

I. Example Classical (Cl.) dSph SB Profile: Carina (Munoz et al. 2018)



II. Example Ultra-faint (UF) dSph SB Profile: Leo T (Munoz et al. 2018)



Projecting 3D Stellar Density → Model SB Profile

I. 3D Stellar Density – Zhao (1996) Profile:

$$\nu(r) = \nu^{\rm Zhao}(r) = \frac{\nu_0}{(r/R_*)^{\gamma_*}[1+(r/R_*)^2]^{(n-\gamma_*)/2}};$$
$$\nu(y) = \frac{\nu_0}{y^{\gamma_*}[1+y^2]^{(n-\gamma_*)/2}}$$

II. Project ν(r) in the plane of the sky → SB profile = I(Y):

$$I(Y) - I_{\rm bck} = I_{\rm net}(Y) = 2\int_Y^\infty \frac{\nu(y) y dy}{\sqrt{y^2 - Y^2}}$$

III. Define $Y_{\rm eff}(n,\gamma_*) = R_{\rm eff}/R_*(n,\gamma_*)$ →
use $R_{\rm eff,\ obs}$ → $R_*(n,\gamma_*)$

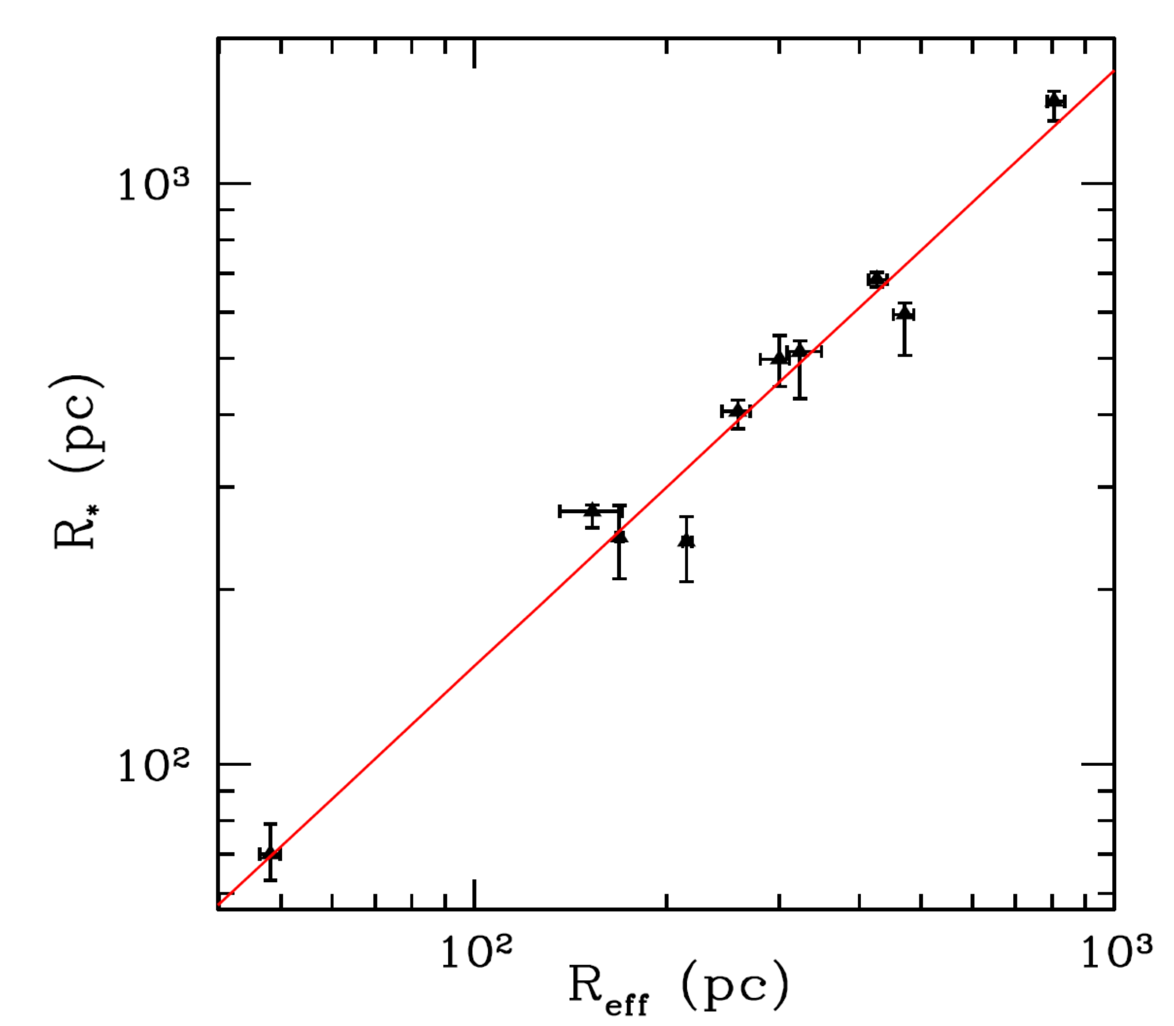
$$\frac{\int_0^{Y_{\rm eff}} I(s) s ds}{\int_0^\infty I(s) s ds} = 0.5.$$

IV. Compare $I_{\rm net}(Y)$ to $I_{\rm net,obs}(Y)$, maximize likelihood function:

$$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2_{\rm SB,i}(Y_i)}} \exp\left(-\frac{(\mathrm{I_{net,obs}(Y_i) - I_{net}(Y_i)})^2}{2\sigma^2_{\rm SB,i}(\mathrm{Y_i})}\right)$$

→ **Obtain best-fit n, γ_*, ν_0, and $R_*(n,\gamma_*)$ via III & IV.**

$R_* (R_{eff})$



$$R_* = (1.89^{+0.18}_{-0.21}) R_{\rm eff}$$

Spherical Jeans Equation (SJE)

Mass Modeling: SJE Analysis I

$$\frac{1}{\nu}\frac{d}{dr}(\nu\overline{v_r^2}) + 2\frac{\beta\overline{v_r^2}}{r} = -\frac{GM(r)}{r^2}$$

Velocity Anisotropy Profile, β(r):

$$\beta(r) \equiv 1 - \frac{\overline{v_t^2}(r)}{\overline{v_r^2}(r)}$$

$$(\beta(r) > 0, \beta(r) < 0) \rightarrow \text{(radial, tangential)}$$

SJE Solution:

$$\overline{v_r^2}(r) = \frac{1}{\nu(r)f(r)}\int_r^{+\infty} f(s)\nu(s)\frac{GM(s)}{s^2}ds,$$

where

$$f(r) = \exp\left[\int^r \frac{2}{q}\beta(q)\,dq\right].$$

SJE Analysis II: Velocity Anisotropy Model

For our velocity anisotropy model (Baes & van Hese 2007):

$$\beta^{\mathrm{Baes}}(r) = \beta(y) = \frac{\beta_0 + \beta_\infty y^2}{1 + y^2}$$

$$f(r) = \exp\left[\int^r \frac{2}{q}\beta(q)\, dq\right].$$

becomes:

$$f(y) = y^{2\beta_0}(1 + y^2)^{\beta_\infty - \beta_0}$$

SJE Analysis III: Dark Matter (DM) Profiles (ρ_{DM})

$$M(r) = 4\pi \int_0^r \rho_{\rm DM}(s)\, s^2 ds$$

Dekel Zhao DM (2020):

$$\rho_{\rm DM}(r) = \frac{\rho_s}{x^\gamma (1+x^p)^2},$$

- $p = (3 - \gamma)/2$
- $x = r/r_s$
- r_s = scale radius
- ρ_s = scale density
- $u = x^p$.

$$M_{\rm DM}(x) = [4\pi r_s^3 \rho_s] \int_0^x \frac{s^2 ds}{s^\gamma (1+s^p)^2} = [M_s] I_{\rm DM}(x) = [M_s] \frac{1}{p} [\ln(1+u) - \frac{u}{1+u}]$$

Spans the range of $0 \leq \gamma \leq 1$ between core and cusp:

Navarro, Frenk, & White (NFW 1996; cusp $\gamma = 1$):

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{x(1+x)^2}$$

- $x = r/r_s$
- r_s = scale radius
- ρ_s = scale density.

Burkert (1996; core $\gamma = 0$):

$$\rho_{\rm B}(r) = \frac{\rho_0}{(1+x_B)(1+x_B^2)}$$

- $x_B = r/r_0$
- r_0 = core radius
- ρ_0 = core density.

SJE Analysis IV: SJE with above choices of $\beta(y)$, $\nu(y)$, & $\rho_{DM}(x)$:

$$y\bar{v_r^2}\left(\frac{\gamma_* + ny^2}{1+y^2} - \frac{d\ln(\bar{v_r^2})}{d\ln(y)} - 2\frac{\beta_0 + \beta_\infty y^2}{1+y^2}\right) = \sigma_M^2 I_{\rm DM}(\alpha y)$$

where $\sigma_M^2 = \frac{GM_{\rm DM}}{R_*}$ $\alpha = R_*/r_{\rm DM}$

Solution:

$$\bar{v_r^2}(y) = \frac{1}{\nu(y)f(y)}\int_y^\infty f(s)\nu(s)\frac{GM(s)}{s^2}ds$$
$$= y^k(1+y^2)^{(l-k)/2}\sigma_M^2\int_y^\infty \frac{I_{\rm DM}(\alpha s)}{s^{k+2}(1+s^2)^{(l-k)/2}}ds,$$

where

$$k = \gamma_* - 2\beta_0;$$
$$l = n - 2\beta_\infty.$$

Velocity Dispersion Profile Analysis

Velocity Dispersion Profiles I

The observable, projected VDP along the line of sight, $\sigma_P(Y)$, is:

$$\sigma_p^2(Y) = \frac{2}{I(Y)} \int_Y^{\infty} \left(1 - \beta(y)\frac{Y^2}{Y^2}\right) \frac{\nu(y)\bar{v_r^2}(y)y}{\sqrt{y^2 - Y^2}} \mathrm{d}y$$

For each set of model parameters, compare to $\sigma_{P,obs}(Y_i)$ and maximize the likelihood function for N data points:

$$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\delta_{p,i}^2}} \exp\left(-\frac{(\sigma_{\mathrm{p,obs}}(\mathrm{Y_i}) - \sigma_{\mathrm{p}}(\mathrm{Y_i}))^2}{2\delta_{\mathrm{p,i}}^2}\right)$$

Virial Shape Parameters (VSPs)

We consider 2 virial shape parameters (VSPs, e.g., Merrifield and Kent 1992; Read, Steger, Walker 2018):

$$\mathrm{VSP\ 1:} \frac{2}{5}\int_0^\infty \mathrm{GM}(\alpha \mathrm{y})(5-2\beta(\mathrm{y}))\nu(\mathrm{y})\bar{\mathrm{v}}_\mathrm{r}^2(\mathrm{y})\mathrm{y}\mathrm{dy}$$

$$= \int_0^\infty \langle \sigma_p^2(Y)\rangle^2 I(Y) Y\, dY$$

$$\mathrm{VSP\ 2:} \frac{4}{35}\int_0^\infty \mathrm{GM}(\alpha \mathrm{y})(7-6\beta(\mathrm{y}))\nu(\mathrm{y})\bar{\mathrm{v}}_\mathrm{r}^2(\mathrm{y})\mathrm{y}^3\mathrm{dy}$$

$$= \int_0^\infty \langle \sigma_p^2(Y)\rangle^2 I(Y) Y^3\, dY;$$

Parameter Constraint Summary

- SBP constraints on v_0, γ_*, n, R_* ;
- 2 VSP constraints for β_0 and β_∞ (plus α, γ, σ_M^2);
- VDP: β_0, β_∞, α, γ, and $\sigma_M^2 = \frac{GM_{DM}}{R_*}$.

$$r_{\rm DM} = R_*/\alpha$$

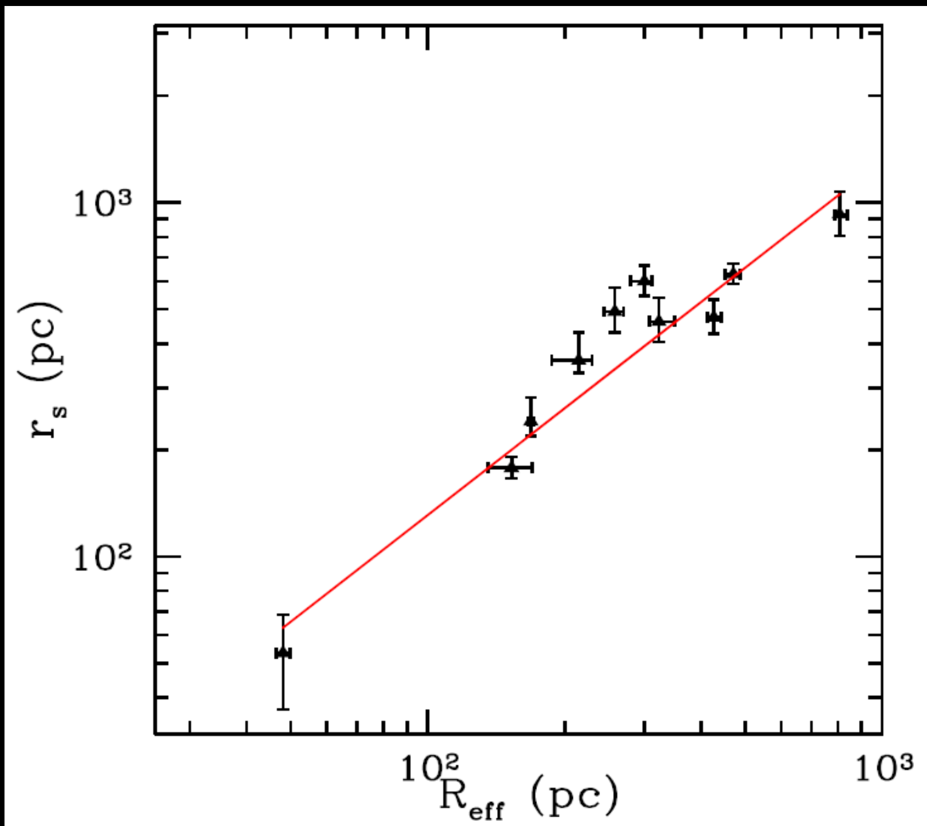
$$M_{\rm DM} = R_* \sigma_M^2 / G$$

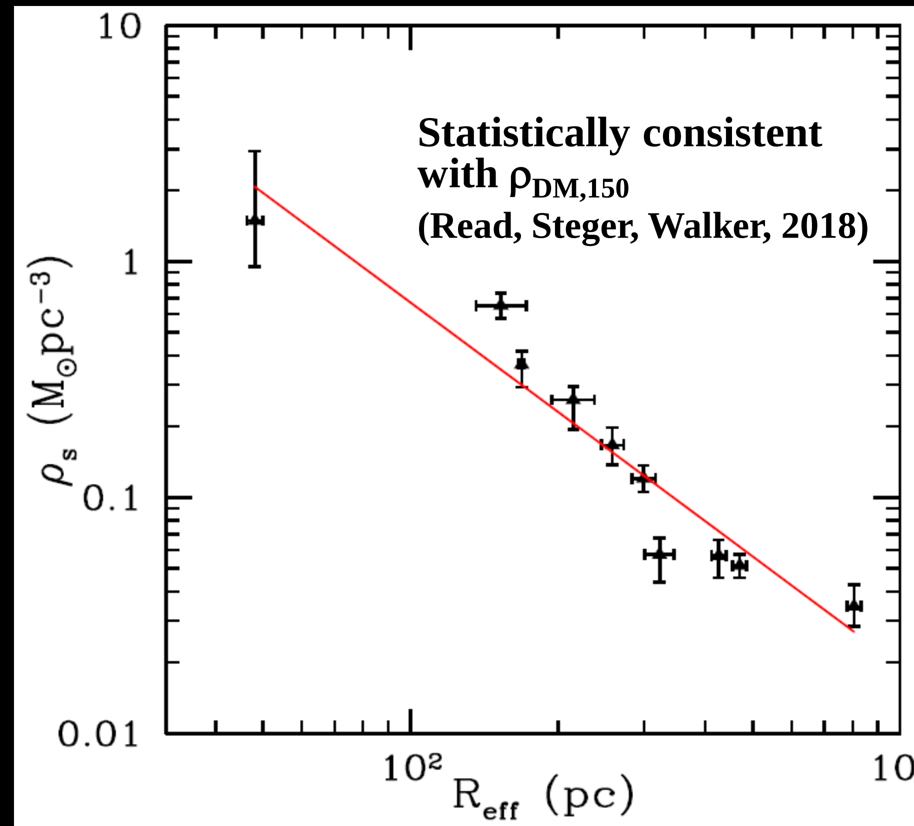
$$\rho_{\rm DM} = \frac{M_{\rm DM}}{4\pi r_{\rm DM}^3}$$

Results I:

R_{eff}, r_s, ρ_s, γ_* & γ Scaling Relations

DZDM: $r_s(R_{eff})$ and $\rho_s(R_{eff})$ Correlations

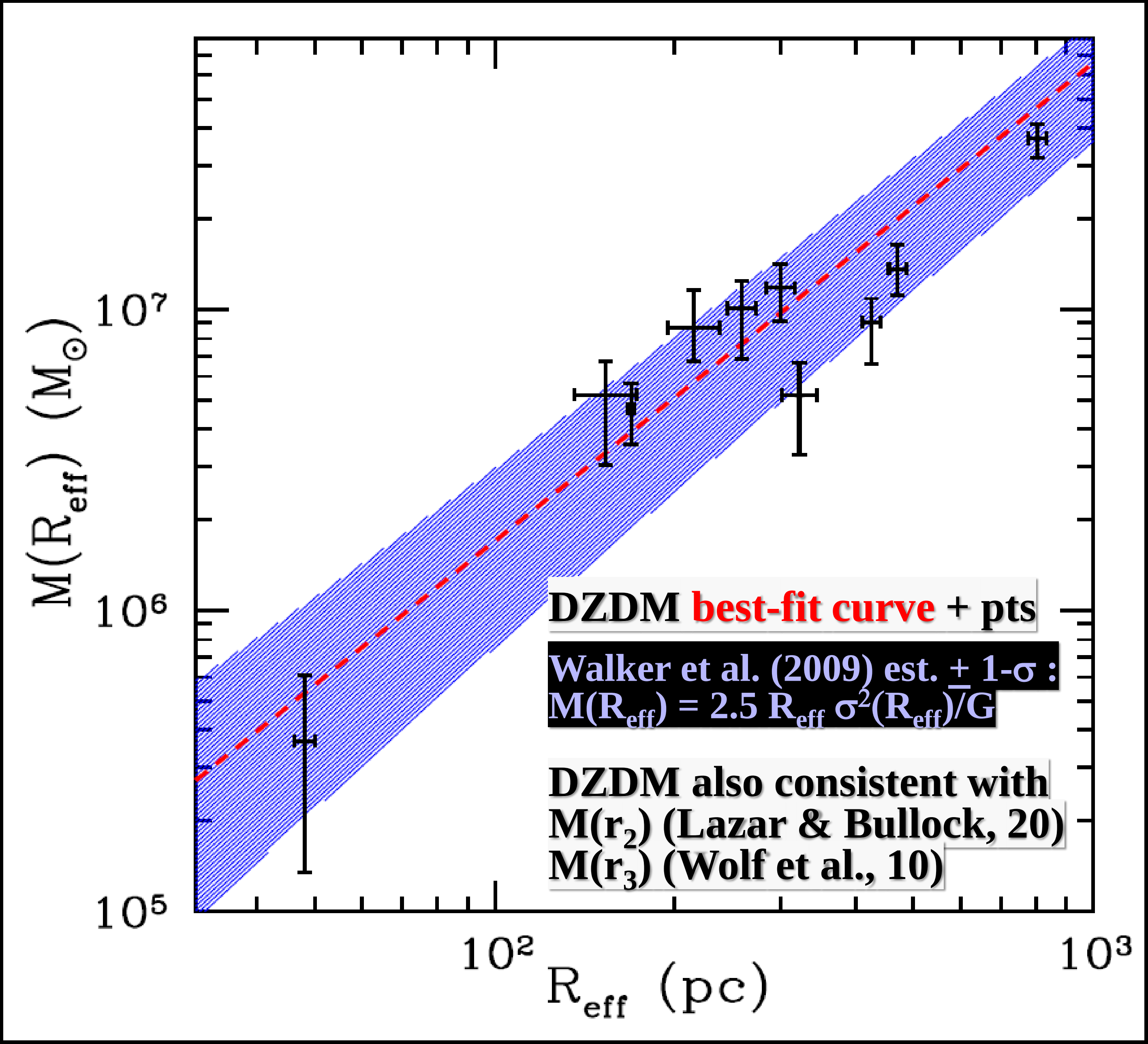




$$r_s(R_{\rm eff}) = (1.31^{+0.14}_{-0.17})R_{\rm eff}$$

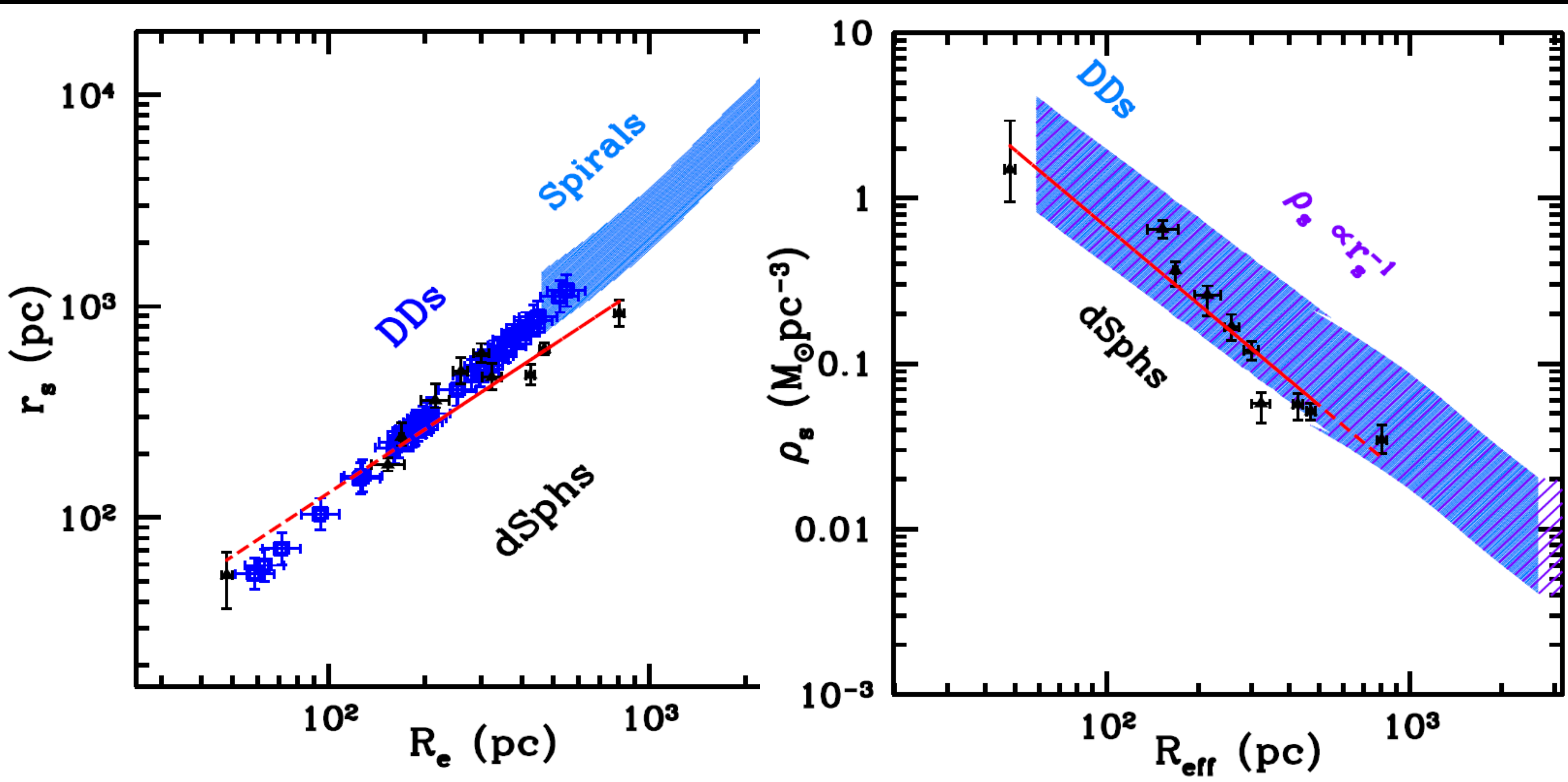
$$\rho_s(R_{\rm eff}) = (0.67^{+0.26}_{-0.17})M_\odot {\rm pc}^{-3}\left(\frac{\rm R_{eff}}{\rm 100pc}\right)^{-1.54\pm0.13}$$

M (R_{eff})

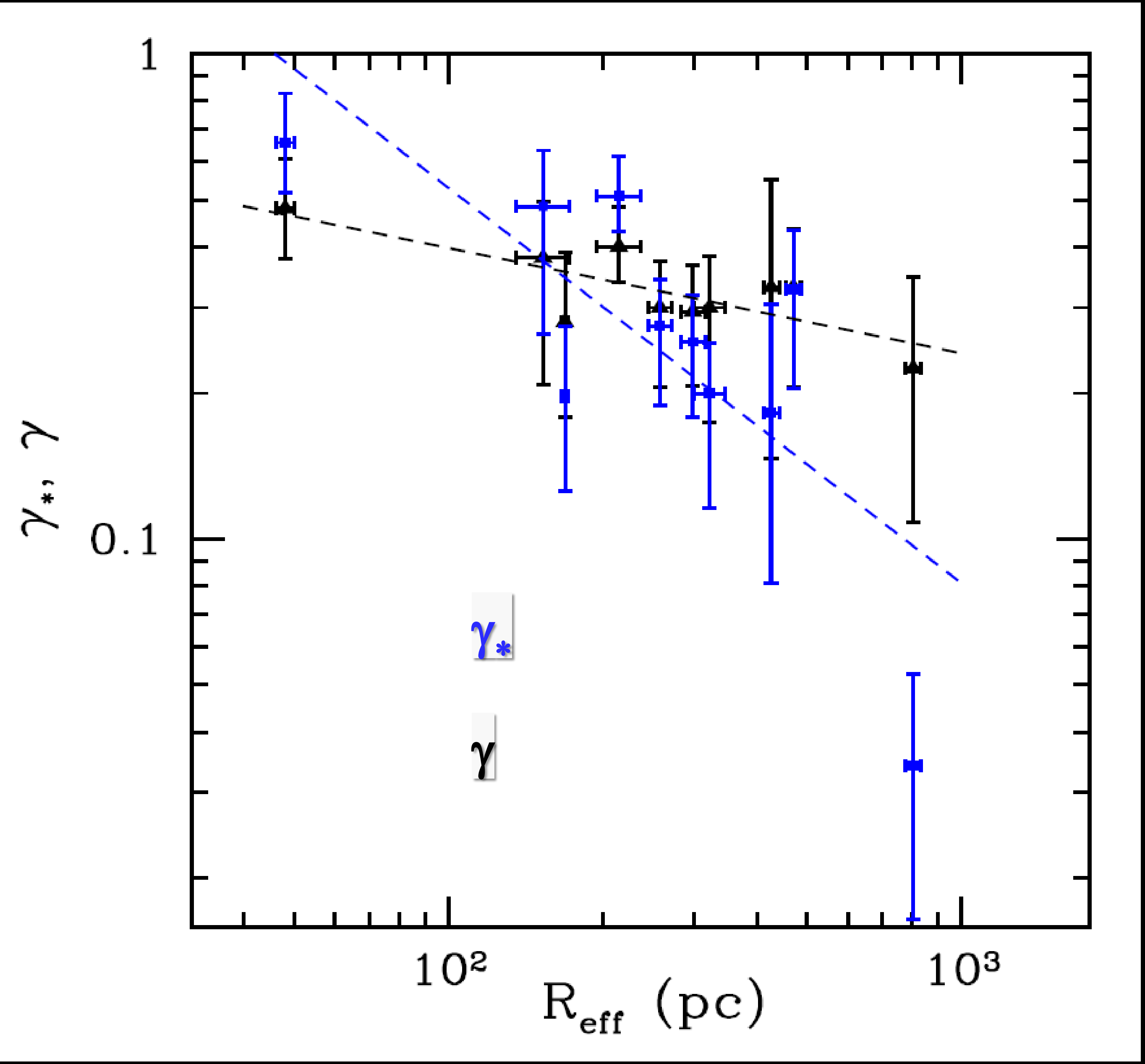


10^7
10^6
10^5
10^2
10^3
$M(R_{eff})$ ($M_\odot$)
R_{eff} (pc)
DZDM best-fit curve + pts
Walker et al. (2009) est. ± 1-σ :
$M(R_{eff}) = 2.5\ R_{eff}\ \sigma^2(R_{eff})/G$
DZDM also consistent with
$M(r_2)$ (Lazar & Bullock, 20)
$M(r_3)$ (Wolf et al., 10)

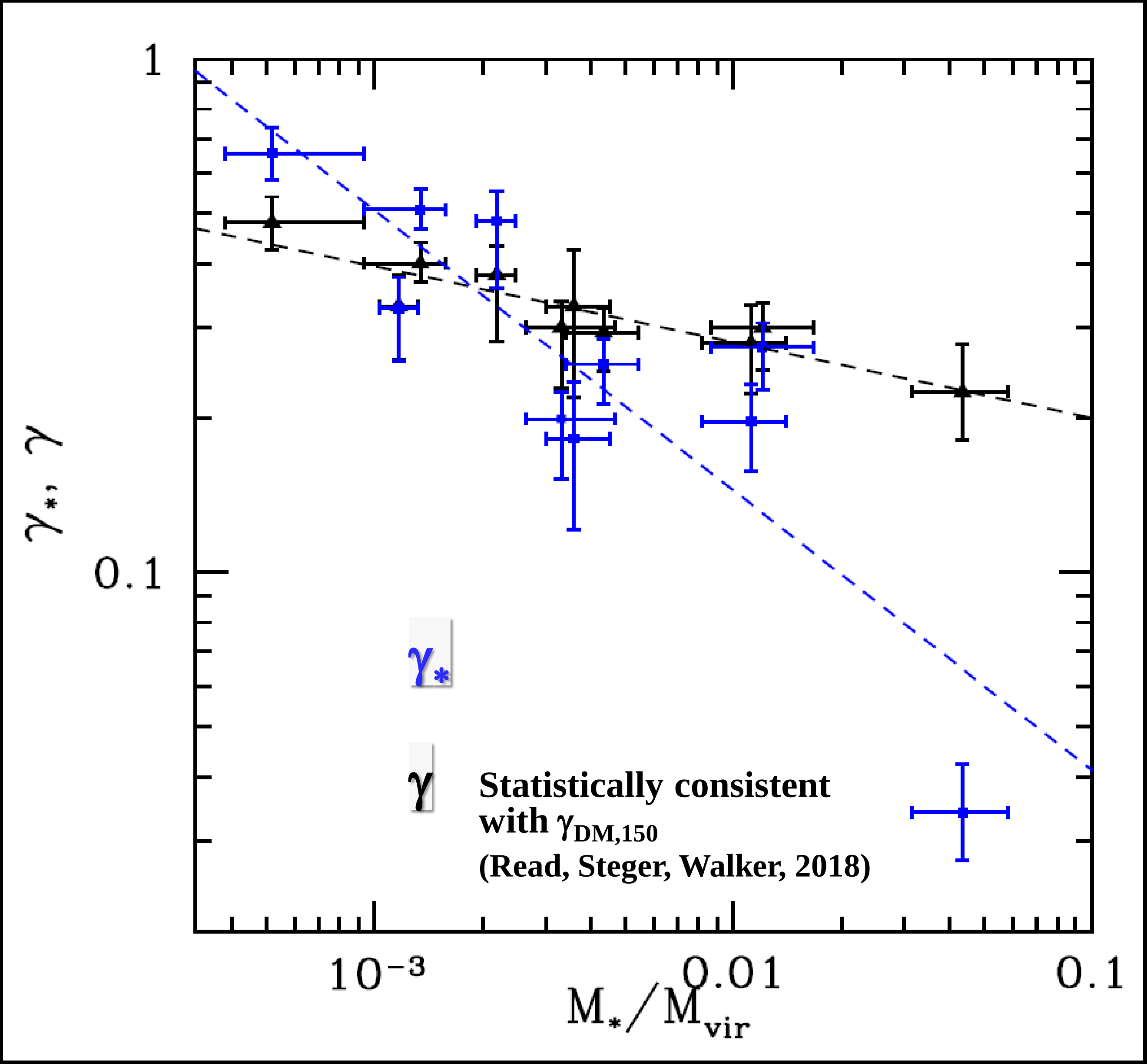
$r_s(R_{eff})$ and $\rho_s(R_{eff})$ Comparisons



γ_* (R_{eff}) and γ (R_{eff})



γ_* (M_*/M_{vir}) and γ (M_*/M_{vir})



Summary and Conclusions

- Used SB data to establish best-fit 3D stellar density → $\nu(\nu_0, \gamma_*, n, R_*, r)$
- VSPs constraints on β_0 and β_∞
- VDP + VSPs
  - velocity anisotropy profile → $\beta(\beta_0, \beta_\infty, r)$
  - DZDM best-fit parameters $(r_{DM}, \rho_{DM}) = (r_s, \rho_s)$
- Scaling Relations with optical effective radius (R_{eff}) &
  - SB scale radius (R_*)
  - $r_2 \sim R_{eff}$ $(M(r_2) = 2 <\sigma_T>^2 r_2/G)$ (Lazar and Bullock, 2020)
  - $r_3 \sim r_{DM} = r_s$ $(M(r_3) = 3 <\sigma_{TOT}>^2 r_3/G)$ (Wolf et al., 2010)
  - (Recall $r_k = R_*[(k-\gamma_*)/(n-k)]^{1/2}$)
  - $(r_{DM}, \rho_{DM}) = (r_s, \rho_s)$
  - $M(R_{eff})$ (Walker et al., 2009)
  - γ_* , γ
- Comparisons to Previous Studies
  - $M(R_{eff})$ (Walker et al., 2009)
  - DDs and Spirals
  - $M(r_2)$ and $M(r_3)$ (Comparison to Wolf et al., 2010 and Lazar and Bullock, 2020)
  - $\rho_{DM, 150}$ (Comparison to Read, Steger, and Walker, 2018)
  - γ_{150} vs. M_*/M_{vir} (Comparison to Read, Steger, and Walker, 2018)
- Future Work
  - Implications of R_{eff} correlations with DM parameters?
    Evidence for baryonic feedback? (Other galaxy types too: DDs, Spirals, etc.)